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# Similarity Requirements for Aeroelastic Models of Helicopter Rotors

*by*

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# SIMILARITY REQUIREMENTS FOR AEROELASTIC MODELS OF HELICOPTER ROTORS

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## SUMMARY

The parameters that determine the dynamic similarity of flexible lifting rotors, when thermal effects are not significant, are identified. Their relative importance is discussed and practical design procedures are developed for aeroelastic models of helicopter rotors.

There are six similarity requirements that a model should satisfy. In practice the full-scale Mach number and the full-scale Froude number cannot be represented at the same time, and the full-scale Reynolds number cannot be represented at all. Hence models will generally be designed to achieve either Mach-number similarity or Froude-number similarity. The uses, limitations and characteristics of each kind of model are examined, and the interpretation of measurements obtained from them is explained.

It is shown that most models are likely to be structural replicas, and some of the problems of making such models are discussed. The quality of construction, necessary to ensure that the models yield reliable experimental data, is shown to be high.

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# 1  INTRODUCTION

Aeroelastic models of helicopter rotors have a wide range of applications. The conventional use of flexible models of aircraft is to investigate flutter, and rotor blade flutter can be examined in this way. However, aeroelastic models are needed also for investigations into other kinds of dynamic phenomena that affect the behaviour of helicopters, and for the measurement of rotor performance.

Two examples of the dynamic phenomena that influence helicopter behaviour are ground resonance and air resonance. Their general nature is understood and helicopters are always designed to prevent their occurrence. However, the methods that are used may, in some cases at least, impose excessive constraints on the design and hence on the performance of the helicopter. In other cases they are demonstrably inadequate, because incidents of ground resonance continue to occur from time to time. If the design of helicopters is to be refined and their performance improved, a more detailed understanding of the mechanisms of blade flutter and of ground and air resonance must be obtained. Experiments with models will contribute to this understanding if the models reproduce the dynamic characteristics of the full-size rotors.

The performance of a flexible fixed-wing aircraft in steady flight may be measured by means of a rigid model which represents the shape to which the aircraft distorts under the action of the steady flight loads. The performance of a helicopter rotor in steady flight cannot be determined accurately from a rigid model because the loads on it are essentially unsteady. The effects of the flexibility of the rotor on its performance can be reproduced only by using a flexible model which responds dynamically to the loads in a similar way.

So far, the experience gained by the RAE in the field of aeroelastic modelling has been confined entirely to fixed-wing aircraft and missiles. This paper makes a preparatory survey of the conditions that govern the design and use of aeroelastic models of helicopter rotors. In places the subject is deliberately treated in some detail, with the object of making the paper helpful to newcomers.

## 2 THE GENERAL CONDITION FOR SIMILARITY

The motion of a helicopter rotor blade in a fluid is likely to be affected by the following independent variables:

| Variable | Symbol | Dimensions | | |
|---|---|---|---|---|
| | | M | L | T |
| Angular velocity of the rotor | $\Omega$ | | | -1 |
| Density of the fluid | $\rho$ | 1 | -3 | |
| Viscosity of the fluid | $\mu$ | 1 | -1 | -1 |
| Fluid velocity | $V$ | | 1 | -1 |
| Velocity of sound in the fluid | $a$ | | 1 | -1 |
| A characteristic length of the rotor blade | $\ell$ | | 1 | |
| Mean 'structural' density of the rotor blade | $\sigma$ | 1 | -3 | |
| Mean modulus of elasticity of the rotor blade | $E$ | 1 | -1 | -2 |
| Acceleration due to gravity | $g$ | | 1 | -2 |

This list does not include the shear modulus because the elastic behaviour of a structure is assumed to be described adequately by the mean elastic modulus E. This implies that the ratio between the shear modulus and the elastic modulus is assumed to be the same for all structural materials. The list also omits any measure of structural damping, because this phenomenon is complex and imperfectly understood. However, its omission does not imply that it is always negligible. For example, structural damping makes an important contribution to the motion of a hingeless rotor without mechanical dampers.

Dimensional analysis collapses the nine independent variables into six non-dimensional parameters that govern the motion of the rotor. Hence, for example, the frequency of a particular mode of the motion is given by

$$\omega = \frac{V}{\ell} F \left\{ \frac{V}{\Omega \ell}, \frac{V\ell\rho}{\mu}, \frac{V}{a}, \frac{\rho}{\sigma}, \frac{E}{\rho V^2}, \frac{V^2}{g\ell} \right\} \tag{1}$$

where the function F is of unknown form. For complete similarity between the dynamic behaviour of a model and a full-size rotor the function F must have

the same value in each system. This implies that each non-dimensional parameter contained in F must have the same value in both systems. There are, therefore, six requirements to be satisfied if complete similarity is to be achieved.

## 3    THE PARAMETERS AFFECTING SCALE

To understand fully the requirements for similarity that are imposed by equation (1) it is necessary to understand the physical meaning of the six non-dimensional parameters. Throughout the rest of this paper it is assumed that the fluid in which the rotors operate is air.

The parameter $V/\Omega\ell$ is equivalent to the advance ratio of the rotor (the ratio of the forward speed of the rotor to its tip velocity) because V may be taken to be the velocity of the undisturbed air relative to the rotor centre and $\ell$ may be the rotor radius. If the relative motion between an element of the model and the undisturbed air is to be similar to that of the corresponding element of the full-size rotor, then both rotors must operate at the same advance ratio.

The parameter $V\ell\rho/\mu$ is equivalent to the Reynolds number of the flow, since $\ell$ can be any characteristic linear dimension of the rotor. It may be rewritten

$$Re = \frac{V\ell\rho}{\mu} \times \frac{V\ell}{V\ell} = \frac{\rho V^2 \ell^2}{V\ell\mu} \quad .$$

The aerodynamic inertia force on the rotor, associated with the distribution of normal pressure over its surface, is proportional to $\rho V^2 \ell^2$; the aerodynamic friction force, due to the distribution of viscous stress over the surface, is proportional to $V\ell\mu$. Hence

$$Re = constant \times \frac{aerodynamic\ inertia\ force}{viscous\ force} \quad .$$

Therefore, if the ratio between the two kinds of aerodynamic force on an element of the model rotor is to be the same as the ratio for the corresponding element of the full-size rotor, both rotors must operate at the same Reynolds number.

The parameter $V/a$ is equivalent to the Mach number of the flow. The velocity of sound in a fluid is the velocity at which infinitesimal pressure

changes can be transmitted through it and hence, when the flow velocity any-where approaches or exceeds the local velocity of sound, the flow pattern becomes sensitive to small changes in Mach number. If the distribution of pressure throughout the flow around the model rotor is to be the same as that around the full-size rotor, both rotors must operate at the same Mach number.

The density ratio $\rho/\sigma$ may be rewritten

$$\frac{\rho}{\sigma} = \frac{\rho}{\sigma} \times \frac{\Omega^2 \ell^4}{\Omega^2 \ell^4} \quad .$$

For a given advance ratio $V/\Omega\ell$ the aerodynamic inertia force on the rotor, associated with the distribution of normal pressure over its surface, is proportional to $\rho\Omega^2\ell^4$; the rotor inertia force, due to its mass distribution, is proportional to $\sigma\Omega^2\ell^4$. Then

$$\frac{\rho}{\sigma} = \text{constant} \times \frac{\text{aerodynamic inertia force}}{\text{rotor inertia force}} \quad .$$

The inertia force due to an element of a rotor is made up of the centrifugal force and a force due to rotor oscillation. The parameter $E/\rho V^2$ may be written as

$$\frac{E}{\rho V^2} = \frac{E}{\rho V^2} \times \frac{\ell^4}{\ell^4} = \frac{E\ell^4/\ell^2}{\rho V^2 \ell^2} \quad .$$

The numerator of this expression is proportional to the elastic force due to bending, since $\ell^4$ is proportional to $I$, and the denominator is proportional to the aerodynamic inertia force. Hence

$$\frac{E}{\rho V^2} = \text{constant} \times \frac{\text{elastic force}}{\text{aerodynamic inertia force}} \quad .$$

The last parameter $V^2/g\ell$ is equivalent to the Froude number of the rotor, and may be rewritten

$$\frac{V^2}{g\ell} = \frac{V^2}{g\ell} \times \frac{\sigma\ell^2}{\sigma\ell^2} = \frac{\sigma\ell^3 \times V^2/\ell}{\sigma\ell^3 g} \quad .$$

For a given advance ratio $V/\Omega\ell$ the numerator is proportional to the rotor inertia force, and the denominator to the rotor weight. Therefore

$$\frac{V^2}{g\ell} = \text{constant} \times \frac{\text{rotor inertia force}}{\text{rotor weight}} \quad .$$

Thus if the four kinds of force that act on a rotor, namely the aerodynamic, elastic, gravitational and elastic forces, are to have the same relative magnitudes for the model as for the full-size rotor, the values of the parameters $\rho/\sigma$, $E/\rho V^2$ and $V^2/g\ell$ must be the same for both rotors*.

## 4    SCALE FACTORS

Each characteristic of a model can be related to the corresponding characteristic of the full-size design by means of a scale factor. The factor will be denoted by $\lambda$ with an appropriate suffix. For example the linear scale of the model is the ratio of model size to full size, and is denoted by $\lambda_\ell$. Throughout this paper it is assumed that the model is smaller than the prototype and hence that $\lambda_\ell < 1$.

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\*    It is worth noting that

$$\frac{\rho}{\sigma} = \frac{\rho}{\sigma} \times \frac{\ell^5}{\ell^5}$$

$$= \text{constant} \times \frac{\rho \bar{c} R^4}{mk^2} \quad .$$

Provided that the blade lift-curve slope is constant, this expression may be written

$$\frac{\rho}{\sigma} = \text{constant} \times \frac{\rho\, a\bar{c} R^4}{\bar{I}}$$

i.e.    $\dfrac{\rho}{\sigma} = \text{constant} \times \text{Lock number}$    .

The Lock number is a non-dimensional parameter affecting the motion of the blades about their flapping hinges.

## 5    PRACTICAL DESIGN REQUIREMENTS FOR MODELS

Some of the independent variables that are included in the table, page 4, are virtually constant in practice. The acceleration due to gravity, for example, may safely be regarded as constant. Viscosity and the velocity of sound are both functions of absolute temperature and cannot be varied by more than a small percentage. They too may be treated as constant. These assumptions are summarised by the following equations:

$$\lambda_g = \lambda_\mu = \lambda_a = 1 \tag{2}$$

and they are regarded as being valid throughout the argument that follows.

When one of the six requirements for similarity is satisfied, the scale factor for the relevant non-dimensional parameter becomes equal to unity. This establishes a relationship between the scale factors for the independent variables which expresses that requirement.

If the advance ratio is the same for both rotors then, since this is equivalent to $V/\Omega \ell$, we have

$$\frac{\lambda_V}{\lambda_\Omega \lambda_\ell} = 1$$

therefore

$$\lambda_\Omega = \frac{\lambda_V}{\lambda_\ell} \quad . \tag{3}$$

If the Reynolds number is the same then, since it is equivalent to $V\ell\rho/\mu$, we have

$$\frac{\lambda_V \lambda_\ell \lambda_\rho}{\lambda_\mu} = 1$$

therefore

$$\lambda_\rho = \frac{1}{\lambda_V \lambda_\ell} \tag{4}$$

If the Mach number is the same then, since it is equivalent to $V/a$, we have

$$\frac{\lambda_V}{\lambda_a} = 1$$

therefore $\qquad\qquad\qquad \lambda_V = 1$ . $\qquad\qquad\qquad$ (5)

If the density ratio is the same

$$\frac{\lambda_\rho}{\lambda_\sigma} = 1$$

therefore $\qquad\qquad\qquad \lambda_\sigma = \lambda_\rho$ . $\qquad\qquad\qquad$ (6)

If the ratio $E/\rho V^2$ is the same

$$\frac{\lambda_E}{\lambda_\rho \lambda_V^2} = 1$$

therefore $\qquad\qquad\qquad \lambda_E = \lambda_\rho \lambda_V^2$ . $\qquad\qquad\qquad$ (7)

If the Froude number is the same then, since it is equivalent to $V^2/g\ell$, we have

$$\frac{\lambda_V^2}{\lambda_g \lambda_\ell} = 1$$ ,

therefore $\qquad\qquad\qquad \lambda_V = \sqrt{\lambda_\ell}$ . $\qquad\qquad\qquad$ (8)

Equations (5) and (8) show that the only way to satisfy these requirements simultaneously is to make the model the same size as the rotor it represents. Since this is not practical, the relative importance of the requirements must be assessed to see whether any of them may be relaxed.

5.1 <u>Requirements that cannot be relaxed</u>

The relative motion between the model rotor and the air must be correct. The instantaneous chordwise velocity of any element of a rotor blade, relative to the undisturbed air, is given by

$$v = \Omega r + V \sin \psi \qquad ,$$

which can be rewritten

$$\frac{v}{\Omega r} = 1 + \frac{V}{\Omega r} \sin \psi \qquad .$$

The quantity $V/\Omega r$ is effectively the advance ratio. If this advance ratio does not have the full-scale value then the cyclic variation in $v$, as a fraction of the tip speed, is incorrect. Therefore equation (3) cannot be relaxed.

It is equally necessary that the principal forces acting on the model should all have a common scale. The dominant forces on a rotor are the aerodynamic, inertia, and elastic forces, and they are subject to the same scale factor only if equation (3) is satisfied and if the density ratio and the parameter $E/\rho V^2$ have their full-scale values under model conditions. Accordingly equations (6) and (7) cannot be relaxed.

## 5.2   Remaining requirements

The remaining requirements express the need to operate the model at full-scale values of Reynolds number, Mach number and Froude number.

A difference in aerodynamic scale is avoided completely by operating the model at full-scale Reynolds number and Mach number simultaneously, i.e. by satisfying both equations (4) and (5). These equations are compatible if

$$\lambda_\rho = \frac{1}{\lambda_\ell}$$

but this requires that small models should be tested under very high pressures and that tests at moderate pressures should be made with large models. For example a 1/10 scale model would require 10 atmospheres and a pressure of 2 atmospheres would suit only a 1/2 scale model. In a heavier gas, such as Freon, proportionately lower pressures would be required. However, there are few facilities anywhere that allow such tests to be done, and none at all in the UK. It must be assumed that all rotor tests are conducted under atmospheric conditions and that

$$\lambda_\rho = 1 \qquad .$$

Then equation (4) reduces to

$$\Lambda_V = \frac{1}{\lambda_\ell} \tag{4a}$$

and cannot be satisfied at the same time as equation (5) (unless $\lambda_\ell = 1$). Thus it is impracticable to test a model rotor at full-scale Reynolds number and full-scale Mach number simultaneously.

This is a common situation in model testing. It is usually argued that, since Reynolds-number effects are confined largely to boundary layers and wakes, they frequently do not have much effect on the normal pressures on a surface and hence do not alter significantly the principal force on that surface. Accordingly, provided that the Reynolds number is high enough to ensure that the viscous flow is of the right type (that the boundary layer is turbulent, for example) model tests are usually made at Reynolds numbers below the full-scale values. Nevertheless it remains desirable to minimise the effect of the difference in Reynolds number by testing as large a model as possible. Subject to these observations, equation (4) may be relaxed.

Above the so-called critical Mach number, and particularly in the transonic speed range, the whole aerodynamic flow pattern is governed by the Mach number. Under these circumstances the Mach number must be correctly represented and equation (5) cannot be relaxed. However, if the flow over the full-size rotor is wholly subsonic, the flow pattern does not vary much with Mach number and there is no need to operate the model at full-scale Mach number. Thus equation (5) may be relaxed as long as the flow on both rotors is everywhere subsonic.

The importance of the Froude number may be judged by omitting the weight altogether from a calculation of the coning angle of a typical rotor. The error is about 0.2 degree, which is likely to be insignificant in many model tests. If this is the case, equation (8) may be relaxed. However, an incorrect coning angle may alter the flow through the rotor and must affect the Coriolis loading induced in the lag plane by blade flapping. Both effects will be very small and will generally be negligible in terms of rotor performance, but either could have a significant effect on the stability of the motion. Therefore equation (8) should not be relaxed when a model is being designed to investigate any kind of instability.

## 5.3   Summary of design requirements

The practical design requirements for an aeroelastic model rotor are now reduced to five.   The three that cannot be relaxed are

$$\lambda_\Omega = \frac{\lambda_V}{\lambda_\ell} \quad , \tag{3}$$

$$\lambda_\rho = \lambda_\sigma \tag{6}$$

and
$$\lambda_E = \lambda_\rho \lambda_V^2 \quad . \tag{7}$$

The remaining two requirements, which are

$$\lambda_V = 1 \tag{5}$$

and
$$\lambda_V = \sqrt{\lambda_\ell} \tag{8}$$

cannot be satisfied simultaneously unless the model is as large as the prototype. Generally, therefore, one of them must be relaxed.   Some of the considerations that determine which requirement is relaxed are discussed in the next section.

## 6   MODEL DESIGN PROCEDURES

The foregoing argument shows that a model may be designed either to operate at full-scale Mach number or at full-scale Froude number.   In each case the design procedure is likely to follow the same course.   First the linear scale of the model is fixed by such considerations as the size of the test facility, a requirement to install specified instrumentation, or a need to make the model to an acceptable standard of accuracy.   The decision on operating conditions for the model determines the velocity scale   and this, in turn, decides the nature of the materials from which the model must be made.

## 6.1   Design for full-scale Mach number

A model which is to be operated at full-scale Mach number must be designed to satisfy equation (5).   This kind of model is essential if the full-size rotor approaches or exceeds its critical Mach number.   It is desirable even when the full-scale flow is subsonic, because it more nearly achieves aerodynamic similarity.   When equation (5) is satisfied,

$$\lambda_V = 1$$

hence, from equation (3),

$$\lambda_\Omega = \frac{1}{\lambda_\ell} \quad .$$

It was pointed out in section 5.2 that, in practice,

$$\lambda_\rho = 1$$

and hence, from equations (6) and (7),

$$\lambda_\sigma = 1$$

and
$$\lambda_E = 1 \quad .$$

Thus the model structure must have the same elastic modulus and structural density as that of the full-size rotor.

Sometimes it is possible to simplify the construction of aeroelastic models by adopting an alternative form of construction. The conventional approach is to design a spar with the correct distributions of stiffness and mass, and to encase this in a light flexible material which has the required external shape. However, the structure of a typical rotor blade is an efficient hollow shell. In spite of the advent of new materials it may be difficult to devise an alternative structure that is not more troublesome to construct. Blade structures are generally simple, and it is probably as easy to make the model a structural replica as to design and construct an alternative. The replica satisfies the design requirements and is more likely to reproduce correctly such characteristics as structural damping and the position of the flexural axis. Some aspects of the design of replica models are discussed in section 8.

The acceptance of incorrect Froude numbers implies that the weight of the model is relatively unimportant. An example shows how small it may be. Consider a rotor 16 m in diameter turning at 25 rad/s. If the centre of gravity of each blade is half-way along its length the ratio between weight and centrifugal force for a full-size blade is

$$\frac{W}{C} = \frac{mg}{m\Omega^2 r} = \frac{g}{\Omega^2 r}$$

$$= \frac{9.81}{25^2 \times 4} \simeq 0.004$$

For a 1/4 scale model of this rotor

$$\lambda_{W/C} = \frac{\lambda_W}{\lambda_C} = \frac{\lambda_g}{\lambda_\Omega^2 \lambda_r}$$

$$= \lambda_\ell = \frac{1}{4}$$

therefore $\qquad \frac{W}{C} \simeq 0.001$ .

Compared with the centrifugal force, therefore, the weight of such a model is entirely negligible and, this being the case, the model may be tested in any attitude relative to the earth.

## 6.2 Design for full-scale Froude number

A model that is intended to be operated at full-scale Froude number must be designed to satisfy equation (8). Models of this kind are necessary for investigations into dynamic phenomena in which gravitational forces are important, such as ground resonance and air resonance. Ground resonance occurs when there is little or no forward speed, and Mach number effects are at their least important. Air resonance can occur at any flight speed, but its essential mechanism may be studied under conditions which are not dominated by Mach number effects.

Aeroelastic rotor models are unacceptable in most high-speed wind tunnels because of the risk of costly damage if a model should fail. They are likely to be confined generally to low-speed tunnels, where they can cause the least harm. It is, therefore, convenient that the fundamental requirement imposed by equation (8) is that a model which is designed to satisfy it must operate at reduced translational velocities. It follows that full-scale flight at a high forward speed may be simulated by designing a model to satisfy equation (8) and operating it at an appropriately reduced speed. (Provided, of course, that the full-scale flow is entirely subsonic.)

When equation (8) is satisfied

$$\lambda_V = \sqrt{\lambda_\ell}$$

and therefore, from equation (3),

$$\lambda_\Omega = \frac{1}{\sqrt{\lambda_\ell}} \quad .$$

As before,

$$\lambda_\rho = 1$$

and therefore, from equation (6),

$$\lambda_\sigma = 1$$

and from equation (7)

$$\lambda_E = \lambda_\ell \quad .$$

In this case, although the model structure must have the same density as that of the full-size rotor, its modulus of elasticity must be reduced by the factor $\lambda_\ell$. This offers considerable freedom to design an alternative structure, of the kind discussed in section 6.1. It is still possible to reproduce the form of the full-size structure in the model, provided that it is made of materials which have the appropriate density and modulus. The ability to do so depends on the availability of suitable materials. The creation of such materials, in the form of composites, is discussed in section 8.

If the model is intended to investigate a phenomenon in which its weight is significant, then obviously the model must be tested in the correct attitude relative to the ground. When the model is used to simulate high-speed flight in a low-speed wind tunnel the weight may not be so important but, since it is correctly scaled, it may not be negligible. Consider again the example discussed in section 6.1. The ratio W/C for a blade of a 16 m rotor turning at 25 rad/s is approximately 0.004. This ratio will have the same value for

any model of the rotor that satisfies equation (8) regardless of the model size.
If there is no doubt that the weight of such a model can be neglected then the
model may be tested in any attitude relative to the earth but, if there is any
doubt, the model should be tested only in the correct attitude.

7    RESPONSE CHARACTERISTICS OF MODELS

In flight, and in hover unless the flow through the rotor is perfectly
axisymmetric, the aerodynamic force on a blade is a function of its angular
position and varies at frequencies that are related to the rate of rotation.
The amplitude and phase of the response to this excitation are functions of the
damping forces and of the ratios between the excitation frequency and the
natural frequencies of oscillation of the blade.  Consequently, if the response
of a model blade is to be similar to that of a full-size blade the model must
have, in each mode, the same fraction of critical damping as the full-size
blade and the same ratio between the natural frequency of the mode and the rate
of rotation.

The frequency in any bending mode of a cantilever such as a helicopter
blade is given by

$$\omega = \text{constant} \times \sqrt{\frac{EI}{m\ell^3}}$$

$$= \text{constant} \times \sqrt{\frac{E\ell^4}{\sigma\ell^3\ell^3}}$$

$$= \text{constant} \times \frac{1}{\ell}\sqrt{\frac{E}{\sigma}} \qquad .$$

Similarly the frequency in any torsion mode is given by

$$\omega = \text{constant} \times \frac{1}{\ell}\sqrt{\frac{G}{\sigma}} \qquad .$$

Therefore, provided that $\lambda_G = \lambda_E$  (see section 2)

$$\lambda_\omega = \frac{1}{\lambda_\ell}\sqrt{\frac{\lambda_E}{\lambda_\sigma}} \text{ for all modes.} \qquad (9)$$

If equations (3), (6) and (7) are all satisfied, equation (9) reduces to

$$\frac{\lambda_\omega}{\lambda_\Omega} = 1$$

and the ratios between the natural frequencies of the model and its rotational speed are the same as those of the full-size rotor.

Damping of the flapping and pitching motion of a rotor blade is almost entirely aerodynamic, whereas the damping in the lag plane is largely structural or mechanical. The correct scaling of the aerodynamic forces, and hence of their contribution to the damping factor  h, is discussed in section 5.2. Structural damping is considered briefly in section 6.1.

## 7.1 Displacement amplitude and phase

The amplitude of the displacement response at any point on a model may be written

$$a = \text{constant} \times \frac{A\ell^3}{EI} f_1\left(h , \frac{\omega}{\Omega}\right) \quad \text{for the bending modes}$$

and

$$\theta = \text{constant} \times \frac{M\ell}{GJ} f_1\left(h , \frac{\omega}{\Omega}\right) \quad \text{for the torsion modes}$$

therefore

$$\frac{\lambda_a}{\lambda_\ell} = \frac{\lambda_A \lambda_\ell^2}{\lambda_E \lambda_\ell^4} \lambda_{f_1}$$

and

$$\lambda_\theta = \frac{\lambda_M \lambda_\ell}{\lambda_G \lambda_\ell^4} \lambda_{f_1} \quad .$$

When  h  and  $\omega/\Omega$  have the same values for the model as for the full-size aircraft,

$$\lambda_{f_1} = 1 \quad .$$

Now substitute for  $\lambda_E$  from equation (7), and for  $\lambda_A$  and  $\lambda_M$. Then, provided that  $\lambda_G = \lambda_E$,

$$\lambda_\theta = \frac{\lambda_a}{\lambda_\ell} = \frac{\lambda_\rho \lambda_V^2 \lambda_\ell^4}{\lambda_\rho \lambda_V^2 \lambda_\ell^4} = 1 \qquad .$$

Thus the angular amplitudes in torsion are the same as those of the full-size rotor, and the scale of the linear amplitudes in bending is the same as the linear scale of the model. Hence the shapes of the modes of vibration of the model are similar to those of the full-size rotor.

The phase of the displacement response of the model in any mode is given by

$$\beta = f_2\left(h, \frac{\omega}{\Omega}\right)$$

and is obviously unchanged as long as h and $\omega/\Omega$ retain their full-scale values. Thus the response of the model has the correct displacement amplitude and phase if equations (3), (6) and (7) are satisfied together and if, in addition, there is no significant difference in aerodynamic scale between the model and the full-size rotor.

## 7.2  Acceleration

Provided that the motion of the model in a given mode can be regarded as simple harmonic, it is described by an equation of the form

$$\ddot{a} = -\omega^2 a \qquad .$$

Hence the scale factor for the oscillatory accelerations is

$$\lambda_{\ddot{a}} = \lambda_\omega^2 \lambda_a \qquad .$$

But, when equations (3), (6) and (7) are satisfied (and, in addition, there is no aerodynamic scale effect)

$$\lambda_\omega = \lambda_\Omega$$

and

$$\lambda_a = \lambda_\ell$$

therefore

$$\lambda_{\ddot{a}} = \lambda_\Omega^2 \lambda_\ell \qquad . \tag{10}$$

The radial acceleration caused by rotation is

$$\ddot{r} = -\Omega^2 r$$

therefore

$$\lambda_{\ddot{r}} = \lambda_\Omega^2 \lambda_r$$

i.e.

$$\lambda_{\ddot{r}} = \lambda_\Omega^2 \lambda_\ell \qquad . \qquad (11)$$

Hence all accelerations are subject to the same scale factor if equations (3), (6) and (7) are satisfied together and if, in addition, there is no significant difference in aerodynamic scale between the model and the full-size rotor.

7.3 Stress

The tensile stress at a point in a rotor blade due to motion in a bending mode is given by

$$\frac{p}{d} = E \frac{d^2 a}{dr^2} \qquad .$$

It is shown in section 7.1 that, provided equations (3), (6) and (7) are satisfied and, in addition, there is no aerodynamic scale effect,

$$\lambda_a = \lambda_\ell \qquad .$$

therefore

$$\lambda_{\left(\frac{d^2 a}{dr^2}\right)} = \frac{1}{\lambda_\ell} \qquad .$$

Hence

$$\lambda_p = \lambda_E \frac{\lambda_d}{\lambda_\ell}$$

i.e.

$$\lambda_p = \lambda_E \qquad . \qquad (12)$$

The shear stress due to motion in a torsion mode is given by

$$\frac{q}{e} = G \frac{d\theta}{dr} \qquad .$$

Hence, provided that $\lambda_\theta = 1$, (see section 7.1)

$$\lambda_q = \lambda_G \frac{\lambda_e}{\lambda_\ell} \quad .$$

Then, provided also that $\lambda_G = \lambda_E$,

$$\lambda_q = \lambda_E \quad . \tag{13}$$

The radial tensile stress due to rotation is given by

$$f = \frac{m\Omega^2 r}{S}$$

therefore

$$\lambda_f = \frac{\lambda_m \lambda_\Omega^2 \lambda_r}{\lambda_\ell^2}$$

therefore

$$\lambda_f = \lambda_\sigma \lambda_\Omega^2 \lambda_\ell^2 \quad .$$

If equations (3), (6) and (7) are satisfied, then

$$\lambda_f = \lambda_E \quad . \tag{14}$$

Therefore all the dynamic stresses are subject to the same scale factor, and this is equal to the scale factor for the modulus of elasticity, if equations (3), (6) and (7) are satisfied together and if, in addition, there is no significant difference in aerodynamic scale between the model and the full-size rotor.

## 7.4   Strain

When the dynamic stresses are all subject to the same scale factor as the elastic modulus (section 7.3 above) the scale factor for the strains induced by these stresses is

$$\lambda_\delta = \frac{\lambda_E}{\lambda_E}$$

i.e.

$$\lambda_\delta = 1 \quad . \tag{15}$$

Then all the dynamic strains measured in the model are the same as the corresponding strains in the full-size rotor.

The static strains due to deflections under weight are given by equation (15) for models that are designed to operate at full-scale Froude numbers, because the weights of these models are subject to the same scale factor as the other forces. Static strains in models designed to operate at full-scale Mach numbers are smaller, because the scale of the weights of these models is smaller than the scale of the other forces by a factor $\lambda_\ell$. Since any strain is proportional to the force that causes it, the static strain in models that are designed to satisfy equation (5) is given by

$$\lambda_\varepsilon = \lambda_\ell \quad . \tag{16}$$

## 8    STRUCTURAL REPLICA MODELS

Models that are intended for operation at full-scale Froude numbers may be structural replicas but, although they must retain the full-scale structural density, they must have a modulus scale of $\lambda_\ell$. Equations (12), (13) and (14) show that the stresses are also reduced by the factor $\lambda_\ell$. In some cases there may be materials available which satisfy these requirements but often suitable materials must be designed in the form of composites. Fibre-reinforced resins are potentially able to meet most requirements in respect of modulus and stress, although careful design of the fibre arrangement is necessary to achieve the appropriate modulus scale in both bending and torsion. The need to match the density of a full-size metal blade is a problem, because of the low densities of most fibre-reinforced resins, but much can be done by adding a heavy filler to the resin.

The specified structural density and modulus of elasticity of a replica model are easier to attain if the thickness of the components of the model structure can be increased arbitrarily. (Obviously the external shape of the model must not be altered.) Then the volume of material in the structure and, to a first approximation, the second moments of areas of the cross-sections are increased in the same ratio. In the case of a thin-walled shell with thin internal membranes the approximation is very close. The density and modulus of elasticity of the material must be reduced in proportion so that the effective density and stiffness of the structure remain correct. The symbols E and $\sigma$ must now be restricted to relate to the structure as a whole. Denote the actual modulus and density of the material by D and $\Delta$ respectively.

All the external linear dimensions of the model must conform to the linear scale $\lambda_\ell$, and hence the mass of the complete model will be scaled by the factor

$$\lambda_m = \lambda_\sigma \lambda_\ell^3 \quad .$$

Now suppose that one of the linear dimensions, say the thickness $t$, of a component of the model does not conform to the model scale so that

$$\lambda_t \neq \lambda_\ell \quad . \tag{17}$$

If the density of the material of which the component is made is $\Delta$, the mass of the component will be subject to a scale factor given by

$$\lambda_{mc} = \lambda_\Delta \lambda_\ell^2 \lambda_t \quad .$$

This will be equal to the scale factor for the mass of the complete model if

$$\lambda_\Delta = \lambda_\sigma \frac{\lambda_\ell}{\lambda_t} \quad . \tag{18}$$

A similar argument relates the modulus of elasticity of the model component to that of the whole model structure. The scale factor for the bending stiffness of the structure is

$$\lambda_{EI} = \lambda_E \lambda_I = \lambda_E \lambda_\ell^4 \quad .$$

If the component of the model is made of a material with a modulus of elasticity $D$, the bending stiffness of the component will have a scale factor

$$\lambda_D \lambda_{Ic} = \lambda_D \lambda_\ell^3 \lambda_t \quad .$$

This will equal the scale factor for the bending stiffness of the whole structure if

$$\lambda_D = \lambda_E \frac{\lambda_\ell}{\lambda_t} \quad . \tag{19}$$

Equations (17), (18) and (19) allow considerable latitude in selecting from available materials or in designing composite materials. The thickness of each component of a compound structure may be adjusted differently. The increase in cross-section areas alters the stresses in the same ratio as the density and the modulus of elasticity, and strains remain unaffected. Since the mass, stiffness and shape of the structure are correct, all other characteristics of the model remain unchanged.

Models designed to operate at full-scale Mach numbers are very likely to be structural replicas. Since they must have the same structural density and modulus of elasticity as the full-size rotors they should, ideally, be made entirely from the same materials. However, this may impose some practical problems. For example a 1/10 scale model of a blade that has a skin of stain- less steel 0.25 mm thick (0.010 inch) would have to be made from foil 0.025 mm thick (0.001 inch) if the same material were used. This might be difficult to handle and the model would be highly vulnerable if it were built successfully. Equations (17), (18) and (19) show that a thicker skin of some other material can be used, if it is not inappropriate for any other reason.

The components of most blade structures are bonded with adhesives, and the same model design requirements apply to the adhesives as to the rest of the structure. Hence a model intended to operate at full-scale Mach numbers should use the same adhesives as the full-size rotor. Models for use at full-scale Froude numbers should use adhesives with full-scale densities but with their moduli of elasticity reduced by the factor $\lambda_\ell$. It may be difficult in practice to make the thickness of the adhesive layers in a model blade conform to the linear scale of the rest of the model. If there is a consistent error in the adhesive thickness, amounting in effect to a difference in scale, the modulus and density of the adhesive should be adjusted in accordance with equations (18) and (19).

## 9  QUALITY OF MODEL CONSTRUCTION

The limits within which the linear dimensions of the components of a model rotor must be held, if it is to represent faithfully the full-scale design, are found by applying the linear scale factor to the full-scale tolerances. If these limits are not observed the mass and stiffness of the model may be significantly out of scale. Also unless the external shape is within limits on dimensions and surface roughness there may be aerodynamic effects which do not occur at full scale.

It is most important that the purpose of the model should be borne in mind throughout its construction. Any dynamic phenomenon which occurs in the model, but does not occur in the full-scale rotor, causes a spurious excitation to which there is a response that tends to confuse the experimental measurements. For example, if a joint in the structure is not perfectly bonded, the opposing faces may be free to tap against each other or to rub. Therefore critical inspection of the model at every stage of construction is vital.

The complete rotor must not only be mass-balanced to the correct standard but the aerodynamic forces on the individual blades must be matched so that the coning angles of all the blades are within appropriate limits under all conditions. The standards are very high. A typical full-size blade is mass-balanced so that its moment about the root is within ±0.02 per cent of its prescribed value. The coning angles of the blades of a typical helicopter in quantity production are matched so that the blade tip paths are not more than ±0.02 per cent of the rotor diameter from their prescribed path.

Particular attention must be paid to the elimination of excessive noise in bearings, not only on the main rotor shaft but in the blade hinges and the control system. The full-size bearings are likely to be of good quality but, ideally, any roughness or clearance should be reduced by the linear scale factor if the model is to be fully representative. This is likely to be difficult to measure or to achieve but, clearly, the bearings in the model must be of superlative quality and must be fitted with great care.

## 10  CONCLUSIONS

(1)    When thermal effects are not significant, there are six non-dimensional parameters that determine the dynamic and aerodynamic similarity between a helicopter rotor and a flexible model that is geometrically similar to it. The parameters are the advance ratio which describes the relative motion between a rotor and the undisturbed air, the Reynolds number and Mach number which describe the nature of the air flow, and the three force ratios $\rho/\sigma$, $E/\rho V^2$ and $V^2/g\ell$, the last of which is the Froude number, describing the relative magnitudes of the forces that act on a rotor. For strict similarity each parameter must have the same value for both rotors. There are, therefore, six design requirements to be fulfilled if the model is to achieve total similarity.

(2)    The displacement amplitude of the model response will conform to the linear scale, and the phase will be correct, only if the advance ratio and the

force ratios $\rho/\sigma$ and $E/\rho V^2$ have their full-scale values under model conditions. Therefore these three design requirements must always be satisfied.

(3)   It is impracticable to test a model rotor at full-scale Reynolds number. Nevertheless the model Reynolds number should be high enough to ensure that the viscous flow is of the right type (that the boundary layer is turbulent, for example).

(4)   The remaining two requirements, that the model should be operated at full-scale Mach number and full-scale Froude number, cannot be satisfied simultaneously. Therefore an aeroelastic model may be used only when it is possible to relax one or other of these requirements.

(5)   Unless the full-scale flow is wholly subsonic, a model rotor must be designed to operate at full-scale Mach number. The model structure must then have the same density and modulus of elasticity as the full-size structure, and this suggests that the model is likely to be a structural replica. The Froude number of the model will be greater than that of the full-size rotor and its weight, in relation to the other forces, will be smaller. Often the weight may be ignored, and then the model may be tested in any attitude relative to the earth.

(6)   A model must be designed to operate at full-scale Froude 'number if it is to be used either to investigate a dynamic phenomenon in which weight is significant or to simulate high-speed flight in a low-speed test facility. The model structure must have the full-scale structural density but a lower modulus of elasticity, and this allows either an arbitrary or a replica structure to be used. For simulation of high-speed flight it may sometimes be possible to test the model in any attitude relative to the earth but when its weight cannot be neglected it must be tested in the correct attitude. The Mach number of such a model will be lower than that of the full-size rotor, and the test may be misleading unless the flow over the full-size rotor is wholly subsonic.

(7)   The similarity requirements prescribe the density and modulus of elasticity of the model structure and, whether the structure is arbitrary or a replica, this implies that materials will be required with specified values of density and modulus. When the available homogeneous materials do not meet these specifications, composite materials must be designed to do so.

(8)   Provided that the three essential design requirements are not violated, all dynamic accelerations have a common scale, and all dynamic strains in the model are identical to the corresponding strains in the full-size rotor.

(9)   In a structural replica model the thickness of any thin component, such as a skin or a layer of adhesive, may be increased if the density and modulus of elasticity of the material from which it is made are reduced in proportion.

(10)   The quality of manufacture of the model is very important.  The dimensional tolerances are reduced by the same scale as the linear dimensions.  It is particularly important to avoid spurious,dynamic effects caused by faulty structural joints, noisy bearings or inadequate balancing.

Appendix A

## TABLE OF SCALE FACTORS

The following table summarises the scale factors relating to model design, test conditions and model response.

It is assumed that equations (3), (6) and (7) are always satisfied, and that $\lambda_\ell$ is given.

For completeness the last column shows the scale factors for a model designed to operate under sea level conditions at full-scale Reynolds number. The resulting unacceptable Mach number scale is discussed in section 5.2. This table shows that there are other reasons why tests of this kind are impracticable. The model is required to have a higher structural modulus of elasticity than the full-size rotor and to withstand higher stresses, and both these requirements are often impossible to meet. It also requires greater power than the full-size rotor (see Appendix B).

| Quantity to be scaled | | | Exact expression for scale factor | Value of scale factor when $\lambda_a = \lambda_g = \lambda_\mu = \lambda_\rho = 1$ | | | |
|---|---|---|---|---|---|---|---|
| | | | | General | Model designed for full-scale Froude numbers (equation 8) | Model designed for full-scale Mach numbers (equation 5) | Model designed for full-scale Reynolds numbers (equation 4) |
| Dimensionless parameters | Reynolds number | Re | $\lambda_V \lambda_\ell \lambda_\rho / \lambda_\mu$ | $\lambda_V \lambda_\ell$ | $\lambda_\ell^{3/2}$ | $\lambda_\ell$ | $1$ |
| | Mach number | M | $\lambda_V / \lambda_a$ | $\lambda_V$ | $\lambda_\ell^{1/2}$ | $1$ | $1/\lambda_\ell$ |
| | Froude number | F | $\lambda_V^2 / \lambda_g \lambda_\ell$ | $\lambda_V^2 / \lambda_\ell$ | $1$ | $1/\lambda_\ell$ | $1/\lambda_\ell^3$ |
| Velocities | linear | V | $\lambda_V$ | $\lambda_V$ | $\lambda_\ell^{1/2}$ | $1$ | $1/\lambda_\ell$ |
| | angular | $\Omega$ | $\lambda_V / \lambda_\ell$ | $\lambda_V / \lambda_\ell$ | $1/\lambda_\ell^{1/2}$ | $1/\lambda_\ell$ | $1/\lambda_\ell^2$ |
| Structure characteristics | density | $\sigma$ | $\lambda_\sigma$ | $1$ | $1$ | $1$ | $1$ |
| | modulus of elasticity | E | $\lambda_\rho \lambda_V^2$ | $\lambda_V^2$ | $\lambda_\ell$ | $1$ | $1/\lambda_\ell^2$ |
| Forces | aerodynamic | A | $\lambda_\rho \lambda_V^2 \lambda_\ell^2$ | $\left.\begin{array}{c}\\\\\end{array}\right\} \lambda_V^2 \lambda_\ell^2$ | $\left.\begin{array}{c}\\\\\\\end{array}\right\} \lambda_\ell^3$ | $\left.\begin{array}{c}\\\\\end{array}\right\} \lambda_\ell^2$ | $\left.\begin{array}{c}\\\\\end{array}\right\} 1$ |
| | elastic | B | $\lambda_E \lambda_\ell^2$ | | | | |
| | inertial (radial and oscillatory) | C | $\lambda_\sigma \lambda_\Omega^2 \lambda_\ell^4$ | | | | |
| | gravitational | W | $\lambda_\sigma \lambda_\ell^3$ | $\lambda_\ell^3$ | | $\lambda_\ell^3$ | $\lambda_\ell^3$ |
| Model responses* | displacement amplitude | a | $\lambda_A / \lambda_E \lambda_\ell$ | $\lambda_\ell$ | $\lambda_\ell$ | $\lambda_\ell$ | $\lambda_\ell$ |
| | frequency | $\omega$ | $(\lambda_E / \lambda_\sigma)^{1/2} / \lambda_\ell$ | $\lambda_\Omega$ | $1/\lambda_\ell^{1/2}$ | $1/\lambda_\ell$ | $1/\lambda_\ell^2$ |
| | acceleration | $\ddot{a}$ | $\lambda_\omega^2 \lambda_\ell$ | $\lambda_\Omega^2 \lambda_\ell$ | $1$ | $1/\lambda_\ell$ | $1/\lambda_\ell^3$ |
| | dynamic stress | p | $\lambda_E$ | $\lambda_V^2$ | $\lambda_\ell$ | $1$ | $1/\lambda_\ell^2$ |
| | dynamic strain | $\delta$ | $1$ | $1$ | $1$ | $1$ | $1$ |
| | static strain | $\epsilon$ | $\lambda_W / \lambda_C$ | $\lambda_\ell \lambda_V^2$ | $1$ | $\lambda_\ell$ | $\lambda_\ell^3$ |

* Provided that there is no significant difference in aerodynamic scale between the model and the full-size rotor.

Practical alternatives

Not practicable

## Appendix B

### POWER REQUIRED

The power required to drive a rotor may be written*

$$P = \rho b \bar{c} R (\Omega R)^3 p_c \qquad \text{(B-1)}$$

where the value of the non-dimensional coefficient $p_c$ is determined by the form of the aerodynamic flow pattern through the rotor. This is determined in turn by the operating conditions. Therefore, if there is aerodynamic similarity between a model and a full-size rotor, they will both have the same value of $p_c$. Then

$$\lambda_{p_c} = \frac{\lambda_P}{\lambda_\rho \lambda_\Omega^3 \lambda_\ell^5} = 1$$

therefore

$$\lambda_P = \lambda_\rho \lambda_\Omega^3 \lambda_\ell^5 . \qquad \text{(B-2)}$$

In practice (see section 5.2)

$$\lambda_\rho = 1$$

and

$$\lambda_P = \lambda_\Omega^3 \lambda_\ell^5 . \qquad \text{(B-3)}$$

Models designed for full-scale Mach number must be rotated at speeds such that

$$\lambda_\Omega = \frac{1}{\lambda_\ell} .$$

Hence, for these models,

$$\lambda_P = \lambda_\ell^2 . \qquad \text{(B-4)}$$

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* This is the usual British form. In the USA it is generally written

$$P = \rho \pi R^2 (\Omega R)^3 C_P$$

where $C_P = p_c \times \frac{b\bar{c}}{\pi R}$ .

In the case of models designed for full-scale Froude numbers,

$$\lambda_\Omega = \frac{1}{\lambda_\ell^{1/2}}$$

and

$$\lambda_P = \lambda_\ell^{7/2} \quad . \tag{B-5}$$

[A model designed for full-scale Reynolds number must rotate faster than the full-size rotor, because

$$\lambda_\Omega = \frac{1}{\lambda_\ell^2} \quad .$$

Hence

$$\lambda_P = \frac{1}{\lambda_\ell} \quad ,$$

showing that the model requires greater power than the full-size rotor. Other reasons why it is impracticable to operate a model rotor at full-scale Reynolds number are discussed in section 5.2 and Appendix A.]

## SYMBOLS

| | |
|---|---|
| A | aerodynamic inertia force, proportional to $\rho V^2 \ell^2$ |
| a | velocity of sound in fluid; linear amplitude in bending; blade lift-curve slope |
| b | number of blades |
| C | rotor inertia force (radial) $= m\Omega^2 r$ |
| c | (suffix) component of a structure |
| $\bar{c}$ | mean chord of a blade |
| D | elastic modulus of a material |
| d | distance normal to the neutral axis of a cross-section |
| E | elastic modulus of a structure |
| e | distance normal to the torsion axis |
| F | a function; Froude number $= V^2/g\ell$ |
| f | radial tensile stress |
| $f_1, f_2$ | functions |
| G | shear modulus |
| g | acceleration due to gravity |
| h | ratio of damping to critical damping |
| I | second moment of area of a cross-section about the neutral axis, proportional to $\ell^4$ |
| $\bar{I}$ | moment of inertia of a blade about its flapping hinge |
| J | polar second moment of area of a cross-section about the torsion axis, porportional to $\ell^4$ |
| k | radius of gyration of a blade about its flapping hinge |
| $\ell$ | a linear dimension |
| M | Mach number $= V/a$; (suffix) model conditions; aerodynamic moment, proportional to $\rho V^2 \ell^3$ |
| m | mass, proportional to $\sigma \ell^3$ |
| P | power required to drive rotor |
| p | tensile stress due to bending |
| $P_c$ | power coefficient $= P/\rho b \bar{c} R (\Omega R)^3$ |
| q | shear stress due to torsion |
| R | rotor tip radius; (suffix) full-scale conditions |
| Re | Reynolds number $= V\ell\rho/\mu$ |
| r | radius from centre of rotor |
| S | area of a cross-section |
| t | thickness of a structural component |
| V | velocity of undisturbed air relative to rotor |
| W | weight |

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| | |
|---|---|
| $\beta$ | phase angle |
| $\Delta$ | density of a material |
| $\delta$ | dynamic strain |
| $\epsilon$ | static strain |
| $\theta$ | angular amplitude in torsion |
| $\lambda$ | scale factor (identified by suffix) e.g. $\lambda_\ell = \ell_M/\ell_R$ |
| $\mu$ | viscosity of air |
| $\rho$ | density of air |
| $\psi$ | angular position of a blade in azimuth |
| $\sigma$ | density of structure |
| $\Omega$ | angular velocity of rotor |
| $\omega$ | $2\pi \times$ frequency of oscillation |

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SIMILARITY REQUIREMENTS FOR AEROELASTIC
MODELS OF HELICOPTER ROTORS

The parameters that determine the dynamic similarity of flexible lifting rotors, when thermal effects are not significant, are identified   Their relative importance is discussed and practical design procedures are developed for aeroelastic models of helicopter rotors

There are six similarity requirements that a model should satisfy   In practice the full-scale Mach number and the full-scale Froude number cannot be represented at the same time, and the full-scale Reynolds number cannot be represented at all   Hence models will generally be designed to achieve either Mach-number similarity or Froude-number similarity   The uses, limitations and characteristics of each kind of model are examined, and the interpretation of measurements obtained from them is explained

It is shown that most models are likely to be structural replicas, and some of the problems of making such models are discussed   The quality of construction, necessary to ensure that the models yield reliable experimental data, is shown to be high.

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DETACHABLE ABSTRACT CARDS

DETACHABLE ABSTRACT CARDS