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Pressures near the Centre - Line
of Leeward Surfaces on Delta Wings
and Conical Bodies at High Supersonic Speeds

Ву

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Pressures near the Centre-Line of Leeward Surfaces on Delta Wings and Conical Bodies at High Supersonic Speeds

- By -

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SUMMARY

This report deals with the manner in which the flow fields over the leeward surfaces of delta wings and conical bodies can be calculated for conditions when the bow shock wave is detached from the leading edges. The significant features and the parameters controlling the flow process are determined. As a first step towards the calculation of the complete flow field on leeward surfaces an accurate semi-empirical method is developed for predicting pressures near the centre-line of wings and bodies for Mach numbers from 2.5 upwards.

	List of Contents	
	***************************************	Page
1.	Introduction	2
2.	Characteristics of the Flow Field	2
3.	Flow at the Centre-Line of Leeward Surfaces	4
4.	Conclusions	8
	References	9

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^{*} Replaces NPL Aero Report 1319 - A.R.C.32 172

1. Introduction

The aims of the present research are to determine the parameters affecting the flows over the leeward surfaces of delta wings and conical bodies at supersonic speeds and to provide a method for predicting surface-pressure distributions and vortex locations. These are of importance in flight at incidence when vortices shed from a wing or body may interfere with control surfaces placed downstream and in yawing flight when asymmetric pressure distributions are also produced.

High heat-transfer rates, approaching those on compression surfaces, have been measured near the centre-line on the leeward surfaces of delta wings at high supersonic speeds. In order to predict the heat transfer accurately a realistic model of the flow process is required and a knowledge of the pressure distribution is a prerequisite for the application of many theoretical methods.

2. Characteristics of the Flow Field

The general appearance of the flow over the upper surface of a conical body at moderate supersonic speeds is sketched in Fig.1. The pressure near the leading edge is low owing to the expansion of the air around the edge from the underlying compression surface. Frequently, the flow separates from the surface near the leading edge either because it cannot negotiate the edge due to the large flow deflection or it cannot withstand the large adverse pressure gradient imposed between the leading edge and the centre-line of the body. In this situation the separated flow may roll up into a vortex due to the presence of the longitudinal velocity before reattaching on the surface and a secondary separation then occurs outboard of the point of reattachment. However, in some situations the flow may reattach to the surface without forming a vortex.

The general practical problem will usually involve flows with detached shock waves at the leading edges of a body principally because of the finite thickness of a body for which the included angle normal to the leading edge may be greater than that for shock attachment. Shock detachment also occurs at surprisingly low incidences for thin swept wings because the incidence normal to the leading edge increases rapidly particularly when a wing is highly swept. The incidence normal to the leading edge (α_n) of a thin delta wing is given by

$$\tan \alpha_n = \tan \alpha \csc \nu$$
 ... (1)

where α is the angle of incidence and ν is the semi-vertex angle.

On a body consisting of flat facets, for example a rhombic body, the primary separation is fixed at the sharp leading edge and advantage can be taken of the reduced number of parameters to be studied. However, depending on the magnitude of the Reynolds number and the Mach number

component normal to the leading edge, an attached-flow region can exist inboard of the leading edge^{2,3} before the occurrence of primary separation.

Significant changes in the pressure distributions occur as the free-stream Mach number is increased4. At low supersonic speeds there is a distinct trough in the pressure distribution (Fig. 2) caused by the vortex core whereas at Mach numbers above about 2.0 this effect diminishes to produce a pressure distribution in which the pressure at the centreline and that at the leading-edge are connected by a region of pressure rise. Internal oblique shock waves have been detected experimentally²,3,5,6 by many authors on bodies with detached shock waves at the leading edges although this type of flow structure was originally proposed by Fowell (for the inviscid flow over a delta wing with attached shock waves at the leading edges. The location of the oblique shock wave is outboard of the vortex and coincides with the position of secondary separation, which indicates that the separation is precipitated or magnified by an interaction between the shock wave and the boundary layer. In an earlier paper the author suggested that the majority of the pressure rise at high supersonic speeds was contributed by the shock wave or by a series of compression waves that coalesced into a discrete shock wave some distance from the surface. Thus, the location of the pressure rise is controlled principally by the external inviscid flow.

A number of flow processes can exist 8,9 depending on the state of the boundary layer and the magnitude of the pressure gradient in a plane perpendicular to the leading edge. The boundary layer can be attached at all times on the upper surface, the pressure rise produced by the shock wave being too small to initiate separation; this is the model proposed by Fowell. The boundary layer can be attached at the leading edge but subsequently separated by the pressure rise and this separated flow may roll up to form a vortex. If the flow is initially separated from the leading edge the pressure rise can magnify this separation inboard of the shock wave or, if the magnitude of the pressure rise is small the initially-separated flow may reattach to the surface ahead of the pressure rise. It has been shown experimentally 10 that the attachment line of the vortex obtained from oil flow photographs does not coincide generally with the top of the pressure rise but tends to be further inboard. The location of the vortex trace, particularly its inboard extent, is controlled, therefore, by viscosity effects which are dependent on the Reynolds number and also on the downstream boundary condition imposed by the pressure along the centre-line of the body. suggested before, the viscosity effects do not alter appreciably the magnitude or shape of the pressure distribution.

The pressure near the centre-line of the body is independent of the pressure near the leading edge outboard of the pressure rise, which is controlled by the characteristics of the leading edge. The low pressure near the leading edge is achieved because of the expansion of the air around the edge from the adjacent compression surface. When the bow shock wave is attached to the leading edge and there is no separation the leading edge pressure and the resulting inboard flow, involving an oblique shock wave, can be calculated analytically 7. However, when the flow is separated and the bow shock wave is detached from the leading

edge a similar process occurs and the leading edge pressures calculated from an inviscid-flow model produce the correct characteristics and trends when compared with experimental measurements.

Bearing in mind the preceding arguments, an idealised pressure distribution is postulated as sketched in Fig.3 as the first step in order to determine the relevant parameters affecting the flow process and to provide an engineering solution for predicting the pressure distribution at high supersonic speeds. This report will be concerned with the pressures near the centre-line of the body.

3. Flow at the Centre Line of Leeward Surfaces

The parameters affecting the magnitude of the pressure at the centre-line are primarily incidence, free-stream Mach number, aspect ratio and the ridge angle or shape of the cross-section. Previous attempts at correlating the available experimental data have proved unsuccessful¹¹. Also, methods available for calculating these pressures such as linearised theory and an axial Prandtl-Meyer expansion are inaccurate.

For the correlation of pressures on compression surfaces one of the most important parameters is the inclination of the ridge line to the free-stream direction 12, and the pressure distribution at zero lift has only a small effect on the development of the pressures at the centre-line with increasing incidence particularly at high Mach numbers. A similar correlation based on experimental data 6,10,13 is shown in Fig.4 for pressures on the centre-lines of leeward surfaces at a Mach number of 4.0 for various models with different aspect ratios and ridge angles. The correlation is reasonable and independent of aspect ratio but the flatter models (ridge angles approaching 180°) have a lower pressure for the same ridge inclination. A theoretical curve based on the application of a two-dimensional Prandtl-Meyer expansion in an axial direction is also included to demonstrate that the pressures are seriously underestimated.

In order to allow for the effects of a variable ridge angle the pressure coefficients are replotted in Fig.5 against the inclination of the surface to the free-stream direction (δ_n) for surfaces adjacent to the ridge line. The data collapse satisfactorily for the leeward surfaces, the effects of the ridge angle now being accounted for in the parameter δ_n which is given by

$$\delta_n = \delta \sin \zeta$$
 ... (2)

where 2% is the total included angle of the ridge in a plane perpendicular to the ridge line.

An interesting feature shown by the data in Fig.5 is that the value of the pressure coefficient (Cp) is not zero when the surface is aligned with the free-stream direction but remains slightly positive

 $(C_p=+0.005 \ {\rm at\ M}=4)$. Part of this positive pressure coefficient can be attributed to the boundary-layer displacement thickness but the remaining effect is still significant and well-defined. The various positive values of C_p at corresponding negative values of δ_n are those for which the total normal force is zero and these values of C_p increase in magnitude as the body or wing thickness is increased. However, the initial value of the pressure coefficient at zero lift seems to have little effect on the value of the pressure coefficient obtained at zero surface inclination. The positive values of C_p are not correlated successfully by the parameter δ sin ζ but a collection of data for compression surfaces showed that the parameter δ (sin ζ) produced a better correlation.

The pressures at the centre-line are shown to be independent of aspect ratio and are therefore independent of the pressures produced near the leading edge because these are altered by changes of the aspect ratio. Thus, the geometry of the lower surface is not expected to affect significantly the pressures produced at the centre-line. The pressure distributions across the leeward surfaces, shown in Fig.6, for three models with different lower-surface geometries and aspect ratios demonstrate that these variables do not influence the pressures at the centre-line. In Fig.7 one model is yawed but the centre-line pressure remains constant despite the fact that the pressure-rise across the span, the separation at the leading edge and the pressures at the leading edge are all changed.

Experimental data^{3,4,14,15} showing the variation of pressure coefficient with Mach number for various surface inclinations are presented in Fig.8. For large surface inclinations and Mach numbers above 2.0 the pressure coefficients vary in the same manner as the pressure coefficient

required to achieve an ultimate vacuum (c_p vacuum = $-\frac{2}{yM^2}$). The value of c_p

for zero surface inclination is always positive but increases in magnitude as the Mach number is reduced and the value of the pressure coefficient then becomes dependent on the thickness of the body. There is insufficient experimental information available to determine whether this thickness effect at low Mach numbers is continued for larger surface inclinations.

The data of Michael ¹⁵ for thin delta wings at a Mach number of 1.9 show that the centre-line pressures are dependent on the aspect ratio. The rate of change of C with incidence (Fig.9) increases as the aspect ratio is increased from 0.35 to 1.0, but remains approximately constant for the range 1.0 to 2.5. For these tests the Mach-number component normal to the leading edge varied from 0.165 to 1.0 and consistency in the centre-line pressures is achieved for Mach-number components normal to the leading edge greater than 0.5 (aspect ratios > 1). The mean values for these latter data points (aspect ratios > 1), which are still considerably underestimated by an axial Prandtl-Meyer expansion, are also included in Fig.8.

In order to predict the variation with Mach number a simple theoretical treatment is proposed in which the pressure coefficients at a

number of 4.0 are used as basis to provide an empirical prediction technique. If it is assumed that the variation of ${\tt C}_{\tt p}$ with Mach number is of the form

$$C_p \sim (M^2 - 1)^{-\frac{1}{2}}$$

then the values of C_{p} at other Mach numbers are given by

$$C_p = C_{p_4} \left(\frac{\mu^2 - 1}{M^2 - 1} \right)^{\frac{1}{2}} \dots (3)$$

where c_{p_4} denotes the value of c_p at a Mach number of 4.0 for a given surface inclination.

Curves derived using this principle are plotted in Fig.8. The correct variation with Mach number is predicted for Mach numbers from 2.5 upward, with the values for zero surface-inclination accurately predicted. Below a Mach number of 2.0 the experimental pressure coefficients increase rapidly whereas equation (3) predicts an infinite value of $-C_p$ when M = 1. Some of the discrepancies between the theory and experiment at the lower Mach numbers may be associated with the non-conicality of the flow; for some of the models tested + at M = 1.3 the bow shock waves could be detached from the vertices as well as the leading edges and produce pressures which vary in an axial direction. At hypersonic Mach numbers equation (3) could predict pressures lower than vacuum for large surface inclinations and is therefore physically unrealistic. The application of the Prandtl-Meyer relations, still based on the data at M = 4, provides a better estimate of the centre-line pressures at hypersonic Mach numbers (Fig. 10) but underestimates these pressures at low Mach numbers. The flow deflection angles required for this calculation are assumed to be those which would achieve the required pressure coefficients at a Mach number of 4.0. These deflection angles, which are considerably less than the surface inclinations, are then used in all subsequent calculations to obtain the pressure coefficients at other Mach numbers. The values of the 'effective' flow deflection angles are plotted in Fig. 11 against the true surface inclinations for free-stream Mach numbers of 2.5, 4.0 and 7.0. The variation for the two lower Mach numbers is linear, indicating that for conical flows a fixed proportion of the turning angle is achieved compared with that for two-dimensional flows. At a Mach number of 7.0 the curve is non-linear at the larger surface inclinations, either due to the influence of the pressure-limit (ultimate vacuum) or due to the large boundary-layer displacement expected at the low Reynolds number of the experimental tests 17.

The slope of each curve $\left(\frac{\varepsilon}{\delta_n}\right)$ is the ratio of the effective flow

deflection angle ϵ to the true surface inclination, δ_n . The values of $\frac{\epsilon}{\delta}$ are plotted in Fig.12 against the free-stream Mach number and increase with increasing Mach number but are considerably less than unity. The oblique shock waves lying across the surface redirect the flow from the leading edges into an axial direction at the centre-line and to satisfy this condition the resulting pressures at the centre-line are higher than those computed from the application of a two-dimensional expansion. However, it is interesting to note that the variation is well-ordered so that the pressures at the centre-line can be computed simply using the equations for a two-dimensional expansion with the deflection angle Δ given by

$$\Delta = k \delta_n + 0.6 \text{ degrees} \qquad \dots (4)$$

where k is the ratio $\left(\frac{\varepsilon}{\delta_n}\right)$ at the required free-stream Mach number and the constant value 0.6 degrees is the offset shown in Fig.11 which arcounts for the positive values of C obtained at zero surface inclination (Figs.8 and 10).

The flow over the upper surface of a thin delta wing with 30° semi-vertex angle has been computed using the method proposed by Fowell? for flows with attached shock waves at the leading edges. This analysis produces pressures at the centre-line with similar characteristics to those shown by the experimental data. The variation of surface inclination with the effective flow-deflection angle (Fig.13) is almost linear over a large range of Mach number and the ratios of $\frac{\varepsilon}{\delta_n}$ increase with increasing Mach number but are larger than those produced by the experimental data in Fig.11.

At high Mach numbers it is expected that the pressures near the centre-line will be determined to a large extent by the condition of ultimate vacuum and the present data conform to this expectation for Mach numbers above 4.0. This effect is demonstrated in Fig. 14 in which the pressure coefficients are non-dimensionalised by C_p vacuum $\begin{pmatrix} C_p/C_p & P_p & P_p \\ P_p & P_p & P_p \end{pmatrix} \text{ and when plotted against the Mach number the data tend to asymptote to particular values of <math>C_p/C_p & P_p & P$

be achieved. The data plotted in Fig.14 refer only to conditions near the centre-line for a limited range of incidence whereas lower values of C_p are experienced near the leading edges and near the centre-line at high incidences thus indicating that the revised limit for C_p/C_p vac. approximately 0.9 proposed in Ref.18 is likely to be achieved.

Further work is in progress on the problems of predicting the pressures near leading edges and the locations of the top and bottom of the pressure rise based on the simplified model of the flow process discussed earlier.

4. Conclusions

A simplified model of the flow process and the resulting pressure distribution on the leeward surface of a delta wings or conical body has been proposed. The model postulates the existence of oblique shock waves or compression waves that provide the majority of the pressure rise between the leading edge and the centre-line.

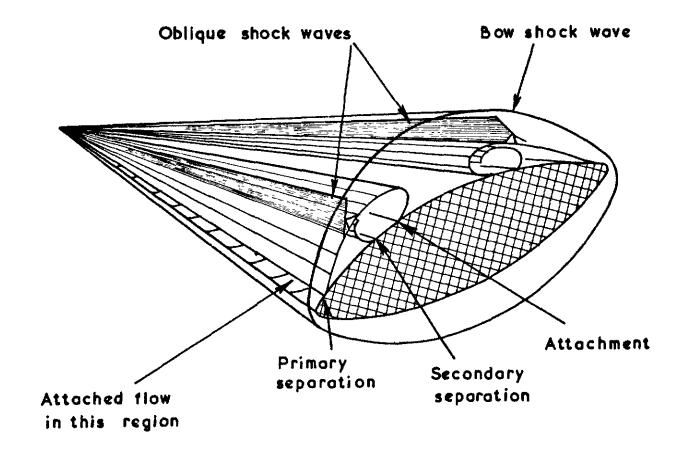
The pressures near the centre-line are found to be a function of only the free-stream Mach number and the surface inclination, and independent of the geometry of the lower surface or the pressure levels near the leading edge, provided the Mach number is greater than 2.5.

Experimental data at a Mach number of 4.0 have been used as the basis for predicting pressures near the centre line for Mach numbers greater than 2.5. The results obtained utilising this method compare favourably with experimental data and provide a first step towards predicting the complete flow field on the leeward surfaces of delta wings.

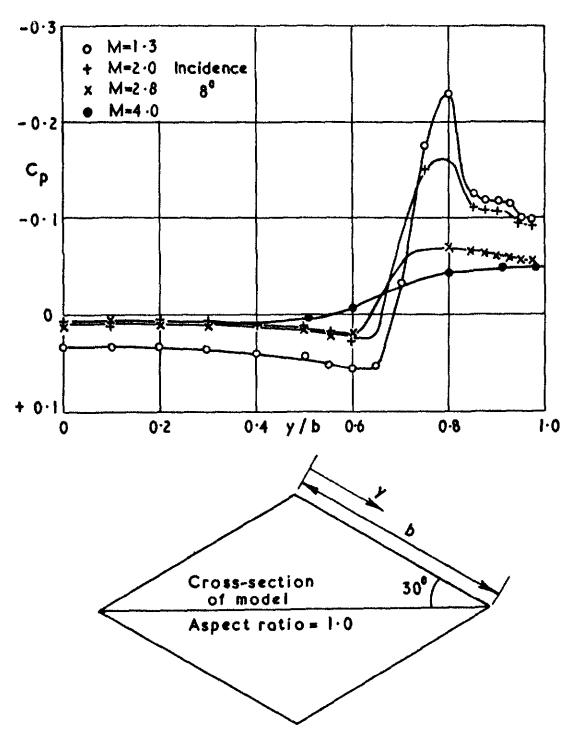
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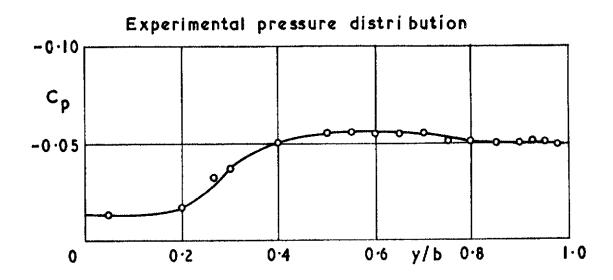
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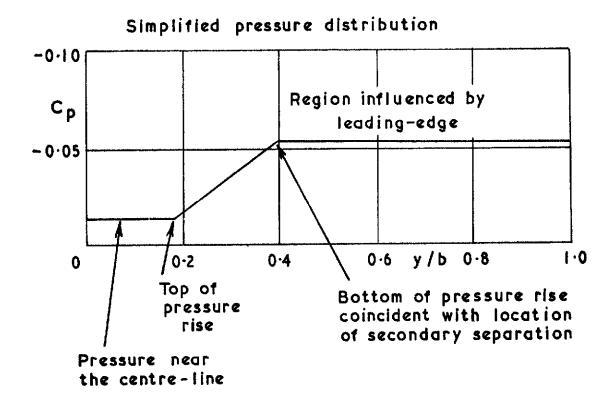


Sketch of flow over the leeward surface of a conical body at moderate supersonic speeds

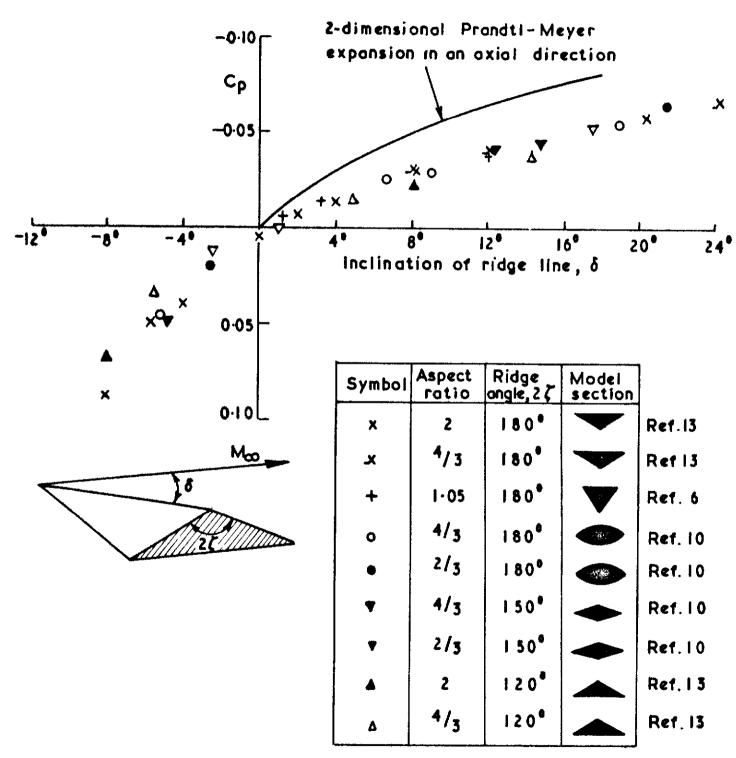


Pressure distributions across the leeward surface of a rhombic body at 8° incidence and various Mach numbers (Refs.4 and 10)

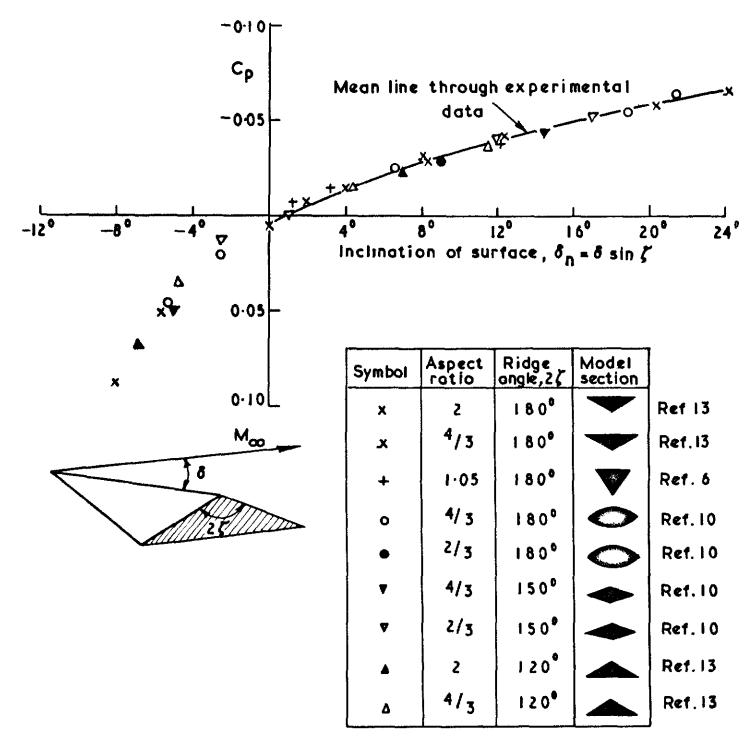




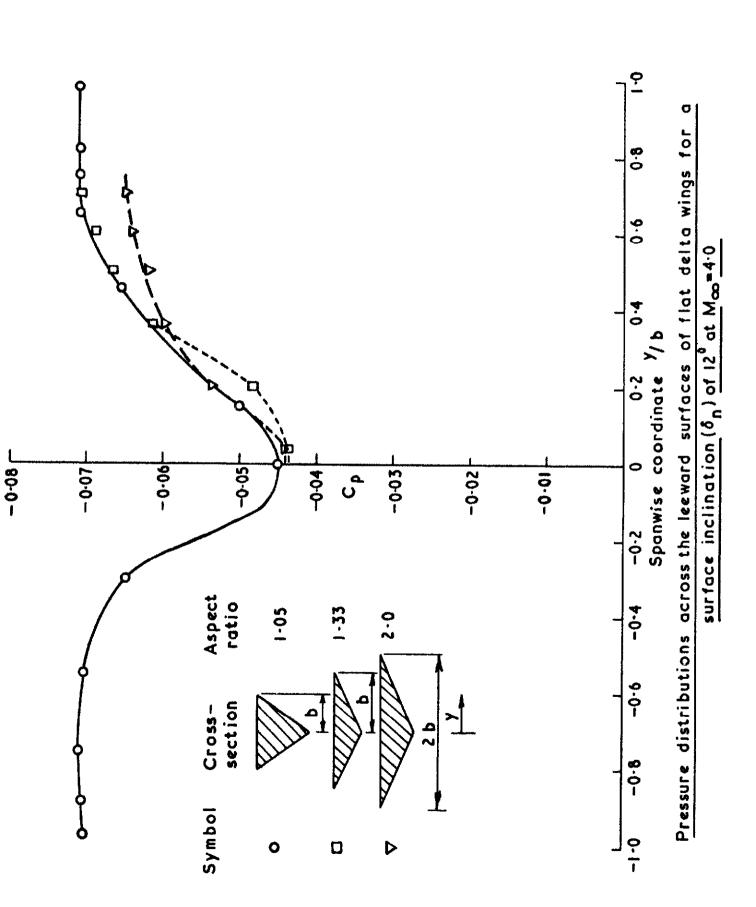
Comparison of simplified pressure distribution with experimental pressure distribution over the leeward surface of a conical body



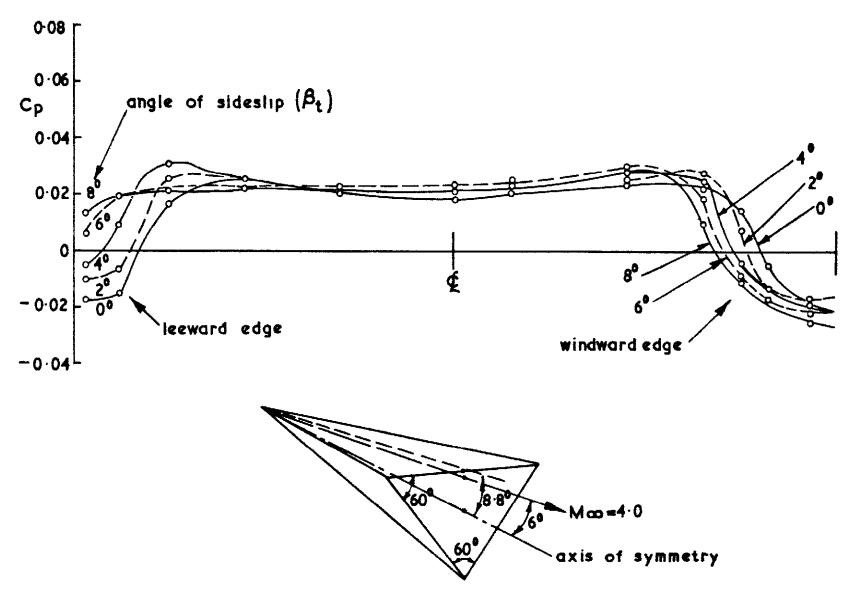
Pressure coefficient at the ridge line on leeward surfaces of conical bodies at $M_{\infty} = 4.0$



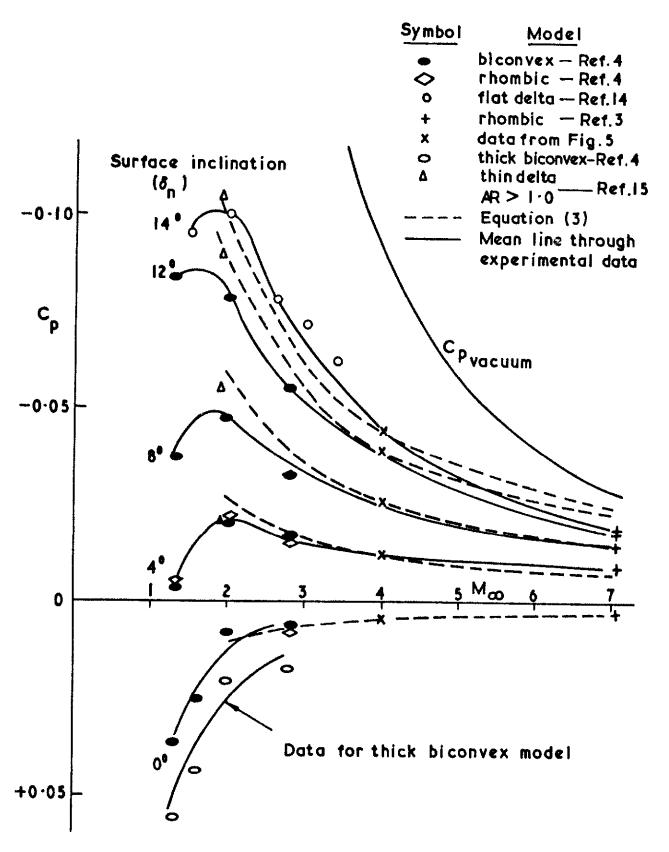
Correlation of pressure coefficient at the ridge line on leeward surfaces of conical bodies at $M_{\infty}=4.0$





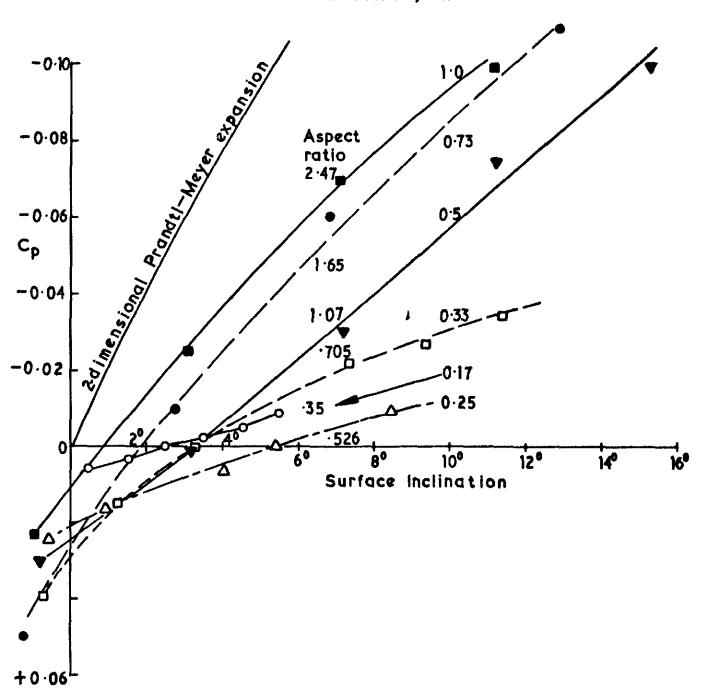


Pressure distributions across the leeward surface of a conical body at 6^0 incidence and various angles of sideslip at $M\infty=4\cdot0$

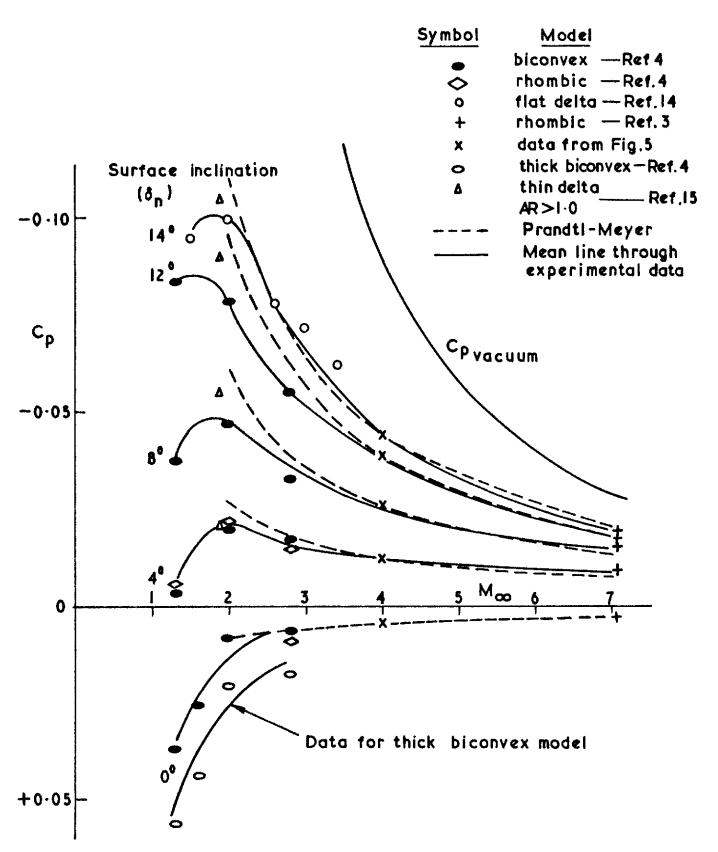


Variation of pressures at the centre-line of leeward surfaces with Mach number for various surface inclinations. Comparison with equation (3)

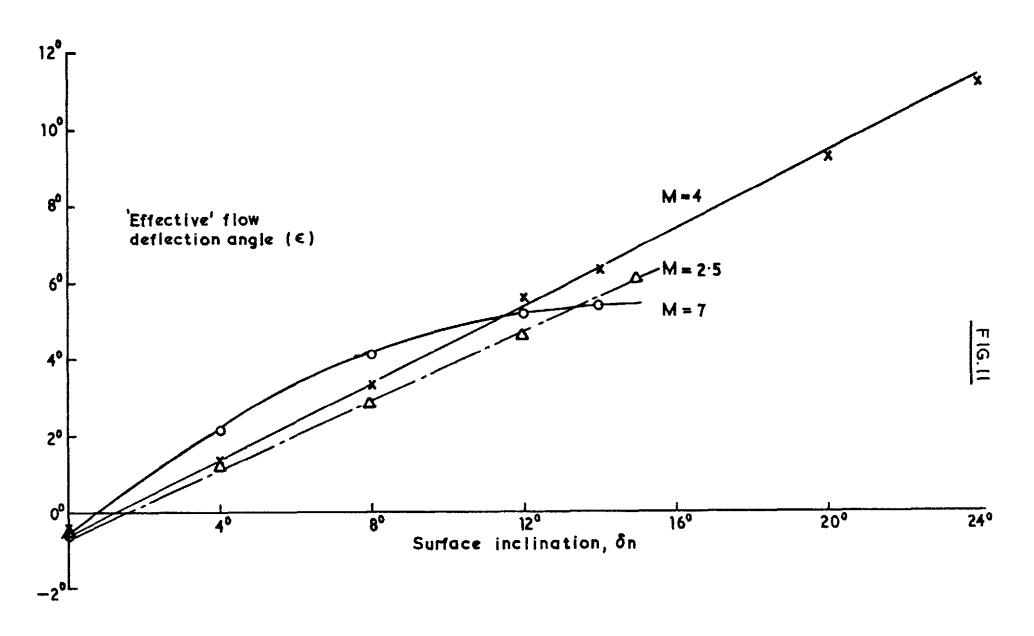
Mach number component normal to the leading edge at zero incidence, Mn



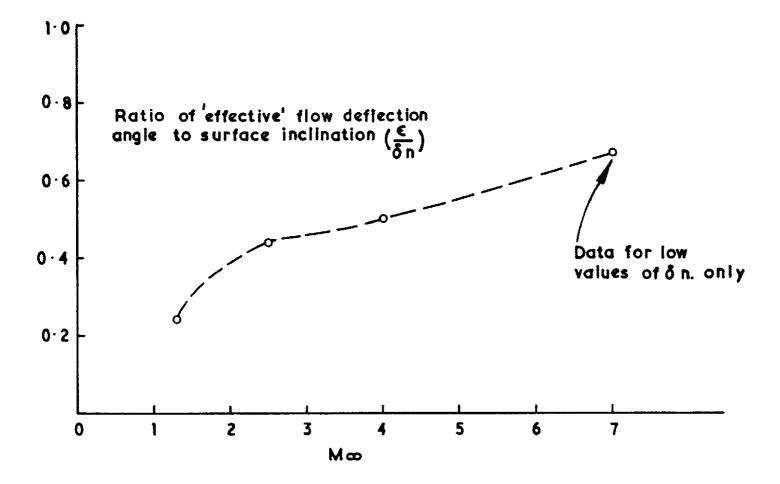
Effects of aspect ratio on the variation of pressure coefficient with surface inclination for thin delta wings at $M \infty = 1.9$ (Ref.15).



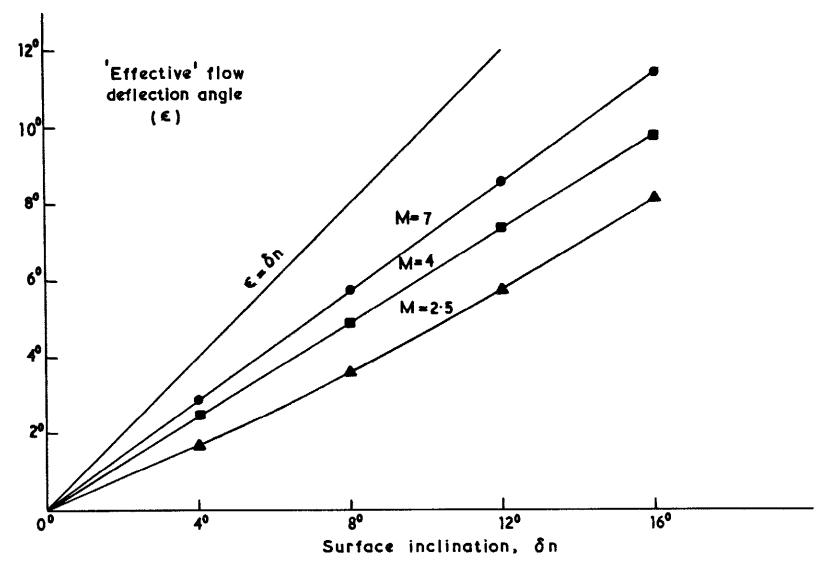
Variation of pressures at the centre-line of leeward surfaces with Mach number for various surface inclinations. Comparison with an axial expansion based on M=4 data.



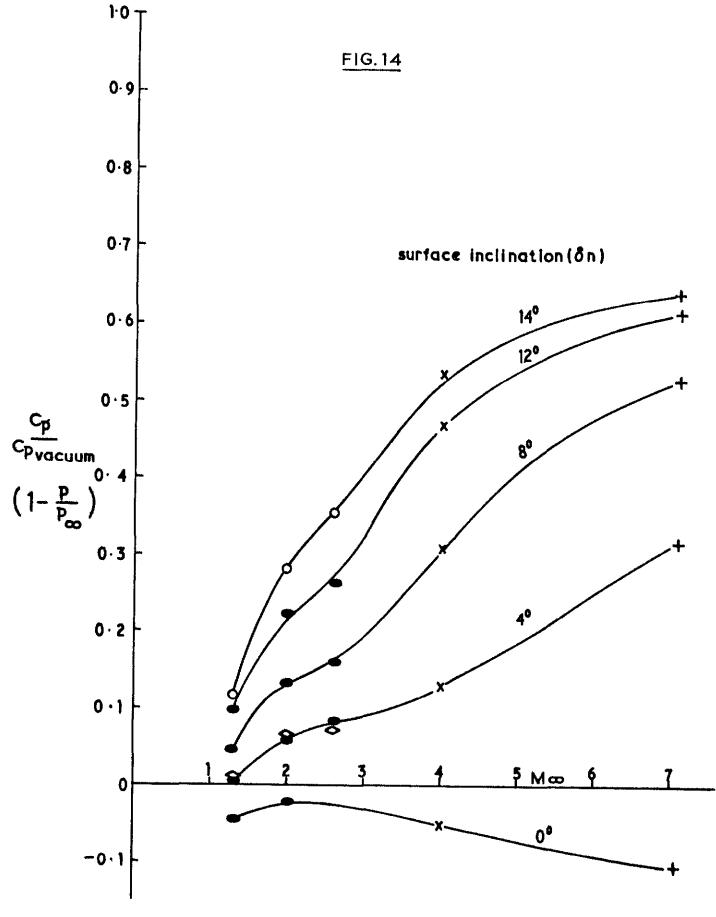
Variation of surface inclination for a delta wing with the 'effective' flow deflection angle for an axial exspansion.



Variation of ratio of effective flow deflection angle to true surface inclination with Mach number for the application of an axial expansion to the leeward surfaces of delta wings.



Computed variation of surface inclination for a thin delta wing of 30° semi-vertex angle (Ref.7) with the 'effective' flow deflection angle for an axial expansion.



Variation of pressures at the centre-line of delta wings with Mach number for various surface inclinations.

A.R.C. C.P. No.1153 June, 1970 M.J.Larcombe

> PRESSURES NEAR THE CENTRE-LINE OF LEEWARD SURFACES ON DELTA WINGS AND CONICAL BODIES AT HIGH SUPERSONIC SPEEDS

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