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By

M. J. Larcombe

Aerodynamics Dept NPL

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A Numerical Procedure for Constructing Shock-Wave Envelopes around Conical Bodies using Data from Schlieren Photographs

- By -
M. J. Larcombe
Aerodynamics Divn., NPL

SUMMARY

The cross-section shape of shock-waves surrounding conical bodies can be determined by schlieren photography from measurements of the tangent planes of the shock-wave. A numerical procedure has been devised to provide a mathematical function describing the shape of the shock-wave utilising the tangent plane data and thus reducing the amount of time spent in graphical constructions as well as providing data for further analysis.

Introduction

In an earlier paper[1] the author showed how shock-wave envelopes surrounding conical bodies could be constructed from data obtained by schlieren photography. The technique produced detailed information on the changes of shock-wave geometry for a variety of models in different attitudes and also the surface pressure distributions were computed from which overall forces could be obtained.

The process originally used for constructing the shock-wave envelopes was time consuming since it was completed graphically; also, measurements of shock-wave inclination, for example, to be used in subsequent calculations were then taken from the graph and therefore subject to larger errors than the original data.

A numerical curve fitting technique is now used to reduce the complexity of the process and also provide an analytic function describing the shape of the shock-wave for subsequent calculations.

Fig. 1/

* Replaces NPL Aero Note 1092 - A.R.C.32 299

Fig. 1 shows a cross-section of a model and its surrounding shock-wave demonstrating the method adopted for constructing the shock-wave envelope from a series of tangents at constant increments of roll angle (ϕ). The original data from the schlieren photographs are given in the form of tangent planes but because both the body and the shock-wave are conical all cross-sections are similar and the problem can be reduced to one in plane geometry.

The conditions for the construction of the shock-wave envelopes are that the inscribed curve should be continuous and also tangential to the original data tangents.

All measurements are made about an origin O (Fig. 2) through which the free stream axis passes and a data tangent is defined by the length of its normal, r, at a roll angle ϕ. The point of tangency on the curve is (x,y) in cartesian co-ordinates or (R,θ) in polar co-ordinates.

The equation of the family of tangents can be written as

$$\frac{r}{R} = \cos(\theta - \phi) \qquad \ldots (1)$$

or

$$r = R(\cos\theta\cos\phi + \sin\theta\sin\phi) \qquad \ldots (2)$$

where r and ϕ are parameters and R and θ are variables.

also,

$$\frac{\partial r}{\partial\phi} = R(-\cos\theta\sin\phi + \sin\theta\cos\phi) \qquad \ldots (3)$$

or

$$\frac{\partial r}{\partial\phi} = R\sin(\theta - \phi) \qquad \ldots (4)$$

Substituting for R from equation (1) gives

$$\frac{\partial r}{\partial\phi} = r\tan(\theta - \phi)$$

so that

$$\theta - \phi = \arctan\left[\frac{\partial r/\partial\phi}{r}\right] \qquad \ldots (5)$$

Thus, referring back to Fig. 2 the angular displacement between the normal to the tangent (r) and the radius vector (R) at the point of tangency is a function of the rate of change of the length of the normal with respect to angular rotation. Thus, for a circular section (when r is constant) $\theta - \phi$ is zero. Solution of the above equations gives the equation to the envelope of the family of tangents provided a continuous function can be found relating r and ϕ.

Because/

Because of the periodic nature of the problem it is convenient to write the equation involving r and ϕ in the form of a Fourier series,

$$r = f(\phi) = a_0 + a_1 \sin\phi + a_2 \sin 2\phi + a_3 \sin 3\phi + \dots a_m \sin m\phi$$
$$+ b_0 + b_1 \cos\phi + b_2 \cos 2\phi + b_3 \cos 3\phi + \dots b_m \cos m\phi \quad \dots (6)$$

also $$\frac{\partial r}{\partial\phi} = a_1 \cos\phi + 2a_2 \cos 2\phi + 3a_3 \cos 3\phi + \dots ma_m \cos m\phi$$
$$- b_1 \sin\phi - 2b_2 \sin 2\phi - 3b_3 \sin 3\phi - \dots mb_m \sin m\phi \quad \dots (7)$$

If there are n data tangents given in terms of the length of the normal r_α and the roll angle ϕ_α then the general equation for one data point is

$$r = a_0 + a_1 \sin\phi_\alpha + a_2 \sin 2\phi_\alpha + a_3 \sin 3\phi_\alpha + \dots a_m \sin m\phi_\alpha$$
$$+ b_0 + b_1 \cos\phi_\alpha + b_2 \cos 2\phi_\alpha + b_3 \cos 3\phi_\alpha + \dots b_m \cos m\phi_\alpha$$

If C_i represent the coefficients of the series $a_0, a_1, a_2 \dots a_m$ and $b_0, b_1, b_2 \dots b_m$ then they can be calculated using a least mean squares approximation[2] to the data,

where $$C_i = \frac{\sum_{\alpha=1}^{n} r_\alpha f_i(\phi_\alpha)}{N_i} \quad \dots (8)$$

and $f_i(\phi_\alpha)$ is the i^{th} function of ϕ_α, e.g. $\sin(i\phi_\alpha)$

N_i for the cosine terms is denoted $N_{ic} = \sum_1^n \cos^2(i\phi_\alpha)$

N_i for the sine terms is denoted $N_{is} = \sum_1^n \sin^2(i\phi_\alpha)$

Hence $$a_i \doteq \frac{\sum_{\alpha=1}^{n} r_\alpha \sin(i\phi_\alpha)}{N_{is}} \quad \dots (9)$$

and $$b_i = \frac{\sum_{\alpha=1}^{n} r_\alpha \cos(i\phi_\alpha)}{N_{ic}} \quad \dots (10)$$

Therefore,/

$$\left.\begin{aligned}
\text{Therefore,}\quad a_0 &= \frac{\sum_{\alpha=1}^{n} r_\alpha}{N_{0s}} = 0 \\
a_1 &= \frac{\sum_{\alpha=1}^{n} r \sin \phi_\alpha}{N_{1s}} \\
a_2 &= \frac{\sum_{\alpha=1}^{n} r \sin 2\phi_\alpha}{N_{2s}} \qquad \text{etc.} \\
\text{and}\quad b_0 &= \frac{\sum_{1}^{n} r_\alpha}{N_{0c}} \qquad \text{is the mean value of } r \\
b_1 &= \frac{\sum_{\alpha=1}^{n} r_\alpha \cos \phi_\alpha}{N_{1c}} \\
b_2 &= \frac{\sum_{\alpha=1}^{n} r_\alpha \cos 2\phi_\alpha}{N_{2c}} \qquad \text{etc.}
\end{aligned}\right\} \quad \cdots (11)$$

Following calculation of the coefficients for the series the values of r and $\partial r/\partial\phi$ from equations (6) and (7) can be calculated at preset increments of ϕ. The angular displacement $\theta - \phi$ can then be calculated from equation (5).

Referring to Fig. 2, the cartesian co-ordinates of the point of tangency on the shock-wave are then given by

$$\begin{aligned}
x &= \frac{r \sin \theta}{\cos(\theta - \phi)} \\
y &= \frac{r \cos \theta}{\cos(\theta - \phi)}
\end{aligned} \quad \cdots (12)$$

For later calculations involving the shape of the shock-wave it is convenient to present the R,θ co-ordinates of the shock-wave in constant increments of θ and to do this requires an iterative process because the value of R is not known 'a priori'; the curve is defined initially in terms of the co-ordinates r,ϕ.

These equations have been programmed in Autokode for the KDF9 computer using a graph plotter output. Some typical plotted shock-wave shapes are presented in Figs. 3, 4 and 5.

The/

The number of coefficients in the series is equal to $(2m+1)$ and must be less than the number of data points (n). Typically the order of the series is 5, giving 11 coefficients for 24 data points. In cases where the shock-wave envelope has discontinuities of curvature, which can occur when the shock-waves are attached to the leading edges of the model, the number of coefficients should be increased. Small errors in the original data are generally smoothed by the least mean squares fit to the data but in some situations, when the errors are large, a physically unrealistic picture of the shock-wave envelope is produced. Consider for example the data tangents drawn in Fig. 6. The central tangent at $\phi = 90°$ lies outside the point of intersection of the adjacent two tangents and could give a shock-wave shape as shown, but by reducing the number of coefficients in the series the curve can be made smooth and not re-entrant. The effect of reducing the number of coefficients in the series is shown successively in Figs. 7a, 7b and 7c where a number of data tangents are seriously in error. However, the number of coefficients cannot be reduced indiscriminately because a distorted shock-wave shape will result tending towards a circular cross-section. Usually the number of coefficients in the series has only a small effect on the resulting shape of the shock-wave but when data tangents are in error the number of coefficients should be reduced until at least two successive realistic and undistorted shock-wave shapes are produced. If this cannot be achieved then the errors in the data tangents are too large for the method to be applicable.

Conclusions

A numerical procedure has been devised to produce a continuous analytic function describing the cross-section shape of a conical shock-wave when the data are supplied in the form of tangent planes. The technique can smooth errors in the original data that would otherwise produce a physically unrealistic shock-wave shape. The mathematical function describing the shape of the shock-wave can be used in subsequent calculation methods for determining the lift and drag of conical bodies.

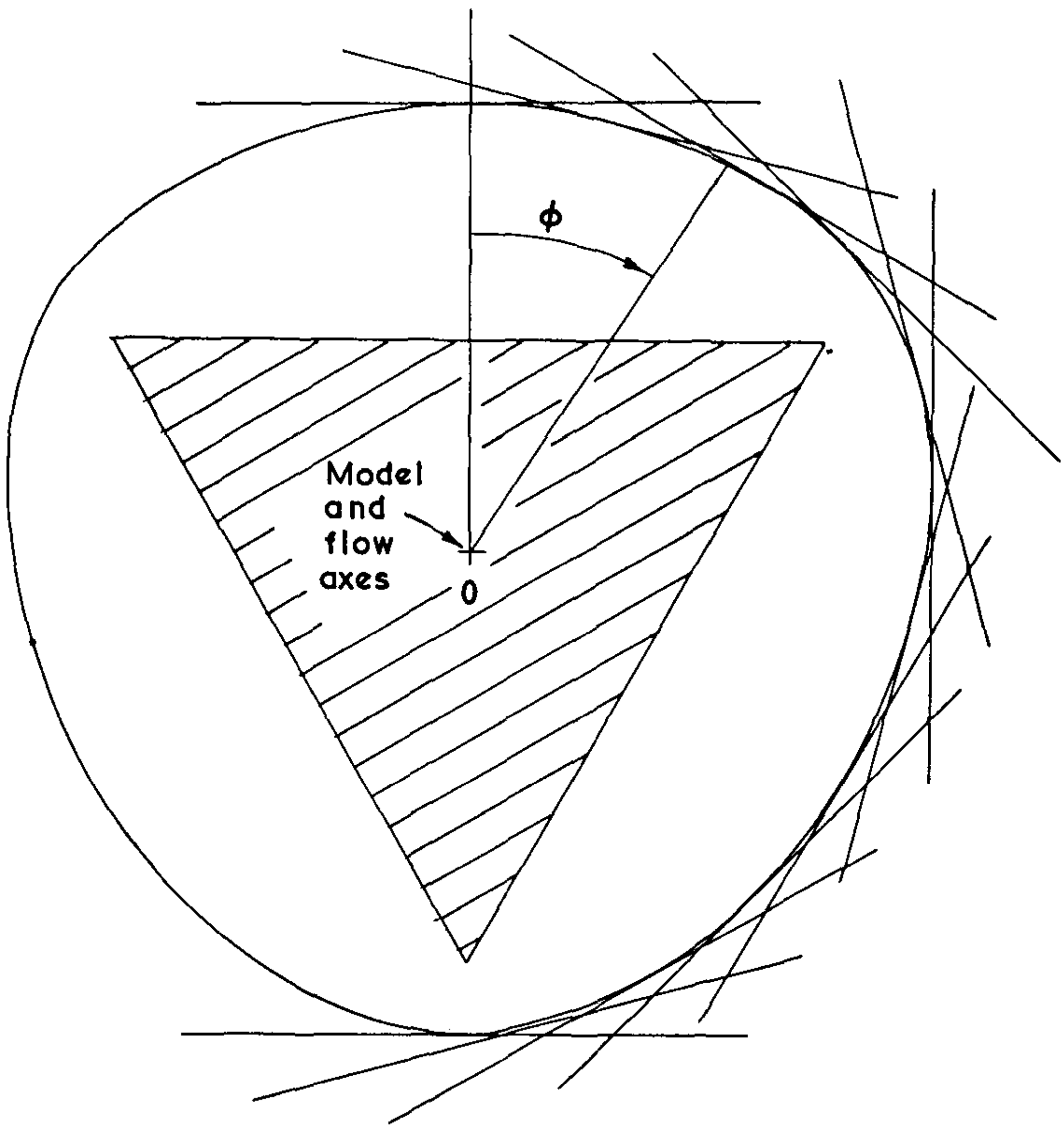
References

No.	Author(s)	Title, etc.
1	M. J. Larcombe	Estimation of surface pressures from observed shock-wave envelopes surrounding conical bodies at $M = 4.0$. NPL Aero Report 1244. August, 1967.
2	Cornelius Lanczos	Applied Analysis. Sir Isaac Pitman and Sons. 1957.

BMG

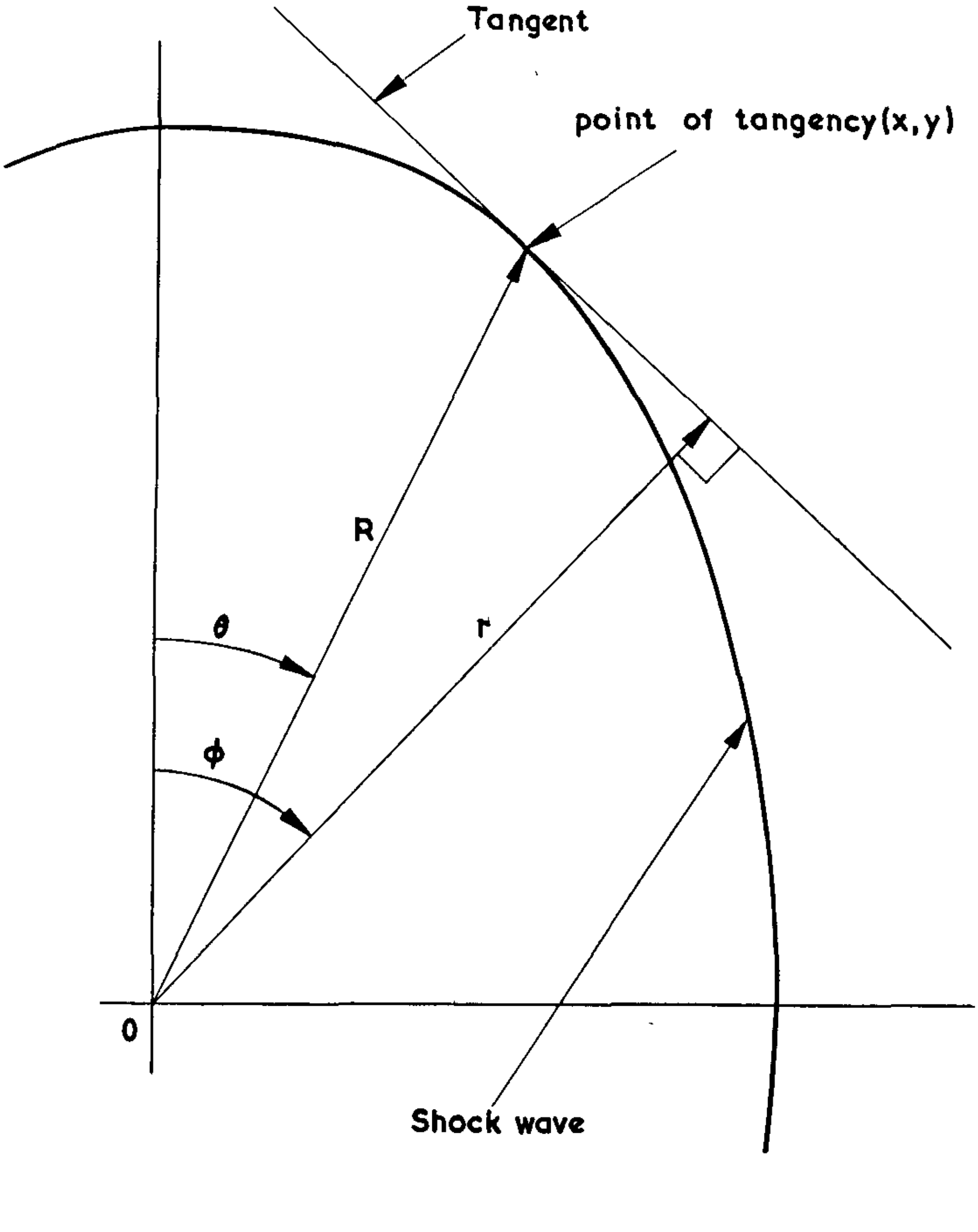
FIG. 1

Aspect ratio 1 05, zero incidence and $M_\infty = 4{\cdot}0$



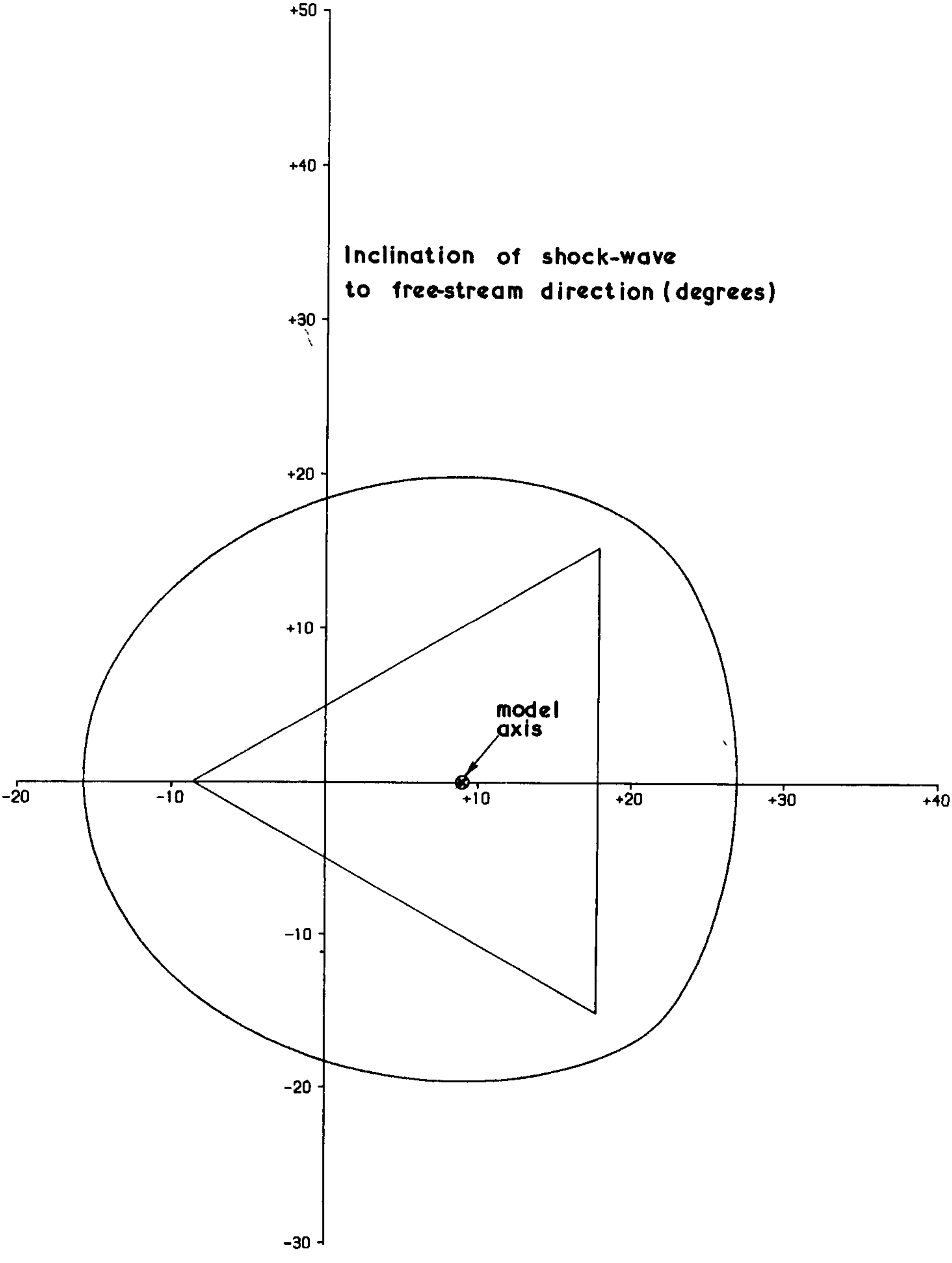
Construction of shock-wave envelope from tangent planes projected onto a plane perpendicular to the free-stream direction.

FIG.2

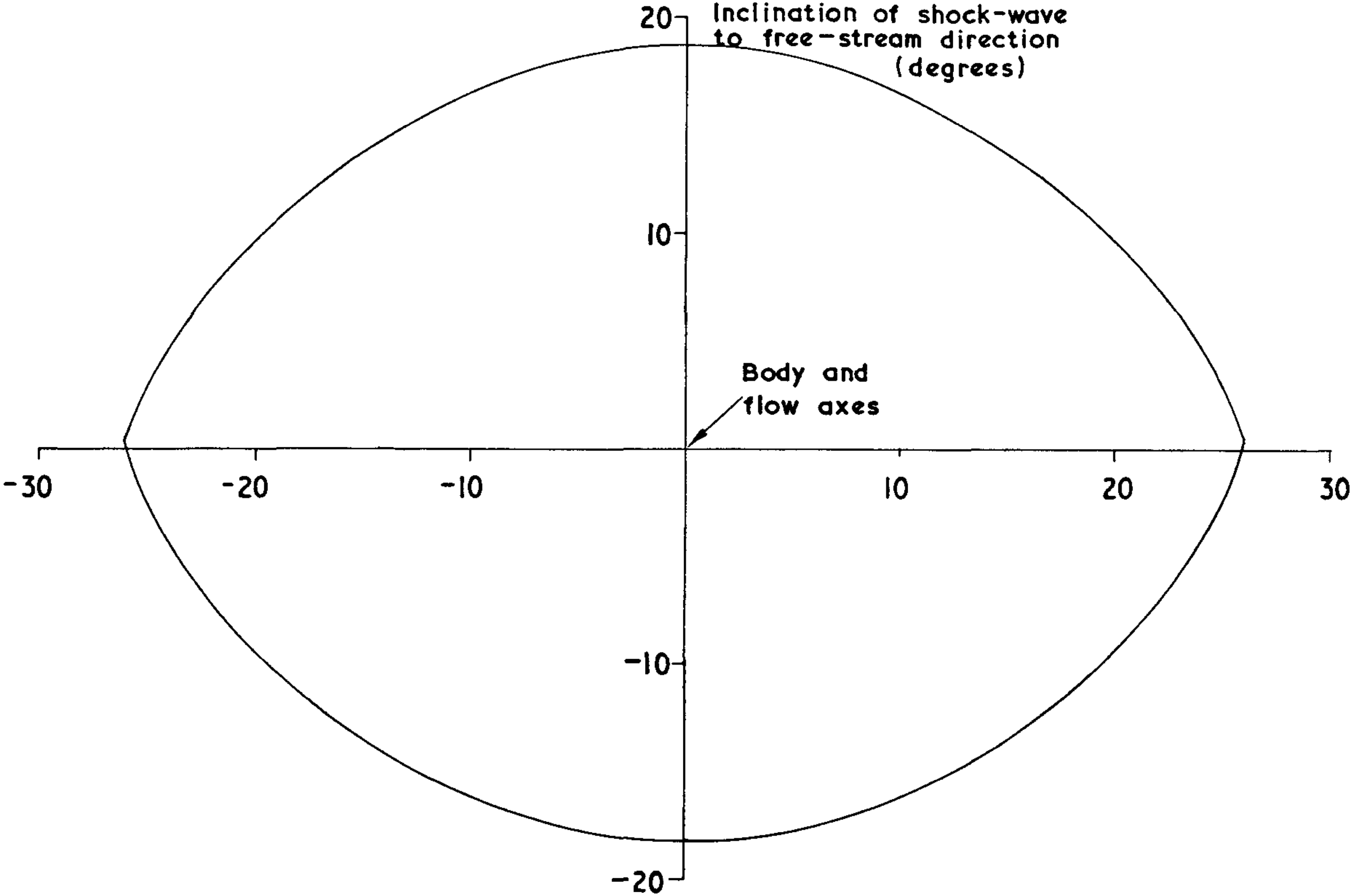


Notation for a tangent to the shock-wave

FIG. 3



Shock-wave shape around a conical body with triangular cross-section and aspect ratio 1·05 at M=4·0 and −8·8° incidence



Shock-wave shape around a conical body with rhombic cross-section and aspect ratio 1·94 at M = 4·0 and 0° incidence

FIG. 4

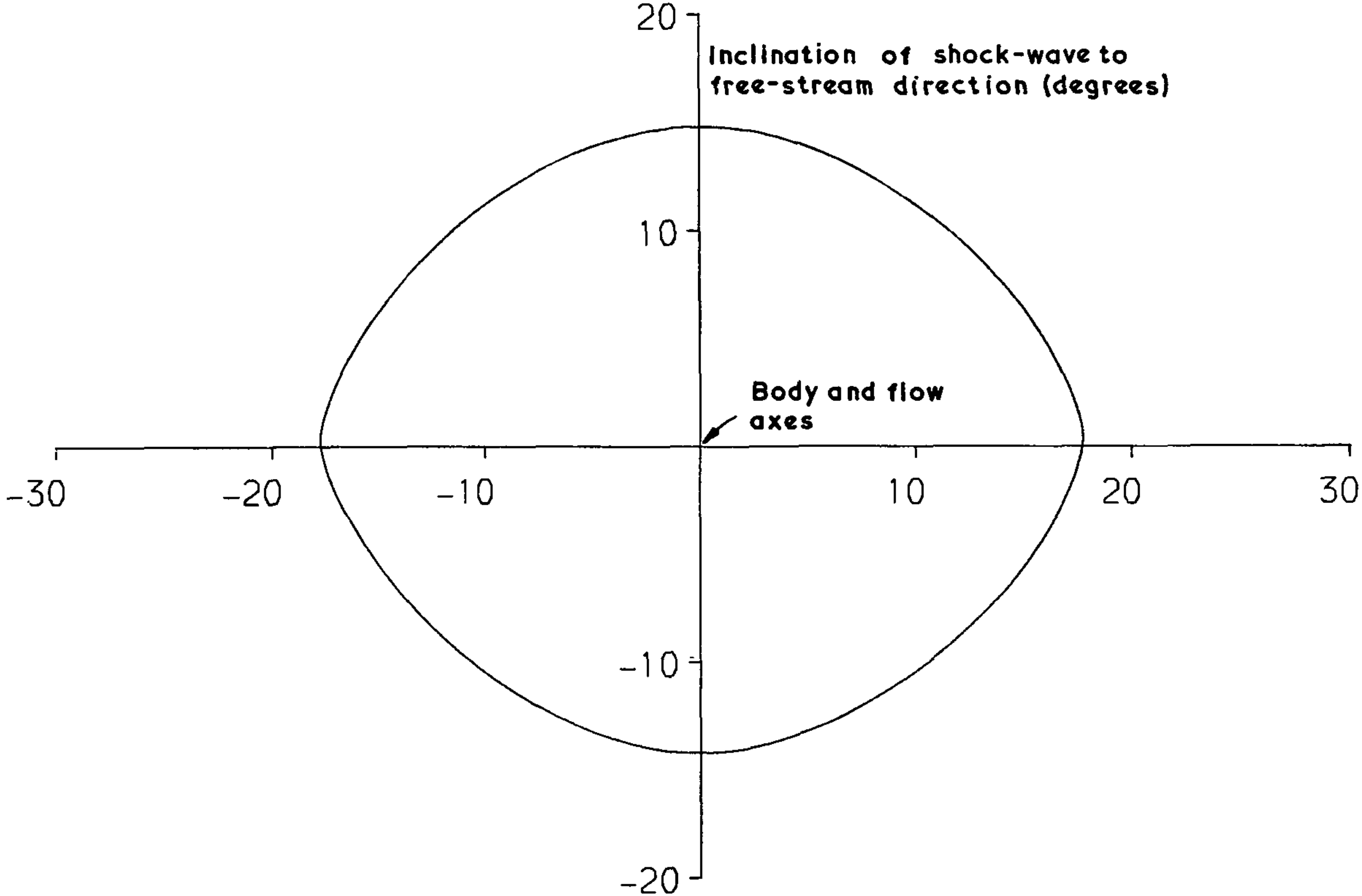
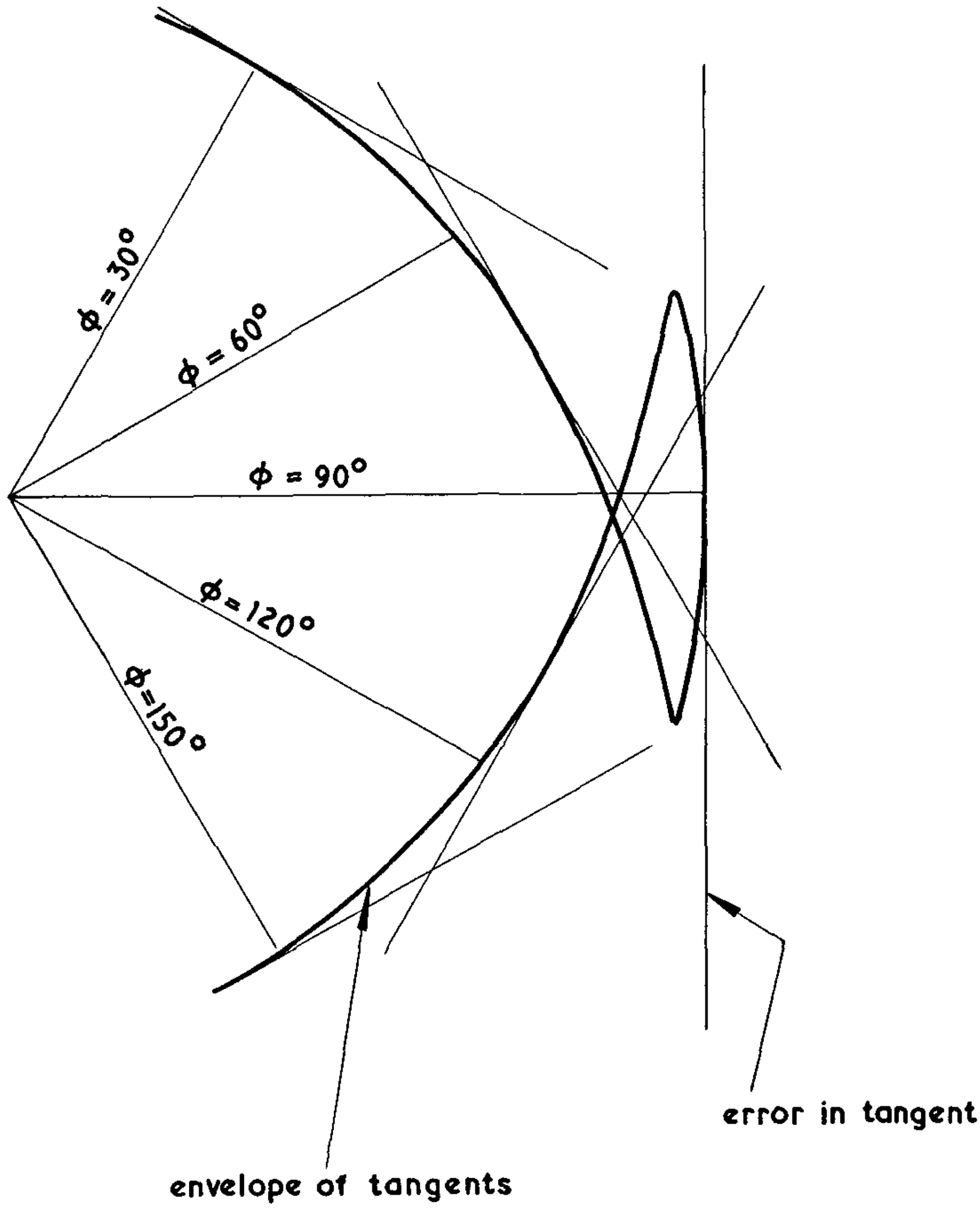


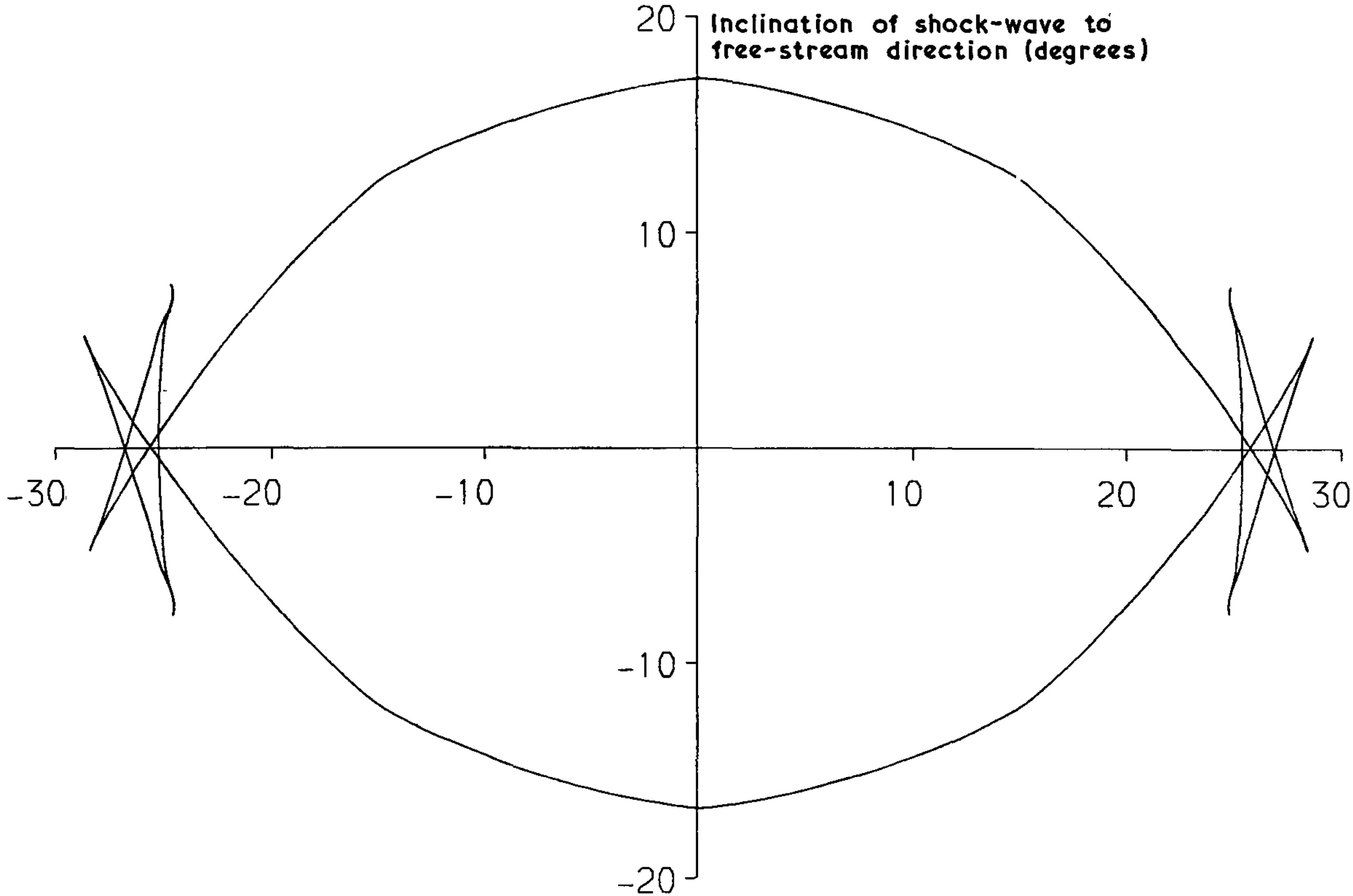
FIG.5

Shock-wave shape around a conical body with rhombic cross-section and aspect ratio $\frac{4}{3}$ at $M = 4 \cdot 0$ and 0° incidence.

FIG. 6

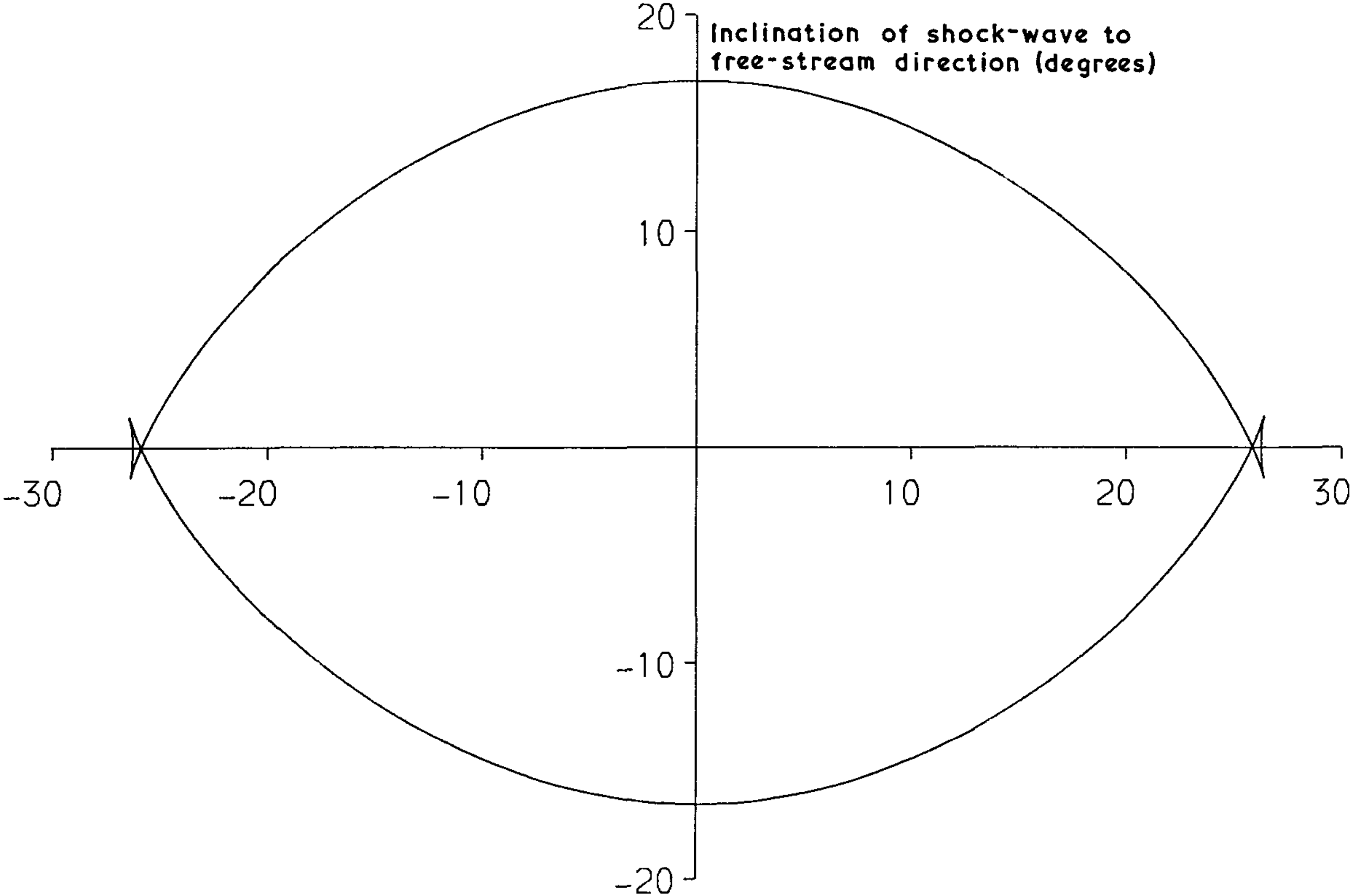


Effect of errors in the data on the construction of a shock-wave envelope



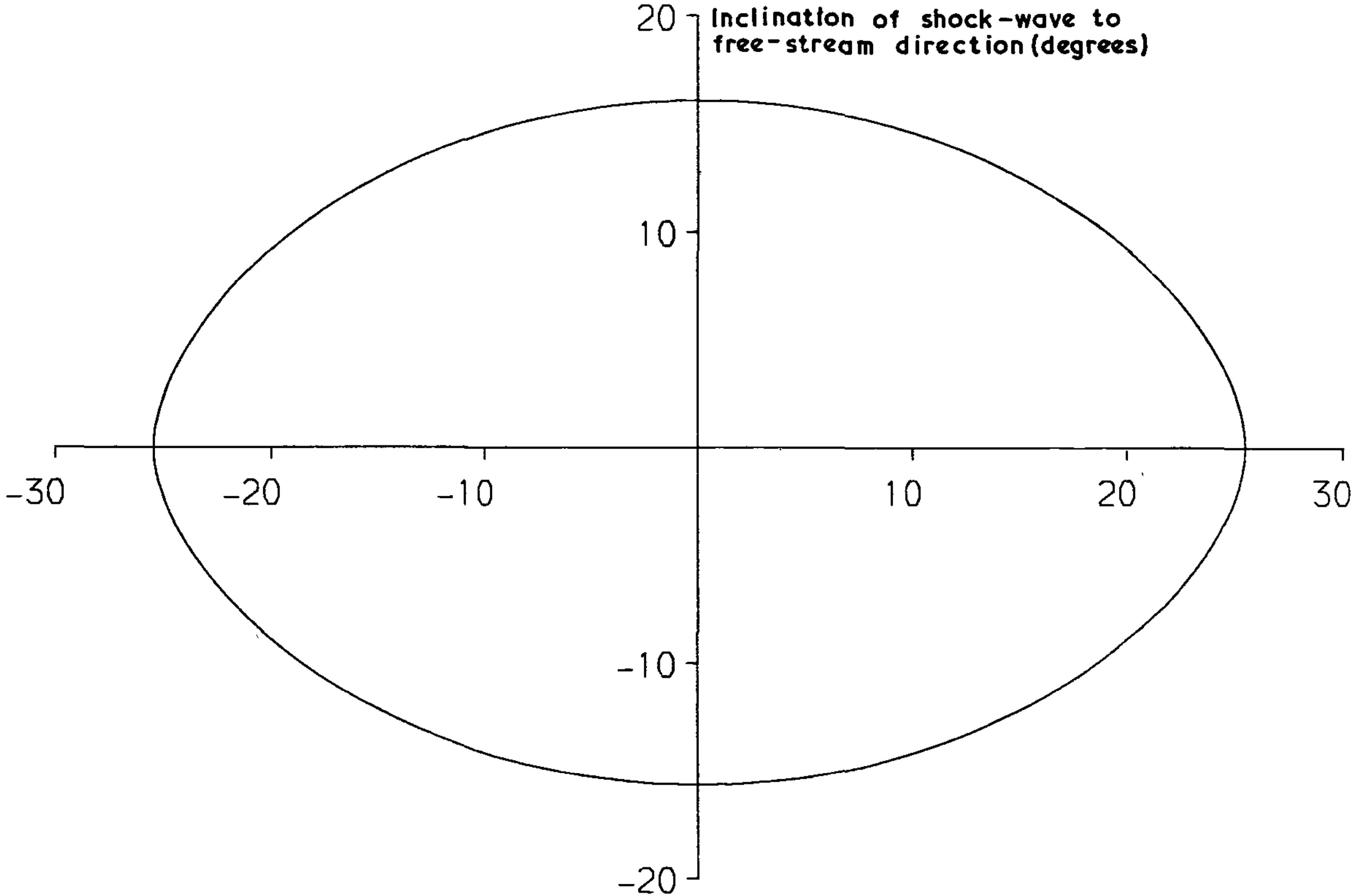
Shock-wave shape around a rhombic body with incorrect data tangents, number of data points (n) is 24 and the number of coefficients (2m+1) is 17

FIG.7a



Shock-wave shape around a rhombic body with incorrect data tangents.
Number of data points (n) is 24 and the number of coefficients (2m+1) is 9.

FIG.7b



Shock-wave shape around a rhombic body with incorrect data tangents

Number of data points (n) is 24 and the number of coefficients (2m + 1) is 7

FIG. 7c

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