C.P. No. 102
(14,138, 14,297)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
CURRENT PAPERS

Swept-wing Loading. A Critical Comparison of Four Subsonic Vortex Sheet Theories

By

H. C. Garner, B.A.,
of the Aerodynamics Division, N.P.L.

With Foreword by

L. W. Bryant, B.Sc.,
of the Aerodynamics Division, N.P.L.

LONDON : HER MAJESTY'S STATIONERY OFFICE

1952

Price 13s 0d net

Foreword

- By -
L. W. Bryant
of the Aerodynamics Division, N.P.L.

11th October, 1951

To appraise the conclusions of this paper it is necessary to distinguish two methods of approach to the problem of wing loading. The first method, actively pursued at the National Physical Laboratory, is to seek an accurate thin wing potential solution, later to be developed to take account semi-empirically of aerofoil thickness and boundary layers, with a final appeal to experiment. The second method seeks at the outset to coordinate experimental results, making use of potential theory in a less rigorous manner than in the first method, to evolve a rapid process of computation reliable enough for practical use.

The present investigation is concerned solely with the first method of approach, the concept of reaching an accurate solution for a thin wing in potential flow and the relative promise of the various computational procedures so far proposed to achieve this. The necessary accuracy to be sought by the basic potential theory depends on the particular wing characteristics which are to be computed. But it appears that for distributions of local lift and aerodynamic centre every effort should be made to arrive at the most reliable solution possible without prohibitive labour. For this purpose the speed and consistency of Multhopp's method promises well, but more work is needed to assess its accuracy with regard to local aerodynamic centre; and this basic investigation is proceeding. However, the general conclusions in this paper seem to be well supported by the evidence of comparative results from the available theories.

An appeal to experiment for a check on the validity of a particular potential theory is not at this stage of real help, because all experimental results have to be "corrected" for thickness and boundary layer effects before they become comparable with theory. The object of seeking a trustworthy theory is precisely to find out how to make these "corrections" for a wide range of plan form, aerofoil section, Reynolds number and Mach number.

From the standpoint of the designer aerodynamicist, these fundamental considerations are of little immediate value, as the desired "corrections" for swept wings are not yet known with sufficient confidence. Thence arises the need to approach the problem of wing loading in the second way mentioned above. Küchemann's valuable work in this direction has succeeded in establishing from experimental sources some very important guiding principles; and his method would appear to be capable of powerful empirical development.

There remains, however, the important goal of establishing a reliable potential theory, in which aerofoil thickness and boundary layers are neglected. Garner's view is that the conditions essential to such a solution are much better satisfied by Multhopp's purely theoretical method than by Küchemann's semi-empirical method.

Swept-wing Loading. A Critical Comparison of Four Subsonic Vortex Sheet Theories

- By -
H. C. Garner, B.A.
of the Aerodynamics Division, N.P.L.

12th July, 1951

Summary

From a systematic series of calculations of swept-wing loading the writer has formed an opinion of the accuracy and most useful application of vortex lattice theory and the vortex sheet theories of Weissinger, Multhopp and Küchemann. The results provide a general picture of the effect of sweep and compressibility on lift slope and aerodynamic centre. It is recommended that:-

(i) An elaborate solution by Multhopp's theory should be used when special accuracy is required.

(ii) It should normally be possible to choose a shorter version of Multhopp's theory which may be expected to provide a potential solution at least as quickly and more accurately than any other given theory.

(iii) Vortex lattice theory is to be preferred when additional calculations of control characteristics or flutter derivatives are required for the same plan form and supreme accuracy is not essential.

(iv) Weissinger's theory (with a modified procedure) is to be preferred when estimating the effects of compressibility and sectional lift slope on suitable plan forms.

(v) Küchemann's theory, being essentially a lifting line theory with a semi-empirical correction for sweep, will roughly tackle a wide range of lateral stability derivatives and may allow for three-dimensional boundary layer characteristics. Its practical value should grow with experience.

This note is intended to prepare the ground for two developments, which require urgent study and in which vortex sheet theory must play an important part:-

(a) The use of sectional data in the prediction of aerodynamic characteristics of swept wings.

(b) A method of calculation of general application to swept wings at high subsonic speeds.

Summary/

Published with permission of the Director, National Physical Laboratory.

Summary of Contents

Tables

Figures/

Figures

---

1. Introduction

In the search for more accurate data on aerodynamic derivatives of swept wings, a reliable theoretical potential solution is an essential calculation. There are many vortex sheet theories which approximate to such solutions for wings of zero thickness in inviscid flow below the critical Mach number. In the absence of an analytically exact potential solution for any swept wing the inconsistency of the various theories is a fundamental drawback, if the effects of wing section and viscosity are to be understood. From this standpoint a theory is not necessarily enhanced by a favourable comparison with experiment. Its intrinsic accuracy must be assured.

The historical threads are gathered from Ref.1 (1949), which reports on a special discussion of the problem of load distribution on finite swept-back wings. At the suggestion of Gates this meeting agreed that studies should be made of families of plan forms related according to the linear perturbation theory of the effect of compressibility.

Three such (δ, λ) families of four plan forms were selected, as shown in Fig.1:-

(a) Delta wings - 1,2,3,4:

(b) Pointed arrowhead wings - 5,6,7,8:

(c) Medium tapered arrowhead wings - 9,10,11,12.

The geometrical parameters are defined in Fig.2. The four wings, shown in Ref.1, Fig.4(a), form a further set:-

(d) Cropped wings of 45° sweep-back - A,B,C,D.

It should be noted that plan forms 7 and D are identical, as are 10 and B.

The recommended programme of calculations in Ref.1 has been revised in accordance with the development of the various theories. The agreed experimental programme is in hand at N.P.L. Consideration of the effects of wing thickness and viscosity will be postponed till the experiments are completed. The present note is mainly concerned with potential vortex sheet theories applied to the swept plan forms of Fig.1.

2. Theoretical Background

A general approach to an accurate potential solution for a finite swept wing is considered in Ref.2; and a solution on that basis has been obtained[3] for a delta wing (Fig.1, Wing 2). The writer has applied a similar scheme to each of the wings in family (d), but serious ill-conditioning of the equations has prevented the use of all the solving points necessary in a reliable check solution envisaged in Ref.1. However, a separate report of this work will be published[+] in due course; and results now being obtained on a simplified basis substantiate the conclusions of this note, as far as can be judged.

Of the routine methods discussed in Ref.1 only the vortex lattice theory has survived in current use. This theory is employed here with and without P functions in order that their value may be assessed (§4.1).

[+]A.R.C. 14,781 (Garner and Acum, April, 1952).

In/

In U.S.A., Weissinger's method is favoured. As a result of comparisons with vortex lattice theory in Ref.7 (Van Dorn and DeYoung, 1947) and subsequent development by DeYoung[8] (1947), the method is shown to be of great value for wings of uniform sweep and taper. The use of a solving point at the three-quarter chord point of the "kinked" central section of a swept wing is open to criticism. It is of interest to compare solutions in which it is omitted (§5.1). Both types of solution have been applied to representative wings in Fig.1.

Since the discussion of Ref.1 two other distinct theories have been published. Multhopp's lifting surface theory[9] (1950) may reasonably claim to be the most accurate routine method; its computation however is elegant but not short. Küchemann's theory[10] (1950) may reasonably claim to be the most rapid method of calculating swept-wing loading, but its limitations in accuracy must be clearly recognized. Calculations by Küchemann's method are provided in Ref.11 (Dee, 1951).

Some comparisons with a theoretical formula for lift slope and the elliptic quarter chord point for aerodynamic centre, as suggested in Ref.12 (Bryant and Garner, 1950), are also included. The six methods of solution are summarized as follows:-

(1) Vortex Lattice 6 point (§4):
(2) Vortex Lattice 8 point (§4.1):
(3) Weissinger 4 point (§5):
(4) Weissinger 3 point (§5.1):
(5) Multhopp 16 point (§6):
(6) Küchemann 8 point (§7).

The basic physical concept, demands of computation, distribution of solving points and advantages of each solution are set out in tabular form in Fig.3 and are more fully discussed later in the appropriate paragraphs.

2.1 Experimental Background

This programme of calculations for the families of wings in Fig.1 will be supported by low-speed tunnel tests on at least one wing in each family:-

i.e., Family (a) Wing 2:
Family (b) Wing 7:
Family (c) Wing 10:
Family (d) All four wings.

Pressure plotting at two sections will provide some information on spanwise loading and local aerodynamic centre. The practical requirements of chordwise loading will be further deduced from measurements of the spanwise distribution of hinge moment on control flaps of two chord ratios, $E = 0.2$ and 0.4. The wing section throughout is R.A.E. 102, on which systematic two-dimensional tests with both control flaps are being carried out at N.P.L. at a Reynolds number, $R = 10^6$, which will be covered in the three-dimensional tests.

From/

From the practical point of view it is desirable to be able to predict the aerodynamic derivatives of wings. The relative merits of vortex lattice theory and Weissinger's theory have been discussed in Ref.13 (Toll and Diederich, 1948). The primary consideration here in relation to the present fundamental approach is whether a vortex sheet theory can be adapted to include wing sectional characteristics. The systematic experimental programme outlined above may be used to judge the practical claims of any theory; but unless that theory can also be substantiated in potential flow it can hardly be used with confidence to estimate the effects of wing sectional modifications and of changes in Reynolds number from model to full scale.

2.2 Future Theoretical Requirements

The need for more precise knowledge of the capabilities of vortex sheet theories is associated with the modern trends towards lower aspect ratios and higher speeds. The characteristics of a wing at a subsonic Mach number M are related to the incompressible flow past a wing with its lateral dimensions reduced by a factor $\sqrt{1 - M^2}$. This effective reduction in thickness chord ratio favours the vortex sheet theories; but their reliability deteriorates with such a decrease in aspect ratio, since two-dimensional considerations are inherent in the assumed chordwise loadings and the choice of solving points. The adaptation of the current subsonic theories to transonic flow is a matter for intensive research.

As aspect ratio decreases the emphasis shifts from spanwise loading to chordwise loading; and various theoretical treatments on this basis for swept wings have appeared recently. A new method[14] (Lawrence, 1951) will assist the practical problem for delta wings at least. Robinson's[15] (1950) theory for swallow tail wings will be a useful guide, when the trailing edge is swept-back. Both these theories are less suitable for the plan forms selected in Fig.1 than for wings of very low aspect ratio. But they should form part of a comprehensive study of the theories available for calculating wing loading in transonic flow.

It is hoped that this note will help to prepare the ground for two developments in which vortex sheet theory must play an important part:-

(a) The use of sectional data in the prediction of aerodynamic characteristics of swept wings:

(b) A method of calculation of general application to swept wings at high subsonic speeds.

3. Results

Potential solutions for each of the wings shown in Fig.1 have been obtained by some of the following theories:-

(1) Vortex Lattice theory (§4)

(2) Weissinger's theory (§5)

(3) Multhopp's theory (§6)

(4) Küchemann's theory (§7)

The solutions by vortex lattice theory for Wings 1,2, ... 12 are presented in Tables I, II, ... XII respectively. Each wing has been calculated with and without P functions (§4.1), and Wings 2 and 10 by a third solution with additional chordwise terms.

Solutions/

Solutions by Weissinger's theory for Wings 1,2,5,7,9 and 10 and family (d) (including Wings 7 and 10) are given in Table XIII. In each case a modified 3 point solution in included, as explained in §5.1.

Solutions by Multhopp's theory for Wing 2 and family (d) are to be found in Table XIV.

Küchemann applied his theory to Wing 2 in his original report[10] and calculations by his method for the four wings A,B,C,D in family (d) are provided in Ref.11.

Theoretical values of the lift slope $\partial C_L/\partial \alpha$ and the aerodynamic centre (measured as a fraction of the mean chord from the leading apex) are summarized in Tables XV and XVI respectively. Comparisons of $\partial C_L/\partial \alpha$ with a simple formula (§8) and of aerodynamic centre with the elliptic quarter chord point (§8.1) are included.

3.1 Lift Slope

As explained in §1 and Fig.1, the selected plan forms are arranged in four families:

(a) Delta wings:

(b) Pointed arrowhead wings:

(c) Medium tapered arrowhead wings:

(d) Cropped wings of 45° sweep-back.

Some curves of $\partial C_L/\partial \alpha$ for each family are given in Fig.4; and all the calculations are summarized in Table XV. Comparisons of the standard 6 point vortex lattice solution and Weissinger's standard 4 point solution reveal that the former gives a consistently higher lift slope, the percentage difference increasing with sweep-back and being of the order 6% for $\Lambda = 45°$. Typical discrepancies of this order were found in Ref.7.

When P functions are introduced into the vortex lattice calculations $\partial C_L/\partial \alpha$ is lowered slightly. The removal of the central three-quarter chord solving point in Weissinger's solution has the effect of raising $\partial C_L/\partial \alpha$. The two theories thus modified are normally within 3%.

It is very probable that Weissinger's standard solution always overestimates the effect of the central kink and that this effect is underestimated when his solution is modified (§5.1). In four cases out of five, lift slopes calculated by Multhopp's theory are within 0.5% of the mean of Weissinger's standard and modified values. Such consistency is encouraging; and having regard to the even better agreement between Multhopp's solution and the check solution[3] for Wing 2 by continuous numerical integration, the available evidence suggests that the accuracy of Multhopp's method is superior to that of the other routine vortex sheet theories.

By comparison Küchemann's method seems inconsistent, giving at times values of $\partial C_L/\partial \alpha$ greater than vortex lattice theory and in the case of the pointed Wing 7 values lower than Weissinger's standard solution.

The/

The approximate formula of equation (14) seems to be accurate within ± 3%, unless either the angle of sweep-back exceeds 60° or the aspect ratio is less than 1.5. Beyond these limits the formula reads high and thus tends to underestimate the effective sweep-back Λ' (§8) and not to allow fully for induced aerodynamic camber.

The general effect of sweep on lift slope is shown in Fig.8. The dotted curves for wings of given sweep are deduced from the four full curves representing the calculations for a family of wings (Fig.1).

3.2 Aerodynamic Centre

The aerodynamic centre by any linearized vortex sheet theory for an uncambered wing occurs at the point of intersection of the axis of zero pitching moment and the centre line of the wing. The quantity a.c. is defined as the distance of the aerodynamic centre downstream of the loading apex measured as a fraction of the mean chord.

Some curves of a.c. for each family of wings (Fig.1) are given in Fig.5; and all the calculations are summarized in Table XVI.

Although Weissinger's method predicts a negligible change of a.c. within a (δ, λ) family, vortex lattice theory suggests an appreciable increase in the quantity a.c. with decreasing aspect ratio. Thus there may be a rearward movement of aerodynamic centre associated with compressibility at subsonic speeds.

Falkner's standard vortex lattice solutions give a central kink in the local aerodynamic centre, but a systematic rounding off is used here (§4). The effect of this is shown in Table XVI to be more important than the introduction of P functions and two extra solving points (Fig.3). There is no apparent improvement in a.c. through taking three chordwise solving points as in the 9 point solutions for Wings 2 and 10.

One drawback to Weissinger's theory is his strict adherence to the two-dimensional type of chordwise loading, which presupposes a local a.c. on the quarter chord locus. However for swept wings the a.c. also depends on the spanwise distribution of lift and this is partly covered in Weissinger's standard 4 point solution. In the modified 3 point solution the conditions at the central section are left free and are accounted for by smooth interpolation (§5.1), which probably exaggerates the chordwise displacement of local a.c. The average of the standard 4 point and modified 3 point solutions is compared with the best vortex lattice solution (8 point rounded) and Multhopp's solution in Table XVI. When these last two agree the comparison with the average value from Weissinger's theory is excellent. The discrepancy between Multhopp's solutions and vortex lattice theory for Wing 7 is due to the pointed tip and its influence on the calculated spanwise loading (§3.3).

Küchemann's method is in fair agreement with Multhopp's theory for Wings 2, B (or 10) and C. But serious differences for Wings A and D suggest that Küchemann's method as a potential theory may only be suitable for medium tapered wings ($0.15 < \lambda < 0.45$).

It is interesting to compare the calculated aerodynamic centres with the goemetrically defined elliptic quarter chord $\bar{h}$ (Fig.2 and §8.1), which is shown as a function of δ and λ in Fig.9(a). Theoretical values of (a.c. - $\bar{h}$) for the plan forms of family (d) are plotted in

Fig.9(b)/

Fig.9(b), where the variations are quite as dependent on taper ratio as on aspect ratio. With this guide and other theoretical comparisons of a.c. and $\bar{h}$ in Fig.9(a) the elliptic quarter chord point may often be used with discretion to obtain rough estimates of the aerodynamic centre within $\pm$ 0.03$\bar{c}$.

As a general conclusion for most swept wings, it is necessary to use one of the more elaborate routine vortex sheet theories to evaluate the position of the aerodynamic centre within 0.02$\bar{c}$. If less accuracy is required Weissinger's method is recommended for wings of uniform sweep and taper as follows:

(1) Complete 4 point solution as set out in Ref.7, App. C;

(2) Repeat the final stages of (1) by modifying one equation as suggested in §5.1 (modified 3 point solution);

(3) Take the average of the a.c.'s calculated from the formulae appropriate to (1) and (2).

The computation should not exceed one day.

3.3 Spanwise Loading

The spanwise distributions of lift have been calculated for Wings 7 and 10 by the six methods summarized in Fig.3. The quantity $c\,C_{LL}/\bar{c}\,C_L$, representing the lift per unit span divided by its average value, is plotted against the spanwise distance η for Wings 7, 10, A and C in Figs. 6(a), 6(b), 6(c) and 6(d) respectively.

It is interesting to note that the two solutions from Weissinger's theory tend to give a fairly wide variation near the central section. A mean curve can be expected to give a good estimate of the distribution of lift for most swept wings.

For the pointed wing in Fig.6(a) Küchemann's method appears untrustworthy, giving a much reduced loading near the tip. The curves corresponding to the methods of Multhopp and Küchemann have in common a steep finite slope at the tip, while the other theories show the more usual infinite slope associated with elliptic loading. By the nature of the distribution of solving points (Fig.3) these two theories are more likely to be correct near the tip and this raises the problem of finite or infinite spanwise pressure gradients near a pointed wing tip. Although conventional wings are not pointed, this behaviour is of more than academic interest since triangular[14] and swallow tail[15] wings of low aspect ratio are more amenable to theoretical treatment, which may yield results of general interest. Further it should be noted that the inconsistent theoretical aerodynamic centres calculated for Wing 7 are primarily due to these differences in the spanwise loading.

The other wings do not reveal such serious discrepancies in spanwise loading by the different theories, but Küchemann's method apparently gives slight excess loading near the tip for the wings of lower aspect ratio. In Fig.6(b) Multhopp's 16 point solution and the 8 point vortex lattice solution (with Falkner's P functions) are indistinguishable.

The calculations for the three (δ, λ) families by vortex lattice theory give an estimate of the effect of compressibility on spanwise loading. The typical decrease in central loading at a given lift coefficient, C_L, may be seen for the delta family (a) in Tables I,II,III,IV,

the/

the 1.4% decrease in C_{LL}/C_L ($\eta = 0$) from Table III to Table II representing a change in Mach number from 0 to 0.661 for Wing 3. This effect is more marked for the pointed family (b) and less marked for the medium tapered family (c).

Care is needed in the calculation of spanwise loading whenever it differs appreciably from elliptic. This occurs for pointed and untapered wings. The treatment of Weissinger's theory recommended in §5.2 for obtaining aerodynamic centres should often suffice.

3.4 Local Aerodynamic Centre

The calculated local aerodynamic centres for Wings 7 and 10 are plotted in Figs. 7(a) and 7(b) respectively.

The deviation of Multhopp's local a.c. near the tip of the medium tapered wing in Fig.7(b) is of interest and suggests considerable uncertainty in the chordwise pressure distribution in that region. Otherwise for both wings the theoretical curves are in close agreement except near the central section, where the standard vortex lattice theory requires a spanwise smoothing.

The assumed wing loading of the vortex lattice theory gives an unacceptable central kink in the locus of the local a.c. for swept wings. By a procedure explained in §4, the locus is rounded in the region $-0.2 < \eta < 0.2$. This effectively improves the comparison between the three theories in Figs. 7(a) and 7(b).

No rigorous routine procedure for incorporating the necessary change in chordwise loading in the central region has been devised. The artifices used by Multhopp (§6) and Küchemann (§7) are no more conclusive than that proposed for vortex lattice theory. The most plausible treatment is given in Multhopp's theory, where the distribution of local a.c. is effectively determined for a rounded wing. As indicated in §2., it is hoped that further work on the lines of Ref.3 for arrowhead wings will provide useful information on the chordwise distribution of lift at the central section and the extent of its influence in a spanwise direction.

4. Vortex Lattice Theory

Falkner's[4] use of a vortex lattice is essentially a technique for evaluating downwash. The question of accuracy has been examined in Ref.5, where it is stated by Falkner that the beneficial coupling effect of the lattice makes it unnecessary to obtain individual values of the downwash to great accuracy. Various methods of calculation are compared and a measure of convergence is obtained, but there is no proof that vortex lattice theory tends to the exact vortex sheet theory as the lattice spacing becomes infinitesimal.

The pressure distribution over a wing is represented by

$$\frac{p_b - p_a}{\rho V^2} = \frac{8s}{c}\left[F_0(\eta)\cot\tfrac{1}{2}\theta + F_1(\eta)\sin\theta + F_2(\eta)\sin 2\theta\right] \qquad \ldots (1)$$

where θ is the usual chordwise angular co-ordinate given by

$$x - x_l = \tfrac{1}{2}c(1 - \cos\theta) .$$

The/

The spanwise variables F_0, F_1, F_2 are determined by satisfying the condition of tangential flow at as many points of the plan form as there are unknowns. Provided that both the leading and trailing edges of the wing are smooth, the downwash corresponding to equation (1) is continuous over the plan form and the necessary boundary conditions are possible. But at the central kink of a swept wing a logarithmically infinite downwash from exact integration cannot be avoided (Ref.2, §4), although the vortex lattice gives finite values there. The singularity in downwash arises where there is a discontinuity in $\frac{\partial}{\partial\eta}\left(\frac{p_b - p_a}{\rho V^2}\right)$; this occurs where $\frac{\partial\theta}{\partial\eta}$ is discontinuous, and shows for example in the kinked distribution of local aerodynamic centre in Fig.7(b). The following smoothing procedure has been used.

Let $D(\eta)$ be the distance of the local a.c. downstream of the leading apex. Firstly values of

$$D_f(\eta) = x_l + \frac{c}{4}\,\frac{2F_0 + 2F_1 - F_2}{2F_0 + F_1} \qquad \ldots (2)$$

are calculated from the pressure distribution in equation (1). It appears that $D_f(\eta)$ is scarcely affected outboard of the solving station $\eta = 0.2$ by the central kink. The spanwise gradient of $D_f(\eta)$ at this station is calculated from the formula

$$0.6\, D_f'(0.2) = D_f(0.10) - 8\, D_f(0.15) + 8\, D_f(0.25) - D_f(0.30).$$

Then for $-0.2 \leqslant \eta \leqslant 0.2$, the local a.c. is assumed to occur where

$$D(\eta) = D_f(0.2) + 2.5\, D_f'(0.2)\, (\eta^2 - 0.04). \qquad \ldots (3)$$

For this range of η, D_f of equation (2) is replaced by D of equation (3)§ the aerodynamic centre of the wing is therefore displaced through a distance

$$\bar{c}\,\Delta(\text{a.c.}) = \frac{4\pi A}{C_L}\int_0^{0.2} (D - D_f)(F_0 + \tfrac{1}{2}F_1)\, d\eta\,, \qquad \ldots (4)$$

where Δ (a.c.) is the correction to the quantity a.c. defined in §3.2. The effect of equation (4) is shown by the comparison of the standard and rounded 6 point vortex lattice solutions for Wings 1,2, ... 12 in Table XVI.

Throughout the present calculations with a 21 × 6 lattice the central horseshoe vortices of spanwise extent $-0.05 < \eta < 0.05$ and the central solving points, if any, are displaced downstream to correspond to the chord line at $\eta = \pm 0.025$. With this small modification the 6 point solutions took boundary conditions at $\frac{1}{2}$ chord and $5/6$ chord (i.e., $\cos\theta = 0$ and $-2/3$) at the three sections $\eta = 0.2, 0.6, 0.8$, the additional section $\eta = 0$ being included for the 8 point solutions (Fig.3).

The 9 point solutions for Wings 2 and 10 were obtained with a 41 × 12 lattice, the central vortices being displaced backward to correspond to the chord line at $\eta = \pm 0.0125$, and the solving points being taken at $\frac{1}{2}$ chord, $^2/_3$ chord and $^5/_6$ chord at each of the sections $\eta = 0.2, 0.6, 0.8$. These solutions did not prove any more accurate than the standard 6 point solutions with a 21 × 6 lattice.

4.1 Use of P Functions

In the standard solutions by vortex lattice theory the spanwise variables in equation (1) for symmetrical loading are of the form

$$F_0 = \sqrt{1 - \eta^2}\,(a_0 + c_0\eta^2 + e_0\eta^4) . \qquad \ldots (5)$$

Experience has shown that a certain improvement in accuracy is obtained when the spanwise load distribution is modified to be consistent with lifting line theory. When dc/dy is discontinuous, lifting line theory requires a continuous downwash with a sudden change of gradient. In the particular problems of this note (Fig.1) such discontinuities are confined to the central section; and in Ref.16 (1947) Falkner has recommended that, instead of equation (5), the following equation should be used:-

$$F_0 = \sqrt{1 - \eta^2}\,(a_0 + c_0\eta^2 + e_0\eta^4) + p_0\,P(\eta) , \qquad \ldots (6)$$

where

$$P = \varepsilon P_a + (1 - \varepsilon)\,P_b ,$$

the quantities P_a and P_b being defined in Ref.17, Table I. The factors of P_a and P_b have been determined to the nearest 0.05 from Ref.17, Fig.2 and the particular values for Wings 1,2, ... 12 are indicated in the respective tables. For each wing equations (5) and (6) with similar expressions for F_1 have been used for the 6 point and 8 point solutions respectively.

However P functions do not affect the general form of equation (1) or the considerations of the logarithmic singularity that follow. Thus the smoothing procedure outlined in §4 is still necessary if a kinked distribution of aerodynamic centre is to be avoided. P functions therefore do not constitute a rigorous treatment of the central region of a swept wing. Based as they are on the lifting line theory they are not convincing for the purpose of calculating aerodynamic centres. The 8 point solutions should be judged in relation to the improvement in accuracy in proceeding from the more economical 6 point solutions. In the opinion of the writer the 16 point solutions by Multhopp's theory (§6) are exact enough to permit a pronouncement of the theoretical value of P functions.

In the case of delta wings (Fig.1, family (a)) P functions have a negligible effect compared with the difference in $\partial C_L/\partial\alpha$ between the 6 point vortex lattice solution and Multhopp's solution, and the change in a.c. effected by the smoothing procedure of §4.

For the arrowhead Wings 7 and 10 the improvement in $\partial C_L/\partial\alpha$ is about 40% of the likely positive error in the 6 point solution and much better accuracy is obtained for the a.c. of the medium tapered Wing 10. The use of P functions and the two additional solving points apparently gives slight but distinct improvements in the spanwise distributions of lift and aerodynamic centre.

It/

It is thought that the case for P functions is strengthened by the existing comparisons. Except for delta wings appreciable accuracy is gained by using 8 point solutions in preference to 6 point solutions when calculating symmetrical swept-wing loading by vortex lattice theory. Against this must be set a 70% increase in computation.

4.2 Future Applications

(i) In calculations of swept-wing loading, as considered in this note, the vortex lattice theory may be used when the plan form of the wing is unsuitable for Weissinger's method and the superior accuracy of Multhopp's theory is not required.

(ii) When vortex lattice theory is used, the modification to aerodynamic centre outlined in §4 should always be included. The improvement in accuracy through the use of P functions occasionally justifies the 70% extra labour involved (§4.1).

(iii) The outstanding feature of vortex lattice theory is its flexibility. At the expense of extreme accuracy it is possible to treat a wide range of problems. The method is being applied to deflected control surfaces of partial span; the special chordwise loadings present no difficulty, and there is, furthermore, freedom of choice of solving points.

(iv) In particular vortex lattice theory gives a simplified treatment of the effect of compressibility[18] (Falkner, 1948) and will determine oscillatory derivatives of high frequency[19] (Jones, 1946). Neither the simpler nor the more rigorous vortex sheet theories can readily be applied thus.

(v) It is tentatively suggested that by dividing a vortex sheet into spanwise strips instead of chordwise ones the technique of vortex lattice theory may be adapted to calculate the loading on low aspect ratio wings of arbitrary plan form.

(vi) The uniform simplicity of a vortex lattice makes for straightforward calculations, which are more likely to be handled successfully by electronic computing engines.

5. Weissinger's Theory

The original L-method of Weissinger[6] (1942) has been expressed more suitably for computation with tabulated constant factors in Ref.7, Appendix C (Van Dorn and DeYoung, 1947), where the standard 4 point solution is described. In this simplified theory the vortex sheet is concentrated at the quarter chord locus; and the downwashes due to this vortex line and its attendant trailing vorticity are used to satisfy the boundary conditions at points on the three-quarter chord locus (Fig.3).

This logical development of the lifting line theory can usefully be applied to a swept wing so long as its aspect ratio is not very low and its semi-leading and semi-trailing edges are straight. This form of Weissinger's theory does not provide an estimate of the departure from two-dimensional chordwise loading, but may be used to determine the spanwise distribution of lift and the aerodynamic centre of a wing[8] (DeYoung, 1947).

Weissinger's standard 4 point solution takes one sixth of the time required for the corresponding 6 point standard solution by vortex lattice theory. The speed of Weissinger's theory justifies its use whenever its accuracy is comparable with that of more elaborate methods. Results have been obtained for Wings 1,2,5,7,9 and 10 and family (d) (including Wings 7 and 10) and are given in Table XIII. These show that $\partial C_L/\partial \alpha$ (Table XV) is consistently underestimated and that there is a tendency to underestimate a.c. (Table XVI). In the spanwise distributions of lift for a given C_L the contributions from the central region are consistently underestimated (Figs. 6(a), 6(b), 6(c), 6(d)).

5.1 Modified 3 Point Solution

It is very probable that Weissinger's standard solution always overestimates the effect of the central kink of a swept wing. The theory is subject to the same fundamental criticism as vortex lattice theory, for the pressure distribution, represented by the first term of equation (1), logically produces a singularity in downwash at the kinked central section. It is also pertinent that the use of a solving point on the three-quarter chord locus is essentially based on two-dimensional considerations, which are of little worth where the locus has a violent kink.

It is therefore interesting to replace the boundary condition at the central section by a relation between the unknown values of the non-dimensional circulation G_ν, defined as in Ref.7 by

$$\frac{\text{Circulation}}{2sV} = \gamma(\eta) = G_\nu ,$$

when

$$\eta = \cos\frac{\nu\pi}{8} \qquad (\nu = 1,2,3,4) .$$

If G_ν is taken in the symmetrical form

$$G_\nu = A_1 \sin\frac{\nu\pi}{8} + A_3 \sin\frac{3\nu\pi}{8} + A_5 \sin\frac{5\nu\pi}{8} \; (\nu = 1,2,3,4) ,$$

it follows that

$$0.3827\, G_1 - 0.7071\, G_2 + 0.9239\, G_3 - 0.5000\, G_4 = 0 . \qquad \dots (7)$$

The modified 3 point solution is obtained by using equation (7) in place of the central boundary condition at $\nu = 4$. If the standard 4 point solution is being calculated, the modified 3 point solution is readily deduced with very little additional computation.

The lift coefficient in the standard solution is

$$C_L = 2A \int_0^1 \gamma \, d\eta$$

$$= \frac{\pi A}{4}\left[0.3827\, G_1 + 0.7071\, G_2 + 0.9239\, G_3 + 0.5000\, G_4\right] . \qquad \dots (8)$$

With/

With the aid of equation (7) this becomes

$$\left.\begin{aligned} C_L &= \frac{\pi A}{4}(0.7654\ G_1 + 1.8478\ G_3) \\ &= \frac{\pi A}{4}(1.4142\ G_2 + G_4) \end{aligned}\right\} . \qquad \ldots (9)$$

The aerodynamic centre in the standard solution is evaluated by concentrating the lift on the quarter chord locus, and is given by the formula

$$\text{a.c.} = \frac{c_0}{4\bar{c}} + \frac{s \tan\Lambda}{\bar{c}}\bar{\eta} ,$$

where

$$\bar{\eta} = \frac{0.3525\ G_1 + 0.5030\ G_2 + 0.3440\ G_3 + 0.0404\ G_4}{0.3827\ G_1 + 0.7071\ G_2 + 0.9239\ G_3 + 0.5000\ G_4} , \qquad \ldots (10)$$

and the other quantities are defined in Fig.2. In a more general solution with a spanwise loading concentrated at a distance $l(\eta).c$ from the leading edge the aerodynamic centre

$$\text{a.c.} = \frac{c_0}{4\bar{c}} + \int_0^1 I\,d\eta \Big/ \bar{c}\int_0^1 \gamma\,d\eta ,$$

where

$$I = \gamma\{\eta s \tan\Lambda + c(l - \tfrac{1}{4})\} .$$

From the modified 3 point solution the integrand I_ν may be evaluated for $\nu = 1,2,3$ by substituting $l = \frac{1}{4}$; its central value I_4 may be determined from an interpolational equation for I_ν similar to equation (7) for G_ν . Then from equations (8) and (9), it follows that

$$\begin{aligned} \text{a.c.} &= \frac{c_0}{4\bar{c}} + \frac{0.7654\ I_1 + 1.8478\ I_3}{\bar{c}(0.7654\ G_1 + 1.8478\ G_3)} \\ &= \frac{c_0}{4\bar{c}} + \frac{s \tan\Lambda}{\bar{c}}\,\frac{0.7071\ (G_1 + G_3)}{1.4142\ G_2 + G_4} . \end{aligned} \qquad \ldots (11)$$

Calculations of $\partial C_L/\partial\alpha$ and a.c. by Weissinger's standard 4 point solution using equations (8) and (10) and also by the modified 3 point solution using equations (9) and (11) have been compared with Multhopp's solutions for Wing 2 and the whole family (d) (Fig.1). The tendencies to underestimate $\partial C_L/\partial\alpha$ and a.c. are usually overcorrected when the 4 point solution is replaced by the 3 point solution. The central solving point cannot safely be ignored; and the formula (11) for aerodynamic centre probably exaggerates the displacement of the central local a.c. from the quarter chord locus. The average of the 3 point

and/

and 4 point solutions is in good agreement with Multhopp's 16 point solution, especially for Wings 2, 10 (or B) and C. Evidence from Figs. 6(a), 6(b), 6(c) and 6(d) suggests that the spanwise loading near the central section lies between Weissinger's two solutions and that their mean will often provide a satisfactory estimate of the spanwise distribution of lift.

5.2 Future Applications

(i) In calculations of swept wing loading Weissinger's theory is recommended, provided that the wing is of constant sweep and taper, that details of chordwise loading are unnecessary and that the superior accuracy of Multhopp's theory is not required.

(ii) The following recommended procedure involves one day's computation:-

(1) Complete the standard 4 point solution, as described in Ref.7, Appendix C.

(2) Repeat the final stages of (1) by replacing the central boundary condition by equation (7) and using equations (9) and (11) (§5.1).

(3) Take the average of (1) and (2) to give lift slope, aerodynamic centre and spanwise loading.

(iii) The speed of Weissinger's method makes it suitable for estimating the effects of compressibility and sectional lift slope, which may be taken into account by an adjustment of plan form (Ref.20, Appendix B; DeYoung, 1950).

(iv) DeYoung has extended Weissinger's theory to problems of antisymmetrical[20] (1950) and symmetrical[21] (1951) spanwise loadings including flap deflections of partial span. In so far as such problems can be treated by considering a spanwise distribution of equivalent incidence without effects of local aerodynamic centre, rolling and lifting characteristics of wings of constant sweep and taper may be calculated.

6. Multhopp's Theory

The most useful contribution to the problem of swept-wing loading of recent years is Multhopp's subsonic vortex sheet theory (Ref.9, 1950). To a very large extent the method is soundly based; and an elegant scheme of computation just brings the 16 point solution into the category of a routine calculation (Fig.3). A simple 4 point solution ($m = 7$, 1 chordwise) would take as long as a corresponding solution by Weissinger's theory, would give at least as good accuracy and is applicable to any plan form. The determination of local aerodynamic centres would require at least an 8 point solution ($m = 7$, 2 chordwise), which would take approximately half as long as a standard 6 point solution by vortex lattice theory and is reasonably expected to reduce the error.

From the mathematical standpoint any given lifting surface problem has a unique solution. Since there is no such explicit solution for any swept wing, it is strictly impossible to obtain an absolute check on any approximate calculation. But the uncertainties in any theory are at most threefold (Ref.2, §2) and concern

(a) the assumed wing loading:

(b) the evaluation of downwash:

(c) the choice of solving points.

No/

No approximate vortex sheet theory is impeccable as regards (c); however Multhopp's theory is undoubtedly the best in this respect from both chordwise and spanwise considerations (Ref.9, §3).

The limitation (b) arises from an assumed vortex configuration or a simplified interpolation; of all routine methods discussed in this note Multhopp's theory again comes nearest to satisfying the requirements. His chordwise integrations are exact and are expressed as influence functions of two variables, which are presented once and for all in a series of charts (Ref.9, Figs. 1,2, ... 6). It is the rapid process for evaluating these influence functions that makes Multhopp's method a practical computing proposition. His method of approximate spanwise integration (Ref.9, §5) is similar to that of his lifting line theory[22] (1938) and is mathematically convergent. The rate of convergence with m is improved by Multhopp's correction for a logarithmic singularity (Ref.9, §5.2).

Another limitation arises in connection with (a). Multhopp effectively assumes a pressure distribution as it appears in vortex lattice theory in equation (1) without the term $F_2(\eta)$: viz.

$$\frac{p_b - p_a}{\rho V^2} = \frac{8s}{c}\left[F_0(\eta)\cot\tfrac{1}{2}\theta + F_1(\eta)\sin\theta\right]$$

$$= \frac{8s}{c}\left[\frac{\gamma}{2\pi}\cot\tfrac{1}{2}\theta + \frac{2\mu}{\pi}\left(\cot\tfrac{1}{2}\theta - 2\sin\theta\right)\right], \qquad \ldots (12)$$

where the unknowns γ and μ are to be determined at m spanwise stations

$$\eta = \sin n\pi/(m+1) \qquad \left[n = 0, \pm1, \pm2, \ldots \pm\tfrac{1}{2}(m-1)\right].$$

The evaluation of downwashes at the kinked central section of a swept wing due to the pressure distribution of equation (12) is strictly meaningless, since the double integrals do not tend to a finite limit. Multhopp's smooth "interpolation polynomials" for spanwise integration break down when the integrand has a sudden change of gradient. Thus for a kinked swept wing they give the wrong wing area and also an untrue finite limiting downwash at the central section. However for a given value of m there is a small unique modification to the ordinates of the central chord (Ref.9, App. VI) such that integration of the "interpolation polynomials" will give correct areas; downwashes may then be evaluated for this slightly modified smooth wing without obvious inconsistencies. This treatment of the central section, though not rigorous, has more to commend it than the artifices used in the other theories:

(i) Vortex lattice theory. Use of P functions (§4.1);

(ii) The recommended procedure for Weissinger's theory (§5.2);

(iii) Küchemann's equivalent lift slope (§7, equation (13)).

These considerations establish the superiority of Multhopp's theory; and in the opinion of the writer this theory may be used to judge the order of accuracy achieved by the other theories discussed in this note.

However/

However there are problems that are easily handled by vortex lattice theory but are not suited to Multhopp's theory. Oscillatory derivatives for any frequency or flutter mode may be treated by vortex lattice theory[19] (Jones, 1946)*, while the application of Multhopp's theory is limited to low frequencies of the first order (Ref.23).

Each influence function depends on two variables of unrestricted range; and each linearly independent chordwise loading requires a different influence function. By way of contrast vortex lattice theory+ requires a single influence function, irrespective of chordwise loading, for which the spanwise variable is restricted to even integral values. This influence function may be evaluated at a glance from critical tables for particular values of the spanwise variable. This simplifying feature of vortex lattice theory involves a loss of accuracy, but it means that the theory may be applied to problems with deflected control surfaces, whereas Multhopp's theory would necessitate an additional influence function+ for each ratio of control chord to wing chord.

6.1 Computation and Accuracy

Calculations by Multhopp's theory have largely been confined to the 16 point solutions (m = 15, 2 chordwise). Results for Wing 2 and the whole family (d) of Fig.1 are given in Table XIV, and have been used to assess the accuracy of other methods of calculation.

An interesting spanwise distribution of lift is found near the tip of the pointed Wing 7 (or D). The curve of spanwise loading in Fig.6(a) suggests that a steep finite gradient may occur near a pointed tip in place of the more usual infinite slope, exemplified by elliptic loading (§3.3). This result requires further investigation and may have an important bearing on calculations for pointed wings.

Multhopp's charts of influence functions (Ref.9, Figs. 1,2, ... 6) have been found difficult to read and check to the required accuracy. To eliminate this drawback these functions of two variables should be available in a tabulated form suitable for double linear interpolation; the speed of computation and accuracy would then be improved.

As suggested near the beginning of §6, it will often be convenient to carry out calculations by Multhopp's method for m = 7 with 4 spanwise stations on the half wing. It should be noted that the values of $a_{\nu n}$ given in Ref.9, Table III, are incorrect and should read as follows:-

$a_{\nu n}$ for m = 7

ν, n	-3	-1	+1	+3
-2	0.3599	0.3879	0.0344	0.0064
0	0.0280	0.3943	0.3943	0.0280
+2	0.0064	0.0344	0.3879	0.3599

The/

*Available N.P.L. calculations for Wing 2 (Fig.1) are given in A.R.C. 14,156 (Miss Lehrian, July, 1951).

+This may be avoided in an approximate calculation with the chordwise loading of equation (12) and suitably modified boundary conditions. (Ref. 9, App. II).

The chief element of uncertainty in the intrinsic accuracy of Multhopp's method is associated with his treatment of the kinked central section of a swept wing. The local aerodynamic centre in this region is most likely to be affected. However comparisons with Ref.3 for a delta wing (Fig.1, Wing 2) are excellent (Ref.9, Fig. VIII). No very significant discrepancies have yet been found when the number of spanwise variables, $\frac{1}{2}(m + 1)$ for a symmetrical problem, is halved from 8 to 4. In particular, calculations for Wing 10 compare favourably as follows:-

Solution	m = 7	m = 7	m = 15
	1 ch.	2 ch.	2 ch.
$\partial C_L/\partial\alpha$	2.701	2.771	2.735
$\bar{\eta}$ (eq. (10))	0.441	0.436	0.436
a.c.	-	0.936	0.956

The extent to which Multhopp's theory can usefully be simplified to calculate aerodynamic centre remains to be investigated further.

It seems likely that the theory will deteriorate in detailed accuracy for wings of very low aspect ratio, unless more than two terms are taken in the chordwise loading. Serious departure from two-dimensional loading is bound to occur and the second term of equation (12) can only approximate to this. A comparison with Lawrence's method for a delta wing (Ref.14) and Robinson's method for a swallow tail wing (Ref.15) would be of interest.

6.2 Future Applications

(i) For the problems that it will tackle Multhopp's theory is distinctly superior to other routine methods of calculating swept wing loading. Except for the smallest aspect ratios an elaborate solution by Multhopp's theory should be used when special accuracy is required.

(ii) It should normally be possible to choose a simpler solution to provide answers at least as quickly and more accurately than other methods. Thus an 8 point solution (m = 7, 2 chordwise) by Multhopp's theory would effectively replace a standard 6 point solution by vortex lattice theory; and a 4 point solution (m = 7, 1 chordwise) by Multhopp's theory would replace a similar solution by Weissinger's theory.

(iii) Computation by Multhopp's method would be improved if the influence functions were available in a tabulated+form suitable for double linear interpolation instead of the charts in Ref.9, Figs. 1,2, ... 6.

(iv) Multhopp has applied his theory to oscillatory derivatives of low frequency (Ref.23). This unpublished work provides a useful method of calculating the pitching derivative $m_{\dot{\theta}}$.

(v) The reliability of vortex sheet theories for wings of low aspect ratio needs a comprehensive study in which Multhopp's theory is likely to play an important part. Comparisons with Lawrence's[14] method for a delta wing and Robinson's[15] method for a swallow tail wing would be a useful first step.

7./

+Such tables (Rep. MA/21/0505) are now available at the N.P.L. (Mathematics Division).

7. Küchemann's Theory

For an unswept wing Küchemann's theory reduces to the lifting line theory[22]. The basis of his modifications to lifting line theory is an effective change in the local lift slope a_c and aerodynamic centre h_c at the central section associated with a given angle of sweep-back Λ as follows:-

$$a_c = 2\pi \left(1 - \frac{2\Lambda}{\pi}\right) , \quad \Delta h_c = \frac{\Lambda c}{2\pi} . \qquad \ldots (13)$$

Corresponding changes in sectional data at the tips are obtained by substituting $-\Lambda$ for Λ in equation (13). The argument put forward in Ref.10, App. I is unconvincing mathematically and the results should be judged entirely by comparisons with the more exact vortex sheet theories or with experiment. Serious inconsistencies with the other theories are clearly recognizable in Figs. 4(d), 6(a), 6(b), 6(c), 6(d) and 9(b); and comparisons with experiment are outside the scope of this note.

The accuracy in local a.c., calculated from equation (13) near the centre of a swept wing, is hard to assess. Multhopp's theory (§6) does not provide an exact treatment of this problem, so that the comparisons in Figs.7(a) and 7(b) are inconclusive. But it is thought that calculations on the lines of Ref.3 for the wings of family (d) in Fig.1 may help in this respect (§2).

Being a modified lifting line theory Küchemann's method must be expected to become inaccurate at low aspect ratios. It is found moreover that the calculated $\partial C_L/\partial\alpha$ for Wing A of aspect ratio 1.714 is 18% higher than Multhopp's value (Table XV). However the allowance for sweep on a semi-empirical basis can give fair accuracy as for Wing C of moderate aspect ratio and conventionally small taper ratio[11] (Dee, 1951). Without empirical modification Küchemann's theory is unlikely to handle an extensive range of plan forms with the desirable accuracy. The method may be applied to swept wings with the advantages and disadvantages of the lifting line theory for unswept wings, of which the outstanding merits are speed and adaptability.

7.1 Future Applications

(i) The practical value of Küchemann's method lies in its simplicity, which permits a rough estimate of the spanwise distributions of lift and aerodynamic centre to be made in the shortest time.

(ii) Serious inaccuracies in lift slope are found for wings of low aspect ratio $(A < 3)$ and favourable comparisons of aerodynamic centre are confined to a very limited range of plan forms (Fig.9(b)). Küchemann's method is untrustworthy as a potential theory for fundamental research.

(iii) Being essentially a lifting line theory Küchemann's method will tackle a wide range of lateral stability derivatives to a rough approximation.

(iv) Küchemann's method is easily adapted to fit empirical results, such as effects of boundary layer on sectional data and three-dimensional characteristics near the centre and tips of swept wings. With experience the method should become a handy tool for designers.

8. Theoretical Formula for Lift Slope

The curves in Fig.3 include lift slopes calculated from an empirical formula based on theoretical results

$$\frac{\partial C_L}{\partial \alpha} = A\left[\frac{A}{2\pi\cos\Lambda'} + 0.339(1+\tau) + 0.064\sqrt{\frac{2\pi\cos\Lambda'}{A}}\right]^{-1}, \qquad \ldots (14)$$

where τ is a taper parameter defined by lifting line theory and given approximately as the product of two functions $f(\lambda)$ and $g\left(\frac{A}{2\pi\cos\Lambda'}\right)$, which may be found in Ref.12, Tables 4(a) and 4(b) respectively, and Λ' is related to Λ, the angle of sweep-back of the quarter chord locus, by

$$\tan\Lambda' = \left\{1 - \frac{0.8}{A(1+\lambda)}\right\}\tan\Lambda. \qquad \ldots (15)$$

The formula (14) has been found to agree well with some experimentally determined lift slopes, but where a reliable estimate of the two-dimensional lift slope a_1 is available the practical formula (Ref.12, §5.2)

$$\frac{\partial C_L}{\partial \alpha} = A\left[\frac{A}{a_1\cos\Lambda'} + \frac{1+\tau}{\pi} + 0.032\sqrt{\frac{a_1\cos\Lambda'}{A}}\right]^{-1} \qquad \ldots (16)$$

is to be preferred.

For the purposes of the present note equation (14) has been used for comparison with the various vortex sheet theories. The theoretical calculations of lift slope are summarized in Table XV, where the theoretical formula is shown to be a useful guide. The most serious discrepancies are between the formula and Weissinger's theory for Wings 5 and 9, but for such extreme wings neither result can be trusted. It appears that the formula (14) is accurate within ± 3% unless either the angle of sweep-back exceeds 60° or the aspect ratio is less than 1.5. Beyond these limits the formula reads high. It is concluded that when Λ is large equation (15) tends to overcorrect for the effective loss of sweep in the central region, so that Λ' is virtually underestimated. Furthermore evidence for rectangular wings[14] of low aspect ratio suggests that the third term in the bracket of equation (14) does not allow fully for aerodynamic camber if $A < 1.5$.

The formula has been used as a basis for estimating the effects of compressibility on lift slope in §8.2.

8.1 Elliptic Quarter Chord Point

The elliptic quarter chord point of a uniformly swept and tapered wing corresponds to its aerodynamic centre when elliptically loaded with lift concentrated along the quarter chord locus; and it occurs at a distance $\bar{h}\bar{c}$ from the leading apex such that

$$\bar{h} = \frac{1}{2(1+\lambda)} + \frac{2A}{3\pi}\tan\Lambda$$

$$= \frac{1}{2(1+\lambda)}\left\{1 + \frac{1-\lambda}{\pi}\left(\frac{1}{4} + \frac{16}{3}\delta\right)\right\} \qquad \ldots (17)$$

as/

as defined geometrically in Fig.2. $\bar{h}$ is shown as a function of the taper ratio λ and the shape parameter δ in Fig.9(a), where it is correlated with the values of the quantity a.c. (§3.2) as calculated for the wings of the three (δ, λ) families (Fig.1) by vortex lattice theory and for Wings 2, A, B (or 10), C, D (or 7) by Multhopp's theory. This comparison suggests that, except for pointed wings, the theoretical aerodynamic centre will normally occur within $\pm$ 0.05 $\bar{c}$ of the elliptic quarter chord point. Except possibly for pointed wings no very great changes in a.c. occur within a (δ, λ) family, although those calculated by vortex lattice theory are not negligible and imply an appreciable increase in a.c. or rearward movement of aerodynamic centre with increase of Mach number.

The variation between a.c. and $\bar{h}$ for the cropped wings of 45° sweep-back in family (d) is shown in Fig.9(b). It should be remembered that the behaviour of (a.c. - $\bar{h}$) is quite as dependent on taper ratio as on aspect ratio. For example (a.c. - $\bar{h}$) will be comparatively large and positive for swept-back wings of constant chord. However it is considered that with discretion the elliptic quarter chord point may often be used to obtain rough estimates of a.c. within $\pm$ 0.03.

8.2 Effect of Compressibility

The calculation of the aerodynamic characteristics of a given wing at a given subcritical Mach number M presents no more difficulty than the corresponding problem for the same wing in incompressible flow. Linear perturbations of velocity are assumed in vortex sheet theories and to this approximation compressibility is taken into account in steady flow by reducing the spanwise dimensions of the wing by a factor $\beta = \sqrt{1 - M^2}$ and by applying the factor $1/\beta$ to the aerodynamic coefficients calculated for the equivalent wing in incompressible flow.

Thus, for example, the characteristics of Wing 2 (Fig.1) at a Mach number $M = 0.745$ $(\beta = 2/3)$, are related to those of Wing 1 in incompressible flow, such that

$$\left.\begin{aligned} (C_L)_{2,M} &= \frac{1}{\beta}(C_L)_{1,0} \\ (C_m)_{2,M} &= \frac{1}{\beta}(C_m)_{1,0} \end{aligned}\right\}$$

and the effect of M on aerodynamic centre for Wing 2 is merely the change in incompressible a.c. in changing from Wing 2 to Wing 1. This quite small effect is not readily calculable. Allowance for compressibility is complicated by the loss of accuracy due to the lower aspect ratio and larger sweep of the equivalent wing. But, as recommended in §5.2, Weissinger's method (Ref.20) is conveniently quick for calculating the effect of compressibility on lift slope, aerodynamic centre and spanwise loading, though the problem needs extensive study. Calculations by vortex lattice theory for the three (δ, λ) families of Fig.1 show the effect of compressibility on the spanwise loading at a given C_L (§3.3). For example, Wing 2 shows a 1% decrease in central loading and a >2% increase in tip loading as M changes from 0 to 0.745.

A rough estimate of the theoretical correction to lift slope for compressibility is suggested qualitatively by the formula (14) in §8, and quantitatively by the results of the present calculations for the related wings of families (a), (b) and (c) and of Lawrence's calculations for rectangular and triangular wings of low aspect ratio.

Generalized/

Generalized to include Mach number, equation (14) becomes

$$\left(\frac{\partial C_L}{\partial \alpha}\right)_M = A\left[p_M + 0.339(1 + \tau_M) + 0.064/\sqrt{p_M}\right]^{-1} , \qquad \ldots (18)$$

where

$$p_M = A\beta/2\pi \cos \Lambda' ,$$

$$\tau_M = f(\lambda) \cdot g(p_M) , \quad \text{(Ref.12, Tables 4(a), 4(b))}$$

$$\tan \Lambda' = \frac{1}{\beta} \tan \Lambda \left[1 - \frac{0.8}{A\beta(1 + \lambda)}\right] .$$

It follows that to the first order in $(1 - \beta)$

$$\frac{(\partial C_L/\partial \alpha)_M}{(\partial C_L/\partial \alpha)_0} = \frac{1}{1 - (1 - \beta) \cdot G \cdot (\partial C_L/\partial \alpha)_0} , \qquad \ldots (19)$$

where

$$G = \frac{1}{A} \frac{\partial}{\partial \beta} \left[p_M + 0.339(1 + \tau_M) + 0.064/\sqrt{p_M}\right]_{\beta = 1} .$$

If the approximate relation

$$\tau_M = f(\lambda) \cdot 0.10(2p_M)^{\frac{3}{4}}$$

is used, it can be shown that

$$G = \frac{1}{2\pi \cos \Lambda'} \left[1 + 0.043f(\lambda)p_0^{-\frac{1}{4}} - 0.032p_0^{-2/3}\right]$$

$$\left[1 + \sin \Lambda' \cos \Lambda'(\tan \Lambda - 2 \tan \Lambda')\right] . \qquad \ldots (20)$$

Equation (20) gives the limiting form

$$G = \frac{1}{2\pi} \cos \Lambda \quad \text{as} \quad A \to \infty \; (\Lambda' \to \Lambda , \; p_0 \to \infty).$$

It is seen from equation (20) that in general G must be expected to depend on λ , but in preparing Fig.10 it has been found that in relation to the uncertainties in the true theoretical values of G the variations with λ are not important. Of the same order is the variation of G , as defined in equation (19), with M. G is therefore regarded as a function of Λ and A , which has in practice been estimated direct from equation (19) using available calculations and a value of $\beta = 0.75$, which corresponds to $M = 0.661$. The curves for very low aspect ratio are speculative and based entirely on the results given by Lawrence in Ref.14, Figs. 2 and 3.

For/

For any swept wing it is possible to use the curves of Fig.10 and equation (19) to estimate the effect of compressibility on lift slope provided that the data for incompressible flow is known. The same procedure may be useful for correcting low speed experimental data.

9. Concluding Remarks

This investigation of theoretical swept wing loading has provided solutions for the plan forms shown in Fig.1. From these a general picture (Fig.8) of the effect of sweep on lift slope and a chart (Fig.9) comparing theoretical aerodynamic centres with corresponding elliptic quarter chord points have been obtained. As explained in §8.2, the approximate effect of compressibility on lift slope at subcritical speeds may be estimated for any swept-back wing from Fig.10. There is a pronounced difficulty in obtaining consistent results for pointed swept wings.

The primary object of this note, however, is to compare the merits of vortex lattice theory and those of Weissinger, Multhopp and Küchemann, and to suggest the part that each should play in the scene of future aerodynamics. A brief summary of the basic physical concepts, demands of computation, distributions of solving points and special advantages of the various methods of solution is set out in Fig.3. The general finding is that the more highly developed methods achieve accuracy at the expense of greater labour and loss of adaptability. By including additional terms in the chordwise loading the application of a method may of course be extended to derivatives of pitching moment and hinge moment. But the more precise evaluation of downwash by Multhopp's theory is obtained by a means which discourages additional chordwise terms and is not very flexible. Thus there are problems for which the less accurate vortex lattice theory is more suitable. Furthermore lifting characteristics associated with phenomena affecting sectional data or with rate of yaw are more satisfactorily estimated by the simpler and quicker theories of Weissinger or Küchemann.

The following recommendations are made:-

(i) An elaborate solution by Multhopp's theory should be used when special accuracy is required.

(ii) It should normally be possible to choose a shorter version of Multhopp's theory which may be expected to provide a potential solution at least as quickly and more accurately than any other given theory.

(iii) Vortex lattice theory is to be preferred when additional calculations of control characteristics or flutter derivatives are required for the same plan form and supreme accuracy is not essential.

(iv) Weissinger's theory (with the procedure suggested in §5.2,(ii)) is to be preferred when estimating the effects of compressibility and sectional lift slope on suitable plan forms.

(v) Küchemann's theory, being essentially a lifting line theory with a semi-empirical correction for sweep, will roughly tackle a wide range of lateral stability derivatives and may allow for three-dimensional boundary layer characteristics. Its practical value should grow with experience.

10. Acknowledgement

The writer is greatly indebted to Mrs. S. D. Burney for her responsible assistance in the calculations by vortex lattice theory and by Multhopp's theory.

References/

11. References

No.	Author(s)	Title, etc.
1	-	Report to the Performance Sub-Committee on a Second Special Meeting held to Discuss the Problem of Calculating the Load Distribution on Swept-back Wings. (18th May, 1949). A.R.C. 12,378.
2	H. C. Garner	Methods of Approaching an Accurate Three-dimensional Potential Solution for a Wing. (5th October, 1948). A.R.C. 11,802.
3	H. C. Garner	Theoretical Calculations of the Distribution of Aerodynamic Loading on a Delta Wing. R. & M. 2819. (March, 1949).
4	V. M. Falkner	The Calculation of Aerodynamic Loading on Surfaces of Any Shape. (August, 1943). A.R.C. R. & M.1910.
5	V. M. Falkner	The Scope and Accuracy of Vortex Lattice Theory. (29th October, 1949). A.R.C. 12,743.
6	J. Weissinger	Über die Auftriebsverteilung von Pfeilflügeln. Forschungsbericht Nr. 1553 (February, 1942). Translation in N.A.C.A. T.M. No.1120.
7	N. H. Van Dorn and J. DeYoung	A Comparison of Three Theoretical Methods of Calculating Span Load Distribution on Swept Wings. N.A.C.A. T.N. No.1476 (June, 1947).
8	J. DeYoung	Theoretical Additional Span Loading Characteristics on Wings with Arbitrary Sweep, Aspect Ratio and Taper Ratio. N.A.C.A. T.N. No.1491 (December, 1947).
9	H. Multhopp	Methods for Calculating the Lift Distribution of Wings. (Subsonic Lifting Surface Theory). R.A.E. Report No. Aero.2353. (January, 1950). Confidential. A.R.C. 13,439.
10	D. Küchemann	A Simple Method for Calculating the Span and Chordwise Loadings on Thin Swept Wings. R.A.E. Report No. Aero.2392. (August, 1950). Confidential. A.R.C. 13,758.
11	F. W. Dee	Calculation of the Spanwise Lift Distribution, Induced Drag Factor and Pitching Moment on a Series on Tapered, 45° Swept Wings. R.A.E. T.M. No. Aero.159. (February, 1951).
12	L. W. Bryant and H. C. Garner	Control Testing in Wind Tunnels. (23rd October, 1950). and Corrigenda - 29th November, 1950. A.R.C. 13,465.

13/

No.	Author(s)	Title, etc.
13	T. A. Toll and F. W. Diederich	Prediction of Wing Characteristics. N.A.C.A. University Conference on Aerodynamics. A.R.C. 12,177. (June, 1948).
14	H. R. Lawrence	The Lift Distribution on Low Aspect Ratio Wings at Subsonic Speeds. I. Ae. Sc. Preprint No.313. (January, 1951).
15	A. Robinson	Aerofoil Theory for Swallow Tail Wings of Small Aspect Ratio. College of Aeronautics Report No.41. (October, 1950). and Corrigendum - January, 1951. A.R.C. 13,549.
16	V. M. Falkner	The Solution of Lifting Plane Problems by Vortex Lattice Theory. (29th September, 1947). A.R.C. 10,895.
17	V. M. Falkner	Calculated Loadings due to Incidence of a Number of Straight and Swept-back Wings. With Appendix by Doris Lehrian. (5th June, 1948). A.R.C. 11,542.
18	V. M. Falkner	A Comparison of Two Methods of Calculating Wing Loading with Allowance for Compressibility. (25th November, 1948). A.R.C. 11,944.
19	W. P. Jones	The Calculation of Aerodynamic Derivative Coefficients for Wings of any Plan Form in Non-uniform Motion. (December 3rd, 1946). A.R.C. 10,142.
20	J. DeYoung	Theoretical Antisymmetric Span Loading for Wings of Arbitrary Plan Form at Subsonic Speeds. N.A.C.A. T.N. 2140. (July, 1950).
21	J. DeYoung	Theoretical Symmetric Span Loading due to Flap Deflection for Wings of Arbitrary Plan Form at Subsonic Speeds. N.A.C.A. T.N. 2278. (January, 1951).
22	H. Multhopp	Die Berechnung der Auftriebsverteilung von Tragflügeln. Luftfahrtforschung, 1938, pp. 153/169. Translation in A.R.C. Report 8516.
23	H. C. Garner	Multhopp's Subsonic Lifting Surface Theory of Wings in Slow Pitching Oscillations. A.R.C. 15,096. (July, 1952).

TABLE I/

TABLE I

Wing 1. Vortex Lattice Theory

126 vortex, 6 point
Standard Solution

$a_o = 0.10572$
$a_1 = 0.03779$
$c_o = 0.06046$
$c_1 = -0.13352$
$e_o = -0.01308$
$e_1 = 0.03586$

$\partial C_L/\partial\alpha = 2.441$
$a.c. = 0.9428\bar{c}$

126 vortex, 8 point
$P = 0.6\ P_a + 0.4\ P_b$

$a_o = 0.11304$
$a_1 = 0.02104$
$c_o = 0.05615$
$c_1 = -0.12273$
$e_o = -0.01137$
$e_1 = 0.03098$
$p_o = -0.10587$
$p_1 = 0.23683$

$\partial C_L/\partial\alpha = 2.445$
$a.c. = 0.9495\bar{c}$

η	C_{LL}/C_L	Local a.c.	C_{LL}/C_L	Local a.c.
0	0.7333	0.3485	0.7391	0.3614
0.05	0.7650	0.3231	0.7700	0.3362
0.10	0.7976	0.3031	0.8008	0.3157
0.15	0.8310	0.2893	0.8329	0.3006
0.20	0.8654	0.2827	0.8661	0.2917
0.25	0.9008	0.2798	0.9008	0.2866
0.30	0.9376	0.2763	0.9369	0.2814
0.35	0.9757	0.2722	0.9746	0.2761
0.40	1.0154	0.2675	1.0140	0.2705
0.45	1.0567	0.2624	1.0550	0.2647
0.50	1.0999	0.2567	1.0981	0.2585
0.55	1.1450	0.2507	1.1430	0.2521
0.60	1.1918	0.2443	1.1897	0.2454
0.65	1.2400	0.2376	1.2379	0.2384
0.70	1.2889	0.2306	1.2865	0.2313
0.75	1.3360	0.2235	1.3335	0.2240
0.80	1.3768	0.2164	1.3741	0.2167
0.85	1.4002	0.2092	1.3971	0.2095
0.90	1.3767	0.2023	1.3736	0.2023
0.95	1.2154	0.1956	1.2122	0.1955
1.00				

TABLE II/

TABLE II

Wing 2. Vortex Lattice Theory

126 vortex, 6 point Standard Solution
$a_0 = 0.09975$
$a_1 = 0.01642$
$c_0 = 0.02844$
$c_1 = -0.07978$
$e_0 = -0.01415$
$e_1 = 0.04033$

$\partial C_L/\partial \alpha = 3.134$
a.c. $= 0.9273\bar{c}$

126 vortex, 8 point
$P = 0.65\, P_a + 0.35\, P_b$
$a_0 = 0.10687$
$a_1 = 0.00011$
$c_0 = 0.02125$
$c_1 = -0.06288$
$e_0 = -0.00944$
$e_1 = 0.02917$
$p_0 = -0.09160$
$p_1 = 0.20406$

$\partial C_L/\partial \alpha = 3.136$
a.c. $= 0.9319\bar{c}$

41 × 12 lattice 9 point solution
$a_0 = 0.10547$
$a_1 = 0.00481$
$a_2 = -0.00268$
$c_0 = -0.01321$
$c_1 = 0.00129$
$c_2 = 0.03389$
$e_0 = 0.03731$
$e_1 = -0.06197$
$e_2 = -0.04797$

$\partial C_L/\partial \alpha = 3.125$
a.c. $= 0.9206\bar{c}$

η	C_{LL}/C_L	Local a.c.	C_{LL}/C_L	Local a.c.	C_{LL}/C_L	Local a.c.
0	0.7421	0.3312	0.7472	0.3413	0.7438	0.3219
0.05	0.7741	0.3052	0.7784	0.3154	0.7759	0.2958
0.10	0.8068	0.2850	0.8093	0.2947	0.8085	0.2756
0.15	0.8400	0.2714	0.8412	0.2798	0.8502	0.2624
0.20	0.8739	0.2654	0.8741	0.2718	0.8755	0.2572
0.25	0.9086	0.2635	0.9081	0.2679	0.9100	0.2563
0.30	0.9443	0.2612	0.9434	0.2642	0.9456	0.2552
0.35	0.9810	0.2585	0.9797	0.2606	0.9820	0.2538
0.40	1.0189	0.2555	1.0175	0.2569	1.0196	0.2522
0.45	1.0581	0.2522	1.0565	0.2533	1.0584	0.2503
0.50	1.0987	0.2488	1.0972	0.2495	1.0986	0.2482
0.55	1.1410	0.2452	1.1393	0.2458	1.1402	0.2456
0.60	1.1843	0.2415	1.1829	0.2420	1.1830	0.2427
0.65	1.2290	0.2379	1.2274	0.2383	1.2268	0.2394
0.70	1.2737	0.2343	1.2722	0.2347	1.2707	0.2357
0.75	1.3164	0.2310	1.3150	0.2313	1.3127	0.2314
0.80	1.3529	0.2280	1.3512	0.2281	1.3483	0.2267
0.85	1.3718	0.2254	1.3699	0.2253	1.3663	0.2214
0.90	1.3454	0.2235	1.3434	0.2228	1.3393	0.2155
0.95	1.1847	0.2223	1.1825	0.2209	1.1787	0.2090
1.00						

TABLE III./

TABLE III

Wing 3. Vortex Lattice Theory

126 vortex, 6 point Standard Solution	126 vortex, 8 point $P = 0.65\,P_a + 0.35\,P_b$
$a_0 = 0.09195$	$a_0 = 0.09881$
$a_1 = 0.00654$	$a_1 = -0.00918$
$c_0 = 0.00391$	$c_0 = -0.00430$
$c_1 = -0.04107$	$c_1 = -0.02188$
$e_0 = -0.00294$	$e_0 = 0.00290$
$e_1 = 0.02150$	$e_1 = 0.00786$
	$p_0 = -0.08121$
	$p_1 = 0.18147$
$\partial C_L/\partial\alpha = 3.634$	$\partial C_L/\partial\alpha = 3.635$
a.c. $= 0.9177\bar{c}$	a.c. $= 0.9214\bar{c}$

η	C_{LL}/C_L	Local a.c.	C_{LL}/C_L	local a.c.
0	0.7527	0.3219	0.7578	0.3307
0.05	0.7850	0.2956	0.7893	0.3044
0.1	0.8177	0.2753	0.8202	0.2836
0.15	0.8506	0.2619	0.8518	0.2690
0.2	0.8840	0.2565	0.8840	0.2615
0.25	0.9177	0.2554	0.9171	0.2585
0.3	0.9519	0.2541	0.9509	0.2559
0.35	0.9868	0.2525	0.9856	0.2536
0.4	1.0227	0.2507	1.0212	0.2513
0.45	1.0591	0.2488	1.0576	0.2491
0.5	1.0967	0.2468	1.0953	0.2469
0.55	1.1355	0.2446	1.1340	0.2448
0.6	1.1750	0.2425	1.1737	0.2427
0.65	1.2151	0.2403	1.2140	0.2406
0.7	1.2552	0.2382	1.2542	0.2385
0.75	1.2932	0.2362	1.2921	0.2364
0.8	1.3247	0.2345	1.3235	0.2344
0.85	1.3390	0.2330	1.3377	0.2325
0.9	1.3096	0.2319	1.3078	0.2308
0.95	1.1505	0.2312	1.1484	0.2292
1.0				

TABLE IV,

TABLE IV

Wing 4. Vortex Lattice Theory

126 vortex, 6 point
Standard Solution

$a_0 = 0.07711$
$a_1 = -0.00024$
$c_0 = -0.01949$
$c_1 = -0.00795$
$e_0 = 0.00987$
$e_1 = 0.00092$

$\partial C_L/\partial \alpha = 4.288$
a.c. $= 0.9061\bar{c}$

126 vortex, 8 point
$P = 0.70\,P_a + 0.30\,P_b$

$a_0 = 0.08376$
$a_1 = -0.01468$
$c_0 = -0.02871$
$c_1 = 0.01301$
$e_0 = 0.01685$
$e_1 = -0.01499$
$p_0 = -0.07732$
$p_1 = 0.15716$

$\partial C_L/\partial \alpha = 4.272$
a.c. $= 0.9092\bar{c}$

η	C_{LL}/C_L	local a.c.	C_{LL}/C_L	local a.c.
0	0.7735	0.3139	0.7729	0.3209
0.05	0.8064	0.2873	0.8055	0.2943
0.1	0.8392	0.2670	0.8380	0.2735
0.15	0.8715	0.2539	0.8702	0.2592
0.2	0.9037	0.2491	0.9023	0.2524
0.25	0.9353	0.2488	0.9341	0.2503
0.3	0.9671	0.2484	0.9660	0.2488
0.35	0.9983	0.2480	0.9976	0.2478
0.4	1.0298	0.2475	1.0296	0.2469
0.45	1.0611	0.2469	1.0611	0.2463
0.5	1.0927	0.2462	1.0933	0.2458
0.55	1.1244	0.2454	1.1254	0.2452
0.6	1.1565	0.2446	1.1579	0.2447
0.65	1.1878	0.2438	1.1899	0.2440
0.7	1.2192	0.2428	1.2217	0.2431
0.75	1.2472	0.2418	1.2499	0.2420
0.8	1.2697	0.2407	1.2726	0.2406
0.85	1.2753	0.2395	1.2779	0.2388
0.9	1.2408	0.2382	1.2428	0.2365
0.95	1.0840	0.2370	1.0853	0.2335
1.0				

TABLE V/

TABLE V

Wing 5. Vortex Lattice Theory

126 vortex, 6 point Standard Solution	126 vortex, 8 point $P = 0.65 P_a + 0.35 P_b$
$a_o = 0.07092$	$a_o = 0.10677$
$a_1 = 0.02630$	$a_1 = -0.04280$
$c_o = -0.05939$	$c_o = -0.10105$
$c_1 = 0.15557$	$c_1 = 0.23451$
$e_o = 0.16435$	$e_o = 0.19180$
$e_1 = -0.42998$	$e_1 = -0.48053$
	$p_o = -0.42298$
	$p_1 = 0.79938$
$\partial C_L/\partial \alpha = 1.625$	$\partial C_L/\partial \alpha = 1.606$
a.c. $= 1.80790\bar{c}$	a.c. $= 1.83550\bar{c}$

η	C_{LL}/C_L	local a.c.	C_{LL}/C_L	local a.c.
0	0.6500	0.4369	0.6356	0.4808
0.05	0.6837	0.3777	0.6714	0.4224
0.1	0.7202	0.3328	0.7122	0.3752
0.15	0.7595	0.3049	0.7549	0.3411
0.2	0.8022	0.2970	0.8004	0.3227
0.25	0.8486	0.3005	0.8487	0.3162
0.3	0.8990	0.3040	0.9004	0.3134
0.35	0.9534	0.3068	0.9559	0.3121
0.4	1.0123	0.3086	1.0159	0.3112
0.45	1.0761	0.3086	1.0804	0.3097
0.5	1.1450	0.3060	1.1501	0.3065
0.55	1.2197	0.3000	1.2255	0.3003
0.6	1.3008	0.2895	1.3073	0.2900
0.65	1.3899	0.2730	1.3973	0.2738
0.7	1.4894	0.2489	1.4977	0.2499
0.75	1.6035	0.2146	1.6135	0.2155
0.8	1.7419	0.1663	1.7540	0.1668
0.85	1.9262	0.0989	1.9418	0.0980
0.9	2.2159	0.0033	2.2383	0.0002
0.95	2.8684	-0.1360	2.9057	-0.1427
1.0				

TABLE VI/

TABLE VI

Wing 6. Vortex Lattice Theory

126 vortex, 6 point
Standard Solution
a_o = 0.06852
a_1 = 0.01704
c_o = -0.02456
c_1 = 0.03206
e_o = 0.05648
e_1 = -0.14986

$\partial C_L/\partial \alpha$ = 2.866
a.c. = 1.7721$\bar{c}$

126 vortex, 8 point
$P = 0.65 P_a + 0.35 P_b$
a_o = 0.09304
a_1 = -0.03072
c_o = -0.05697
c_1 = 0.09631
e_o = 0.07989
e_1 = -0.19550
p_o = -0.28121
p_1 = 0.51589
$\partial C_L/\partial \alpha$ = 2.813
a.c. = 1.7924$\bar{c}$

η	C_{LL}/C_L	local a.c.	C_{LL}/C_L	local a.c.
0	0.6756	0.4254	0.6572	0.4525
0.05	0.7100	0.3654	0.6939	0.3929
0.1	0.7461	0.3194	0.7348	0.3452
0.15	0.7838	0.2897	0.7765	0.3112
0.2	0.8234	0.2795	0.8195	0.2938
0.25	0.8652	0.2802	0.8638	0.2878
0.3	0.9096	0.2807	0.9102	0.2843
0.35	0.9568	0.2809	0.9590	0.2820
0.4	1.0073	0.2805	1.0111	0.2802
0.45	1.0616	0.2791	1.0670	0.2783
0.5	1.1202	0.2766	1.1270	0.2758
0.55	1.1845	0.2724	1.1929	0.2720
0.6	1.2553	0.2660	1.2654	0.2662
0.65	1.3355	0.2568	1.3474	0.2576
0.7	1.4283	0.2441	1.4424	0.2452
0.75	1.5407	0.2267	1.5568	0.2279
0.8	1.6843	0.2035	1.7039	0.2038
0.85	1.8863	0.1725	1.9103	0.1710
0.9	2.2183	0.1308	2.2489	0.1263
0.95	2.9741	0.0749	3.0200	0.0652
1.0				

TABLE VII/

TABLE VII

Wing 7. Vortex Lattice Theory

126 vortex, 6 point
Standard Solution

a_o = 0.06371
a_1 = 0.00922
c_o = -0.01411
c_1 = -0.01303
e_o = 0.00827
e_1 = -0.03060

$\partial C_L/\partial \alpha$ = 3.688
a.c. = 1.7363$\bar{c}$

126 vortex, 8 point
P = 0.7 P_a + 0.3 P_b

a_o = 0.08112
a_1 = -0.02465
c_o = -0.03895
c_1 = 0.03748
e_o = 0.02716
e_1 = -0.06889
p_o = -0.20074
p_1 = 0.36242
$\partial C_L/\partial \alpha$ = 3.626
a.o. = 1.7510$\bar{c}$

η	C_{LL}/C_L	local a.c.	C_{LL}/C_L	local a.c.
0	0.6983	0.4152	0.6804	0.4329
0.05	0.7336	0.3546	0.7179	0.3723
0.1	0.7697	0.3080	0.7590	0.3247
0.15	0.8063	0.2774	0.7998	0.2908
0.2	0.8448	0.2660	0.8410	0.2743
0.3	0.9249	0.2647	0.9254	0.2653
0.4	1.0124	0.2622	1.0164	0.2606
0.5	1.1105	0.2561	1.1176	0.2567
0.6	1.2263	0.2511	1.2366	0.2513
0.7	1.3751	0.2400	1.3892	0.2412
0.8	1.6017	0.2222	1.6206	0.2222
0.9	2.0944	0.1930	2.1202	0.1876
1.0				

TABLE VIII/

TABLE VIII

Wing 8. Vortex Lattice Theory

126 vortex, 6 point
Standard Solution

a_o = 0.05803
a_1 = 0.00485
c_o = -0.01497
c_1 = -0.02224
e_o = -0.00684
e_1 = 0.00840

$\partial C_L/\partial\alpha$ = 4.232
a.c. = $1.7078\bar{c}$

126 vortex, 8 point
$P = 0.7\ P_a + 0.3\ P_b$

a_e = 0.07113
a_1 = -0.02042
c_o = -0.03452
c_1 = 0.01782
e_o = 0.00842
e_1 = -0.02301
p_o = -0.14722
p_1 = 0.26212
$\partial C_L/\partial\alpha$ = 4.169
a.c. = $1.7195\bar{c}$

η	C_{LL}/C_L	local a.c.	C_{LL}/C_L	local a.c.
0	0.7180	0.4086	0.7009	0.4215
0.05	0.7540	0.3476	0.7392	0.3606
0.1	0.7903	0.3007	0.7803	0.3126
0.15	0.8271	0.2700	0.8205	0.2794
0.2	0.8640	0.2584	0.8606	0.2636
0.25	0.9017	0.2575	0.9006	0.2589
0.3	0.9400	0.2563	0.9408	0.2556
0.35	0.9792	0.2549	0.9817	0.2532
0.4	1.0198	0.2533	1.0239	0.2512
0.45	1.0618	0.2516	1.0674	0.2495
0.5	1.1060	0.2496	1.1132	0.2481
0.55	1.1534	0.2474	1.1621	0.2465
0.6	1.2046	0.2449	1.2148	0.2450
0.65	1.2624	0.2423	1.2740	0.2430
0.7	1.3298	0.2393	1.3428	0.2405
0.75	1.4119	0.2361	1.4266	0.2372
0.8	1.5203	0.2326	1.5373	0.2326
0.85	1.6778	0.2288	1.6969	0.2267
0.9	1.9454	0.2247	1.9678	0.2184
0.95	2.5773	0.2198	2.6044	0.2072
1.0				

TABLE IX/

TABLE IX

Wing 9. Vortex Lattice Theory

126 vortex, 6 point
Standard Solution

a_o = 0.08803
a_1 = 0.07762
c_o = 0.17721
c_1 = -0.29621
e_o = -0.03775
e_1 = 0.04457

$\partial C_L/\partial \alpha$ = 1.722
a.c. = 0.9698$\bar{c}$

126 vortex, 8 point
$P = 0.55\ P_a + 0.45\ P_b$

a_o = 0.10578
a_1 = 0.04165
c_o = 0.16119
c_1 = -0.26335
e_o = -0.02872
e_1 = 0.02612
p_o = -0.24031
p_1 = 0.44125

$\partial C_L/\partial \alpha$ = 1.696
a.c. = 0.9775$\bar{c}$

η	C_{LL}/C_L	local a.c.	C_{LL}/C_L	local a.c.
0	0.8485	0.4125	0.8346	0.4376
0.05	0.8746	0.3754	0.8622	0.4005
0.1	0.9012	0.3460	0.8922	0.3697
0.15	0.9282	0.3254	0.9219	0.3460
0.2	0.9555	0.3143	0.9517	0.3301
0.25	0.9831	0.3076	0.9810	0.3188
0.3	1.0105	0.2996	1.0099	0.3077
0.35	1.0375	0.2903	1.0381	0.2962
0.4	1.0638	0.2799	1.0655	0.2842
0.45	1.0884	0.2684	1.0912	0.2715
0.5	1.1107	0.2560	1.1144	0.2582
0.55	1.1296	0.2427	1.1343	0.2443
0.6	1.1434	0.2285	1.1489	0.2298
0.65	1.1500	0.2137	1.1562	0.2147
0.7	1.1469	0.1983	1.1539	0.1991
0.75	1.1298	0.1823	1.1372	0.1829
0.8	1.0924	0.1658	1.1002	0.1662
0.85	1.0252	0.1489	1.0327	0.1488
0.9	0.9089	0.1314	0.9160	0.1309
0.95	0.6994	0.1135	0.7052	0.1124
1.0				

/TABLE X/

TABLE X

Wing 10. Vortex Lattice Theory

126 vortex, 6 point Standard Solution	126 vortex, 8 point $P = 0.65\,P_a + 0.35\,P_b$	41 × 12 lattice 9 point solution
$a_0 = 0.09224$	$a_0 = 0.10794$	$a_0 = 0.10238$
$a_1 = 0.02483$	$a_1 = -0.00707$	$a_1 = 0.00750$
$c_0 = 0.06623$	$c_0 = 0.04782$	$a_2 = -0.00955$
$c_1 = -0.10009$	$c_1 = -0.06180$	$c_0 = -0.00135$
$e_0 = 0.00966$	$e_0 = 0.02268$	$c_1 = 0.02576$
$e_1 = -0.01717$	$e_1 = -0.04405$	$c_2 = 0.07808$
	$p_0 = -0.20381$	$e_0 = 0.09160$
	$p_1 = 0.37477$	$e_1 = -0.17430$
		$e_2 = -0.09951$
$\partial C_L/\partial\alpha = 2.836$	$\partial C_L/\partial\alpha = 2.796$	$\partial C_L/\partial\alpha = 2.855$
a.c. $= 0.9482\bar{c}$	a.c. $= 0.9571\bar{c}$	a.c. $= 0.9422\bar{c}$

η	C_{LL}/C_L	local a.c.	C_{LL}/C_L	local a.c.	C_{LL}/C_L	local a.c.
0	0.8501	0.3695	0.8361	0.3880	0.8563	0.3606
0.05	0.8762	0.3314	0.8639	0.3500	0.8825	0.3224
0.10	0.9025	0.3025	0.8935	0.3198	0.9086	0.2937
0.15	0.9286	0.2831	0.9222	0.2979	0.9344	0.2749
0.20	0.9549	0.2747	0.9508	0.2853	0.9602	0.2674
0.25	0.9811	0.2719	0.9787	0.2787	0.9856	0.2658
0.30	1.0069	0.2685	1.0060	0.2728	1.0103	0.2637
0.35	1.0324	0.2644	1.0327	0.2672	1.0347	0.2611
0.40	1.0572	0.2598	1.0586	0.2614	1.0580	0.2578
0.45	1.0804	0.2545	1.0829	0.2555	1.0797	0.2539
0.50	1.1019	0.2486	1.1055	0.2493	1.0998	0.2492
0.55	1.1207	0.2420	1.1253	0.2426	1.1170	0.2436
0.60	1.1350	0.2349	1.1407	0.2354	1.1299	0.2371
0.65	1.1439	0.2270	1.1503	0.2276	1.1372	0.2294
0.70	1.1442	0.2186	1.1514	0.2191	1.1364	0.2206
0.75	1.1316	0.2096	1.1396	0.2099	1.1234	0.2105
0.80	1.1010	0.1999	1.1092	0.1999	1.0925	0.1990
0.85	1.0412	0.1897	1.0496	0.1889	1.0331	0.1860
0.90	0.9321	0.1788	0.9398	0.1771	0.9253	0.1714
0.95	0.7265	0.1673	0.7327	0.1642	0.7216	0.1552
1.00				—	—	

TABLE X

TABLE XI

Wing 11. Vortex Lattice Theory

126 vortex, 6 point Standard Solution	126 vortex, 8 point $P = 0.65\ P_a + 0.35\ P_b$
$a_o = 0.08421$	$a_o = 0.09757$
$a_1 = 0.00820$	$a_1 = -0.01877$
$c_o = 0.02484$	$c_o = 0.00705$
$c_1 = -0.03734$	$c_1 = 0.00003$
$e_o = 0.02110$	$e_o = 0.03454$
$e_1 = -0.02264$	$e_1 = -0.05084$
	$p_o = -0.16019$
	$p_1 = 0.29342$
$\partial C_L/\partial\alpha = 3.559$	$\partial C_L/\partial\alpha = 3.515$
a.c. $= 0.9412\bar{c}$	a.c. $= 0.9479\bar{c}$

η	C_{LL}/C_L	local a.c.	C_{LL}/C_L	local a.c.
0	0.8575	0.3526	0.8434	0.3669
0.05	0.8835	0.3144	0.8711	0.3286
0.1	0.9093	0.2853	0.9003	0.2984
0.15	0.9347	0.2668	0.9284	0.2776
0.2	0.9598	0.2594	0.9556	0.2666
0.25	0.9844	0.2581	0.9819	0.2620
0.3	1.0085	0.2565	1.0074	0.2585
0.35	1.0320	0.2546	1.0322	0.2553
0.4	1.0548	0.2523	1.0561	0.2524
0.45	1.0759	0.2496	1.0785	0.2494
0.5	1.0953	0.2465	1.0990	0.2463
0.55	1.1124	0.2429	1.1171	0.2429
0.6	1.1258	0.2389	1.1314	0.2391
0.65	1.1342	0.2343	1.1406	0.2348
0.7	1.1350	0.2293	1.1421	0.2298
0.75	1.1245	0.2237	1.1324	0.2240
0.8	1.0969	0.2176	1.1050	0.2174
0.85	1.0420	0.2109	1.0500	0.2099
0.9	0.9381	0.2038	0.9456	0.2015
0.95	0.7362	0.1961	0.7420	0.1920
1.0				

TABLE XII/

TABLE XII

Wing 12. Vortex Lattice Theory

126 vortex, 6 point Standard Solution	126 vortex, 8 point $P = 0.7\,P_a + 0.3\,P_b$
$a_0 = 0.07438$	$a_0 = 0.08475$
$a_1 = 0.00348$	$a_1 = -0.01717$
$c_0 = 0.00882$	$c_0 = -0.00577$
$c_1 = -0.01979$	$c_1 = 0.01085$
$e_0 = 0.02145$	$e_0 = 0.03260$
$e_1 = -0.01298$	$e_1 = -0.03641$
	$p_0 = -0.12289$
	$p_1 = 0.22186$
$\partial C_L/\partial \alpha = 4.050$	$\partial C_L/\partial \alpha = 4.007$
$a.c. = 0.9380\bar{c}$	$a.c. = 0.9433\bar{c}$

η	C_{LL}/C_L	local a.c.	C_{LL}/C_L	local a.c.
0	0.8661	0.3471	0.8522	0.3572
0.05	0.8921	0.3086	0.8799	0.3186
0.1	0.9175	0.2795	0.9088	0.2886
0.15	0.9425	0.2610	0.9365	0.2683
0.2	0.9664	0.2544	0.9625	0.2588
0.25	0.9894	0.2536	0.9870	0.2555
0.3	1.0119	0.2526	1.0110	0.2530
0.35	1.0332	0.2514	1.0335	0.2509
0.4	1.0532	0.2499	1.0547	0.2491
0.45	1.0724	0.2482	1.0750	0.2474
0.5	1.0894	0.2462	1.0929	0.2456
0.55	1.1042	0.2439	1.1087	0.2436
0.6	1.1164	0.2414	1.1218	0.2414
0.65	1.1235	0.2385	1.1298	0.2388
0.7	1.1241	0.2352	1.1310	0.2357
0.75	1.1151	0.2316	1.1225	0.2320
0.8	1.0893	0.2277	1.0968	0.2277
0.85	1.0369	0.2235	1.0442	0.2227
0.9	0.9377	0.2190	0.9447	0.2170
0.95	0.7396	0.2142	0.7452	0.2104
1.0				

TABLE XIII

Weissinger's Theory: Standard 4 Point Solutions

n	η_n	Values of γ_n							
		Wing 1	Wing 2	Wing 5	Wing 7	Wing 9	Wing 10	Wing A	Wing C
1	0.9239	0.2891	0.2454	0.1574	0.1008	0.3194	0.2734	0.3157	0.2208
2	0.7071	0.5271	0.4500	0.3352	0.2439	0.5696	0.4776	0.5675	0.3774
3	0.3827	0.6895	0.5984	0.4674	0.3619	0.7047	0.5907	0.7107	0.4795
4	0	0.7429	0.6501	0.4905	0.3893	0.7259	0.6098	0.7444	0.4980
$\partial C_L/\partial\alpha$ =		2.343	3.039	1.5305	3.487	1.5955	2.681	2.088	3.129
a.c. =		0.9158	0.9128	1.7462	1.7102	0.9354	0.9363	0.6924	1.2375

a.c. (as a fraction of $\bar{c}$ from the leading apex)

$$= \frac{c_o}{4\bar{c}} + \frac{s \tan\Lambda}{\bar{c}} \cdot \frac{0.3525\gamma_1 + 0.5030\gamma_2 + 0.3440\gamma_3 + 0.0404\gamma_4}{0.3827\gamma_1 + 0.7071\gamma_2 + 0.9239\gamma_3 + 0.5000\gamma_4}$$

Weissinger's Theory: Modified 3 Point Solutions

n	η_n	Values of γ_n							
		Wing 1	Wing 2	Wing 5	Wing 7	Wing 9	Wing 10	Wing A	Wing C
1	0.9239	0.2909	0.2467	0.1617	0.1024	0.3236	0.2767	0.3178	0.2247
2	0.7071	0.5308	0.4529	0.3424	0.2462	0.5785	0.4846	0.5719	0.3853
3	0.3827	0.6976	0.6056	0.4888	0.3715	0.7231	0.6067	0.7201	0.5008
4	0	0.7610	0.6673	0.5427	0.4169	0.7656	0.6476	0.7650	0.5524
$\partial C_L/\partial\alpha$ =		2.375	3.082	1.613	3.605	1.642	2.764	2.113	3.291
a.c. =		0.9577	0.9559	1.8437	1.8143	0.9768	0.9787	0.7211	1.3016

a.c. (as a fraction of $\bar{c}$ from the leading apex)

$$= \frac{c_o}{4\bar{c}} + \frac{s \tan\Lambda}{\bar{c}} \cdot \frac{0.7071\,(\gamma_1 + \gamma_3)}{0.7654\gamma_1 + 1.8478\gamma_3}$$

TABLE XIV/

TABLE XIV

Multhopp's Theory. 2 Chordwise, 15 Spanwise

Wing 2

$\partial C_L/\partial\alpha$ = 3.050 a.c. = 0.9327$\bar{c}$

n =	0	1	2	3	4	5	6	7
η_n =	0	0.1951	0.3827	0.5556	0.7071	0.8315	0.9239	0.9808
γ_n =	0.65869	0.64281	0.59977	0.53319	0.44890	0.35227	0.24539	0.12659
μ_n =	-0.04064	-0.01728	-0.00494	0.00133	0.00553	0.00951	0.01401	0.01382
Local a.c. =	0.3309	0.2769	0.2582	0.2475	0.2377	0.2230	0.1929	0.1408

Wing A

$\partial C_L/\partial\alpha$ = 2.1360 a.c. = 0.6838$\bar{c}$

n =	0	1	2	3	4	5	6	7
γ_n =	0.7619	0.7552	0.72825	0.6698	0.58025	0.4620	0.3204	0.1637
μ_n =	-0.04841	-0.01677	0.00421	0.02074	0.03442	0.04225	0.03926	0.02372
Local a.c. =	0.3135	0.2722	0.2442	0.2190	0.1906	0.1585	0.1275	0.1051

Wing B (or 10)

$\partial C_L/\partial\alpha$ = 2.7347 a.c. = 0.9561$\bar{c}$

n =	0	1	2	3	4	5	6	7
γ_n =	0.62309	0.62291	0.60066	0.55517	0.48739	0.39523	0.27841	0.14328
μ_n =	-0.05508	-0.02500	-0.00844	0.00157	0.01077	0.02108	0.02717	0.01962
Local a.c. =	0.3664	0.2901	0.2640	0.2472	0.2279	0.1967	0.1524	0.1131

Wing C/

TABLE XIV (Continued).

Wing C

$\partial C_L/\partial a$ = 3.2039 a.c. = 1.26230̄

n =	0	1	2	3	4	5	6	7
γ_n =	0.51331	0.51270	0.43889	0.44415	0.38472	0.31364	0.22686	0.11901
μ_n =	-0.04757	-0.01825	-0.00484	0.00160	0.00488	0.00831	0.01405	0.01502
Local a.c. =	0.3783	0.2856	0.2599	0.2464	0.2373	0.2235	0.1881	0.1238

Wing D (or 7)

$\partial C_L/\partial a$ = 3.5521 a.c. = 1.70160̄

n =	0	1	2	3	4	5	6	7
γ_n =	0.40869	0.40465	0.37151	0.31457	0.24368	0.16601	0.08660	0.01980
μ_n =	-0.03456	-0.00676	0.00410	0.00766	0.00675	0.00301	-0.00029	-0.00053
Local a.c. =	0.3806	0.2667	0.2390	0.2256	0.2223	0.2319	0.2533	0.2768

Table XV/

TABLE XV

Calculated Values of $\partial C_L/\partial \alpha$

Wing	A	λ	Λ	Vortex Lattice Theory 21/6 6 point	Vortex Lattice Theory 21/6 8 point	Vortex Lattice Theory 41/12 9 point	Weissinger's Theory Standard 4 point	Weissinger's Theory Modified 3 point	Weissinger's Theory Mean	Multhopp m = 15 2 chordwise	Küchemann Refs. 10, 11	Formula Eq.14
1	2	0.143	48.4	2.441	2.445		2.343	2.375	2.359			2.370
2	3	0.143	36.9	3.134	3.136	3.125	3.039	3.082	3.060	3.050	3.23	3.040
3	4	0.143	29.4	3.634	3.635							3.540
4	6	0.143	20.6	4.288	4.272							4.208
5	2	0	71.6	1.625	1.606		1.530	1.613	1.572			1.778
6	4	0	56.3	2.866	2.813							2.723
7	6	0	45.0	3.688	3.626		3.487	3.605	3.546	3.552	3.31	3.438
8	8	0	36.9	4.232	4.169							3.951
9	1.32	0.389	63.4	1.722	1.696		1.595	1.642	1.618			1.708
10	2.64	0.389	45.0	2.836	2.796	2.855	2.681	2.764	2.722	2.735	2.89	2.733
11	3.96	0.389	33.7	3.559	3.515							3.452
12	5.28	0.389	26.6	4.050	4.007							3,958
A	1.714	0.556	45.0				2.088	2.119	2.104	2.136	2.52	2.170
B	2.640	0.389	45.0	2.836	2.796	2.855	2.681	2.764	2.722	2.735	2.89	2.733
C	3.818	0.222	45.0				3.129	3.291	3.210	3.204	3.15	3.156
D	6.000	0	45.0	3.688	3.626		3.487	3.605	3.546	3.552	3.31	3.440

TABLE XVI/

TABLE XVI

Calculated Values of Aerodynamic Centre

Wing	A	λ	Λ	Vortex Lattice Theory: 6 point standard	Vortex Lattice Theory: 6 point rounded	Vortex Lattice Theory: 8 point rounded	Vortex Lattice Theory: 9 point rounded	Weissinger's Theory: Standard 4 point	Weissinger's Theory: Modified 3 point	Weissinger's Theory: Mean	Multhopp m = 15 2 chordwise	Küchemann Refs. 10, 11	Elliptic Quarter Chord
1	2	0.143	48.4	0.9338	0.9428	0.9495		0.9158	0.9577	0.9368			0.9150
2	3	0.143	36.9	0.9179	0.9273	0.9319	0.9206	0.9128	0.9559	0.9344	0.9327	0.929	0.9150
3	4	0.143	29.4	0.9081	0.9177	0.9214							0.9150
4	6	0.143	20.6	0.8960	0.9061	0.9092							0.9150
5	2	0	71.6	1.7821	1.8079	1.8355		1.7462	1.8437	1.7950			1.7732
6	4	0	56.3	1.7455	1.7721	1.7924							1.7732
7	6	0	45.0	1.7088	1.7363	1.7510		1.7102	1.8143	1.7623	1.7016	1.639	1.7732
8	8	0	36.9	1.6796	1.7078	1.7195							1.7732
9	1.32	0.389	63.4	0.9598	0.9698	0.9775		0.9354	0.9768	0.9561			0.9202
10	2.64	0.389	45.0	0.9376	0.9482	0.9571	0.9422	0.9363	0.9787	0.9575	0.9561	0.965	0.9202
11	3.96	0.389	33.7	0.9304	0.9412	0.9479							0.9202
12	5.28	0.389	26.6	0.9272	0.9380	0.9433							0.9202
A	1.714	0.566	45.0					0.6924	0.7211	0.7068	0.6838	0.722	0.6352
B	2.640	0.389	45.0	0.9376	0.9482	0.9511	0.9422	0.9363	0.9787	0.9575	0.9561	0.965	0.9202
C	3.818	0.222	45.0					1.2375	1.3016	1.2696	1.2623	1.244	1.2193
D	6.000	0	45.0	1.7088	1.7363	1.7510		1.7102	1.8143	1.7623	1.7016	1.639	1.7732

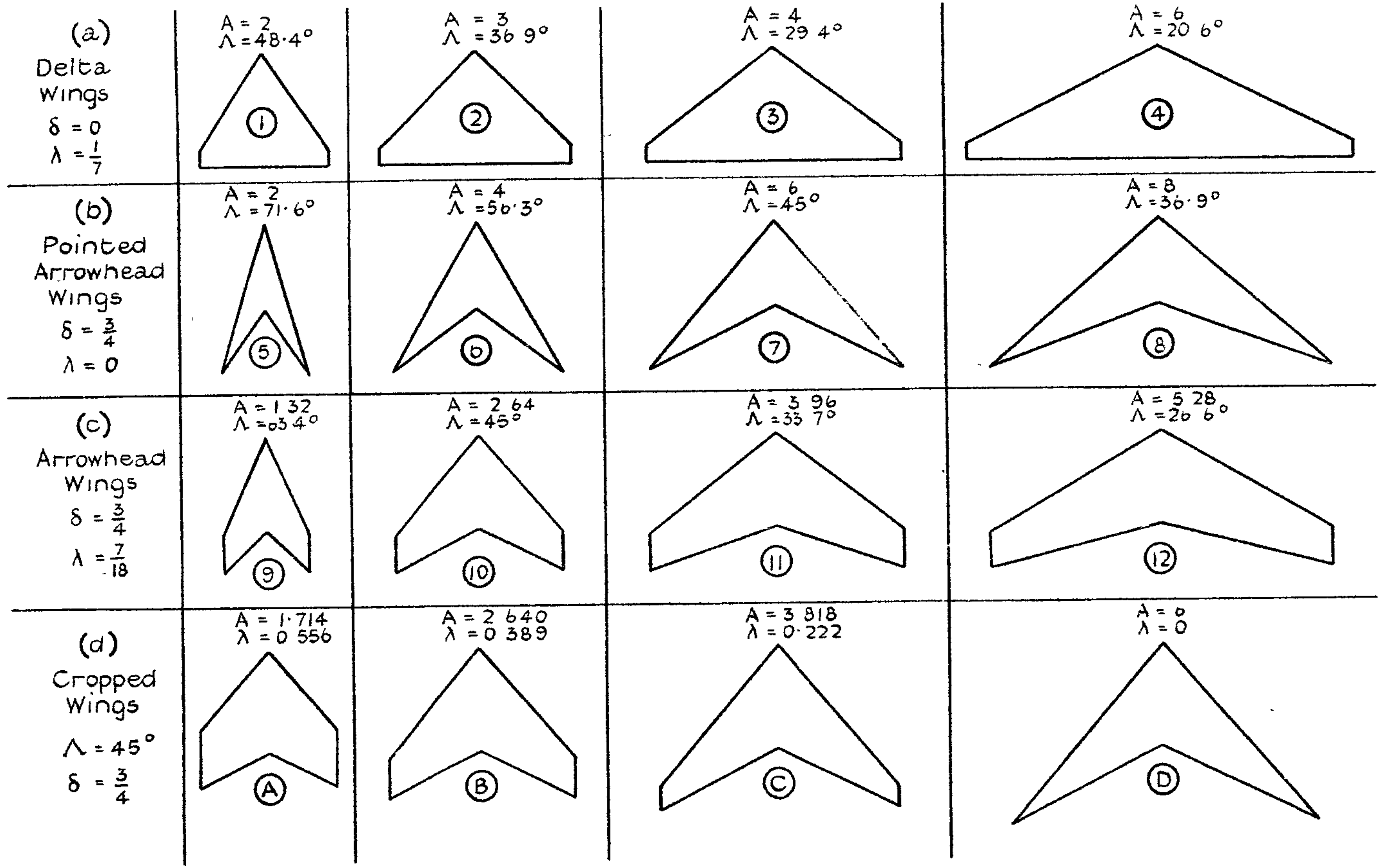
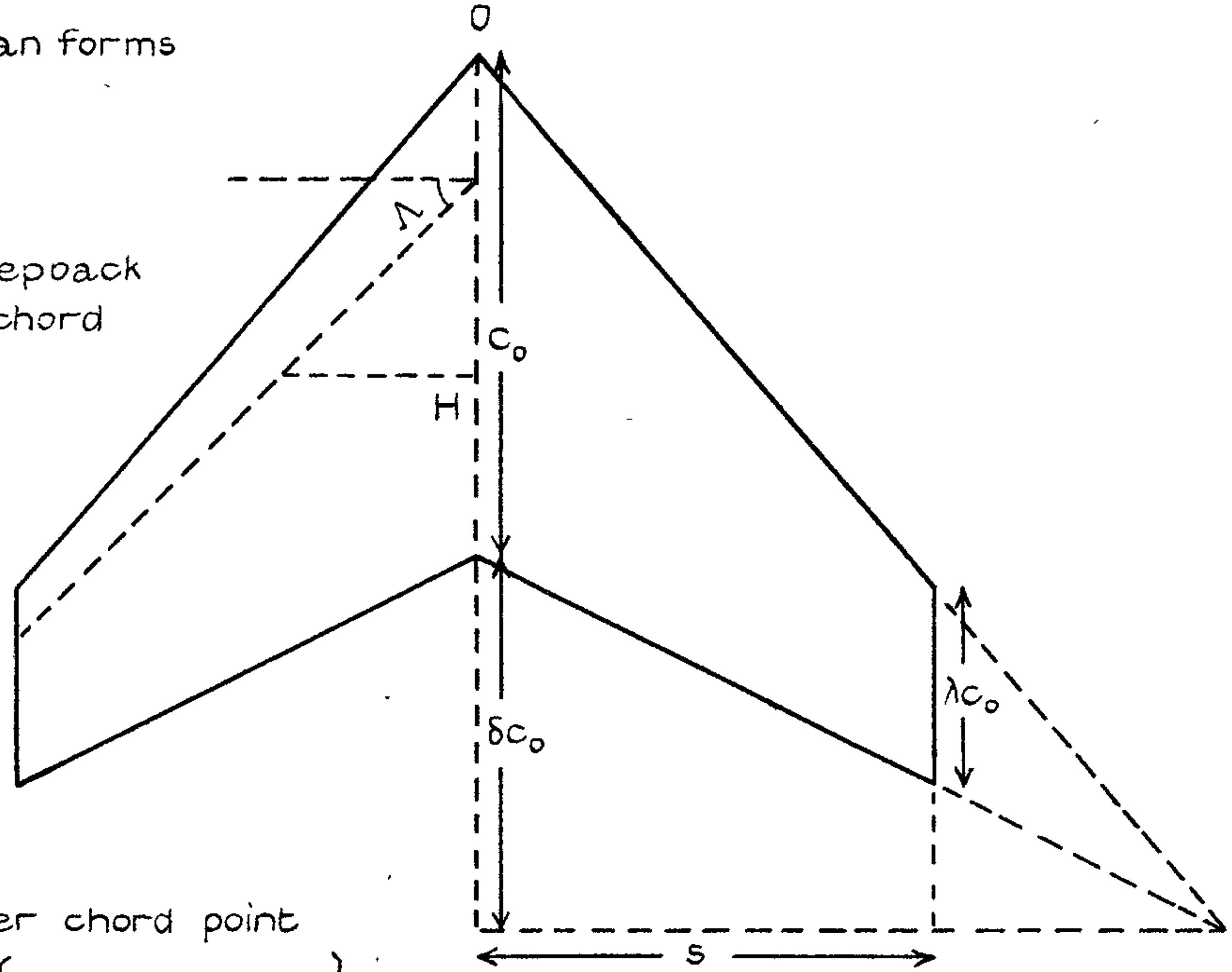


FIG. 1

Four series of swept plan forms

Fig 2.



(δ, λ) Family of plan forms

A Aspect ratio $= 2s/\bar{c}$

Λ Angle of sweepback of quarter-chord

$$\tan\Lambda = \frac{1-\lambda}{1+\lambda} \cdot \frac{3+4\delta}{A}$$

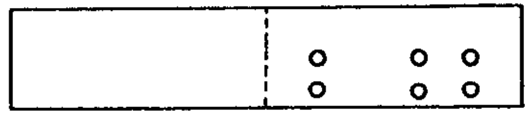
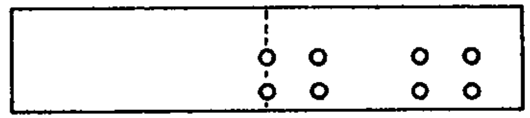
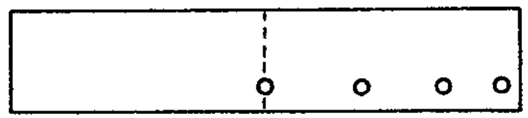
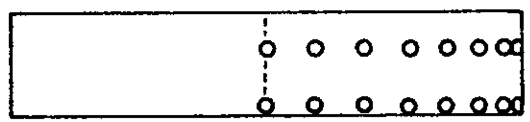
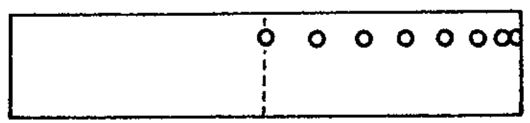
$\bar{c}$ Mean chord $= \frac{1}{2} c_0 (1+\lambda)$

s = Semi-span

H Elliptic quarter chord point

$$\bar{h} = \frac{OH}{\bar{c}} = \frac{1}{2(1+\lambda)}\left\{1 + \frac{1-\lambda}{\pi}\left(4 + \frac{16}{3}\delta\right)\right\}$$

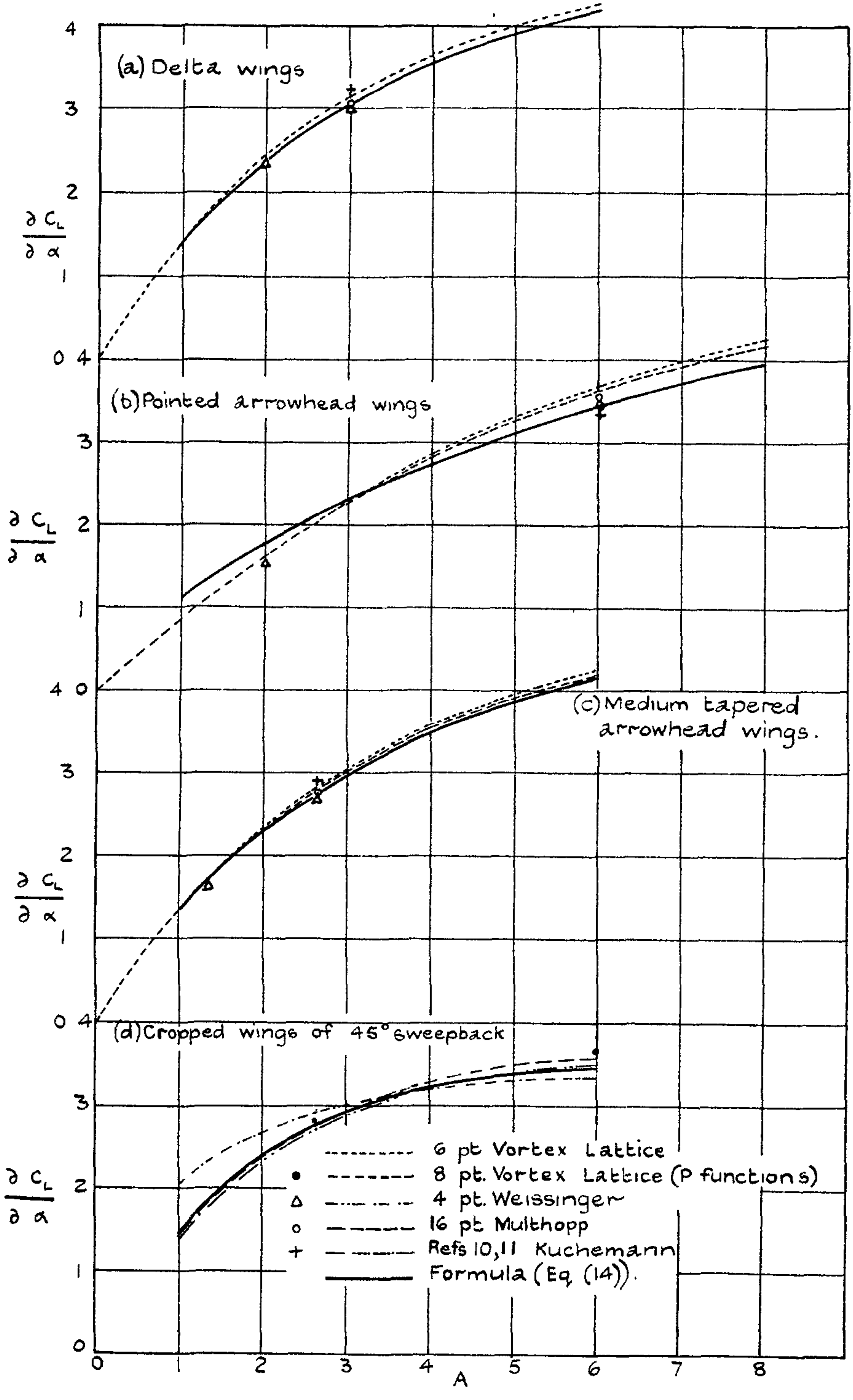
Definition of Parameters.

Method	Concept	Computation	Solving Points	Advantage of Method
Vortex Lattice 6 pt.	21/6 Vortex Lattice	4-6 days		Wide Applicability Limited Accuracy at Centre and Tips
Vortex Lattice 8 pt.	Ditto with P Functions	7-10 days		Slightly Improved Accuracy at Centre
Weissinger 4 pt	Simplified Lifting Surface	½-1 day		Rapid Evaluation of Spanwise Loading
Weissinger 3 pt.	Ditto without Central Point	½-1 day		Overcorrection for Inaccuracy at Central Kink
Multhopp	Continuous Lifting Surface	8-12 days		Superior Accuracy Ideal Solving Points
Küchemann 8 pt.	Modified Lifting Line	< ½ day		Speed and Adaptability Not Good Accuracy

Summary of Current Vortex Sheet Theories

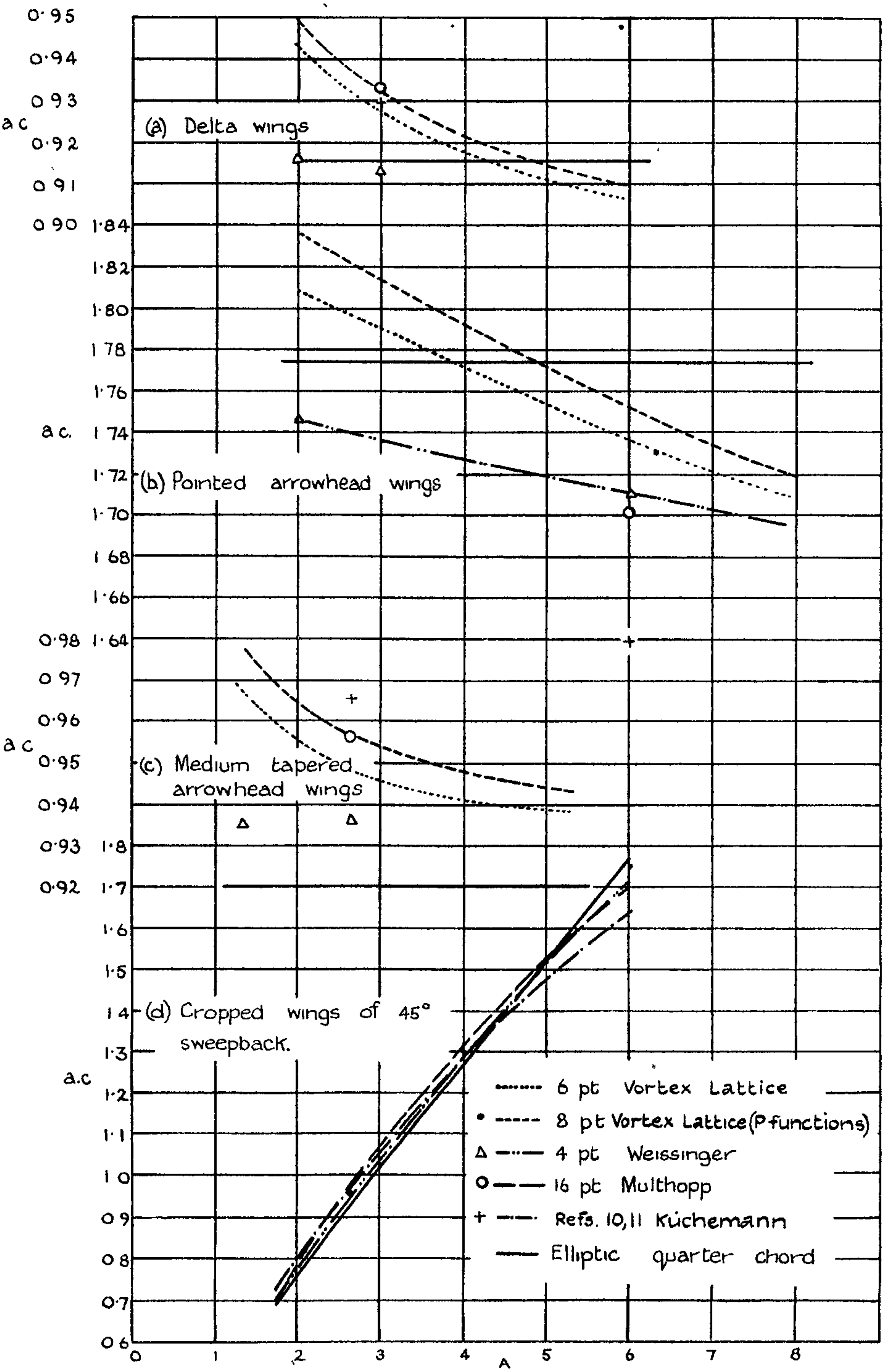
FIG 3.

FIG 4



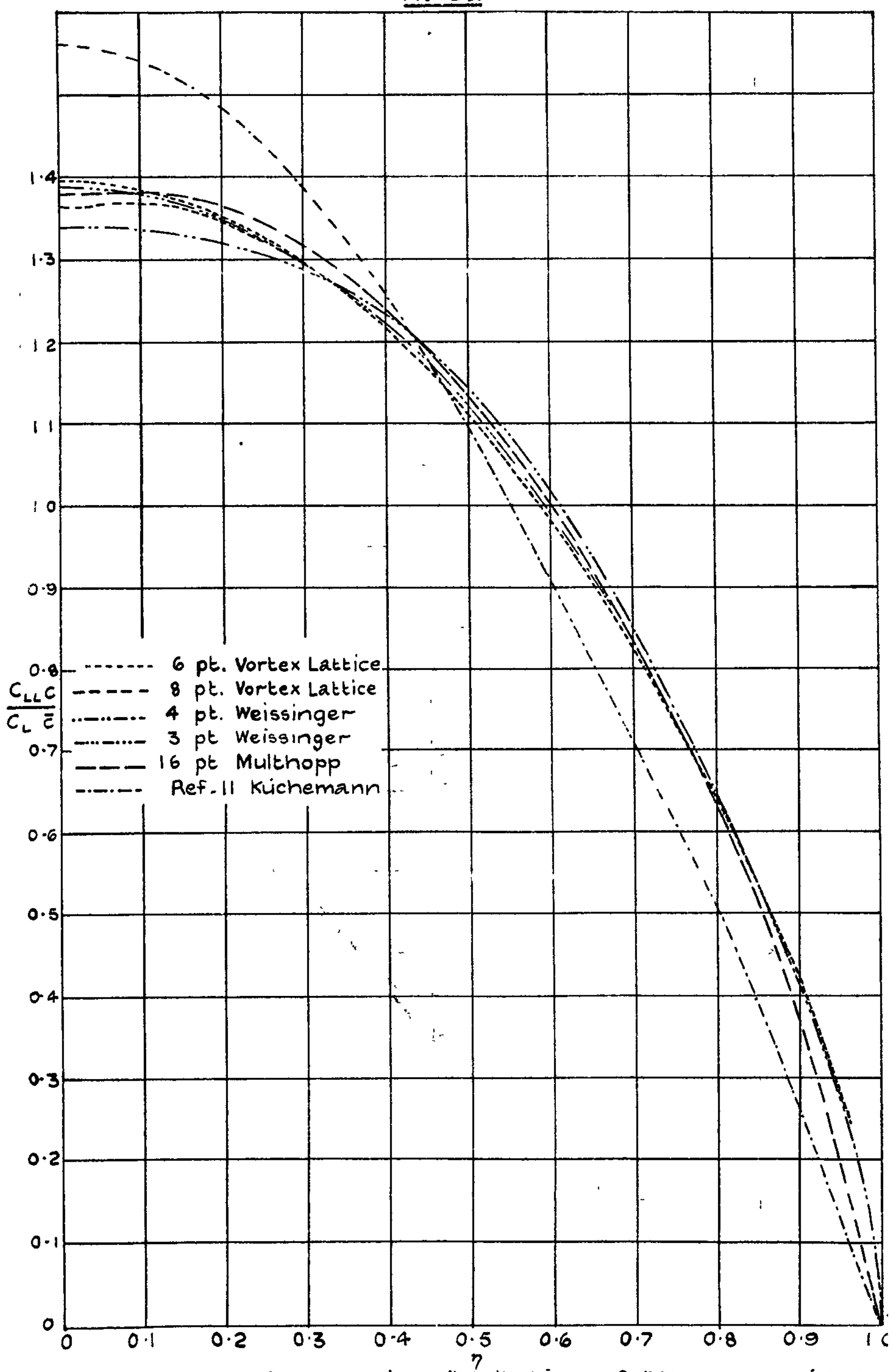
Comparative theoretical curves of $\frac{\partial C_L}{\partial \alpha}$ for swept wings.

Fig 5.



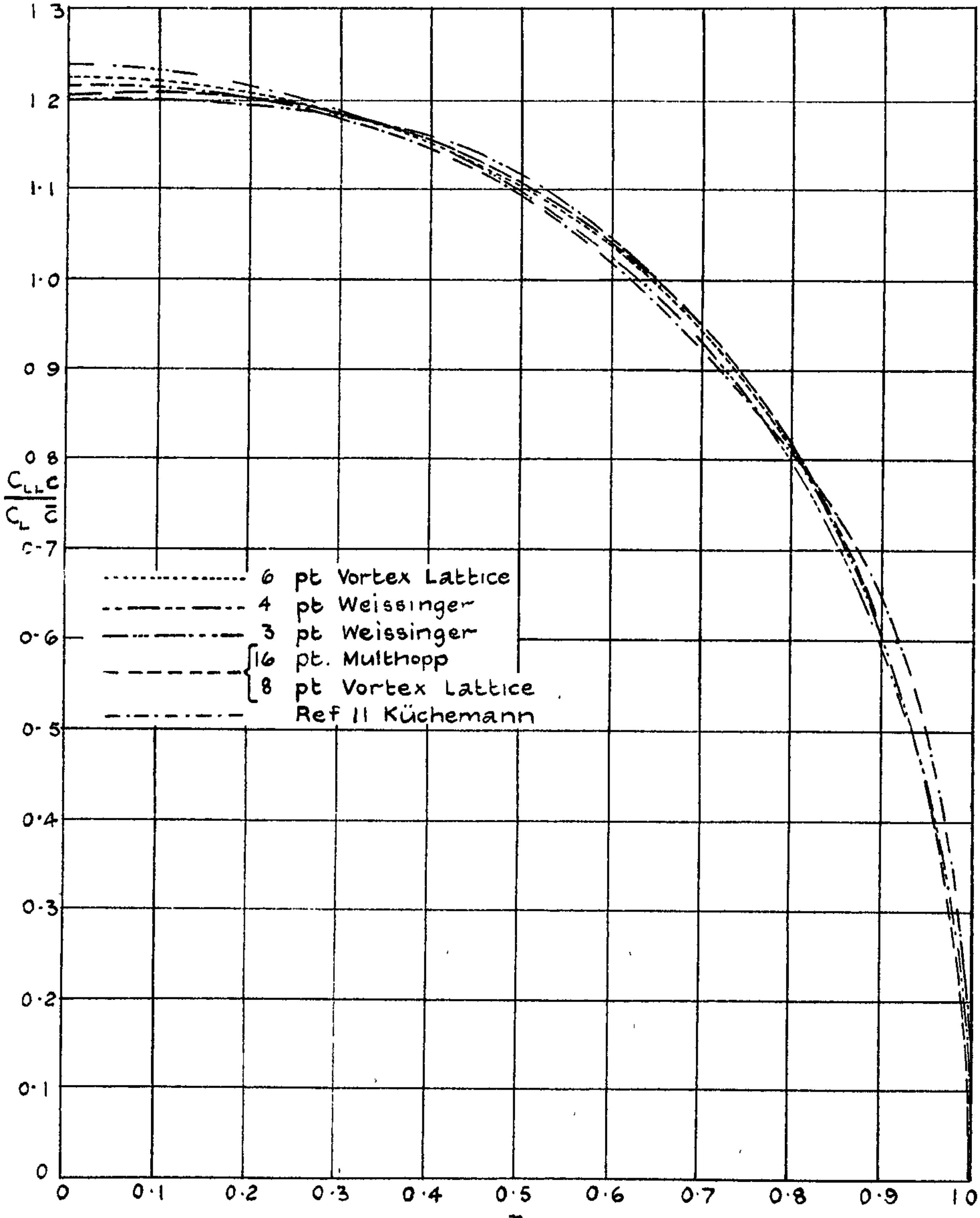
Comparative theoretical curves of aerodynamic centre for swept wings
a c denotes the position as a fraction of the mean chord from the apex of the plan form

Fig 6a



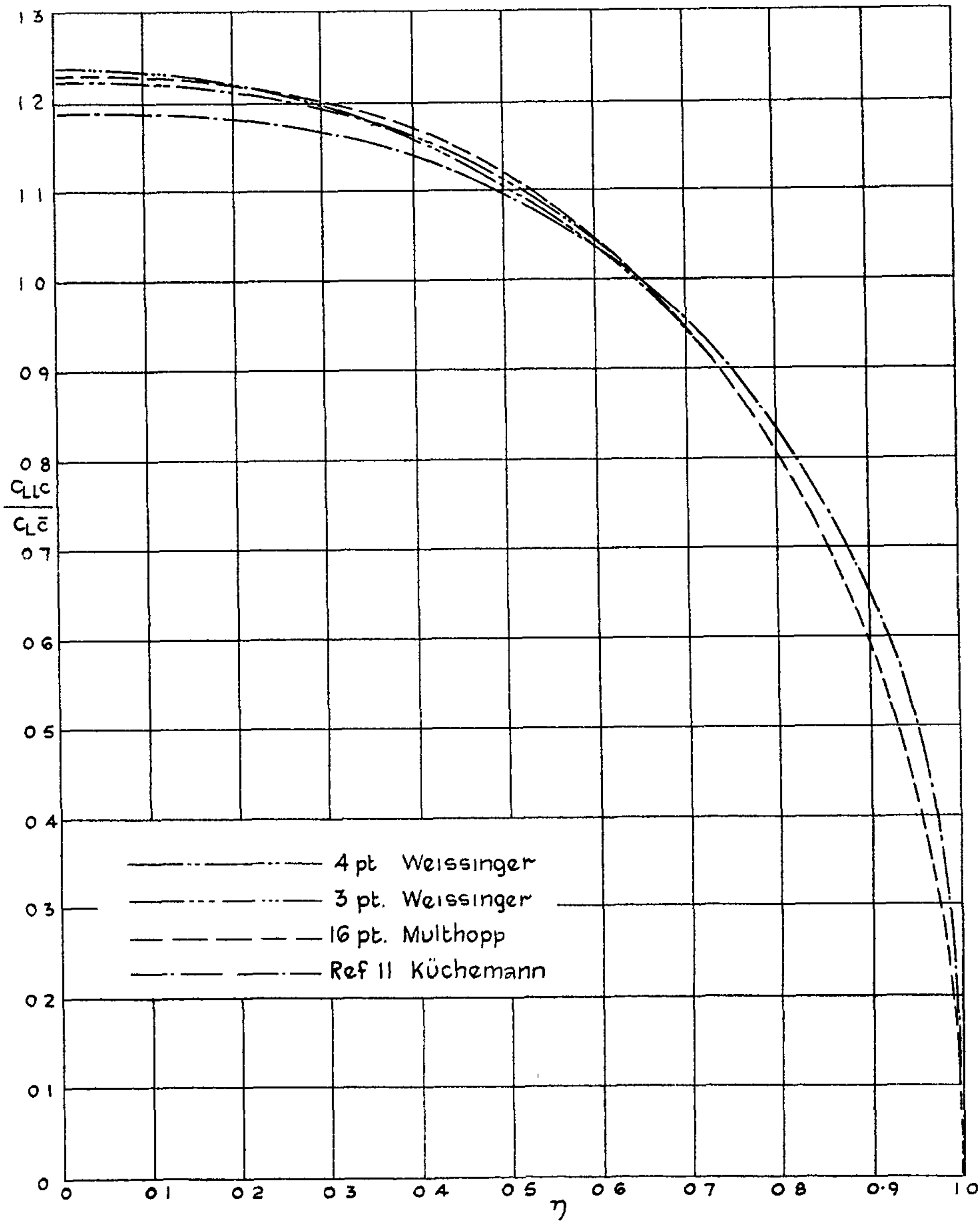
Comparative theoretical spanwise distributions of lift on a pointed wing of 45° sweepback and aspect ratio 6.

Fig 4b.



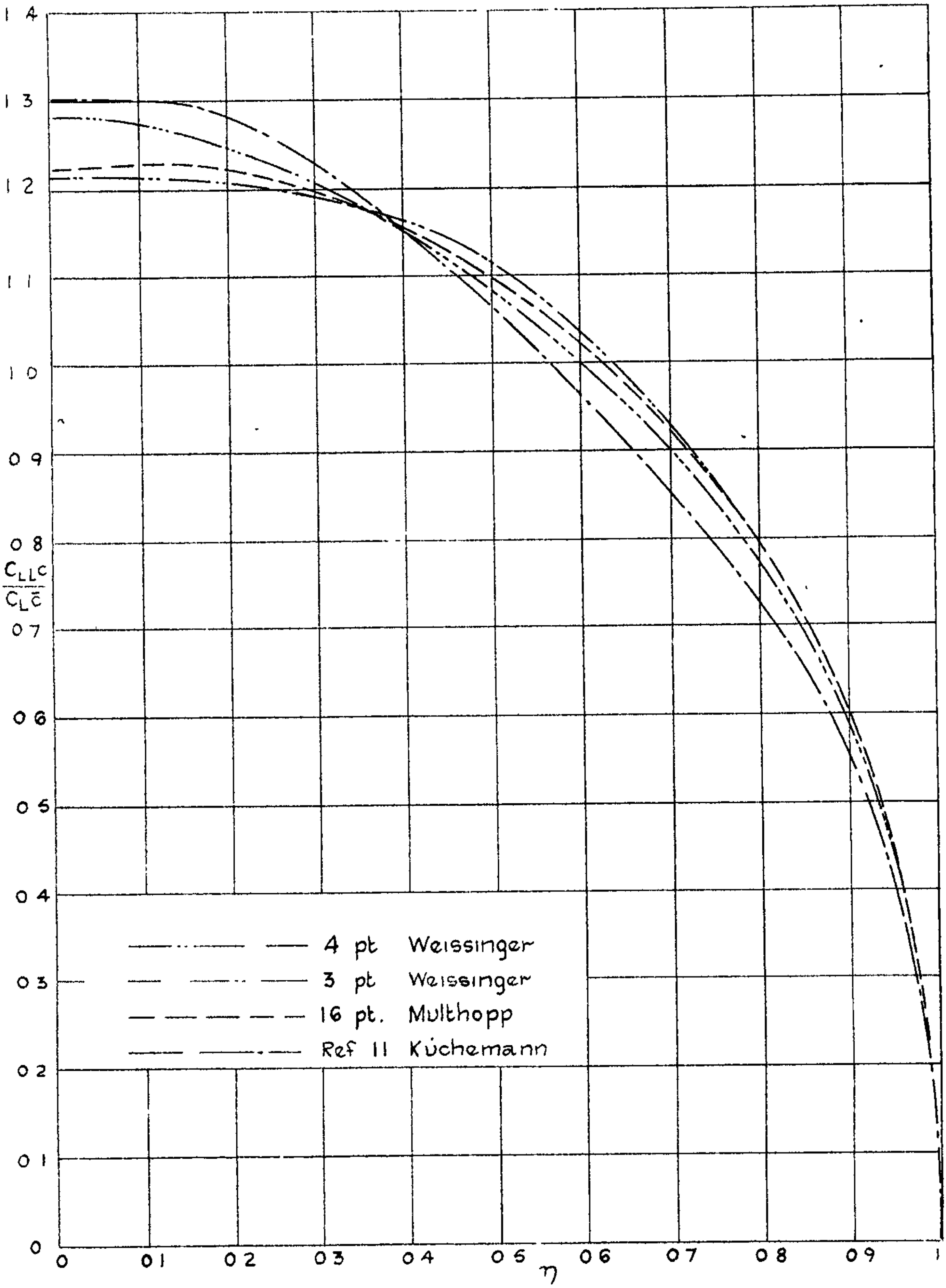
Comparative theoretical spanwise distributions of lift on a medium tapered wing of 45° sweepback and aspect ratio 2.64.

FIG 6c.



Comparative Theoretical Spanwise Distributions of Lift on Wing A of 45° Sweep-back and Aspect Ratio 1·714

FIG 6 d



Comparative Theoretical Spanwise Distributions of Lift on Wing C of 45° Sweep-back and Aspect Ratio 3·818

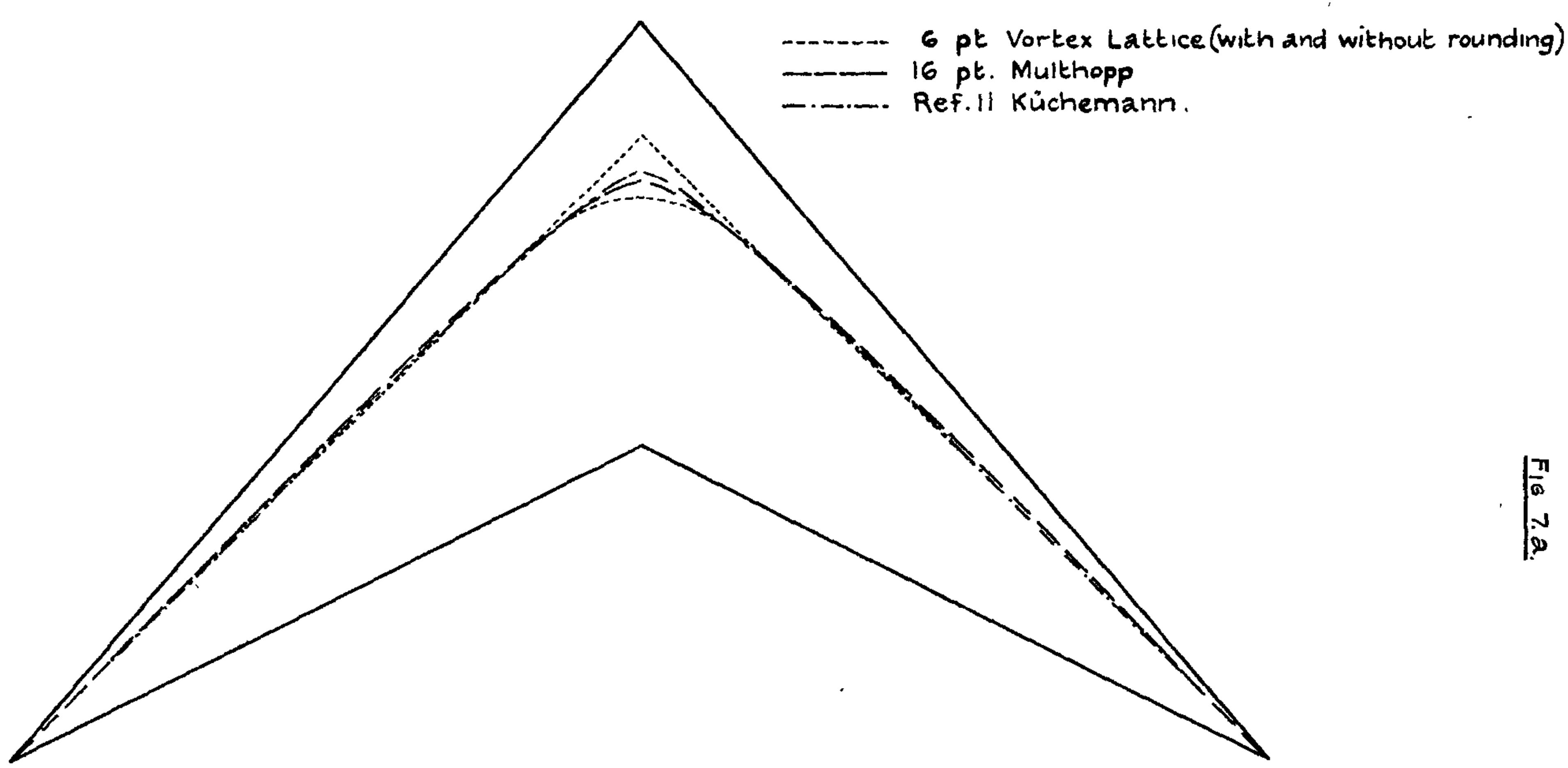
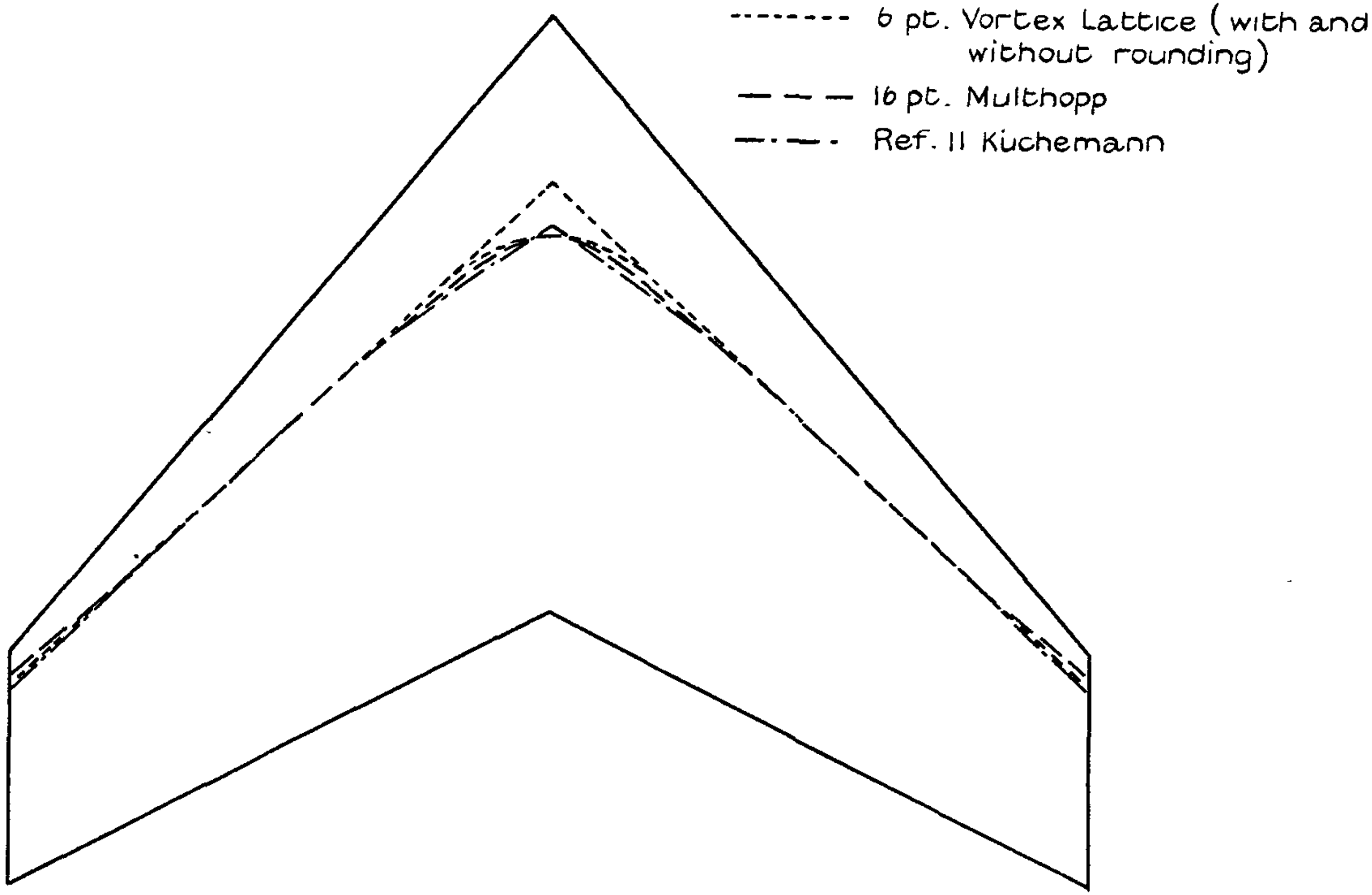


Fig 7.a.

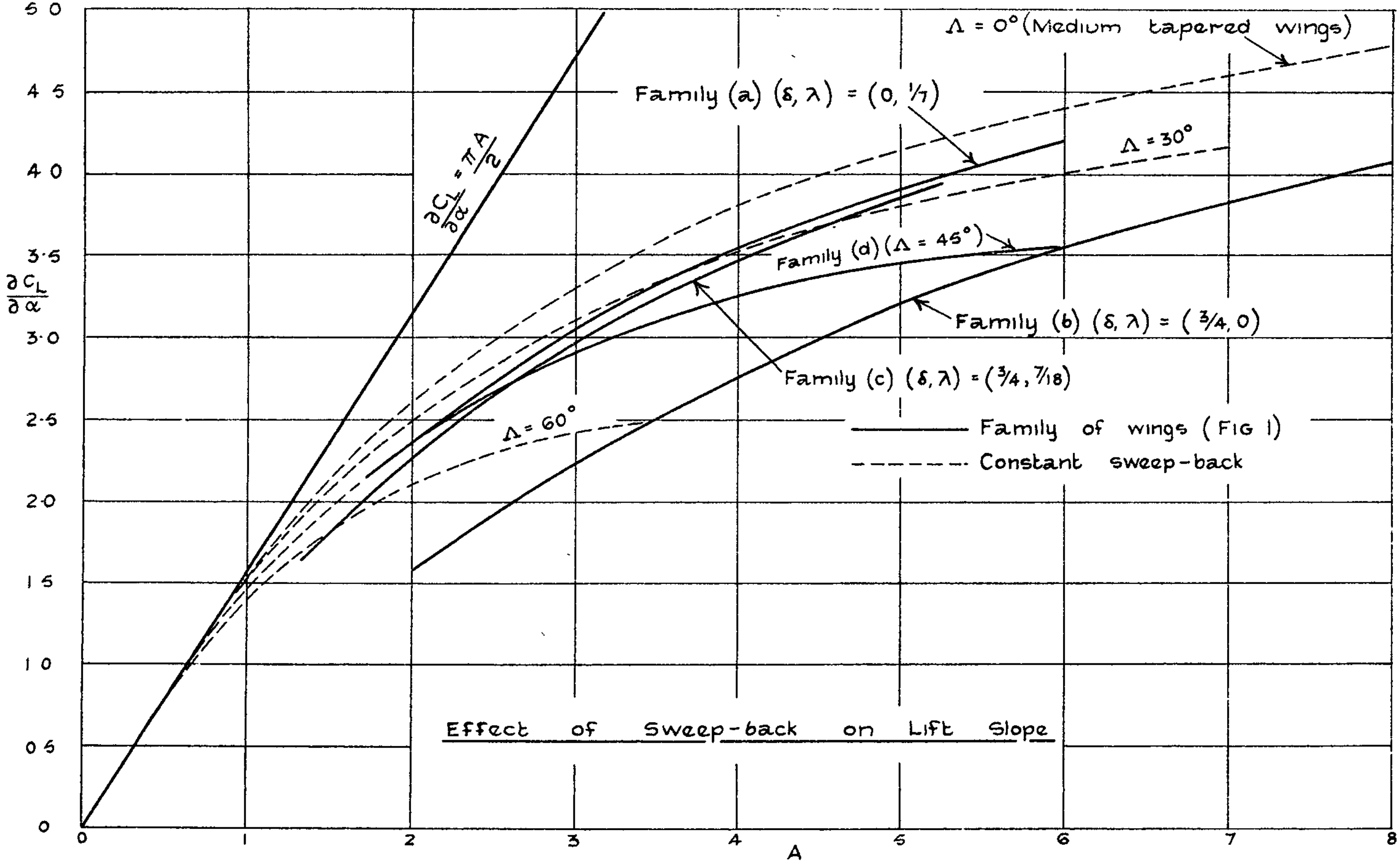
Comparative theoretical local aerodynamic centres on a pointed wing of 45° sweepback and aspect ratio 6.



Comparative theoretical local aerodynamic centres on a medium tapered wing of 45° sweepback and aspect ratio 2.64

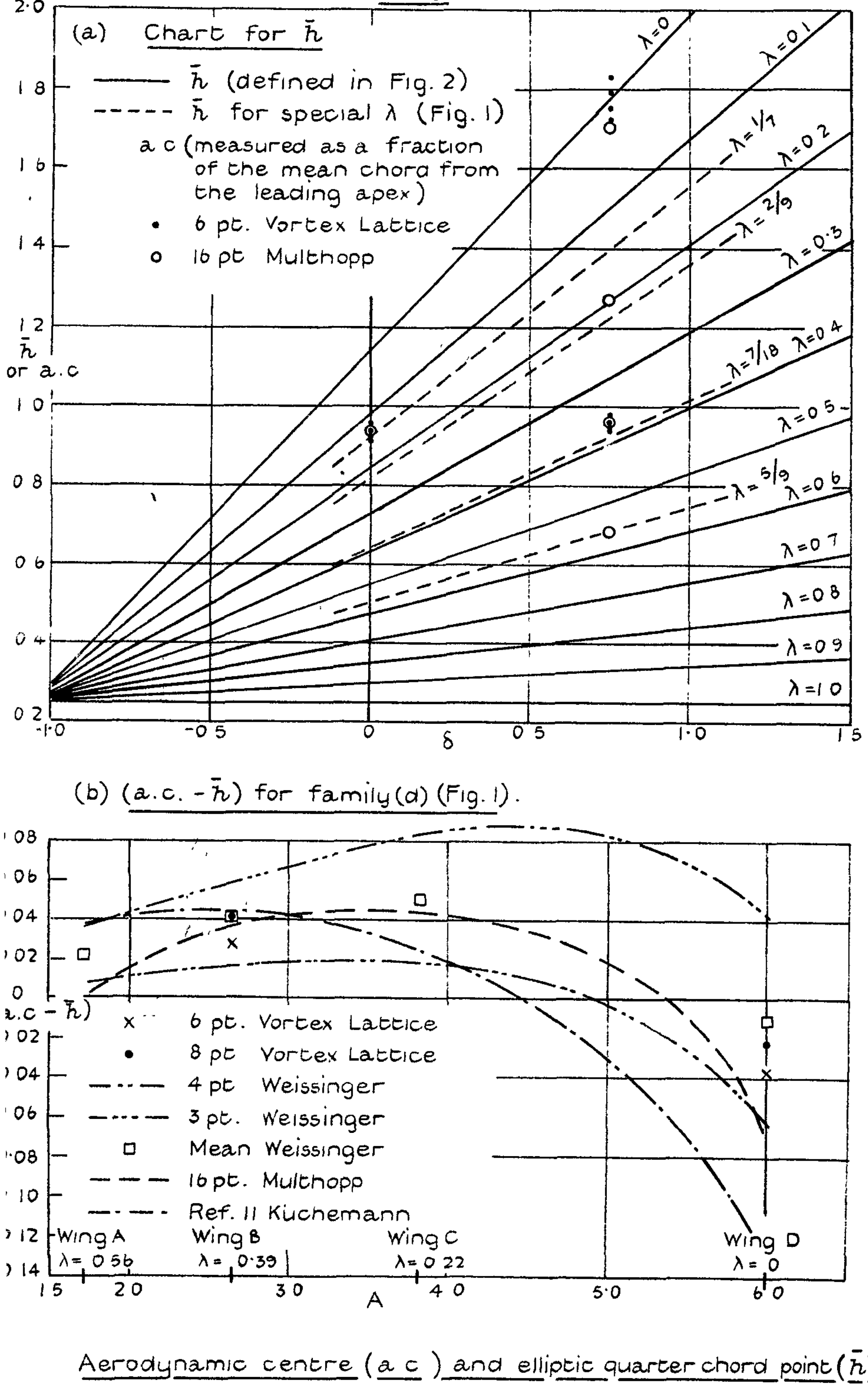
FIG 7b

FIG 8



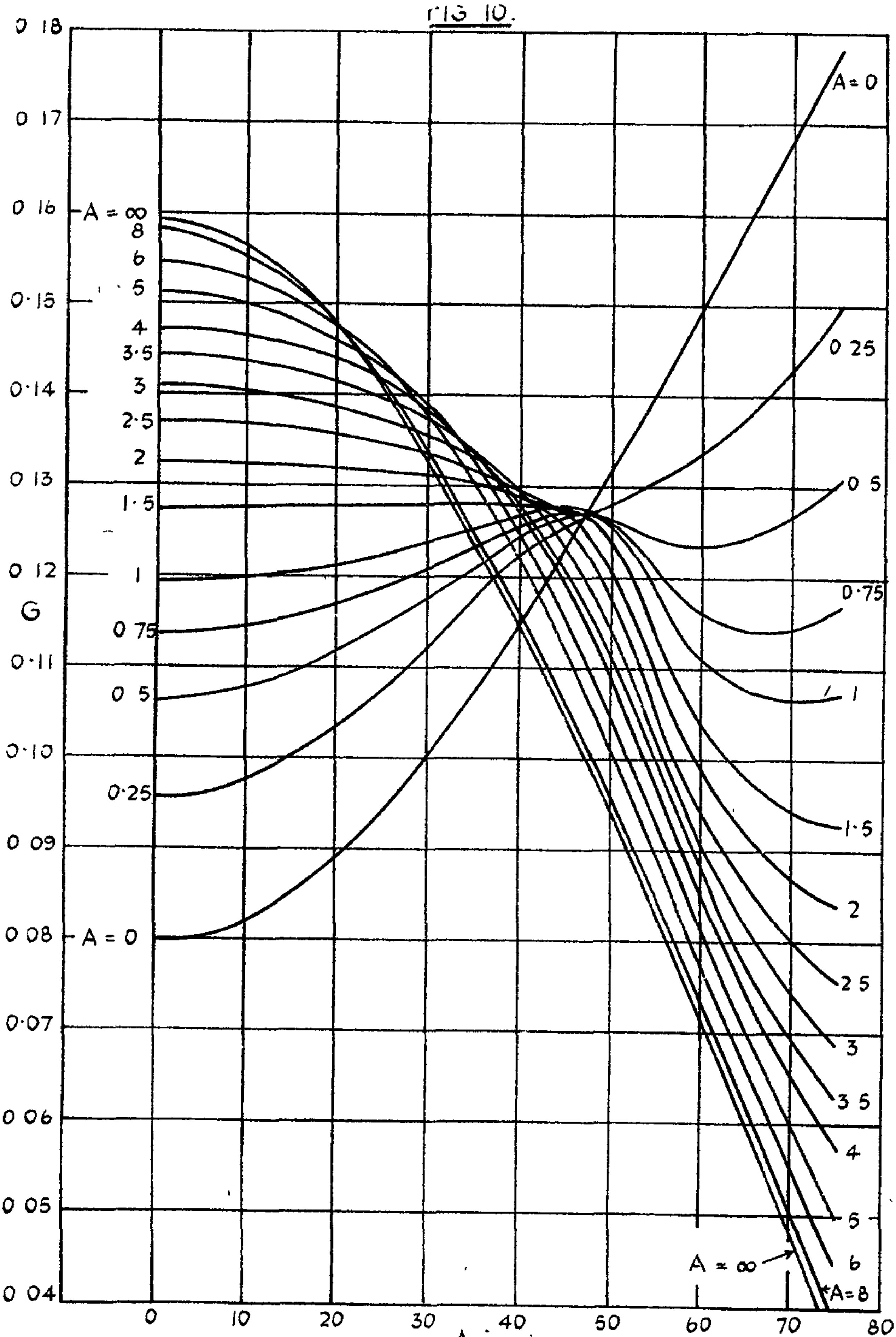
Λ = 0° (Medium tapered wings)
Family (a) (δ, λ) = (0, 1/7)
Λ = 30°
Family (d) (Λ = 45°)
Family (b) (δ, λ) = (3/4, 0)
Family (c) (δ, λ) = (3/4, 7/18)
Λ = 60°
$\frac{\partial C_L}{\partial \alpha} = \frac{\pi A}{2}$
Family of wings (FIG 1)
Constant sweep-back
Effect of Sweep-back on Lift Slope
$\frac{\partial C_L}{\partial \alpha}$
A

FIG 9.



Aerodynamic centre (a c) and elliptic quarter chord point ($\bar{h}$).

FIG 10.



The effect of compressibility on $\partial C_L / \partial \alpha$ at subcritical speeds.

$$\left(\frac{\partial C_L}{\partial \alpha}\right)_M = \left(\frac{\partial C_L}{\partial \alpha}\right)_0 \Bigg/ \left[1 - G\left(1 - \sqrt{1 - M^2}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)_0\right],$$

where G is given approximately as a function of aspect ratio A and quarter chord sweepback Λ. $\left(\frac{\partial C_L}{\partial \alpha}\right)_0$ denotes the lift slope at Mach number $M = 0$.

ADDENDUM

28th September, 1951

For reasons expressed in §6, the writer considers that of the available vortex sheet theories Multhopp's method will provide the most reliable solution for swept wing loading in potential subsonic flow.

At the outset of a calculation by Multhopp's theory it is necessary to prescribe:

(i) the number of spanwise solving stations $m = 3, 5, 7, 11, 15, 23$ or 31.

(ii) the number of chordwise pivotal points per station, $N = 1$ or 2.

The consistency of the theory and the extent to which it can usefully be simplified by reducing m has been investigated further by obtaining three solutions:

$$m(N) = 7(1),\ 7(2),\ 15(2)$$

for each of five wings [Fig.1, Wings 2, A,B,C,D].

The following derivatives are tabulated below:

$\dfrac{\partial C_L}{\partial \alpha}$ (α in radians)

a.c. (measured as a fraction of $\bar{c}$ from the leading apex)

$\bar{\eta}$ (spanwise centre of pressure on the half wing).

Table/

Derivative	Solution	Wing 2	Wing A	Wing B	Wing C	Wing D
$\frac{\partial C_L}{\partial \alpha}$	7(1) 7(2) 15(2)	3.064 3.071 3.050	2.130 2.152 2.136	2.701 2.771 2.735	3.184 3.251 3.204	3.606 3.471 3.552
a.c.	7(1) 7(2) 15(2)	0.9142 0.9180 0.9327	0.6955 0.6846 0.6838	0.9411 0.9363 0.9561	1.2459 1.2355 1.2623	1.6952 1.6493 1.7016
$\bar{\eta}$	7(1) 7(2) 15(2)	0.4244 0.4224 0.4212	0.4369 0.4316 0.4324	0.4407 0.4363 0.4361	0.4388 0.4347 0.4328	0.3989 0.3855 0.3938

Multhopp (Ref.9, App.VII) has stated that a reasonable choice of the number of spanwise stations is

$$m > 3A\sqrt{1 - M^2} .$$

For the five wings calculated above in incompressible flow, $M = 0$, m should thus exceed the values given in the following table:

Wing	2	A	B	C	D
3A	9.0	5.1	7.9	11.5	18.0

The table of calculated derivatives suggests that, if this criterion is satisfied, $\frac{\partial C_L}{\partial \alpha}$ may be obtained within 1%, the aerodynamic centre within about 0.015 $\bar{c}$, and, provided that $N = 2$, the spanwise centre of pressure within about 0.001s. In the special case of a pointed wing, e.g., Wing D, the accuracy may not fall quite within these limits, but the use of Multhopp's theory is still recommended as it permits a concentration of pivotal points near the pointed tip. The most significant discrepancies arise in connection with the aerodynamic centre, but the interesting conclusion is that these are not primarily associated with the central kink.

Three spanwise loadings and local aerodynamic centres, calculated for each of the five wings, are given in detail in Table XVII, where $X_{a.c.}$ denotes the position of the local a.c. as a fraction of the local chord from the leading edge. It is advisable to include two chordwise terms when calculating spanwise loading as none of the 7(1) solutions is fully satisfactory. Naturally it is necessary to take $N = 2$ for the purpose of obtaining local a.c., but there is nothing in Table XVII to suggest that more than two chordwise terms are needed. In fact when the criterion for m is approximately satisfied (as for Wings 2, A and B), the agreement at the crucial 'kinked' central section $\eta = 0$ is impressive. Furthermore except for the pointed wing the notable feature of solutions with insufficient spanwise stations is that $X_{a.c.}$ is underestimated on the outer half of the wing. This explains the underestimate of a.c. exhibited in the table of derivatives for the 7(2) solutions.

The/

The use of a chordwise pressure distribution from equation (12), viz.:

$$\frac{p_b - p_a}{\rho V^2} = \frac{8s}{c}\left[\frac{\gamma}{2\pi}\cot\tfrac{1}{2}\theta + \frac{2\mu}{\pi}\left(\cot\tfrac{1}{2}\theta - 2\sin\theta\right)\right]$$

in potential flow across the outer sections is just as valid for swept wings as for unswept wings. It is concluded that Multhopp's theory is of general application, but that it may be unduly laborious to obtain extreme accuracy for wings of high aspect ratio $A > 5$.

As a further indication of the degree of accuracy in the central region the downwash from the 7(2) and 15(2) solutions have been evaluated at the displaced root three-quarter chord points, i.e., at pivotal points used respectively in 7(1) and 15(1) solutions. The values of the ratio of downwash at 0.75 c to the angle of attack α are given below, corresponding values of unity being already satisfied at 0.3455 c and 0.9045 c.

	Solution	Wing 2	Wing A	Wing B	Wing C	Wing D
$\left(\frac{w}{V\alpha}\right)_{0.75c}$	7(2)	0.968	0.959	0.986	1.009	1.016
at $\eta = 0$	15(2)	0.949	0.962	0.969	0.964	0.971

These comparisons inspire confidence in the intrinsic accuracy of Multhopp's theory, especially as it seems unnecessary to satisfy the boundary conditions along more than two loci to establish values of the local lift and aerodynamic centre.

TABLE XVII/

TABLE XVII

Multhopp's Theory

Calculated Spanwise Loading and Local Aerodynamic Centre

η	Values of $\frac{C_{LL}\, c}{C_L\, \bar{c}}$ Solution 7(1)	7(2)	15(2)	WING	Values of $X_{a.c.}$ Solution 7(1)	7(2)	15(2)
0	1.271	1.289	1.296	2	0.291	0.333	0.331
0.3827	1.181	1.177	1.180		0.250	0.239	0.258
0.7071	0.892	0.887	0.883		0.250	0.220	0.238
0.9239	0.496	0.489	0.483		0.250	0.187	0.193
0	1.197	1.228	1.223	A	0.293	0.353	0.334
0.3827	1.164	1.169	1.169		0.250	0.243	0.244
0.7071	0.943	0.928	0.931		0.250	0.187	0.191
0.9239	0.537	0.511	0.514		0.250	0.121	0.127
0	1.178	1.206	1.203	B	0.308	0.368	0.366
0.3827	1.158	1.159	1.160		0.250	0.241	0.264
0.7071	0.951	0.943	0.941		0.250	0.207	0.228
0.9239	0.561	0.537	0.538		0.250	0.134	0.152
0	1.188	1.219	1.224	C	0.324	0.366	0.378
0.3827	1.165	1.163	1.166		0.250	0.229	0.260
0.7071	0.934	0.923	0.917		0.250	0.220	0.237
0.9239	0.564	0.548	0.540		0.250	0.179	0.188
0	1.327	1.399	1.381	D	0.346	0.360	0.381
0.3827	1.273	1.293	1.255		0.250	0.228	0.239
0.7071	0.856	0.811	0.823		0.250	0.250	0.222
0.9239	0.266	0.205	0.293		0.250	0.277	0.253

C.P. No 102

A.R.C Technical Report

PRINTED AND PUBLISHED BY HER MAJESTY'S STATIONERY OFFICE
To be purchased from
York House, Kingsway, LONDON, W C 2 423 Oxford Street, LONDON, W 1
P O Box 569, LONDON, S E.1
13a Castle Street, EDINBURGH, 2 1 St Andrew's Crescent, CARDIFF
39 King Street, MANCHESTER, 2 Tower Lane, BRISTOL, 1
2 Edmund Street, BIRMINGHAM, 3 80 Chichester Street, BELFAST
or from any Bookseller

1952

Price 13s 0d net

PRINTED IN GREAT BRITAIN

S O. Code No. 23-9006-10-2

DS 32475/4/K3