C.P. No. 97
(13,675)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
CURRENT PAPERS

Fluid Dynamic Notation in Current Use at N.G.T.E.

By

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LONDON : HER MAJESTY'S STATIONERY OFFICE
1952
FIVE SHILLINGS NET

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Addendum

C.P. No. 97

July, 1952.

Recommendations of the Engine-Aerodynamics Sub-Committee of the Aeronautical Research Council

In discussions, for which this paper was prepared, the Sub-Committee reviewed various notations used in engine-aerodyanmics work. A limited degree of standardization was thought desirable to reduce the variety of symbols used for certain common quantities and to conform with accepted practice over a wider technical field. The Sub-Committee therefore recommended:-

> "That the following notation should be adopted as a standard for A.R.C. and Establishment publications:-
>
> | | | | |
> |---|---|---|---|
> | Total pressure | P_t | or | H |
> | Static pressure | P | | p |
> | Total temperature | T_t | | |
> | Static temperature | T | | |
> | Velocity of sound | a | | |
> | Mach number | M | | |
> | Reynolds number | Re | | |
> | Gas constant | R | | |
>
> For mass flow W is a tentative suggestion and an alternative to ω for loss coefficient should be found."

The notation covered by this recommendation should be substituted for that given in the present paper to bring the record of N.G.T.E. usage up to date. The omission of the word "head" in the expressions "total-head pressure" and "total-head temperature" should be noted.

C.P. No. 97

Memorandum No. M.93.

July, 1950.

NATIONAL GAS TURBINE ESTABLISHMENT

Fluid Dynamic Notation in Current Use at N.G.T.E.

- by -

S. Gray.

SUMMARY

This Memorandum records and defines the current system of notation which is in general use, at the National Gas Turbine Establishment, for work on axial flow compressors and cascade investigations in general, and which is being applied to some extent to the work on turbines. Heat transfer and supersonic flow aspects and other specialized treatments are excluded.

Detailed definitions and explanations are given in classified lists, illustrated by figures, and alphabetical and numerical lists of the symbols, suffixes and indices are included.

CONTENTS

ILLUSTRATIONS

1.0 Introduction

This Memorandum, which has been prepared at the request of the Engine Aerodynamics Sub-Committee of the Aeronautical Research Council, records and defines the fluid dynamic notation in current use at the N.G.T.E. in the field of axial flow turbo-machines, excluding the specialized heat transfer and supersonic flow aspects.

The notation is defined and illustrated as applied to both compressor and turbine blading and is the established practice, in the N.G.T.E., for cascade investigations in general and for work on axial flow compressors. It is normally employed in the generalized, fundamental treatment of both compressors and turbines, and the present tendency in turbine design and performance work is to adopt, as far as possible, the same basic conventions as for compressors. The two machines have a common fluid dynamic basis, though they possess individual characteristics, and a common notation would be desirable.

The lists do not give a complete record of the notation which has been employed, being limited to the symbols commonly used in design and performance work and in general analysis. Additional notation is required for work of a specialized nature, such as boundary-layer and three-dimensional flow analysis and potential flow theory. Existing common practice is followed in such cases, where possible, and while this sometimes involves duplication with symbols listed here, there is usually little difficulty in avoiding confusion.

The notation, which is derived primarily from references 1 and 2, corresponds to a large extent with that used in most of the basic works on the development of axial flow compressor and cascade theory in this country and within this field much of the notation is in common use in industry. Some changes and additions have been made, since the issue of these reports, as is always necessary for original work in a developing subject.

In section 2.0, classified lists are given of the basic notation, i.e. the general symbols for fundamental and frequently occurring quantities. The symbols and terminology are defined in detail in section 3.0 and some of the principal relations are quoted. Finally, alphabetical and numerical lists are presented in section 4.0, covering the symbols, suffixes and indices appearing in the previous sections.

2.0 Basic Notation

2.1 General

ℓ	length
d	diameter
r	radius
z	perimeter
A	area
N	speed of revolution
U	blade speed
g	acceleration due to gravity
W	mass flow
η	efficiency

2.2 Blade geometry

c	blade chord (length)
h	blade height
s	pitch (or blade spacing)
β	blade angle (from axial)
ζ	blade stagger angle
θ	blade camber angle

2.3 Fluid velocities and angles

V	fluid velocity
α	fluid flow angle (from axial)
i	incidence
δ	deviation
ε	deflection

2.4 Fluid state, etc.

P	static pressure (absolute)
P_{tot}	total-head pressure (absolute)
$\Delta P, \Delta P_{tot}$	pressure difference or change in pressure
R	pressure ratio
ω	loss of total-head pressure

T'	static temperature (absolute)
T_{tot}	total-head temperature (absolute)
$\Delta T, \Delta T_{tot}$	temperature difference or change in temperature
ρ	density
σ	relative density

2.5 Thermodynamic and aerodynamic properties and conditions

J	mechanical equivalent of heat
K	gas constant
K_p	specific heat at constant pressure
K_v	specific heat at constant volume
γ	ratio of specific heats = K_p/K_v
μ	viscosity
ν	kinematic viscosity
V_c	acoustic velocity
M_n	Mach number = V/V_c
R_n	Reynolds number
C_f	skin friction coefficient
C_D	drag coefficient
C_L	lift coefficient

3.0 Classified Notation and Definitions

3.1 The base profile (Fig. 1)

(a) Notation

c	chord (length)
t	maximum thickness
L.E.	leading edge
T.E.	trailing edge
r_1	radius of curvature of L.E.
r_2	radius of curvature of T.E.

(b) Specification

In specifying the details of a base profile the following are quoted:-

(i) a series of ordinates, to the upper and lower surfaces, as measured perpendicular to the straight base line at different stations from the L.E. Stations are given as distances from the L.E. expressed as percentages of the blade chord, the usual values being:-

0, 1.25, 2.5, 5.0, 7.5, 10, 15, 20, 30, 40, 50, 60, 70, 80, 90, 95,100.

Ordinates are expressed as percentages of the chord, usually for a maximum thickness (t/c) of 10%.

(ii) L.E. radius as a percentage of the maximum thickness.

(iii) T.E. radius as a percentage of the maximum thickness.

(iv) maximum thickness as a percentage of chord (t/c normally 10%).

(v) station of maximum thickness as a percentage of chord.

Note:- The L.E. and T.E. radii are normally varied linearly with maximum thickness, for a given base profile, though this is not strictly accurate.

(c) Base profile code

The base profile is designated by a letter and number code, e.g. C4, T6. The number is a simple serial number, which is prefixed by the letter C or T to signify that the profile was originated for compressor or turbine use.

3.2 Camber line details (Fig. 1)

(a) Notation

The general notation is illustrated in Fig. 1.

a	distance of point of maximum camber from L.E.
b	maximum camber
c	chord

θ camber angle = $\chi_1 + \chi_2$

χ_1 camber inlet angle

χ_2 camber outlet angle

(b) Sign convention

θ, χ_1 and χ_2 are always considered as positive.

(c) Camber line forms

Circular or parabolic-arc camber lines are normally used. These are illustrated in Fig. 1, which gives the principal geometrical relations.

(d) Camber line code

The camber line details are specified in code form by quoting the following particulars, which completely define the geometry, in the order given:-

(i) camber angle (θ) in degrees,

(ii) form of camber, denoted by C for circular-arc or P for parabolic-arc,

(iii) distance of maximum camber from L.E. (a) as a percentage of chord,

Example:- 25P40

3.3 The cambered blade section (Fig. 2)

(a) Construction

A blade section is produced by superimposing a base profile of the required thickness on to a camber line of the required form and camber angle, the construction being as follows:-

(i) the base profile ordinates are multiplied by the ratio of the required maximum thickness to the base profile maximum thickness and are also scaled in proportion to the design chord,

(ii) stations are marked off along the camber line (not the chord line) at proportional distances from the leading edge as quoted for the base profile,

(iii) the ordinates are measured off, normal to the camber line, at the appropriate stations,

(iv) leading and trailing edge circles are drawn to pass through the end points of the camber line, with centres on the camber line,

(v) the ordinates are joined by smooth curves blending tangentially into the leading and trailing edge circles.

(b) Blade chord

The notation differs from common isolated aerofoil practice. The leading and trailing edges of the blade are defined by the points of intersection of the camber line with the profile and the straight line joining those points is the "chord" line. Then the chord can be briefly defined as:-

c blade chord
= length of straight line joining the leading and trailing edges of the camber line.

(c) Blade code

The complete blade section is specified by combining the base profile and camber line codes, with the addition of the thickness/chord ratio expressed as a percentage, and is quoted as follows:-

$$\left(\frac{t}{c}\%\right)\left(\begin{matrix}\text{base}\\\text{profile}\end{matrix}\right)\Big/\left(\theta^{\circ}\right)\left(\begin{matrix}\text{camber}\\\text{form}\end{matrix}\right)\left(\frac{a}{c}\%\right)$$

Example:- 12C4/25P40.

3.4 The blade in cascade

(a) Sign convention

The convention of signs, for the measurement of angles in cascades, can be derived from the isolated blade, as follows:-

Angles are measured positive in the direction of rotation defined by following the blade camber line from the trailing edge to the leading edge (see Fig. 2).

Note:- In considering a common sign convention for both compressors and turbines, it is clear that whatever system is adopted the normal compressor blade will have inlet and outlet angles of the same sign, while the corresponding angles for the normal turbine blade will have opposite signs. It is reasonable therefore to adopt the convention which makes the compressor angles both positive.

(b) Stagger and pitch (Fig. 2)

For the isolated blade the chord line forms an obvious axis of reference, but in cascade the "axial" direction (normal to the line of the cascade) is used. The blade stagger can be considered as the shift of the axis of reference from the isolated blade to the cascade. The notation was originally adopted to conform with normal biplane aircraft practice.

ζ	blade stagger angle, = angle of inclination of the blade measured from the chord line to the axial direction. (Normally negative for compressor cascades and positive for turbine cascades)
s	pitch (or blade spacing) = distance between corresponding points on adjacent blades, measured parallel to the cascade. (Always considered positive)

Note:- Stagger and pitch, in conjunction with the blade code details, completely define the cascade.

(c) General geometry (Fig. 3)

h	blade height
A.R.	aspect ratio = h/c
A_t	throat area - compressor blading (normally per unit blade height and therefore also used as throat width)
o	blade opening - turbine blading.

(d) Blade angles (Fig. 3)

All angles are measured from the axial direction.

β	blade angle = angle measured from the axial direction to the tangent to the camber line at the leading or trailing edge
β_1	blade inlet angle
β_2	blade outlet angle

$$\beta_1 = -\zeta + \chi_1$$

$$\beta_2 = -\zeta - \chi_2$$

$$\theta = \chi_1 + \chi_2 = \beta_1 - \beta_2$$

(e) Fluid flow angles (Fig. 4)

α	fluid flow angle = angle measured from the axial direction to the flow direction
α_1	inlet flow angle
α_2	outlet flow angle
i	incidence = angle measured from the blade inlet direction to the inlet flow direction $= \alpha_1 - \beta_1$ (Note:- this differs from isolated aerofoil practice)
δ	deviation (of flow from blade outlet direction) = angle measured from blade outlet direction to flow direction $= \alpha_2 - \beta_2$
ε	deflection $= \alpha_1 - \alpha_2$ (always positive) $= \theta + i - \delta$

(f) Fluid velocities (Fig. 4)

Sign convention - Fluid velocities resolved normal to the cascade, i.e. in the axial direction, are considered positive in the direction of flow through the cascade. The sign of velocities resolved along the cascade direction follows from the angle convention.

V	fluid velocity
V_a	axial velocity = component resolved normal to the cascade
V_w	whirl velocity = component resolved parallel to the cascade

Suffixes 1 and 2 are used on the above to denote inlet and outlet values respectively.

Then:-

$$V_{w_1} = V_{a_1} \tan \alpha_1$$

$$V_{w_2} = V_{a_2} \tan \alpha_2$$

(g) Vector mean values (Fig. 4)

V_{a_m}	mean of inlet and outlet axial velocities
V_m	vector mean of fluid inlet and outlet velocities
α_m	vector mean of fluid inlet and outlet flow angles

From the velocity triangles in Fig. 4:-

$$V_{a_m} = \tfrac{1}{2}\left(V_{a_1} + V_{a_2}\right)$$

$$V_m = V_{a_m} \sec \alpha_m$$

$$\tan \alpha_m = \tfrac{1}{2}\left(\frac{V_{a_1}}{V_{a_m}} \tan \alpha_1 + \frac{V_{a_2}}{V_{a_m}} \tan \alpha_2\right)$$

$$= \tfrac{1}{2}\left(\tan \alpha_1 + \tan \alpha_2\right) \text{ if } V_{a_1} = V_{a_2}$$

3.5 The cascade in two-dimensional flow

(a) Flow losses

ω	loss of total head pressure
$\bar{\omega}$	mean loss of total head pressure through a cascade

Loss coefficients are frequently used, and are obtained by expressing the loss as a fraction of a velocity head, usually of the highest flow velocity. For convenience in dealing with compressible flow the "dynamic pressure", i.e. the difference between total and static pressures, is normally employed.

$\bar{\omega}/\frac{1}{2}\rho V_1^2$ and $\bar{\omega}/(P_{tot_1} - P_1)$ - on inlet conditions (common form for compressor cascades)

$\bar{\omega}/\frac{1}{2}\rho V_2^2$ and $\bar{\omega}/(P_{tot_2} - P_2)$ - on outlet conditions (common form for turbine cascades)

$\bar{\omega}/(P_{tot_1} - P_2)$ has also been used for turbine cascades. Here the dynamic pressure corresponds to the theoretical outlet velocity, with no losses.

(b) Continuity in cascade testing

For two-dimensional flow, the continuity relation gives:-

$$\rho_1 V_{a_1} = \rho_2 V_{a_2} = \rho_1 V_1 \cos \alpha_1 = \rho_2 V_2 \cos \alpha_2$$

As a measure of the approach to two-dimensional flow in a cascade tunnel, the following coefficient is used:-

ξ contraction coefficient

$$= \frac{\rho_1 V_{a_1}}{\rho_2 V_{a_2}} = \frac{\rho_1 V_1 \cos \alpha_1}{\rho_2 V_2 \cos \alpha_2}$$

(c) Change in pressure through the cascade

ΔP	change in pressure $= P_2 - P_1$ $= \frac{1}{2}\rho V_a^2(\tan^2\alpha_1 - \tan^2\alpha_2) - \bar{\omega}$ (Incompressible flow) (Normally positive for compressor cascades and negative for turbine cascades)
ΔP_{th}	theoretical change in pressure (no losses) $= \frac{1}{2}\rho V_a^2(\tan^2\alpha_1 - \tan^2\alpha_2)$ (Incompressible flow)

Pressure-rise coefficients are obtained by expressing the pressure rise as a fraction of the velocity head, as with the losses.

(d) Forces on the blade in cascade (Fig. 5)

The force exerted by the fluid on the blade is given by the rate of inflow of momentum over the control surfaces shown in Fig. 5. Resolving normal and parallel to the cascade, for unit blade height, gives:-

Axial force on blade

$= -s.\Delta P + W(V_{a1} - V_{a2})$
$= -s.\Delta P$ (Incompressible flow)
(Normally negative for compressor cascades and positive for turbine cascades)

Tangential force on blade

$= W(V_{w1} - V_{w2})$
$= s\rho V_a^2(\tan\alpha_1 - \tan\alpha_2)$ (Incompressible flow)
(Normally positive for both compressor and turbine cascades)

(e) Lift and drag (incompressible flow)

The forces resolved normal and parallel to the vector mean fluid velocity are known as the "lift" and "drag" respectively.

L	lift (per unit length of blade) = component of resultant force on blade resolved normal to the vector mean fluid velocity (positive in direction of camber)
D	drag (per unit length of blade) = component of resultant force on blade resolved parallel to the vector mean fluid velocity (positive in direction from leading edge to trailing edge)

(f) Coefficients of lift and drag

Non-dimensional lift and drag coefficients are obtained by expressing lift and drag in terms of the vector mean velocity head and the blade chord.

C_L	lift coefficient (on vector mean velocity) $= L/\frac{1}{2}\rho V_m^2 . c$ $= 2\frac{s}{c}(\tan\alpha_1 - \tan\alpha_2)\cos\alpha_m - C_D\tan\alpha_m$

C_D	drag coefficient (on vector mean velocity) $= D/\frac{1}{2}\rho V_m{}^2 . c$ $= \frac{s}{c} . \frac{\overline{\omega}}{\frac{1}{2}\rho V_m{}^2} . \cos \alpha_m$ $= \frac{s}{c} . \frac{\overline{\omega}}{\frac{1}{2}\rho V_1{}^2} . \frac{\cos^3 \alpha_m}{\cos^2 \alpha_1}$ (Common form for compressor cascades) $= \frac{s}{c} . \frac{\overline{\omega}}{\frac{1}{2}\rho V_2{}^2} . \frac{\cos^3 \alpha_m}{\cos^2 \alpha_2}$ (Common form for turbine cascades)

Special lift and drag coefficients are obtained by the use of the velocity head based on the outlet velocity instead of the vector mean. Their use is of great advantage in the analysis of theoretical cascade performance, in conjunction with blade pressure distributions also referred to outlet conditions. This practice gives zero relative pressure at the trailing edges of all aerofoils, whether isolated or in cascade (both compressor and turbine). The lift coefficient based on the outlet velocity is known, for convenience, as the "loading factor".

$\psi = C_L(V_2)$	= loading factor = lift coefficient based on outlet velocity $= L/\frac{1}{2}\rho V_2{}^2 . c \quad = \quad C_L . \frac{\cos^2 \alpha_2}{\cos^2 \alpha_m}$
$C_D(V_2)$	drag coefficient based on outlet velocity $= D/\frac{1}{2}\rho V_2{}^2 . c \quad = \quad C_D . \frac{\cos^2 \alpha_2}{\cos^2 \alpha_m}$

Since the drag is normally small compared with the lift, theoretical lift coefficients, for flow with no losses, are frequently used. They are distinguished by the suffix "th", where necessary.

$C_{L_{th}}$	theoretical lift coefficient (on vector mean velocity) $= 2 \frac{s}{c} (\tan \alpha_1 - \tan \alpha_2) \cos \alpha_m$

Frequent use is made of the ratio of lift to drag, which can also be expressed in terms of the lift and drag coefficients.

L/D	lift/drag ratio $= C_L/C_D = C_L(V_2)/C_D(V_2)$

(g) Stalling conditions

A cascade is said to be "stalled" when the deflection ceases to rise steadily with increasing incidence and the loss coefficient starts to increase rapidly. In the past, the stalling incidence was empirically defined as that corresponding to maximum deflection, cascade properties at this point being denoted by the suffix "m"; but where the point of maximum deflection was indefinite it was the practice to assume that stalling occurred when the total-head loss coefficient had risen to twice its minimum value. This latter definition has now been adopted, because of its more general application, and the corresponding cascade properties are denoted by the suffix "s".

i_s	stalling incidence = incidence at which the loss-coefficient has risen to twice its minimum value
ε_s	stalling deflection

(h) "Nominal" performance values

For design purposes it is inadvisable to use values close to the stalling conditions and "nominal" values are therefore defined as those corresponding to a deflection of 0.8 of the stalling deflection. These values are distinguished by an asterisk.

ε^*	nominal deflection $= 0.8\,\varepsilon_s$
δ^*	nominal deviation
m	empirical factor (for nominal deviation) $= \delta^* / \theta \sqrt{s/c}$

(i) "Optimum" performance values

For use in cascade performance analysis, "optimum" values are defined as those corresponding to maximum lift/drag ratio. These values are distinguished by the suffix "opt".

(j) Relation of cascade performance to the isolated aerofoil

In generalized cascade performance analysis, the isolated aerofoil is considered as a cascade of infinite pitch and values for this condition are denoted by the suffix "∞".

(k) Scale effects

R_n	Reynolds number (based on blade chord) $= \rho_1 V_1 c/\mu_1$ on inlet conditions (common form for compressor cascades) $= \rho_2 V_2 c/\mu_2$ on outlet conditions (common form for turbine cascades)

The critical Reynolds number range is defined by the relatively large increase in maximum deflection obtained with increasing Reynolds numbers in this range.

T.F. turbulence factor

Effective Reynolds number = Test Reynolds Number x T.F.
= value of Rn at which similar test data would be obtained in a turbulence-free tunnel.

(l) High-speed performance

M_n	Mach number $= V/V_c$ Note:- This normally refers to the inlet Mach number for compressor cascades. For turbine cascades the outlet Mach number is also used and the two can be distinguished by the suffixes 1 and 2 respectively.

For compressor cascades, an empirical critical Mach number is used as a measure of the onset of compressibility effects on performance. This was previously defined by the commencement of a fall-off in the pressure rise coefficient with increasing Mach number at a given incidence, but it is now related to a specified increase in loss in order to fix a more definite value. To avoid confusion of terminology, this empirical value is now known as the "drag critical Mach number".

Critical Mach Number - The stream Mach number at which sonic velocity is first reached locally at some point on the blade surface.

M_{n_c}	Drag Critical Mach Number - The stream Mach number at which the total-head loss coefficient has risen to 1.5 times its minimum value at the same incidence.
M_{n_m}	Maximum Mach Number (Compressor Cascades) - The stream Mach number corresponding to zero pressure rise across the cascade. Theoretical Maximum Mach Number - The stream Mach number corresponding to sonic mean velocity in the blade throat.

3.6 The cascade or blade row in three-dimensional flow

The losses, and other quantities, have been defined in section 3.5 as applied to two dimensional flow. These ideal conditions are never fully realized, even in cascade work, and in the actual machine stage it is necessary to take into account the losses due to skin friction at the annulus walls and secondary effects at the blade ends etc. In this case the basic symbols are used as applying to the total loss and suffixes are used to distinguish the component losses and derivatives.

For example:-

$\bar{\omega}_p$	mean profile loss of total head pressure, for two-dimensional flow (as in section 3.5)
C_{D_p}	profile drag coefficient (on vector mean velocity) for two dimensional flow (as defined in section 3.5)
C_{D_a}	annulus drag coefficient (on vector mean velocity) from the losses due to skin friction on the annulus walls.
C_{D_s}	secondary drag coefficient (on vector mean velocity). Used to include induced and other losses due to flow conditions at the blade ends, tip clearances, wake interference etc. which cannot, as yet, be separated.
C_D	total drag coefficient $= C_{D_p} + C_{D_a} + C_{D_s}$
λ	an empirical factor, used in connection with secondary losses. $= C_{D_s}/C_{L_{th}}^2$
η_p	profile efficiency = ratio of the actual pressure rise through the cascade, with two-dimensional flow, to the theoretical pressure rise with no losses (i.e. efficiency based on profile loss only).
η_b	blade efficiency = ratio of the actual pressure rise through the cascade to the theoretical pressure rise with no losses (i.e. efficiency based on total loss)

3.7 The compressor or turbine stage

(a) Definition

The stage comprises a rotor, or moving blade row with its associated stator, or fixed blade row, and in general it is considered as occurring in the middle of a multi-stage machine. Fig. 6 shows the velocity triangles and notation in the normal compressor form and the turbine stage is also shown to illustrate the application of the same notation and conventions.

(b) Sign conventions

In the normal compressor stage velocity triangles, all flow angles and velocities are considered positive as shown in Fig. 6. In effect, each blade row is considered separately as a cascade and its flow angles and velocities follow the normal cascade convention of signs, as previously defined. The inlet guide blades to a compressor are an exceptional case, as they form a positive stagger cascade, but the use of the positive sign for the absolute inlet angle to the first stage is in keeping with the general consideration of a stage as placed in the middle of a compressor.

The blade speed is also considered as positive in a compressor stage and does not follow the sign convention of the rotor row as a cascade. Apart from its obvious convenience, some justification for this convention can be found in the fact that the stage work is determined from the blade speed and the rate of change of absolute moment of momentum of the fluid. The latter is related to the stator row in sign and it is reasonable, therefore, to refer the blade speed to the same convention.

(c) Notation and velocity triangle relations

N	speed of revolution of rotor
U	blade velocity (at any given radius). (Positive in compressors and negative in turbines).

The fluid angles and velocities are distinguished by suffixes, as follows:-

o	upstream stator outlet (absolute)
1	rotor inlet (relative)
2	rotor outlet (relative)
3	downstream stator inlet (absolute)
4	downstream stator outlet (absolute)

Then from the velocity triangles

$$U_1/V_{a_1} = \tan \alpha_o + \tan \alpha_1$$

$$U_2/V_{a_2} = \tan \alpha_2 + \tan \alpha_3$$

(d) The stage performance

ΔT_{tot_s}	stage total-head temperature rise = total-head temperature rise over rotor row.

The estimation of stage work or temperature rise is usually based on the cascade performance and velocity triangles at a given design diameter. Equating the increase in internal energy to the work <u>input</u>, obtained from momentum considerations and the velocity triangle relations, (the process being assumed adiabatic) gives:-

$$K_p \, \Delta T_{tot_s} \text{ (theoretical)} = U_2 \, V_{a_2} \tan \alpha_3 - U_1 \, V_{a_1} \tan \alpha_o$$

In practice the stage work or temperature rise in compressors is found to be less than this calculated value, and an empirical correlation factor is defined as:-

Ω work done factor

$$= \frac{\text{actual work or temperature rise}}{\text{calculated work or temperature rise}}$$

Then the actual stage temperature rise is given by:-

$$K_p \, \Delta T_{tot_s} = \Omega \left(U_2 \, V_{a_2} \tan \alpha_3 - U_1 \, V_{a_1} \tan \alpha_o \right)$$

(Positive for compressors and negative for turbines)

If $U_1 = U_2 = U$ and $V_{a_1} = V_{a_2} = V_a$, as is often the case,

$$K_p \, \Delta T_{tot_s} = \Omega \, U \, V_a \, (\tan \alpha_3 - \tan \alpha_o)$$

$$= \Omega \, U \, V_a \, (\tan \alpha_1 - \tan \alpha_2)$$

(Note:- For turbines no such correction factor is necessary, i.e. $\Omega = 1.0$)

The following coefficients are used:-
Flow coefficient = V_a/U
Pressure rise coefficient = $\Delta P_s / \frac{1}{2}\rho U^2$
Temperature rise coefficient = $K_p \, \Delta T_{tot_s} / \frac{1}{2} U^2$

3.8 Overall conditions for the compressor or turbine

In general the suffix "o" is applied to quantities relating to overall performance and the suffixes "I" and "II" are sometimes used to distinguish fluid conditions at inlet and outlet respectively.

n	number of stages
η_o	overall efficiency Normally adiabatic efficiency based on total-head conditions, but suffixes "ad" and "pol" are used to distinguish adiabatic and polytropic efficiencies where necessary.

4.0 Alphabetical and Numerical Lists of Symbols, Suffixes and Indices

4.1 English symbols

a		distance of point of maximum camber from leading edge.
b		maximum camber
c		blade chord (length)
d		diameter
	i/d	inner diameter of flow annulus
	m/d	mean diameter of flow annulus
	o/d	outer diameter of flow annulus
g		acceleration due to gravity
h		blade height
i		incidence
	i_s	stalling incidence
ℓ		length (usually axial)
m		empirical factor for deviation $= \delta^*/\theta \sqrt{s/c}$
n		number of stages
o		blade opening (turbine blading)
r		radius
	r_1	radius of curvature of leading edge
	r_2	radius of curvature of trailing edge
s		pitch (or blade spacing)
t		maximum thickness of blade section
z		perimeter

A		area
	A_a	annulus area
	A_t	throat area - compressor blading, (normally per unit blade height, therefore used also as throat width)
C_f		skin friction coefficient
$C_D, C_{D(V_2)}$		drag coefficients based on vector-mean and outlet velocities respectively. Used without suffixes they normally refer to the total drag. Suffixes a, p, s, refer to annulus, profile and secondary drags respectively.
$C_L, C_{L(V_2)}$		lift coefficients based on vector-mean and outlet velocities respectively. (Note:- $C_{L(V_2)}$ is also known as the "loading factor", ψ)
D		drag
J		mechanical equivalent of heat

K		gas constant
K_p		specific heat at constant pressure
K_v		specific heat at constant volume
L		lift
M_n		Mach number = V/V_c
	M_{n_c}	critical or drag critical Mach number
	M_{n_m}	maximum Mach number
N		speed of revolution
P		static pressure (absolute)
P_{tot}		total-head pressure (absolute)
	$\Delta P, \Delta P_{tot}$	pressure difference or change in pressure
R		pressure ratio
R_n		Reynolds number
T		static temperature (absolute)
T_{tot}		total-head temperature (absolute)
	$\Delta T, \Delta T_{tot}$	temperature difference or change in temperature
U		blade speed
V		velocity (fluid)
	V_a	axial velocity
	V_c	acoustic velocity
	V_m	vector-mean velocity
	V_w	whirl velocity
$\overline{V}$		mean velocity
W		mass flow

A.R.	aspect ratio
L.E.	leading edge
T.E.	trailing edge
T.F.	turbulence factor

4.2 Greek symbols

α		fluid flow angle (from axial)
	α_m	vector-mean fluid angle
	α^*	nominal fluid angle
β		blade angle (from axial)
γ		ratio of specific heats = K_p/K_v
δ		deviation
	δ^*	nominal deviation
ε		deflection
	ε^*	nominal deflection
	ε_s	stalling deflection

ζ		blade stagger angle
η		efficiency
	η_{ad}	adiabatic efficiency
	η_{pol}	polytropic efficiency
θ		blade camber angle
λ		empirical factor for secondary drag $= C_{D_S}/C_L{}^2{}_{th}$
μ		viscosity
ν		kinematic viscosity
ξ		contraction coefficient
ρ		density
σ		relative density
χ_1		blade camber inlet angle
χ_2		blade camber outlet angle
ψ		loading factor = lift coefficient based on outlet velocity, $C_L(V_2)$
ω		loss of total-head pressure
	$\bar{\omega}$	mean loss of total-head pressure
	$\bar{\omega}_p$	mean profile loss of total-head pressure
Ω		work done factor

4.3 Suffixes

The suffixes listed below are those which are considered as "separable", i.e. they are applied to a number of variables or serve to distinguish values (by place etc.) without modifying the basic meaning of the symbol.

4.3.1 Alphabetical

b	blade
o	overall
p	profile
s	stage
t	throat
opt	optimum (values at maximum L/D condition)
th	theoretical (no losses)
tot	total-head

4.3.2 Numerical

o	upstream stator blade outlet (in a stage)
1	blade inlet, or leading edge (in cascade) rotor blade inlet (in a stage)
2	blade outlet or trailing edge (in cascade) rotor blade outlet (in a stage)
3	downstream stator inlet (in a stage)
4	downstream stator outlet (in a stage)
∞	infinity (relating to cascade conditions at infinite pitch)
I	inlet conditions (overall)
II	outlet conditions (overall)

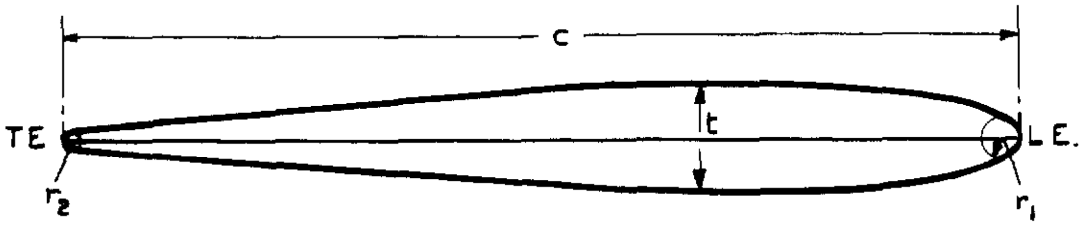
4.4 Indices

*	nominal values
'	values with isentropic flow.

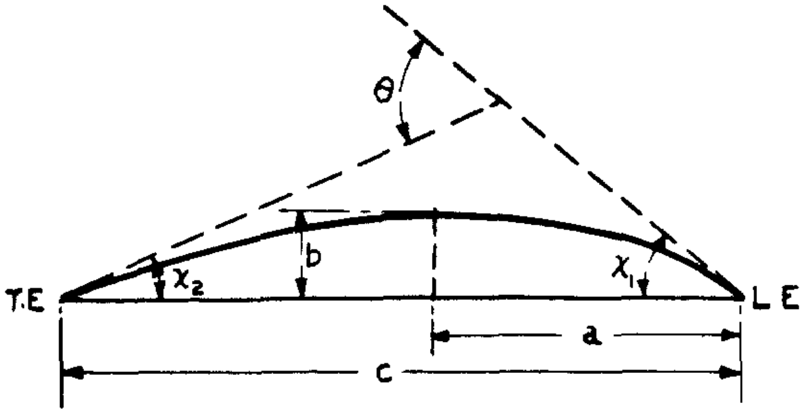
REFERENCES

No.	Author	
1	Howell, A.R.	"The present basis of axial flow compressor design. Part I - Cascade theory and performance". R. & M. 2095. June, 1942.
2	Howell, A.R.	"The present basis of axial flow compressor design. Part II - Compressor theory and performance". R.A.E. Report No. E. 3961 (1942)

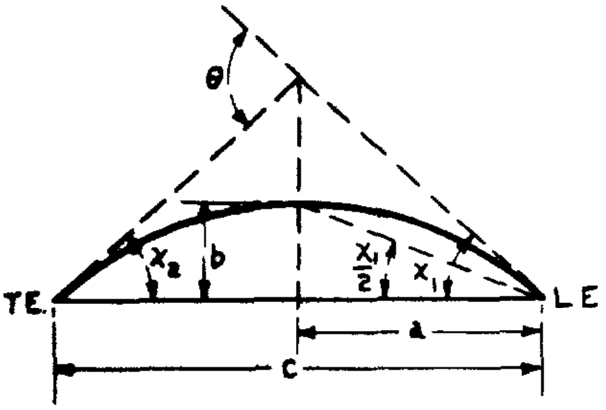
FIG.1.



THE BASE PROFILE



GENERAL CAMBER LINE DETAILS



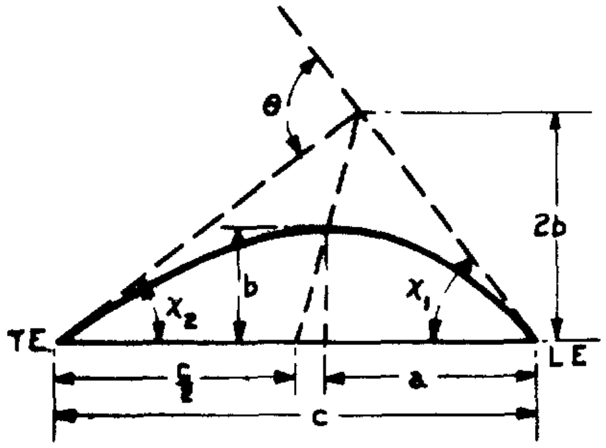
$a = c/2$

$\chi_1 = \chi_2 = \theta/2$

$\tan\frac{\chi_1}{2} = \tan\frac{\chi_2}{2} = \frac{b}{a} = \frac{2b}{c}$

Radius of Curvature $= \frac{c}{2\sin\theta/2}$

CIRCULAR ARC CAMBER LINES



$$\frac{b}{c} = \frac{1}{4\tan\theta}\left(\sqrt{\left[1 + (4\tan\theta)^2\left\{\frac{a}{c} - \left(\frac{a}{c}\right)^2 - \frac{3}{16}\right\}\right]} - 1\right)$$

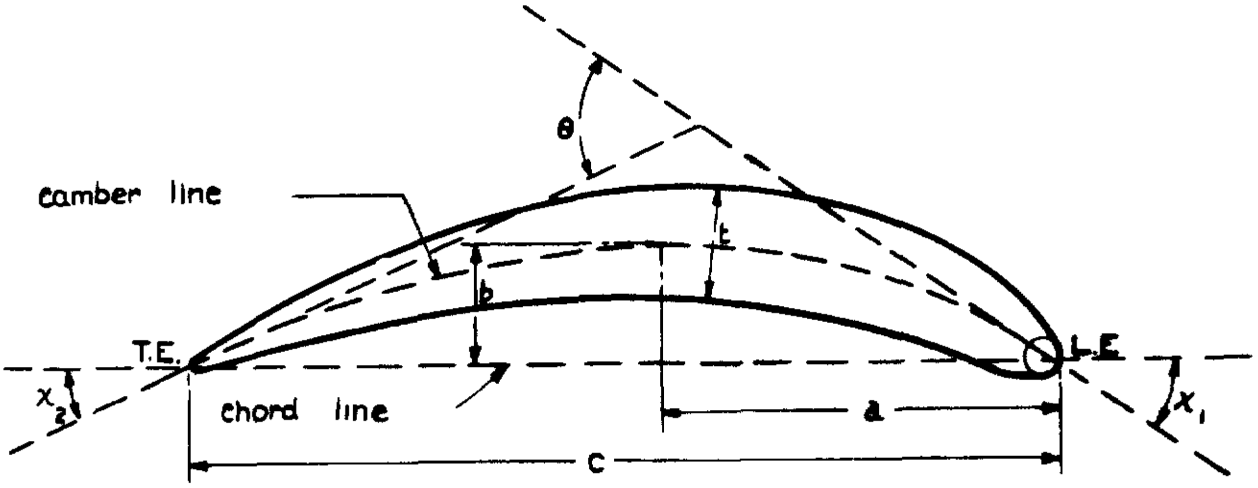
$$\tan\chi_1 = \frac{4b/c}{4a/c - 1}$$

$$\tan\chi_2 = \frac{4b/c}{3 - 4a/c}$$

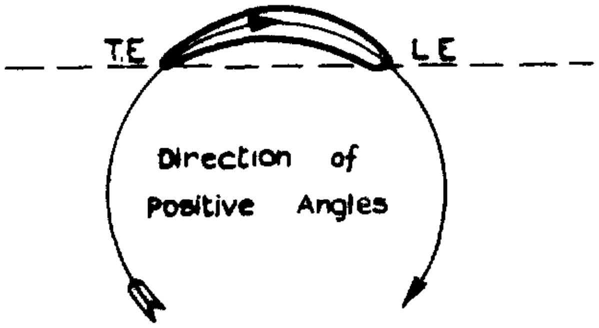
PARABOLIC ARC CAMBER LINES

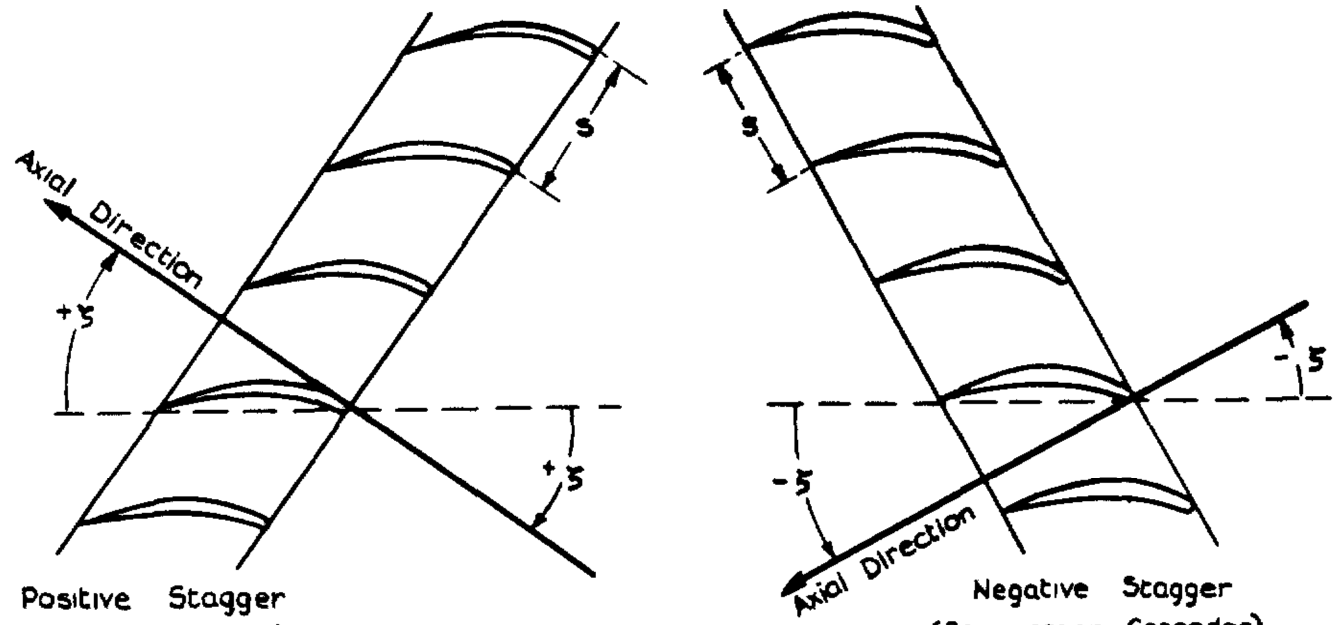
FLUID DYNAMIC NOTATION.

FIG. 2.



THE CAMBERED BLADE SECTION





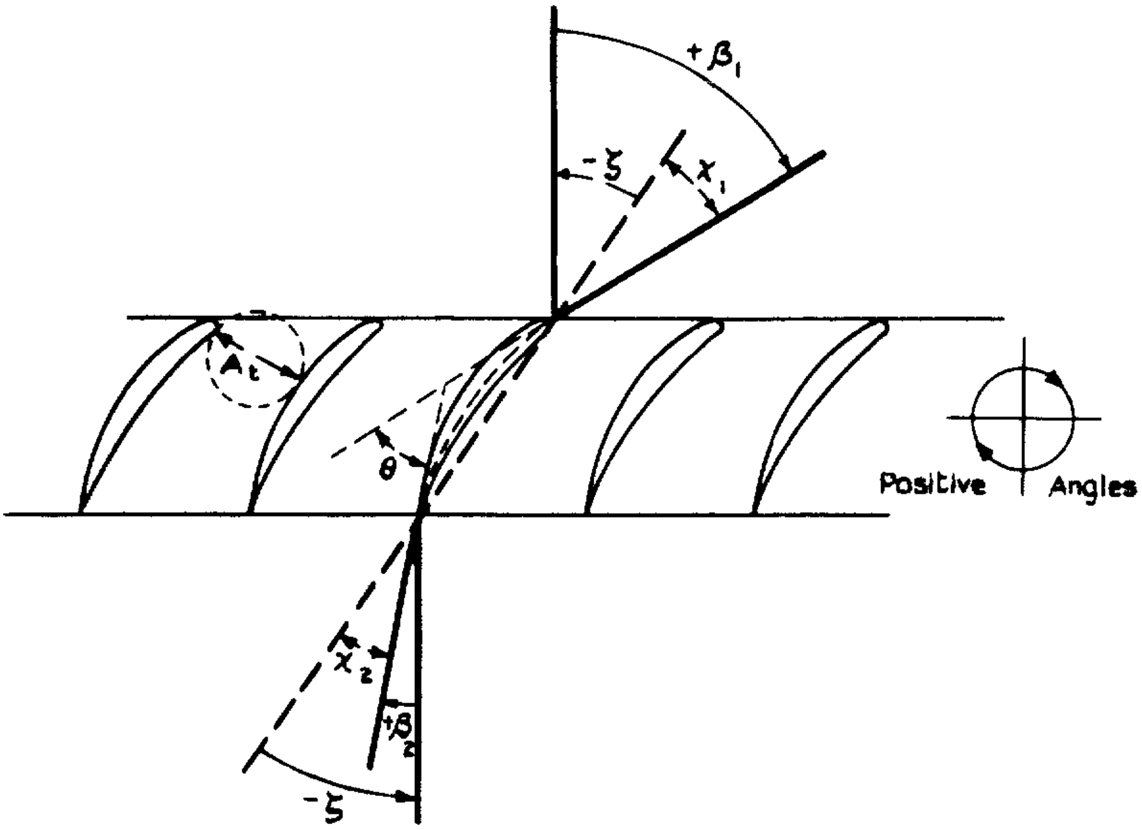
Positive Stagger
(Turbine Cascades)

Negative Stagger
(Compressor Cascades)

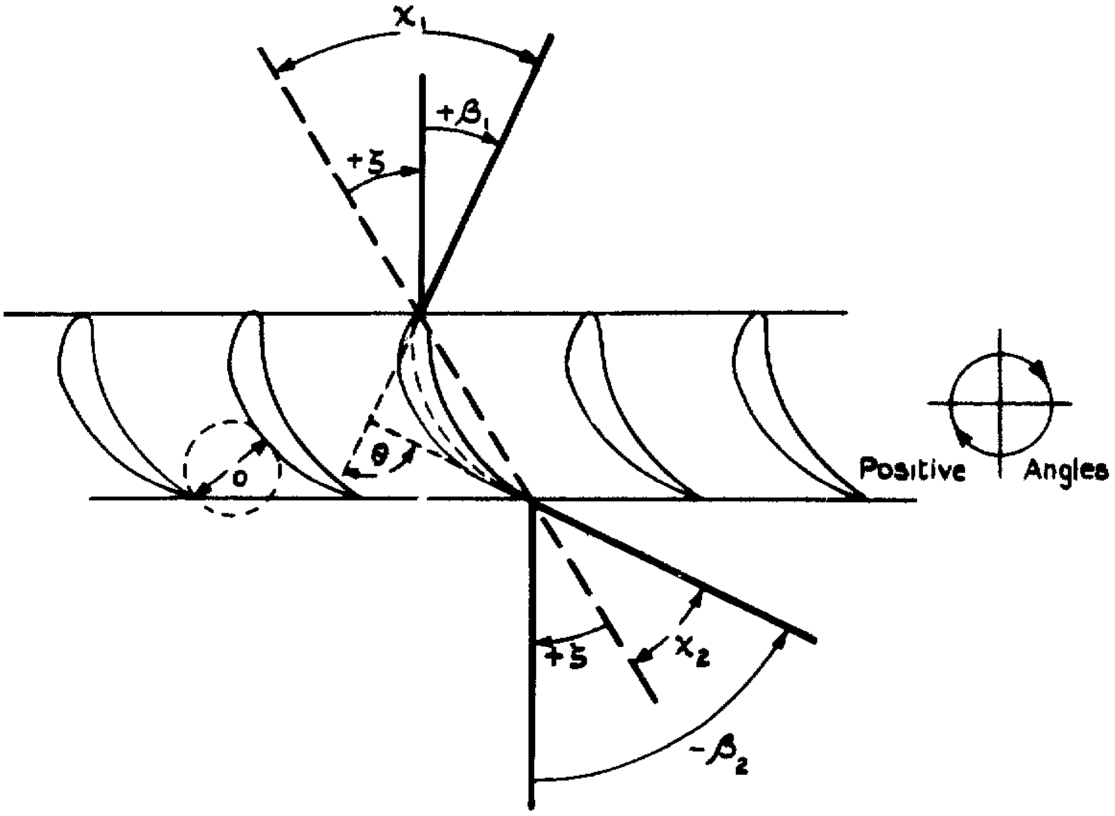
THE BLADE IN CASCADE

FLUID DYNAMIC NOTATION.

FIG. 3.



NEGATIVE STAGGER — COMPRESSOR BLADING

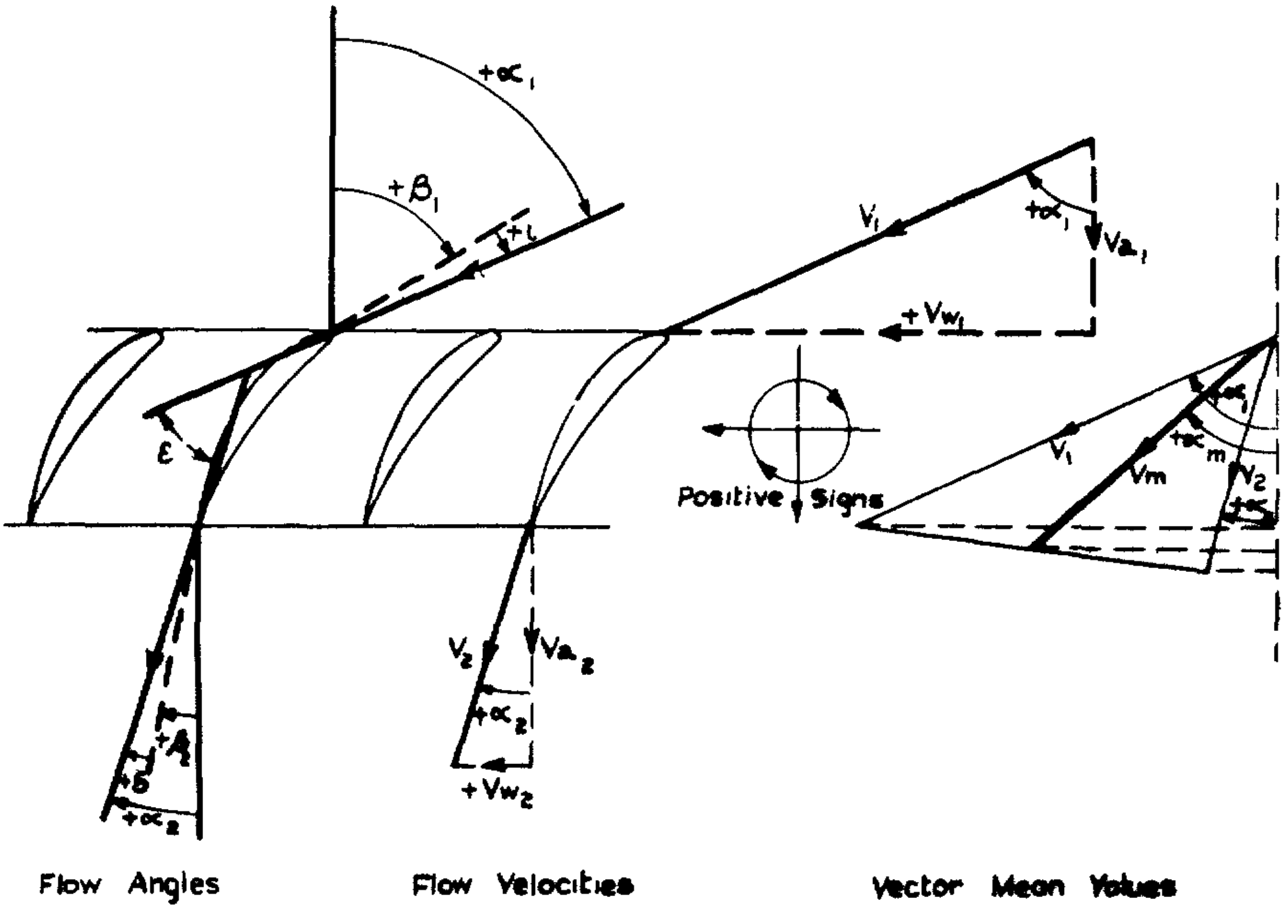


POSITIVE STAGGER — TURBINE BLADING

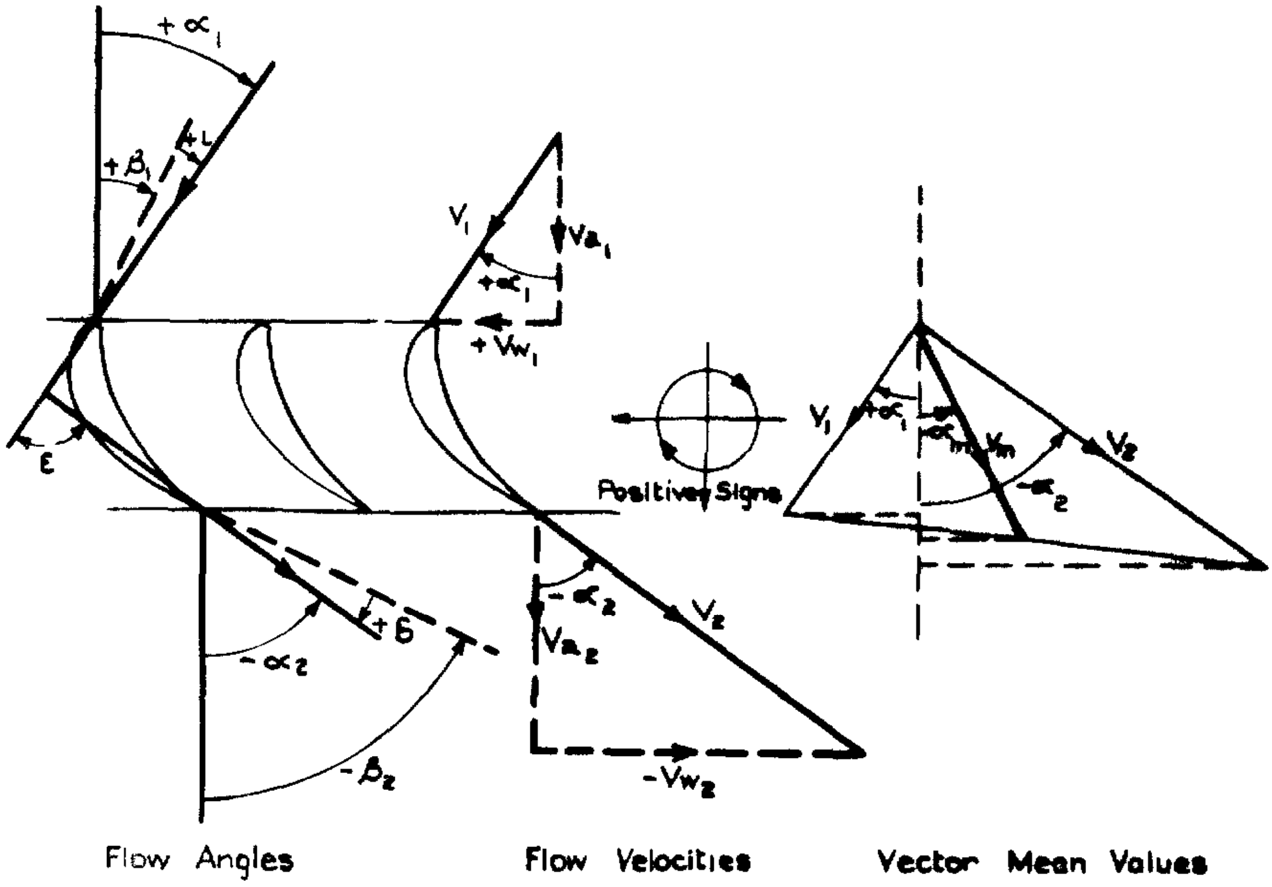
BLADE ANGLES IN CASCADES

FLUID DYNAMIC NOTATION.

FIG.4



NEGATIVE STAGGER — COMPRESSOR BLADING

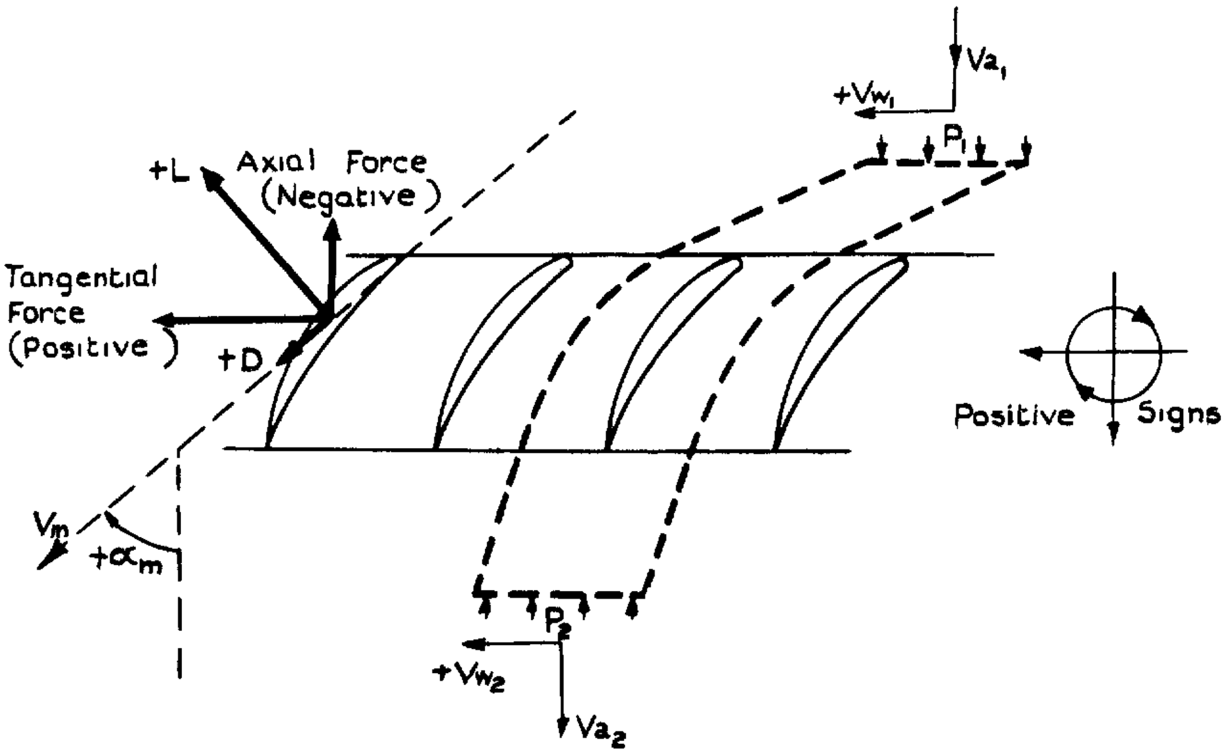


POSITIVE STAGGER — TURBINE BLADING

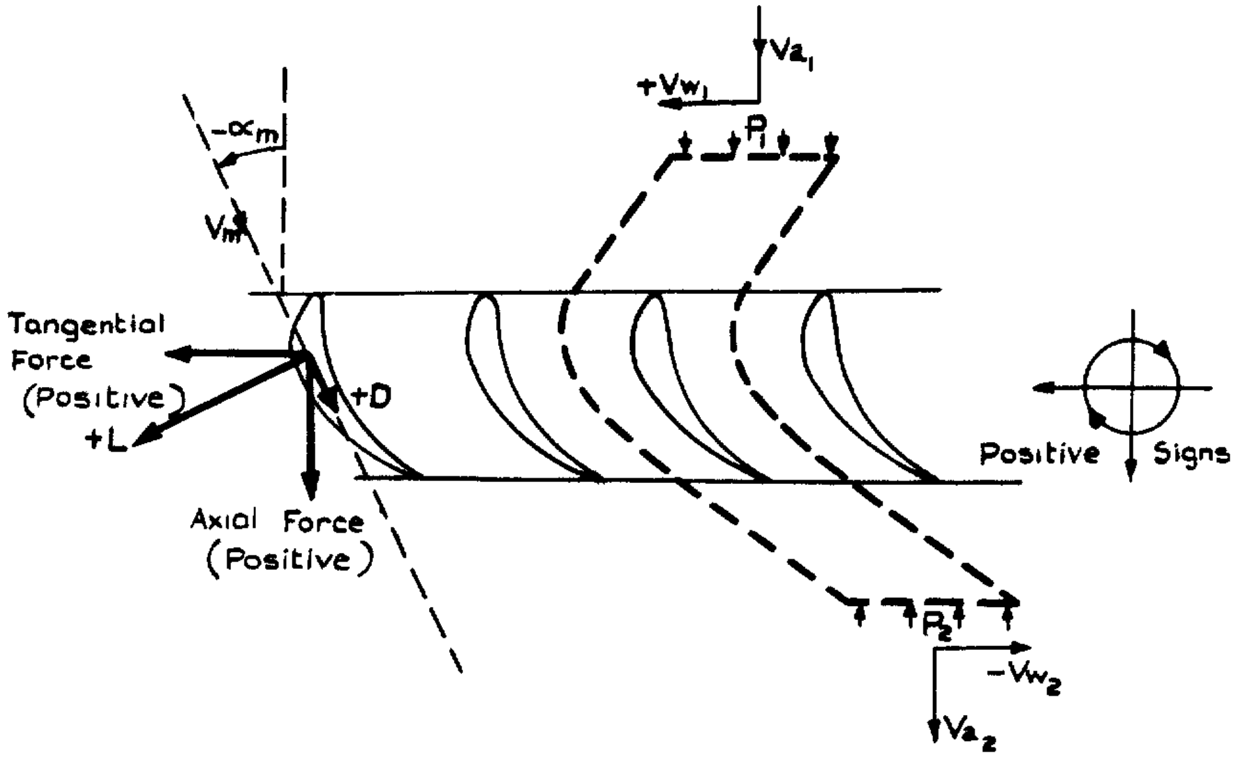
FLOW ANGLES AND VELOCITIES IN CASCADES

FLUID DYNAMIC NOTATION.

FIG. 5.



NEGATIVE STAGGER — COMPRESSOR BLADING

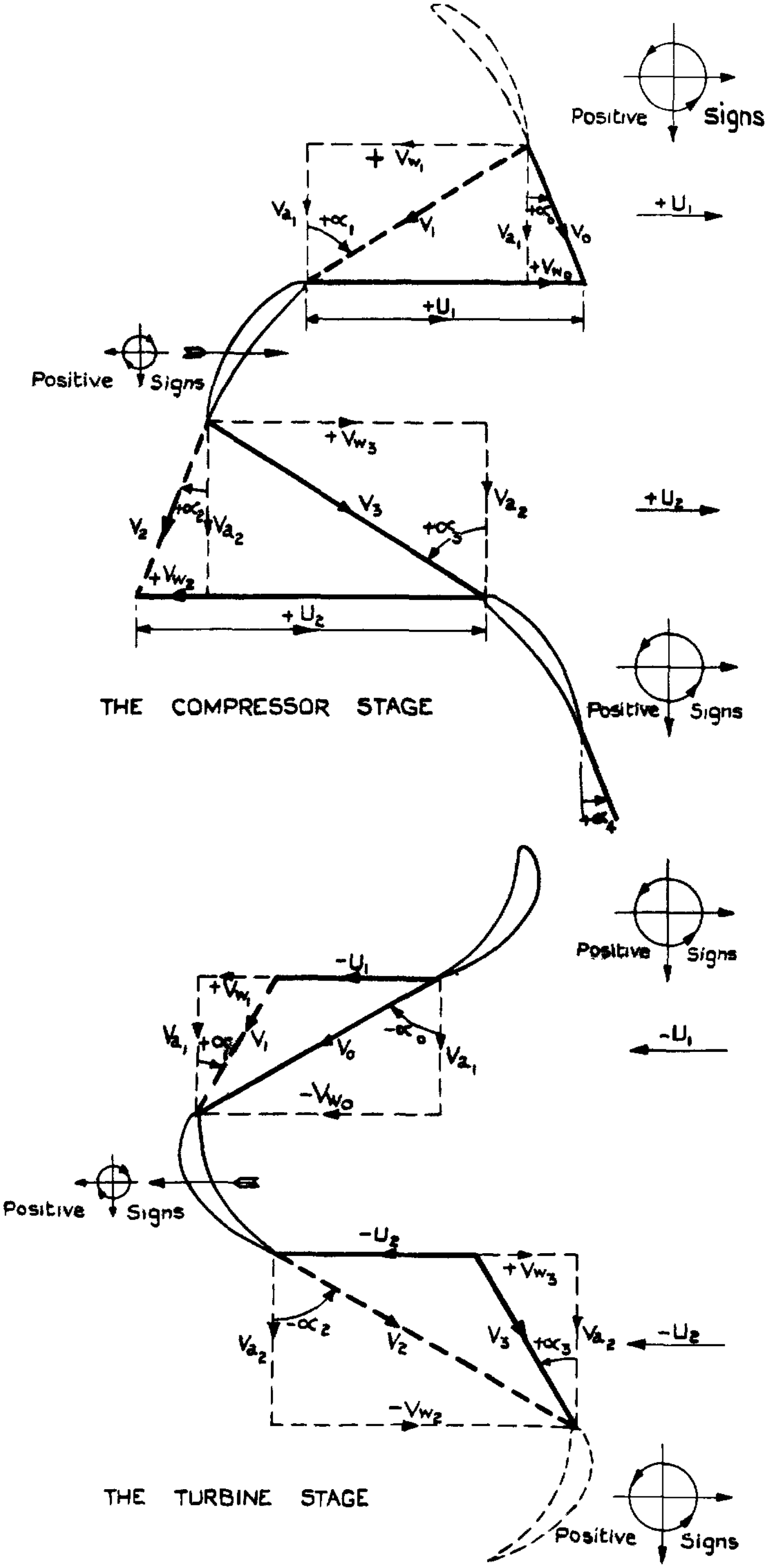


POSITIVE STAGGER — TURBINE BLADING

FORCES ON THE BLADE IN CASCADE

FLUID DYNAMIC NOTATION

FIG. 6.



THE COMPRESSOR STAGE

THE TURBINE STAGE

THE COMPRESSOR OR TURBINE STAGE

FLUID DYNAMIC NOTATION.

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C.P. No. 97
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