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by

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Estimation of Surface Pressures from Observed Shock-Wave Envelopes surrounding Conical Bodies at $M = 4.0$

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SUMMARY

An optical method utilising a standard schlieren system is used to locate the position of the shock wave surrounding a conical body at a free stream Mach number of 4.0. An analysis of the conditions at the shock wave enable the surface pressure distribution to be calculated. Good agreement with experimental pressure distributions is obtained except in regions dominated by viscous effects.

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*Replaces NPL Aero Report 1244 - A.R.C.29 349

1. Introduction

Experimental measurements of the location of the bow shock-wave surrounding a conical body permit the use of alternative methods for analysing the flow field which are complementary to the measurement of properties on the surface of the body. These detailed investigations of the flow about conical bodies furnish essential data for the study of more complex three-dimensional flows. A knowledge of the shock wave envelope is of interest since the overall forces on the forebody are closely associated with the geometry of the shock wave, particularly at high free stream Mach numbers. The proximity of the shock wave to a circular cross-section can be used as an indication of the extent of regions of circularly conical flow. Furthermore, the validity of shock layer theories as well as those that predict the shape of the envelope [1,2,3] can be verified by experimental measurements of both surface pressures and shock wave envelopes.

2. The Model and Test Conditions

The conical model used in the present tests has a cross-section consisting of an equilateral triangle, each of the three facets having a semi-vertex angle of 15° when projected onto a plane parallel to the longitudinal axis of the model. The dimensions and associated nomenclature are given in Fig. 1. The model is equipped with twelve surface pressure holes which were used to obtain the pressure distributions referred to in Section 4.

The tests were conducted in the NPL 7 in. x $4\frac{1}{2}$ in. (17.8 cm x 11.4 cm) blowdown wind tunnel at a stagnation pressure (p_0) of 75 p.s.i.a. (517 kN/m²), a stagnation temperature (T_0) of 280°K and a free stream Mach number of 4.0 .

3. Construction of the Shock-Wave Envelope

A standard schlieren system with the image plane set parallel to the flow axis is used to obtain photographs of the shock wave (Fig. 2). The image is a projection of a part of the shock wave that is tangential to the light beam passing through the test-section. Successive photographs are taken with the model rotated about the free stream axis and the complete shock wave envelope can then be constructed geometrically from the tangent planes. In Fig. 3 a construction for the model at zero incidence is shown in a plane perpendicular to the free stream axis; all cross-sections are alike since the shock wave is also conical. A section of the shock wave is sketched in Fig. 4 in which the radial distance from the flow axis to a position on the shock wave is given by

$$r(\phi_s) = k \tan \theta_s \quad ,$$

where $r(\phi_s)$ is the radial distance from the flow axis to a point on the shock wave determined by the meridional angle ϕ_s

θ_s is the angle between a generator in the shock surface and the free stream direction

and k is a scale factor.

The/

The tangent plane obtained from the schlieren photograph for the model rotated through an angle ϕ_m is tangent to the shock wave at a meridional angle ϕ_s. Thus,

$$\tan \theta_m = \tan \theta_s \cos (\phi_m - \phi_s) \qquad \ldots (1)$$

where θ_m is the inclination of the tangent plane to the free stream axis.

A section of the shock surface at a meridional angle ϕ_s can be considered as equivalent to a section of a plane oblique shock wave or of a conical shock wave with circular cross-section. The inclination of the section of shock wave is then θ_m, given by equation (1).

4. Calculation of Surface Pressures from the Shock Wave Envelope

At high Mach numbers Newtonian theory and other shock layer theories are commonly used. These theoretical treatments implicitly assume constant flow properties between the shock wave and the body surface and are local in their application; thus any change in the geometry of the model surface does not affect the flow properties at an adjacent unchanged section. For these theories to be applicable it is generally assumed that the shock wave should be close to the surface and the free stream Mach number relatively large, i.e. $M_\infty >> 1$ and $M_\infty \delta_n > 1$, where δ_n is the local inclination of the surface.

The question arises whether a shock layer theory can be applied to bodies similar to that of the present investigation at Mach numbers as low as 4. The value of $M_\infty \delta_n$ is approximately unity; however, the shock wave cannot be regarded as close to the surface (Figs. 3, 7 and 8). It has been demonstrated elsewhere [4,5] that the simple local theories such as tangent cone and Newtonian, are inadequate when applied to the surface of the body. Since the shock wave is formed by interactions of various parts of the flow field a local theory applied to the shock wave, rather than the body, can furnish information about the flow properties at the surface of the body providing a suitable technique can be established for deciding which part of the body is affected by a particular section of shock wave.

The straightforward analysis presented here assumes that

i) the shock wave is constructed from arcs of shock waves produced by circular cones at zero incidence, and

ii) a section of the shock surface affects a part of the body where it is intersected by the normal to the shock wave - with reference to Fig. 4, a point P on the shock wave affects point N on the body. At least close to the shock wave the velocity vector downstream of the shock wave is in a plane containing points P, N and the vertex of the body.

An explicit relationship between the shock angle, θ_m and the surface pressure does not exist for a cone at zero incidence, therefore an expression has been introduced which closely approximates to the relationship.

An/

An equation for the pressure coefficient, C_p, in terms of the free stream Mach number, M_∞, and the shock angle θ_m, can be obtained from the oblique shock relations as

$$C_p = \frac{4}{(\gamma + 1)} \left\{ \sin^2 \theta_m - \frac{1}{M_\infty^2} \right\}$$

An approximation for conical flow can be made from the two-dimensional flow if

$$C_p = k \left\{ \sin^2 \theta_m - \frac{1}{M_\infty^2} \right\} \qquad \ldots (2)$$

The value of k can be estimated from the conditions when the shock wave is perpendicular to the free stream direction $(\theta_m = \pi/2)$ in which case the pressure coefficient is given by

$$C_{p_t} = \frac{p_t - p_\infty}{\frac{1}{2} \gamma p_\infty M_\infty^2}$$

where p_t is the total pressure of the free stream flow measured down stream of the shock wave. It is therefore possible to find the value of k such that $C_p = C_{p_t}$ when $\theta_m = \pi/2$; thus

$$k = \frac{p_t - p_\infty}{\frac{1}{2} \gamma p_\infty M_\infty^2 \left[1 - \frac{1}{M_\infty^2}\right]} \qquad \ldots (3)$$

For $M_\infty = 4$, $k = 1.92$ and the surface pressure can be written

$$p/p_o = 0.142 \left\{\sin^2 \theta_m - 0.016\right\}, \qquad \ldots (4)$$

where p_o is the stagnation pressure measured in the settling chamber of the tunnel.

Equation (4) is plotted in Fig. 5 and a comparison with the conical flow solution demonstrates the acceptable accuracy of the explicit relationship.

The surface pressure distribution calculated from the shock wave envelope for the model at zero incidence is presented in Fig. 6; favourable agreement with the experimental results is achieved.

The shock wave envelope for the model at incidence can be obtained using a similar technique to that already described. A cranked adaptor, set at the required incidence, is attached and locked to the model; the complete assembly is then rotated in 15° increments about the axis of the sting mount which is set parallel to the free stream axis. The vertex of the model therefore describes a circular path about the longitudinal axis of the tunnel. The construction of the shock wave envelope from the schlieren photographs, although more tedious, can be accomplished using the same method as for the model at zero incidence.

Cross-sections/

Cross-sections of the envelopes in a plane perpendicular to the free stream direction are shown in Figs. 7 and 8 for the model at +8.8° incidence and -8.8° incidence. Comparison of the three envelopes (Figs. 3, 7 and 8) reveals that small distortions of the envelope occur with respect to the model axes but the complete envelope is displaced with the body as the incidence is changed. This effect is in accordance with the first order theoretical result [7] for circular cones at small angle of attack.

The pressure distributions calculated from the shock wave envelopes utilising equation (4) are shown in Figs. 9 and 10 for the model at +8.8° and -8.8° incidence. The calculated values show reasonable agreement with the experimental pressure distributions except for regions that are dominated by leading edge effects. The calculated values are radically in error on surfaces that support an expansion and subsequent separation at the leading edge. The regions affected by separated flow are marked in Figs. 9 and 10 and simple criteria are available [8] for identifying these regions. The reduction in pressure outboard of the stagnation line on a compression surface is reproduced by the method but the location of the stagnation line is further outboard than the calculated pressure distributions indicate. A large proportion of the differences must be attributed to small errors in the measurement of the shock wave angle and the geometric construction of the shock wave envelope since the surface pressure is very sensitive to small changes of the shock wave angle.

In high Mach number short-duration facilities (i.e., shock tunnels, etc.) it is often difficult to obtain surface pressure measurements. The optical technique used to obtain the location of the shock wave envelope and subsequent calculation of the surface pressures should therefore prove useful. For free stream Mach numbers in excess of 4 the shock wave approaches the surface of the body and the assumption that fluid properties are constant along meridional planes perpendicular to the shock wave becomes more realistic. The accuracy of the method should therefore increase for higher free stream Mach numbers. More experimental evidence is required to justify use of the technique at Mach numbers significantly lower than 4.0.

5. Conclusions

The optical technique described in the present note enables the shape of the shock wave surrounding a conical body to be obtained in a simple and quick fashion utilising a standard schlieren system.

Surface pressure distributions calculated from the shock wave envelope are in good agreement with experimental results for a triangular-section conical body at zero lift for a free stream Mach number of 4.0. At other incidences the predominance of viscous effects near the leading edges makes the method unsuitable although the calculated pressures on surfaces free from excessive viscous effects are in good agreement with experiment.

This technique should prove especially useful in conjunction with high Mach number short duration facilities by providing an estimate of surface pressure distributions.

Acknowledgement

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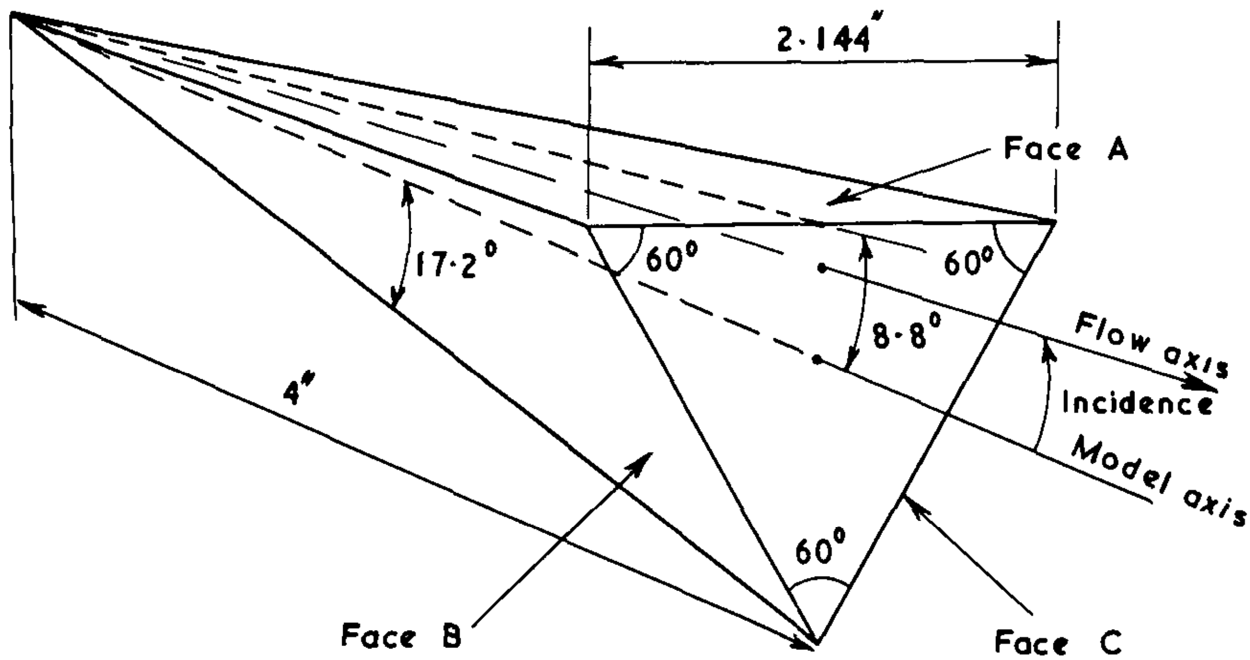
References

No.	Author(s)	Title, etc.
1	L. C. Squire	Applications of linearised characteristics to sharp-edged conical bodies. A.R.C.25 725, March, 1964.
2	Anthony Martellucci	An extension of the linearised characteristics method for calculating the supersonic flow around elliptic cones. J.Aerospace Sciences, Vol.27, No. 9, September, 1960.
3	Robert L. Chapkis	Hypersonic flow over an elliptic cone: theory and experiment. J.Aerospace Sciences, Vol.28, No. 11, November, 1961
4	L. C. Squire	Pressure distributions and flow patterns on some conical shapes with sharp edges and symmetrical cross-sections at M = 4.0. ARC R.& M.3340 June, 1962.
5	A. Akers	Some studies of pressure distributions on the windward surfaces of conical bodies at high supersonic speeds. A.R.C. C.P. No. 723. September, 1963.
6	Ames Research Staff	Equations, tables and charts for compressible flow. NACA Report 1135.
7	Joseph L. Sims	Tables for supersonic flow around right circular cones at small angle of attack. NASA SP-3007.
8	M. J. Larcombe	NPL work to be published.

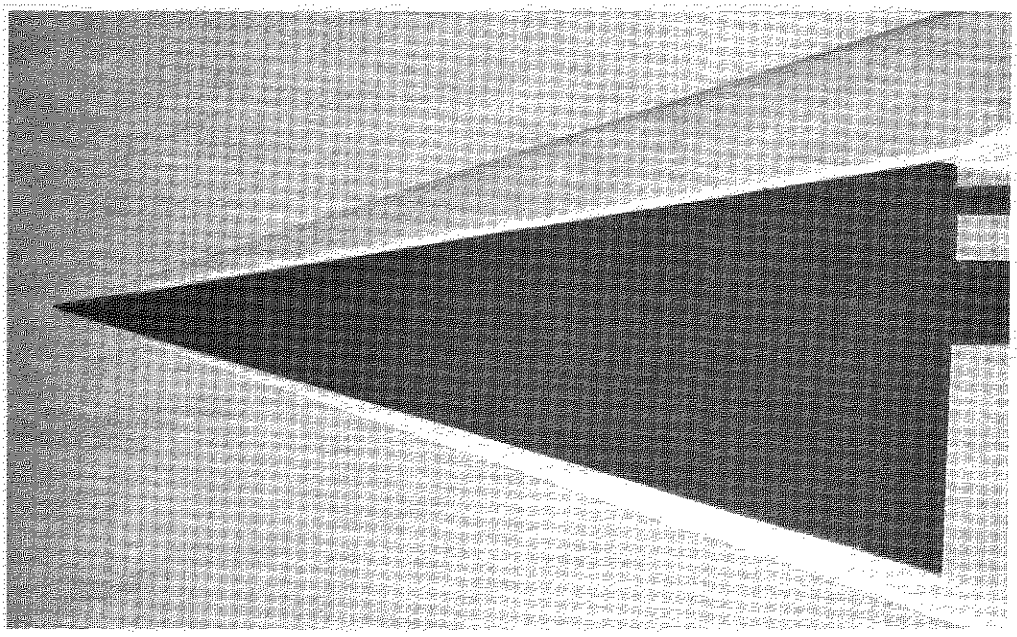
FIG 1



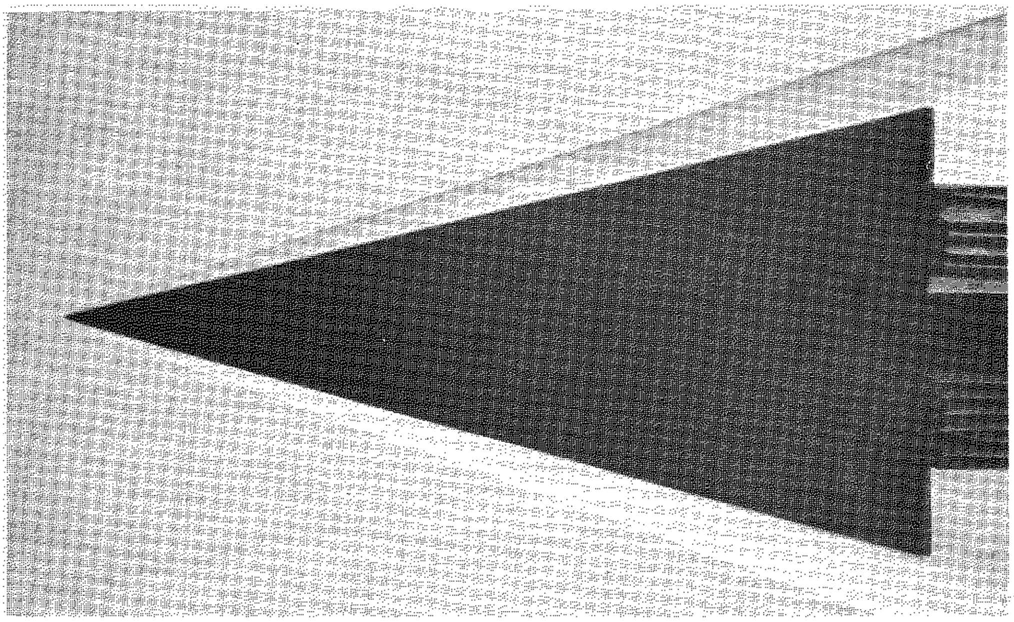
Details of model

FIG. 2

Roll angle, $\phi = 0^{0}$



Roll angle, $\phi = 90^{0}$

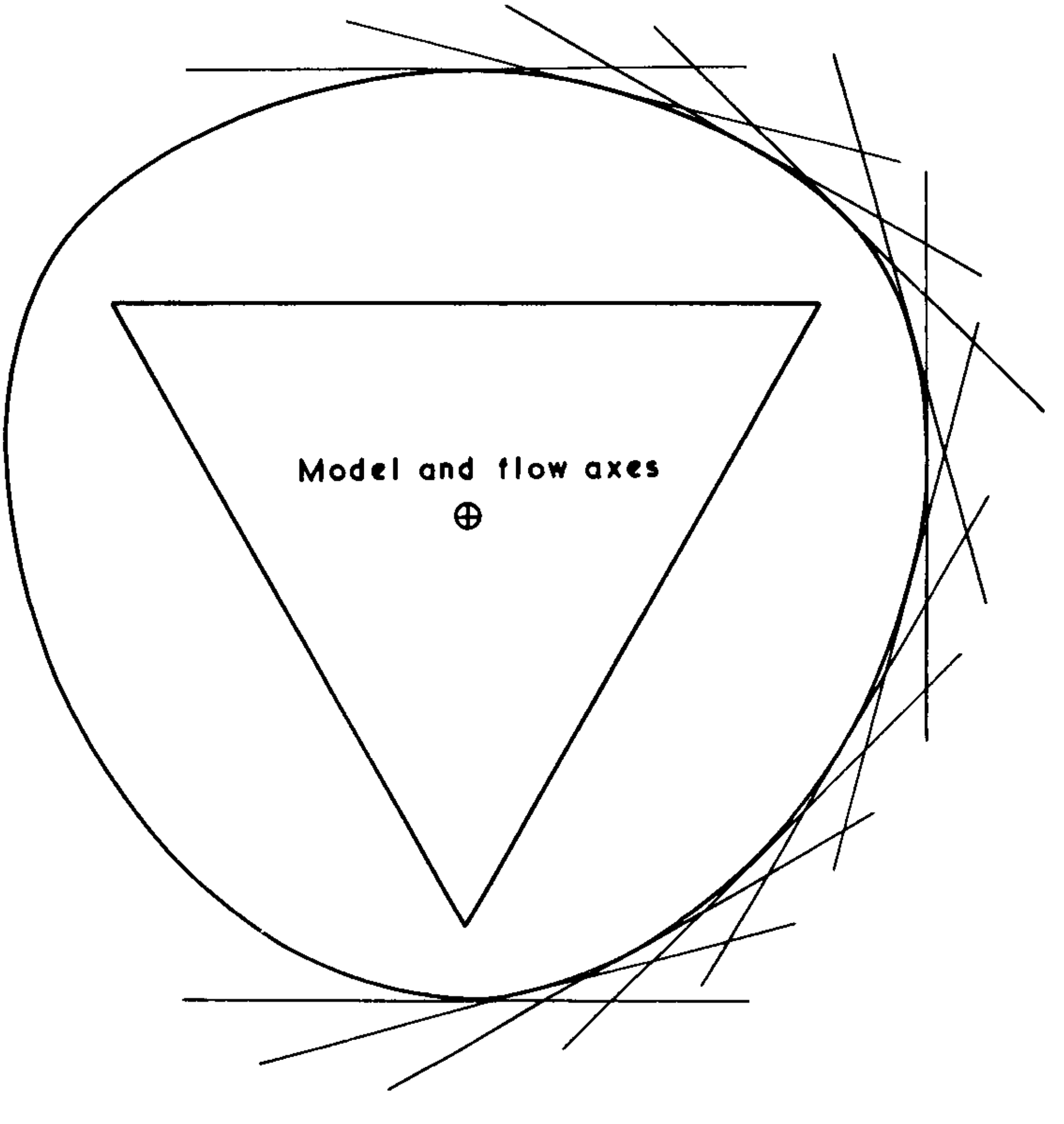


Schlieren photographs of model at zero incidence

FIG. 3

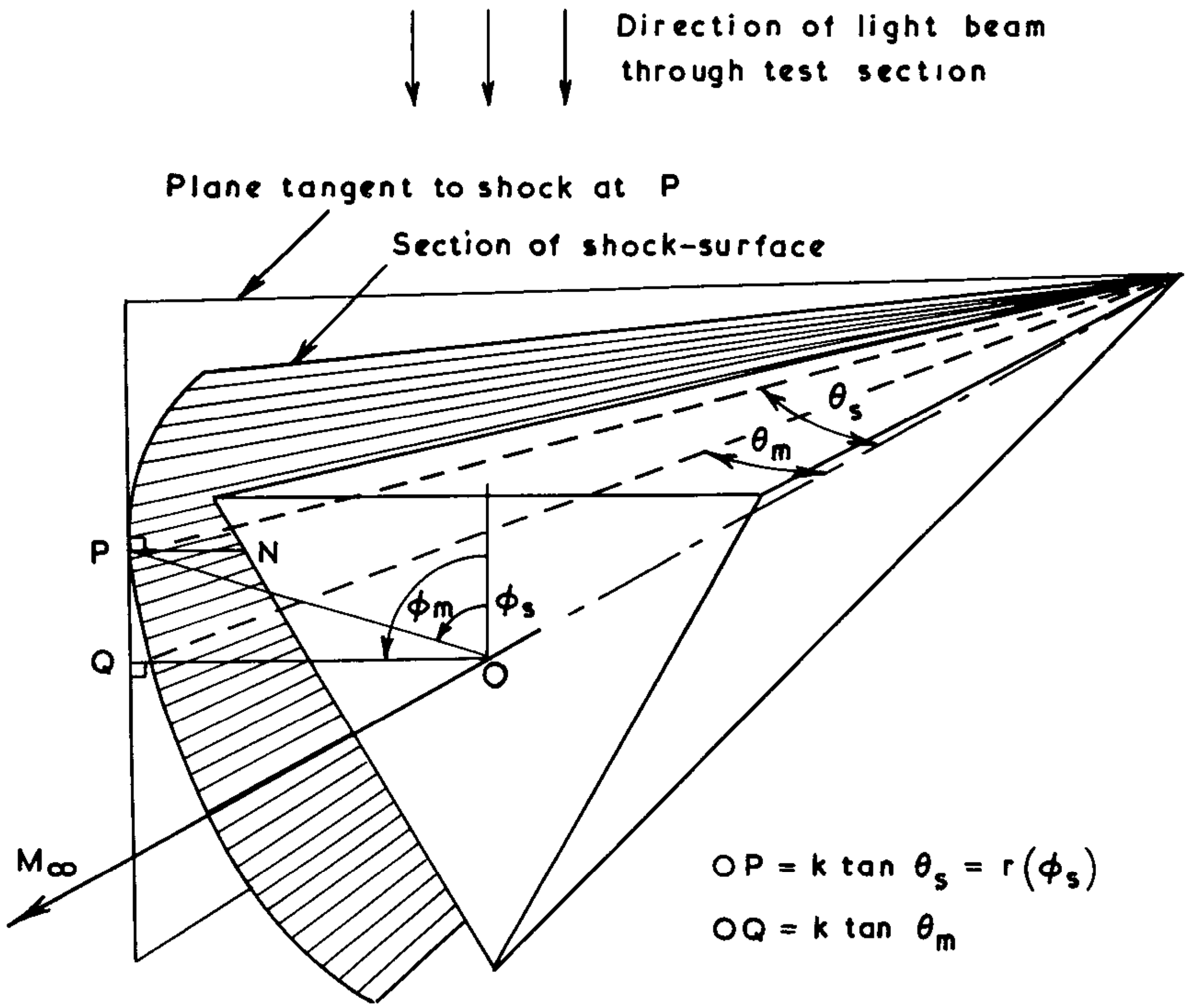
Aspect ratio = 1·05 $M_\infty = 4 \cdot 0$

0° incidence



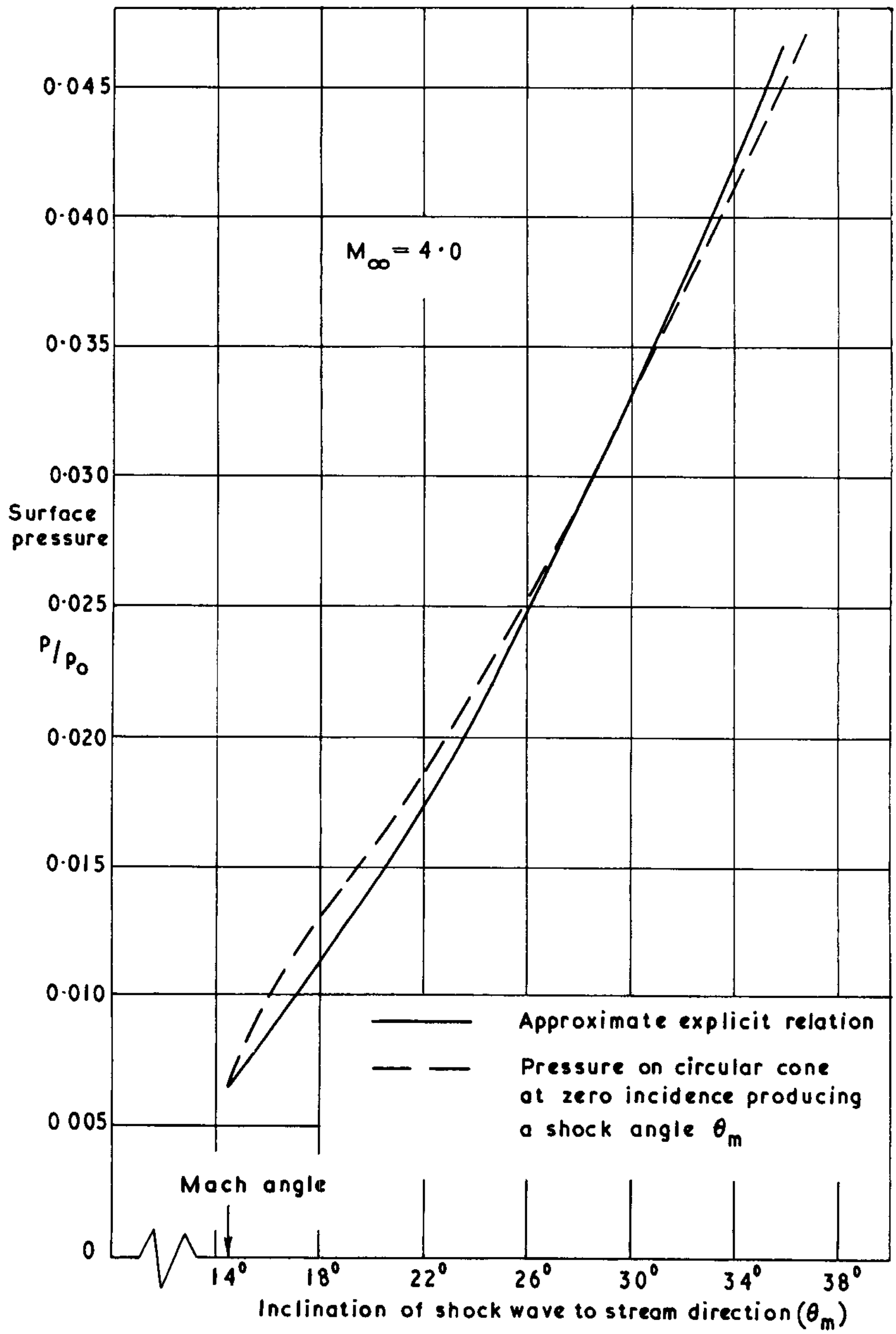
Construction of shock wave envelope from tangent planes projected onto a plane perpendicular to the free stream direction.

FIG. 4

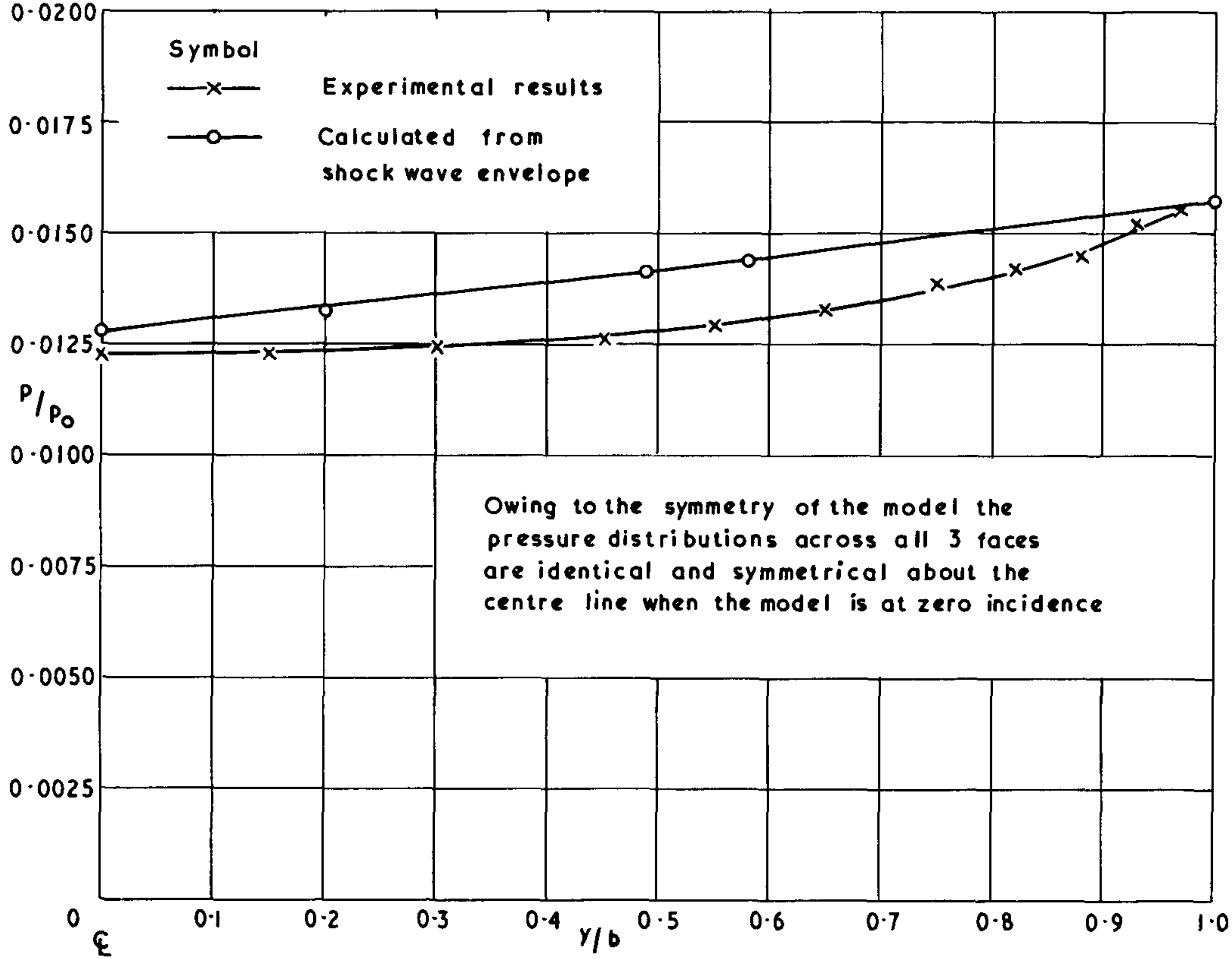


Construction for location of the tangent plane

FIG. 5



Comparison of exact conical flow solution with approximate explicit relation

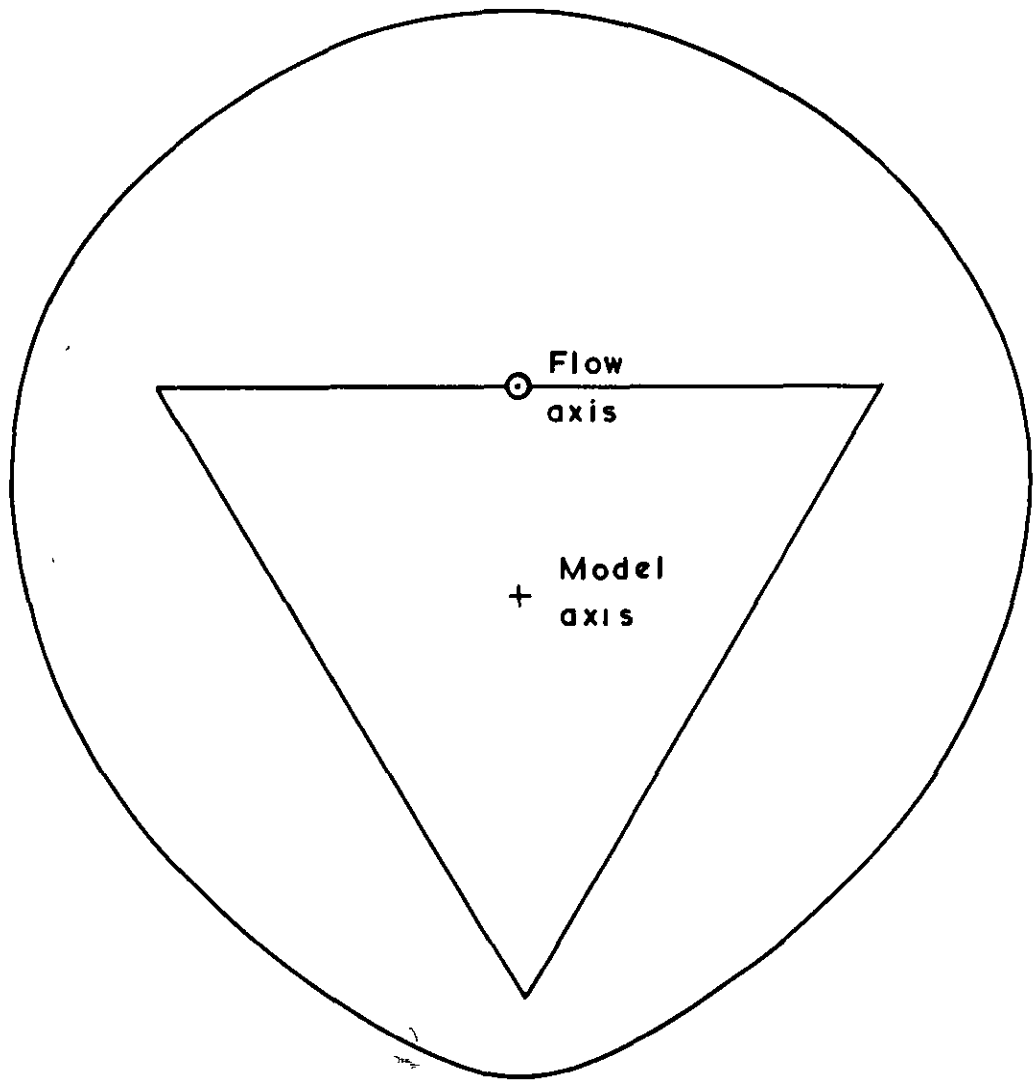


Comparison of experimental pressure distribution with that calculated from the shock wave envelope for the model at zero incidence

FIG. 6

FIG. 7

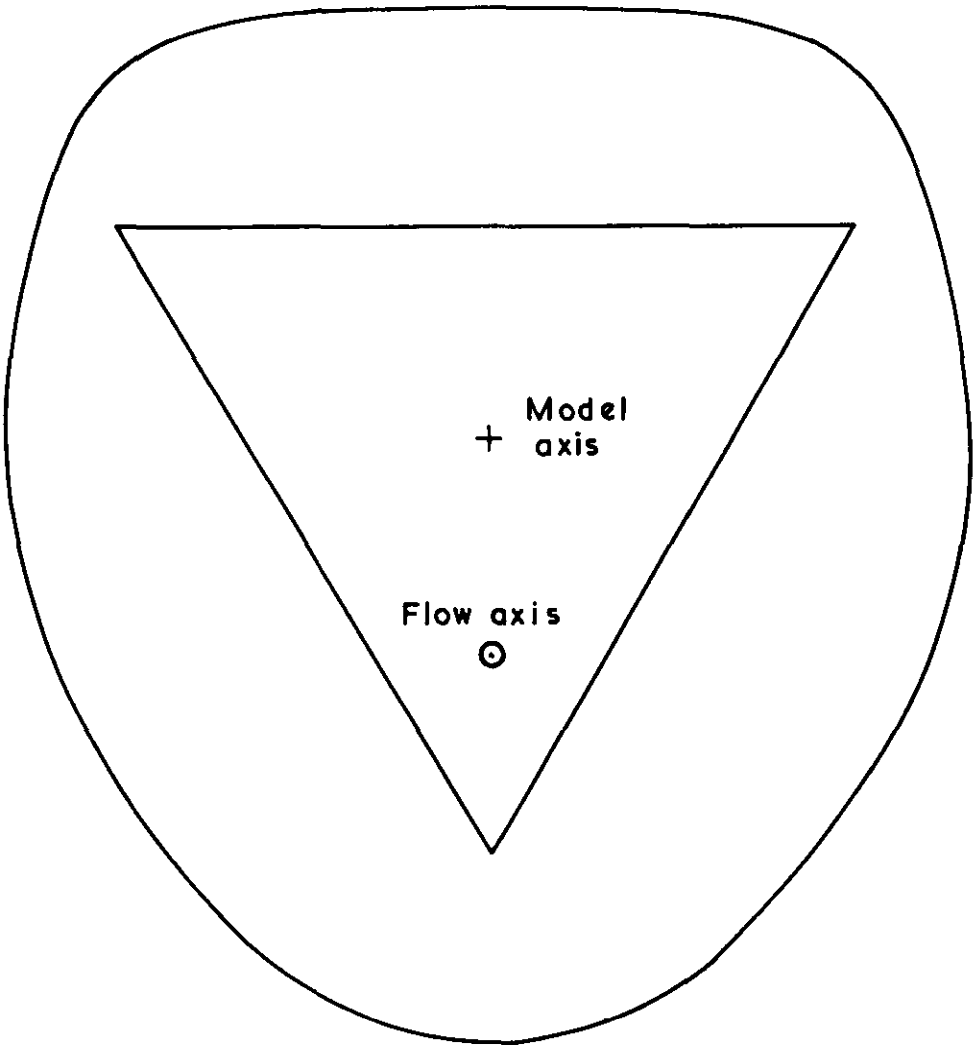
+ 8·8° incidence



Cross-section of shockwave in a plane perpendicular to the free stream direction.

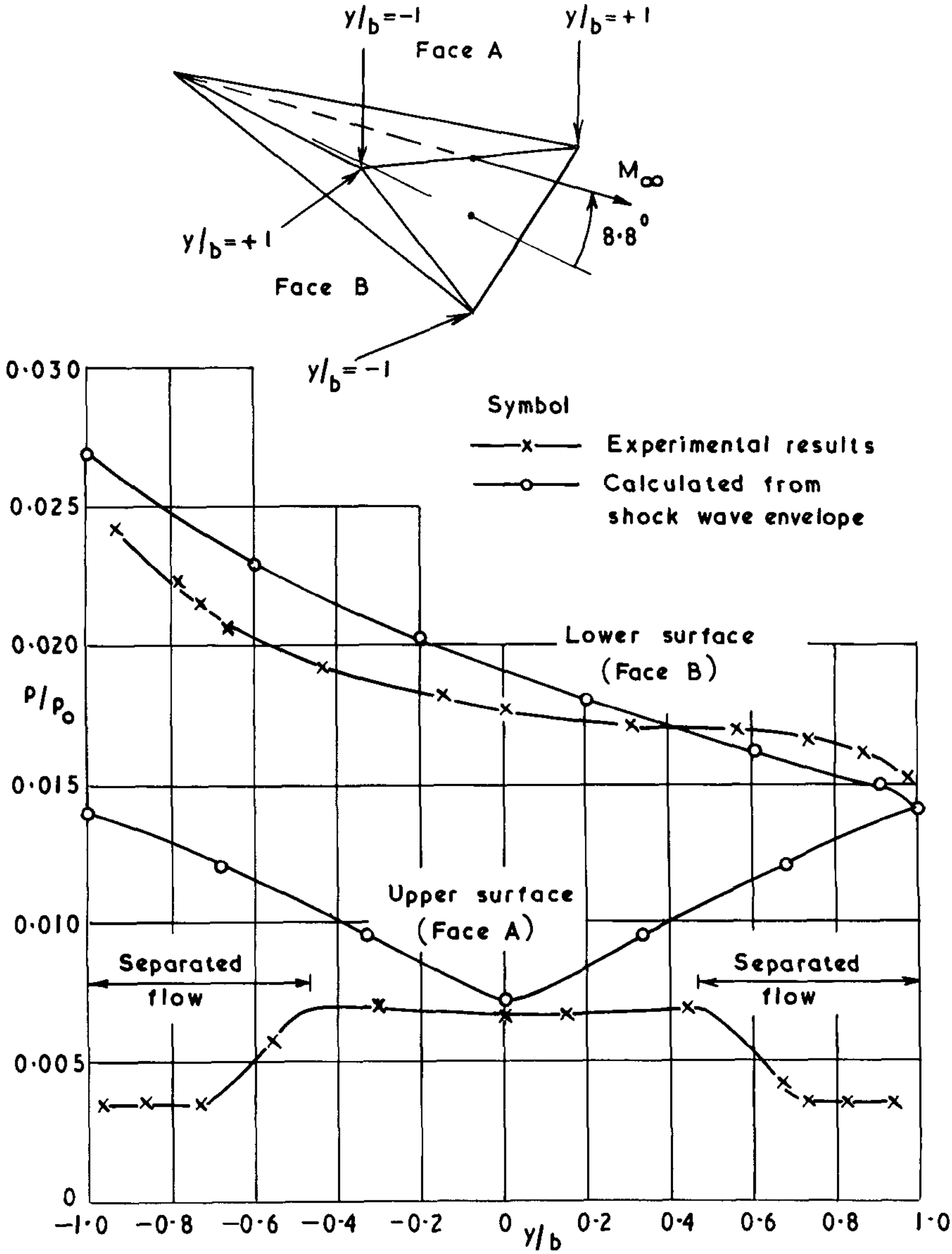
FIG. 8

$-8{\cdot}8^{0}$ incidence



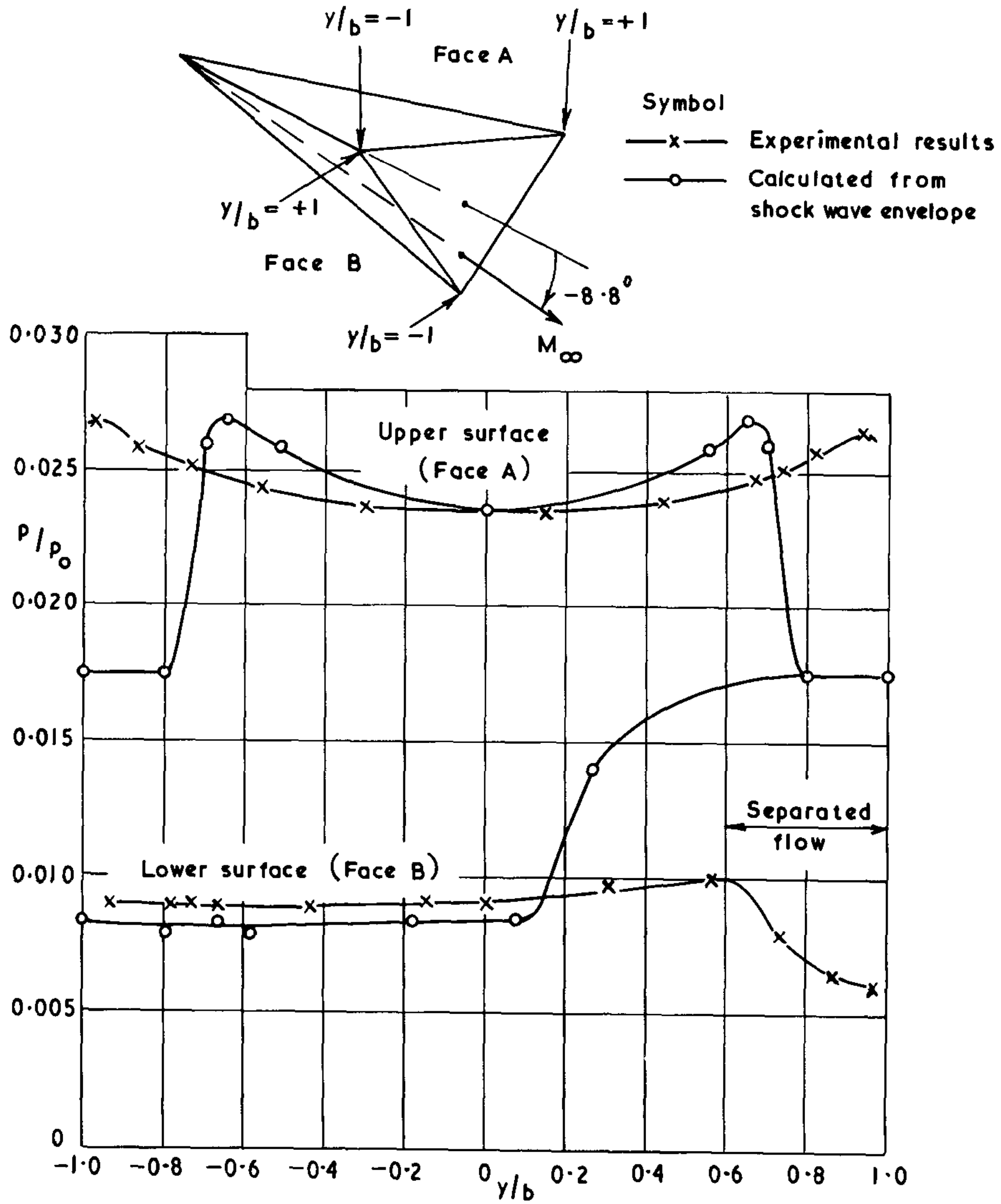
Cross-section of shock wave in a plane perpendicular to the free stream direction.

FIG 9



Pressure distributions calculated from shock wave envelope for model at $+8\cdot8^{0}$ incidence and $M_{\infty}=4\cdot0$

FIG 10



Pressure distributions calculated from shock wave envelope for model at $-8 \cdot 8^0$ incidence and $M_\infty = 4 \cdot 0$.

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ESTIMATION OF SURFACE PRESSURES FROM OBSERVED SHOCK-WAVE ENVELOPES SURROUNDING CONICAL BODIES AT M = 4.0*

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