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A Model for the Aquaplaning of Tyres on Wet Runways

by

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SUMMARY

An attempt is made to estimate pressure distribution, tyre deflection and lift on a fully aquaplaning smooth tyre assuming an idealized non-viscous flow in front of the tyre followed by Stokes flow under the tyre. The problem is reduced to solving two coupled partial differential equations, namely Rayleigh's lubrication equation for the water and the membrane equation for the tyre. It appears that viscosity is not important except for the no-slip condition it implies. The conclusion is reached that it will be essential to solve a three-dimensional free stream surface problem in order to make further progress.

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** Now at University of Bristol.

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1 INTRODUCTION

We consider an aeroplane travelling over a smooth runway on which there is a thin film of water. Through a combination of inviscid and viscous effects there is an upward force on the tyre which may indeed lift it completely off the ground leaving a thin film of water underneath. Even if it is not completely lifted the area of contact with the ground may be significantly reduced, part of the weight being taken by fluid pressure. Thus there may be a drastic reduction in the apparent coefficient of friction and indeed it may be practically zero. In addition the tyre is distorted and there may be pressure forces tending to oppose rotation and this may be sufficient to bring the wheel to rest even though it is not completely off the ground.

There has been some discussion as to whether viscous or non-viscous forces are the more important in this connection. Thus the standard criterion[1] normally in use assumes that aquaplaning begins when the stagnation pressure of the water becomes equal to the tyre pressure multiplied by a factor somewhat greater than unity. Neither viscosity nor the weight of the aeroplane nor the depth of the water is taken into account in this criterion. On the other hand there seems to be no doubt that immediately under the tyre the problem is of "squeeze film" type, more of a lubrication problem, where viscosity is all important[2].

The explanation one can give for the criterion mentioned above is roughly that when the stagnation pressure in front of the tyre reaches tyre pressure, the tyre gives and allows water to penetrate underneath so that there is now a larger area at stagnation pressure to lift the tyre off the ground. This is a great oversimplification since the tyre will begin to distort as soon as there is some outside pressure on it, long before this pressure has become equal to the tyre pressure.

An attempt has been made here to provide a model for what happens under drastic simplifying assumptions. The first of these is that the tyre and ground are smooth. This is indeed drastic though when the tyre is lifted completely off the ground, which is the only case we shall consider here, the effects of ordinary roughness may only be slight. Let us suppose that the aeroplane is at rest and the ground and water are moving under it. In this study we shall suppose that the wheel is actually off the ground and that it is either rotating fully or not at all. The water film will be supposed to be very thin and the flow laminar. For thick films vortices seem to appear under the wheel and the flow may be turbulent[3], but we shall not consider such cases. The next and possibly the crudest assumption that we shall make is that the tyre is completely flexible, rather like a balloon, and yet we shall suppose that the tread is flat along its width, so that there must be thicker material on the tread towards the sides than at the middle. Strictly one should take stiffness into account and this will be variable, being presumably greater towards the sides than in the middle owing to the thicker material there.

The fluid dynamic assumptions we shall make are also crude. If the wheel is off the ground and rotating at the speed it would have if it were in contact one might expect all the water in front to go under, (Fig.1) dragged in by the moving tyre and moving ground, and as it encounters reduced height it will flow out at the sides and the back; there will be no stagnation point and the pressure everywhere outside the tyre will be zero. (By "pressure" we shall always mean the excess over atmospheric pressure, just as we do when we talk of the pressure inside a tyre as being so many pounds per square inch.) This seems to be a problem in which purely viscous Stokes flow need only be taken into account, and it is quite straightforward to pose and solve under the assumptions made.

If the wheel is stationary the water is not dragged in by the wheel though it is still dragged along by the moving ground. In this case there must be a stagnation point on the tyre, not at the surface of the water, but at some distance underneath, (see Fig.2). In this case we divide the problem into two parts. Near the stagnation point we assume inviscid flow and stagnation pressure, dying away quadratically to zero towards the sides and the front. From the stagnation point region onwards we assume viscous Stokes flow with zero pressure at the sides and back. We find in these circumstances that the lift on the wheel is very much greater than when the wheel is rotating, since there is stagnation pressure in the front and viscosity does not allow the water to get away easily towards the sides and back and so relieve this pressure. However, in this case, though viscosity is important in that it leads to the no-slip condition its actual magnitude is not very important as far as pressure and lift are concerned, though it will affect the velocity of the water under the tyre.

The problem in either case reduces to the solution of two coupled elliptic equations. The first is an equation for the pressure in terms of the local height of the tyre above the ground. This is Rayleigh's lubrication or squeeze film equation. The second is an equation for the deflection of the tyre due to the pressure on it. This is a Poisson equation. The equations are solved simultaneously by a Gauss-Seidel iteration and from the solution the pressure forces, the lift force and the deflection of the tyre are calculated.

It would seem that the main problem as yet unsolved is to find a correct solution to the inviscid part of the calculation. The problem here is basically a three-dimensional free stream surface one which seems to be extremely difficult to tackle, especially when the boundary represented by the tyre is not fixed. It appears that only two-dimensional approaches to this problem have been made (e.g. Martin[4]). In Ref.4 the tyre shape was to some extent controlled by the complexities of the problem, in that a certain function had to be assumed and the tyre shape came out in the analysis. So the approach was to try different functions until a tyre shape turned out to be something which looked reasonable. When this was done the "lift" and the pressure distribution were reasonably close to those observed. Nevertheless one would have thought the fact that the flow is really three-dimensional would make an enormous difference, because the water really does escape from the sides, which it cannot do in any model which replaces the tyre by an infinite cylinder (see Fig.3).

We must admit, however, that the present study may not be near to reality either, since it relies on very much of a guess for the pressure distribution of the inviscid part and on perfectly smooth surfaces and a very thin film. It is put forward in an attempt to gain further understanding of the problem.

2 RAYLEIGH'S LUBRICATION EQUATION

We take as origin O the point of the ground immediately below the lowest point of the tyre at its centre line. Ox is measured forwards from this point, Oy perpendicular to Ox along the ground and Oz vertically upwards. u, v and w are the velocity components of the water in the x, y and z directions, and h is the height of a point of the tyre above the ground. Since the film is thin the value of h at any wetted part of the tyre will be small compared to the radius of the tyre.

Inside the "squeeze film" we neglect the rates of variation of u, v with respect to x and y everywhere in comparison with their rates of variation with respect to z. Also, assuming that $w = 0$ everywhere we have $\partial p/\partial z = 0$ that is, p is a function of x and y only.

We may then write

$$\mu u_{zz} = p_x \tag{1}$$

$$\mu v_{zz} = p_y \tag{2}$$

as in Lamb[5], subscripts denoting partial derivatives.

If the wheel is rotating at full speed and the ground is moving then both the upper and lower surfaces $z = 0$ and $z = h$ have speed $-U$ in the x-direction, i.e. $u = -U$ at $z = 0$ and $z = h$, where U is the speed of the aeroplane.

Hence we have on integrating (1) and (2) and supposing also that $v = 0$ at $z = 0$ and $z = h$

$$u = \frac{1}{2\mu} p_x z(z - h) - U \quad , \qquad v = \frac{1}{2\mu} p_y z(z - h) \quad , \tag{3}$$

and if we take an element of height h on a base δx, δy then the amounts of fluid flowing into the element in the x and y-direction are

$$\delta y \int_0^h u \, dz = \left(- \frac{h^3 p_x}{12\mu} - Uh \right) \delta y$$

$$\delta x \int_0^h v \, dz = - \frac{h^3 p_y}{12\mu} \delta x$$

and the excess of fluid flowing in over that flowing out is

$$\frac{\partial}{\partial x}\left(- \frac{h^3 p_x}{12\mu} - Uh \right) \delta x \, \delta y + \frac{\partial}{\partial y}\left(- \frac{h^3 p_y}{12\mu} \right) \delta y \, \delta x$$

and this must be zero by continuity.

Hence we have

$$\frac{\partial}{\partial x}(h^3 p_x) + \frac{\partial}{\partial y}(h^3 p_y) = -12\mu U \frac{\partial h}{\partial x} \quad . \tag{4}$$

If only the bottom surface is moving whilst the wheel is stationary then the boundary conditions in u are $u = -U$ for $z = 0$, $u = 0$ for $z = h$ and so we have

$$u = \frac{p_x}{2\mu} z(z - h) + \frac{Uz}{h} - U$$

and equation (4) becomes

$$\frac{\partial}{\partial x}(h^3 p_x) + \frac{\partial}{\partial y}(h^3 p_y) = -6\mu U \frac{\partial h}{\partial x} \quad . \tag{5}$$

This equation is a special case ($V = 0$) of an equation given by Lamb[4] and forms the foundation of the work of Michell on lubrication.

We define a velocity U_1 by the relation $P = \frac{1}{2}\rho U_1^2$, where P is the tyre pressure and we non-dimensionalize the equations by writing

$$h = aH \ , \quad x = aX \ , \quad y = aY \ , \quad p = \tfrac{1}{2}\rho U_1^2 Q \quad ,$$

where a is the radius of the tyre. We note that $p = PQ$.

The equation becomes

$$\frac{\partial}{\partial X}(H^3 Q_X) + \frac{\partial}{\partial Y}(H^3 Q_Y) = -N \frac{\partial H}{\partial X} \tag{6}$$

where
$$N = \frac{n U\mu}{a\rho U_1^2} = \frac{n}{R}\left(\frac{U}{U_1}\right)^2 \ , \quad R = \frac{Ua}{\nu} \quad , \tag{7}$$

and n = 24 when both surfaces are moving or n = 12 when the wheel is at rest.

3 DEFLECTION OF THE TYRE

The tyre is assumed to be perfectly flexible and its weight is neglected compared to the tension so that its tension per unit length, T, will be the same everywhere. Considering a part far from the ground it can be shown that for a surface with curvature $1/a$ in one direction and zero in the other

$$T = Pa \quad .$$

It can also be shown (see Appendix A) that if the water pressure on the tyre is p, then for small h

$$\frac{P - p}{T} = \nabla^2 h \quad . \tag{8}$$

Hence we have

$$H_{XX} + H_{YY} = 1 - Q \quad . \tag{9}$$

We shall assume that the undistorted tyre, lifted off the ground a distance h_o at its lowest point, has a shape given by

$$H = H_o + \tfrac{1}{2} X^2 \qquad (H_o = h_o/a)$$

which is sufficiently accurate if h is small. We take h_1 to be the depth of water on the runway, and $H_1 = h_1/a$.

If the wheel is rotating the pressure of the water at all edges must be zero and the whole of the footprint is assumed to be a "squeeze film" and so the boundary condition for Q is $Q = 0$ at the edges. That for H is not easy to formulate. Originally it was supposed that outside the area where there is water the height of the tyre had its undisturbed value, but finally it was decided that it is probably more realistic to suppose that the slope of the tyre at the boundaries is equal to its undisturbed slope. This is probably not quite correct either but may be sufficiently accurate for our purpose. Consequently the boundary condition for H is

$$H_X = 0 \quad \text{for} \quad X = 0 \quad , \quad H_Y = 0 \quad \text{for} \quad Y = 0 \ ,$$

$$H_X = X \quad \text{for} \quad X = X_1 \quad , \quad H_Y = 0 \quad \text{for} \quad Y = Y_1 \ ,$$

where X_1 is the value of X at C (see Fig.1) and Y_1 is the non-dimensional half width of the tyre.

If the tyre is lifted off the ground it always seems to cease to rotate unless it is driven, and in this case there will be a stagnation point at the front, (see Fig.1). It is difficult to know how to deal with this case and we have assumed that the pressure drops quadratically from stagnation pressure at B to zero at H and K (see Figs.2 and 4) and also drops quadratically from the known value on HBK to zero on DCG. The quadratic variation in pressure is taken since in non-viscous flow near to a stagnation point the velocity varies as the distance and hence the pressure varies as the square of the distance.

Thus in Fig.4 the pressure is supposed known completely and permanently in the rectangle DHKG, being equal to $\frac{1}{2}\rho\ U^2$ at B and dropping off to zero as just described, and being zero everywhere on the boundary of full rectangle DEFG. Thus Q is to be determined from equation (6) inside HEFK, using calculated values of H in this region, whilst H_X or H_Y are known on the boundary DEFG and H is to be determined by equation (9) inside this rectangle, using values of Q in this area either known (in DHKG) or calculated (in HEFK). H is taken to be H_o at F and H_1 at C.

In the calculations AC was divided into 20 intervals and B was taken at the end of the seventeenth interval from A. EF was also divided into 20 intervals. Thus the flow is supposed to be a known simplified inviscid flow up to the line HBK and to be Stokes flow between HK and EF. There cannot of course be such a sudden transition and there should be an intervening region where viscous and inertia forces are of the same order of magnitude - that is, a flow of boundary layer type.

4 LIFT AND MOMENT ON THE TYRE

Ignoring squares and products of H, H_X and H_Y, we can write the lift L on the wheel as

$$L = \iint p\ dx\ dy$$

taken over the wetted area. This gives

$$L = Pa^2 \iint Q\ dX\ dY \quad .$$

To the same order of magnitude in H we have for the moment M about the centre of the wheel

$$M = Pa^3 \iint Q(X - H_X)\, dX\, dY \quad ,$$

M being defined to be positive if its direction is such as to oppose rotation of the wheel.

5 SOLUTION OF THE EQUATIONS

Equations (6) and (9) are linear in their own dependent variables Q and H respectively, but they are coupled. The method of solution used was the Gauss-Seidel with successive over-relaxation. At each point of the rectangular 20 × 20 mesh the finite difference form of the equations was written down and the solution of both equations was effected in succession, using when available the latest computed values and running through all the points. The sequence was repeated until the process caused no change, the criterion for stopping the iteration was chosen by calculating the root mean square of the changes in pressure that have taken place for one complete cycle of iteration. When this became less than a pre-arranged small value the process was stopped.

The optimum over-relaxation parameter for (9) is well-known. It is

$$\frac{2 - 2 \sin (\pi/N')}{\cos^2 (\pi/N')}$$

for an $N' \times N'$ mesh. In default of finding an optimum one for the more complicated equation (6) the same parameter was also used in this equation.

6 RESULTS

6.1 Wheel rotating

Two calculations were made for this situation. It was found that for speeds in the range of interest that the Reynolds number for water was so high that the term on the right hand side of (6) was very small indeed. In view of the fact that the boundary condition for Q in this problem is Q = 0 at the edges, this small right hand term means that Q is never very far from zero, and indeed the calculated lift did come out negligibly small in the first case when we took $H_0 = 0.000833$, $H_1 = 0.00417$. It was then decided to try $H_0 = 0.0000833$, $H_1 = 0.00417$, $N = 10^{-5}$. As regards actual values in terms of physical variables we can interpret these parameters in many ways. Taking $\nu = 1.228 \times 10^{-5}$ ft^2/sec for water at 15°C, a = 1 foot,

$P = 30$ lb/sq in we find $U_1 = 66.5$ ft/sec, $U = 150$ ft/sec, $h_o = 0.001$ in, $h_1 = 0.05$ in; for these figures we found that the lift in the wheel was only 15 lb wt. This then is the value of the lift even when the wheel is as close as 0.001 in off the ground and the depth of the water on the runway is 0.05 in. Thus we see that the purely viscous effect is quite small. It is of some interest to repeat the calculation for a completely rigid wheel. This is simpler to do, since H is known and fixed. It leads to a lift more than double. This is to be expected since no "give" in the tyre leads to a thinner film of water in places, even though the thickness of the film at the lowest point is the same.

We show in Fig.5 the distribution of Q over the footprint. This Fig. shows half the footprint area. The other half is the mirror image of this in the centre line of the tyre. The horizontal and vertical scales in this figure are not the same. For a rigid wheel the pressure distribution is as in Fig.6. It is differently distributed and has much larger values.

6.2 Wheel not rotating

In this case it was found from some sample calculations in realistic situations that the right hand side of equation (6) was so small that there was little change if it was put equal to zero. In other words the only effect of viscosity as regards pressure and tyre deflection comes from the no-slip condition. Varying viscosity will vary the actual velocity of the water under the wheel but not the pressure, which is tied down by the outside boundary conditions. This is analogous to fluid flowing along a straight pipe with inlet and outlet pressure fixed, where viscosity only affects the velocity and not the pressure at intermediate points.

Results found are given in Table 1.

Table 1

Case	Y_1	H_o	H_1	$(U/U_1)^2$	L/Pa^2
1	0.12	0.0025	0.025	4	0.0430
2	0.12	0.0025	0.025	2.78	0.0299

If we take $P = 60$ lb/sq in, for which $U_1 = 94.1$ ft/sec = 55.7 knots, we have $U = 188.2$ ft/sec = 111.4 knots in the first case and $U = 156.9$ ft/sec = 92.9 knots in the next case. This gives the results shown in Table 2.

Table 2

Case	b in	h_o in	h_1 in	U knots	Lift lb/wt
1	5.76	0.03	0.3	111.4	1486
2	5.76	0.03	0.3	92.9	1033

Here b is the total width of the tyre.

Tyre pressure = 60 lb/sq in; radius = 2 ft, width of tyre = 5.76 in.

It should be stressed that the value of lift force given represents what it is when the wheel is actually off the ground. It bears no relation to conditions where there is only partial aquaplaning. This problem will be discussed (but not solved) in section 7.

Isobars and shape of tyre are shown in Figs.7 and 8 for case 2. The kinks in the isobars are due to the assumption of purely non-viscous flow to the right and purely viscous flow to the left. Presumably these kinks will be smoothed out in the boundary layer type of flow in which inertia terms and viscous terms are of the same order of magnitude. What is perhaps of greater interest is the deflection ΔH of the tyre from its original circular shape. This is shown in Fig.9.

The calculations for a rigid wheel show very little change in lift, as was perhaps to be expected here, since the film is thicker and most of the lift arises from the stagnation pressure.

To show the effect of the Stokes flow in delaying the escape of the water let us imagine that the water impinges at the same stagnation point and flows away freely to all edges, the pressure dying away quadratically to zero at all edges. It is found that the lift in cases 1 and 2 would be reduced to 622 and 431 lb wt respectively.

7 PARTIAL AQUAPLANING

The speed at which aquaplaning begins is what is really required for practical purposes and the present calculation is unable to determine this. A perfectly smooth tyre running along a perfectly smooth road would presumably aquaplane immediately if there was any water, since it is known that a smooth surface such as a square plate, for instance, pushed down on to a wet surface would take in theory an infinite time to reach complete contact. This applies even if it is initially at an angle[6], see Fig.10. The fact that in reality such a plate can be pushed down into "contact" in a finite time is due to roughness which allows water to escape, and "contact" here means contact between the excrescences on the rough surfaces and not complete contact, with all the water pushed out.

We must therefore make allowance for roughness in all cases where a wheel is in so-called contact with wet ground. One mode of attack on the problem might be to consider the contact to be made rather as in Fig.11 in which the individual roughness elements, thought to be of very fine mesh, are supposed to offer no barrier to the flow of water between them. Then as regards the calculation of Q, equation (6) would still be used, the h between A and B being taken as simply h_o or at least never less than h_o though it might in places (e.g. the point C) be more if Q were large enough. The difficulty here is that we do not know in advance what the region of contact is; attempts to solve this problem were unsuccessful.

8 DISCUSSION

It is not claimed that the numerical results are accurate physically, but it is believed that they show the mechanism that is operating. It may be helpful to discuss the assumptions that have been made and to speculate on their effects.

(a) Smoothness

Once the tyre is really off the ground it is not believed that distributed roughness, such as that of a concrete surface, will make a very great difference, and may only correspond to small changes in h_o. Tyres usually have longitudinal grooves and these will obviously make a great difference. If the grooves are not too deep one could in theory use the same general method that has been used here, but with a great deal of extra complication. However, the Stokes approximation may not be valid in the grooves, and also the flow in them may well be turbulent, and this would make any calculation on the present lines out of the question.

(b) Stiffness of the tyre

Presumably the effect of this is to resist changes in h. On the limited results so far available this would not appear to make very much difference to the lift. It may be possible to incorporate the effects of stiffness into the equation for h.

(c) The hydrodynamical assumptions

While it is probably true that close under the tyre the assumption of Stokes flow is justifiable, that regarding the pressure distribution outside this region is far more questionable. The assumed position of the stagnation point has been entirely arbitrary and the assumption of quadratic falling off in pressure towards the sides and front needs further examination. Moreover there is always the possibility of hydrodynamic jump in front of the tyre as suggested by Gadd[7]. In addition there must be a region where viscous and inertia forces are of the same magnitude and this has been omitted from the theory. It is difficult to see how to improve the situation here, and there is probably not much point in attempting refinements suggested in (b) until an improved fluid flow model has been incorporated.

(d) Inertia of the tyre

We have assumed that the tyre is weightless, whilst in fact the loss in vertical momentum as a tyre is distorted close to the ground can lead to a considerable amount of lift. For instance if a tyre is completely deflated it is still possible at high speeds for the "centrifugal force" to hold the wheel rim off the ground. An attempt was made in Ref.3 to take account of this lift (which must manifest itself as a pressure between tyre and ground, as the rim lifts off) but the details in that work have been criticised; though existence of the effect is admitted, some critics believe that it is not so great as forecast. An observer looking at such a tyre would be under the impression that it was in fact inflated to some extent and so it is possible that allowance could be made for it in a crude way by assuming a somewhat greater tyre pressure than is actually the case.

9 CONCLUSIONS

If the tyre is off the ground but is still rotating at full speed there are only appreciable lift forces on the wheel if the film of water is very thin indeed, so thin in fact that roughness probably cannot be ignored, and in these circumstances a rigid wheel instead of a flexible one may increase the lift considerably. In this situation viscosity is all important, but the case can scarcely arise in practice, because of the smoothness it demands.

If the wheel is at rest there is stagnation pressure at some point on the tyre and the lift forces are very much greater. The magnitude of viscosity is not important in determining the lift force and the effect of viscosity only appears in the no-slip condition. For the circumstances considered in this Report there is very little difference in the results for the lift whether the tyre is flexible or completely rigid. All the above remarks are made on the assumption that the tyre is off the ground and is smooth, though ordinary roughness (not grooves) may not make a great difference in these circumstances.

These results have been obtained for an idealized situation in which the flow has been divided into two parts - a non-viscous flow in front of the tyre and Stokes flow underneath, and the non-viscous part has been "guessed". What would seem to be required for a more complete study is a "free streamline" calculation in three dimensions so as to ensure that the inviscid part of the calculation is improved, instead of being very much of a guess as it is here. Two-dimensional calculations on these lines have been made[4] and give fair results, which is surprising in view of the fact that the situation cannot by any stretch of the imagination be considered as two-dimensional.

In the case of "partial" aquaplaning, when the wheel receives some lift but is not completely off the ground, roughness effects cannot be ignored since a perfectly smooth tyre would be lifted off the ground at any speed. In this case the two-dimensional inviscid calculations would be bound to be unrewarding since they would imply that <u>all</u> of the water in front of the tyre would be pushed forward and none would escape at the sides or underneath (Fig.3).

Appendix A

TYRE DEFLECTION

The formula for the deflection of a membrane under a pressure difference P is

$$P = T\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$$

where T is the tension per unit length and ρ_1 and ρ_2 are the principal radii of curvature of the surface.

We have[8] for the mean curvature K

$$K = \frac{1}{\rho_1} + \frac{1}{\rho_2} = \frac{g_{11}\, d_{22} + g_{22}\, d_{11} - 2g_{12}\, d_{12}}{g}$$

where g_{rs} and d_{rs} are the coefficients in the first and second fundamental ground forms of the surface and g is the determinant of the g_{rs}

Hence for a surface with equation z = h(x, y) we find

$$K = \frac{(1 + h_y^2)\, h_{xx} + (1 + h_x^2)\, h_{yy} - 2h_x\, h_y\, h_{xy}}{(1 + h_x^2 + h_y^2)^{3/2}}$$

and when h_x and h_y are small this becomes

$$K = h_{xx} + h_{yy} \quad .$$

Hence if P is the inflation pressure and p the external pressure we have

$$\frac{P - p}{T} = \nabla^2 h \quad .$$

SYMBOLS

a	outside radius of the tyre
b	width of the tyre
h	height of tyre above the ground
h_o	height of lowest point of tyre
h_1	depth of water on runway
H, H_o, H_1	non-dimensional forms of h, h_o, h_1
K	mean curvature of tyre surface
L	lift force on tyre
M	moment of forces on tyre about wheel centre
n	12 or 24 according as wheel is at rest or rotating
N	$\frac{n}{R}\left(\frac{U}{U_1}\right)^2$
N'	number of intervals in finite difference mesh
p	pressure
P	tyre pressure
Q	given by $p = \frac{1}{2}\rho U_1^2 Q$
R	Ua/ν
T	tension per unit length in tyre
U	velocity of aeroplane
U_1	defined by $P = \frac{1}{2}\rho U_1^2$
u, v	velocity components of water
x, y, z	Cartesian coordinates, x forwards, y sideways, z upwards
X, Y	non-dimensional forms of x, y
μ	coefficient of viscosity
ν	kinematic viscosity
ρ	density
ρ_1, ρ_2	principal radii of curvature of tyre

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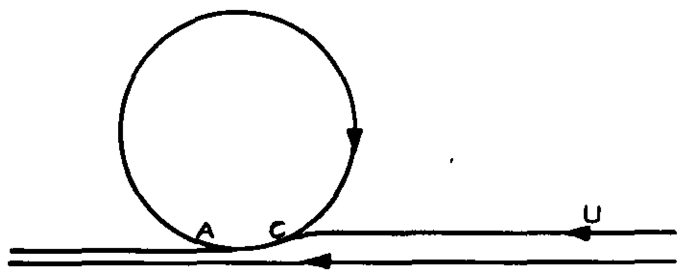


Fig. 1 Rotating wheel

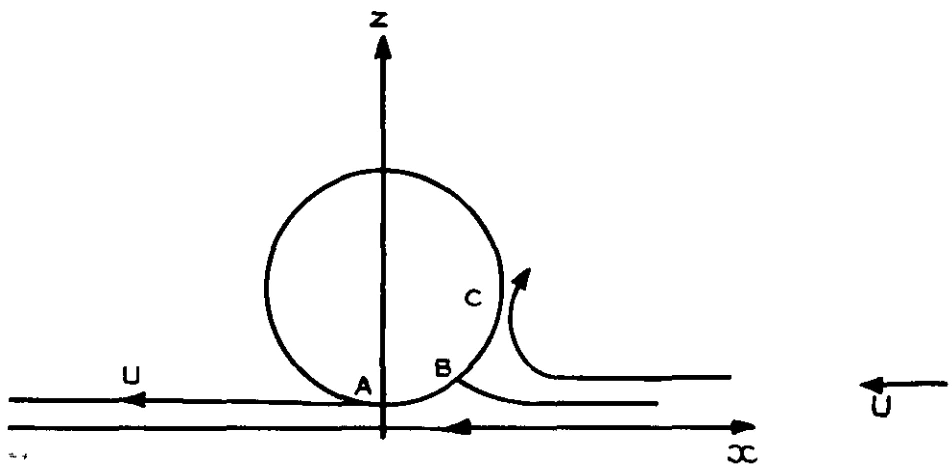


Fig. 2 Non-rotating wheel

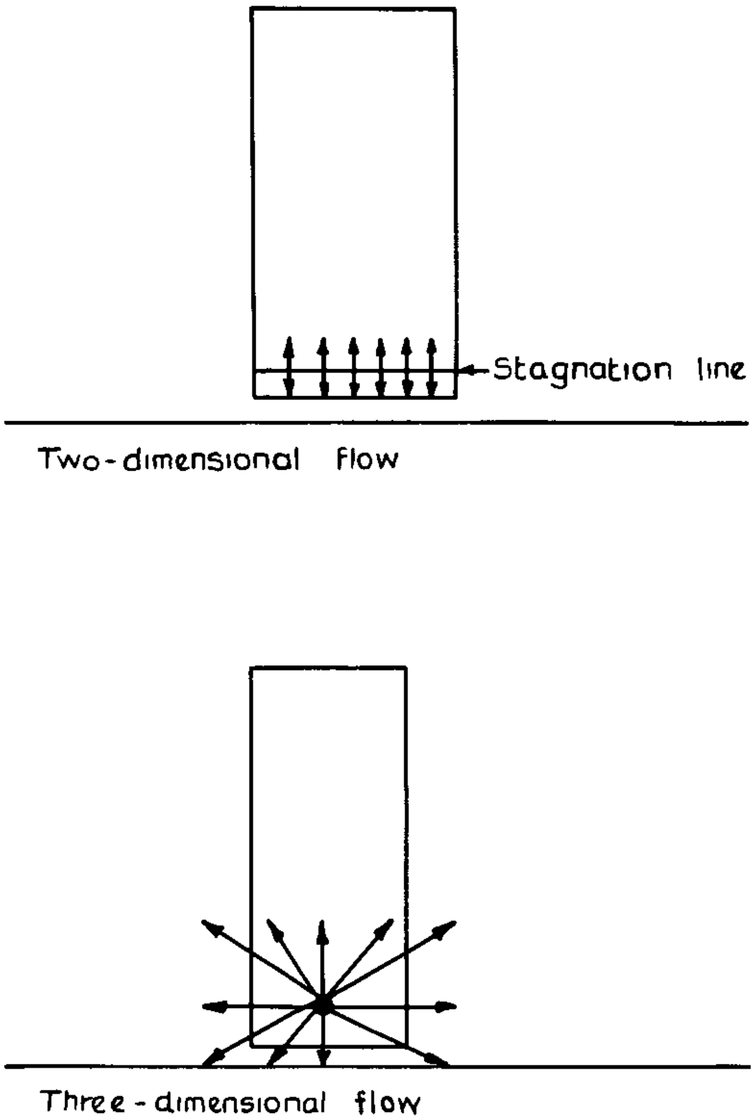


Fig. 3 View of water flow from the front

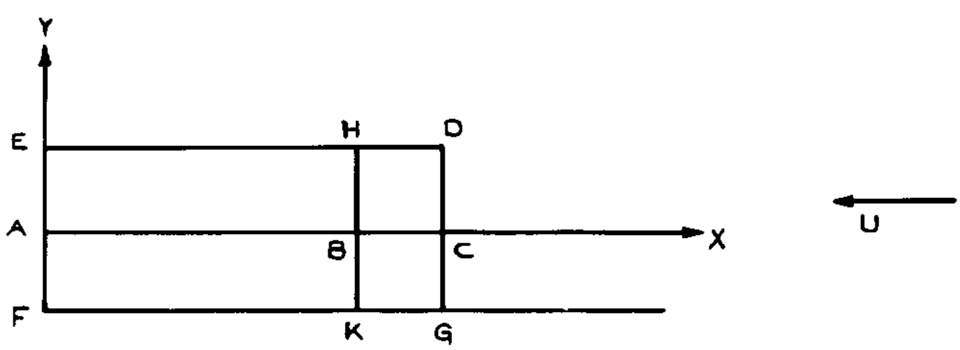


Fig. 4 Footprint from above

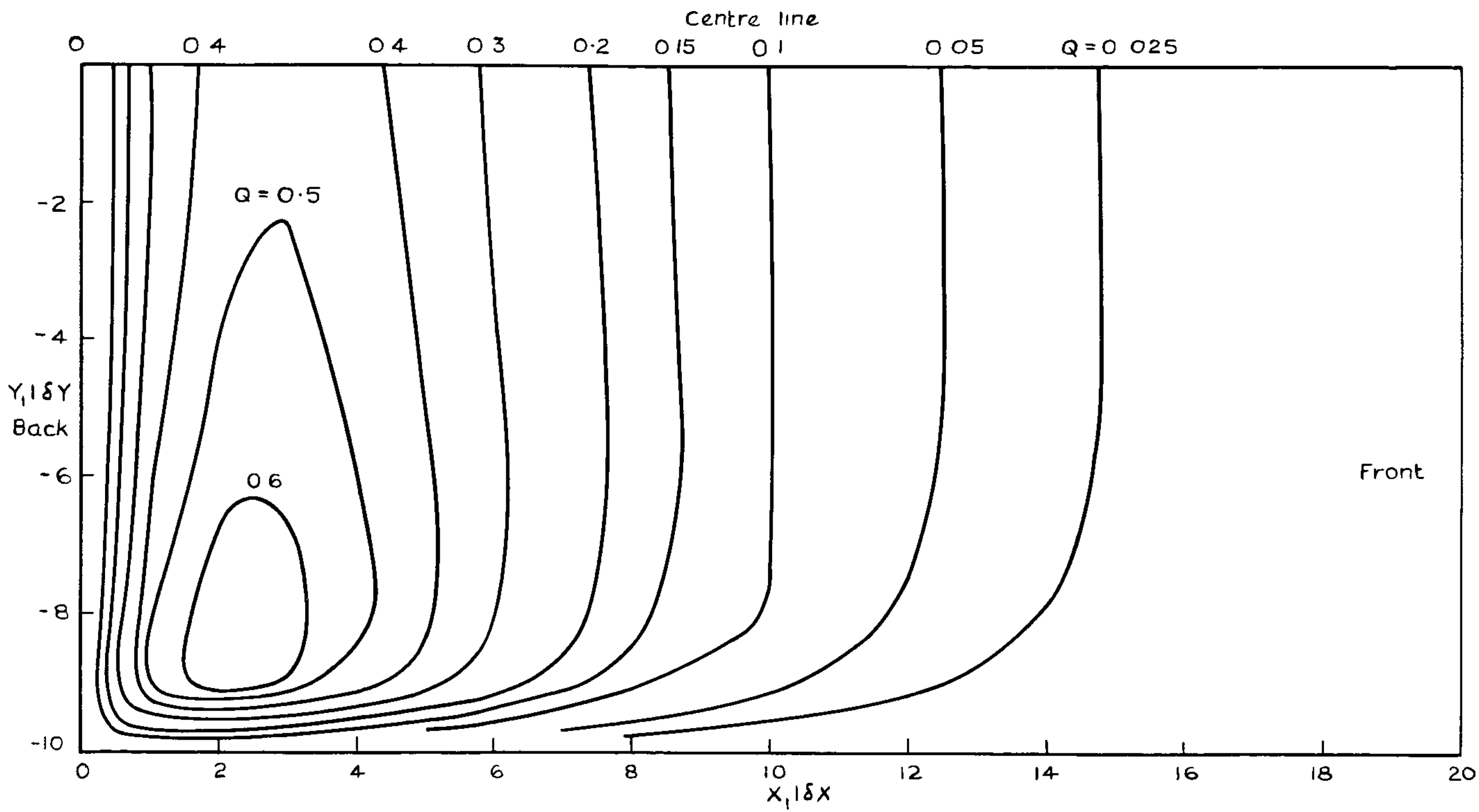


Fig. 5 Isobars Flexible wheel rotating $H_o = 8\ 333 \times 10^{-5}$

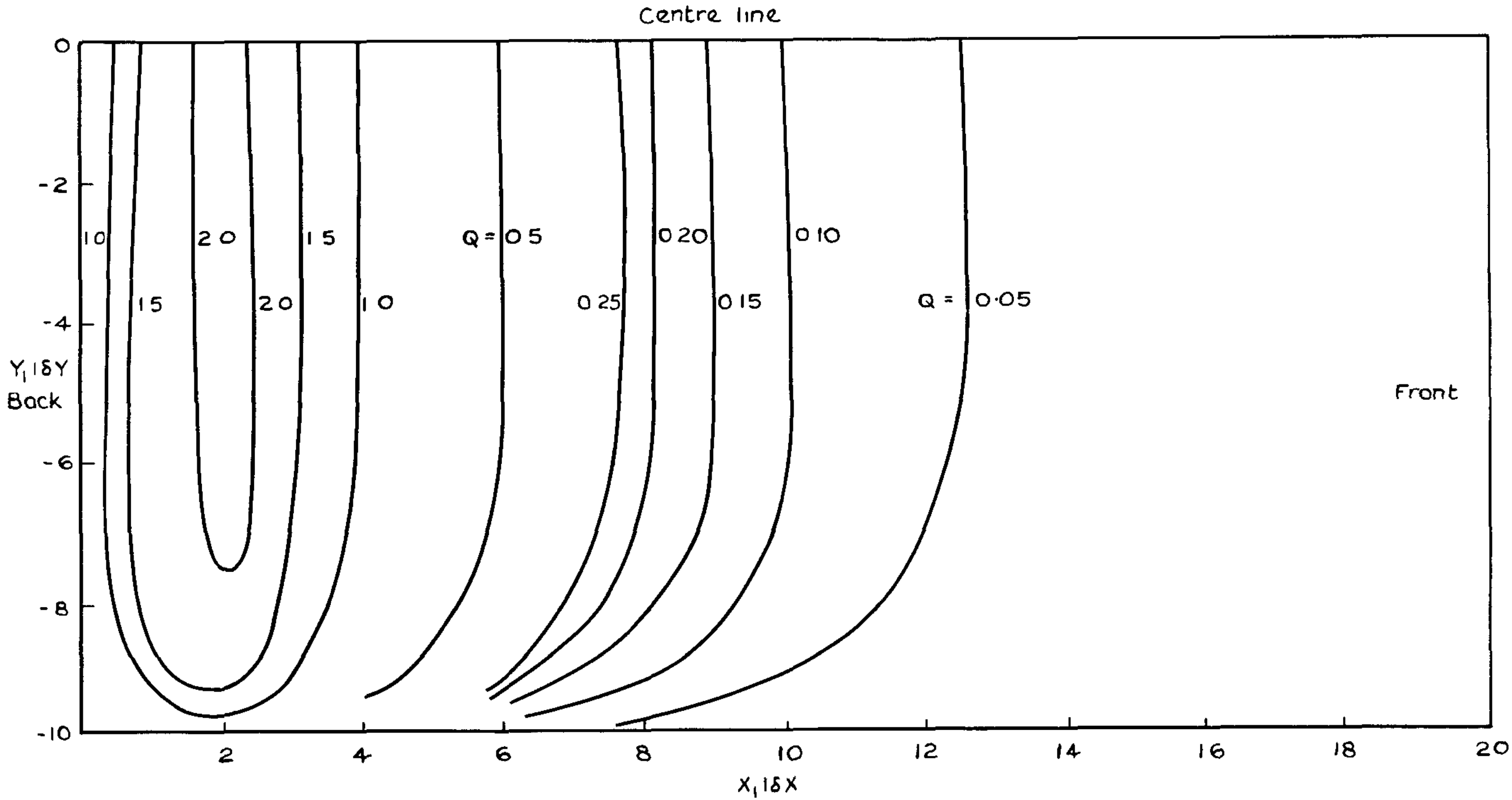


Fig 6 Isobars. Solid wheel rotating

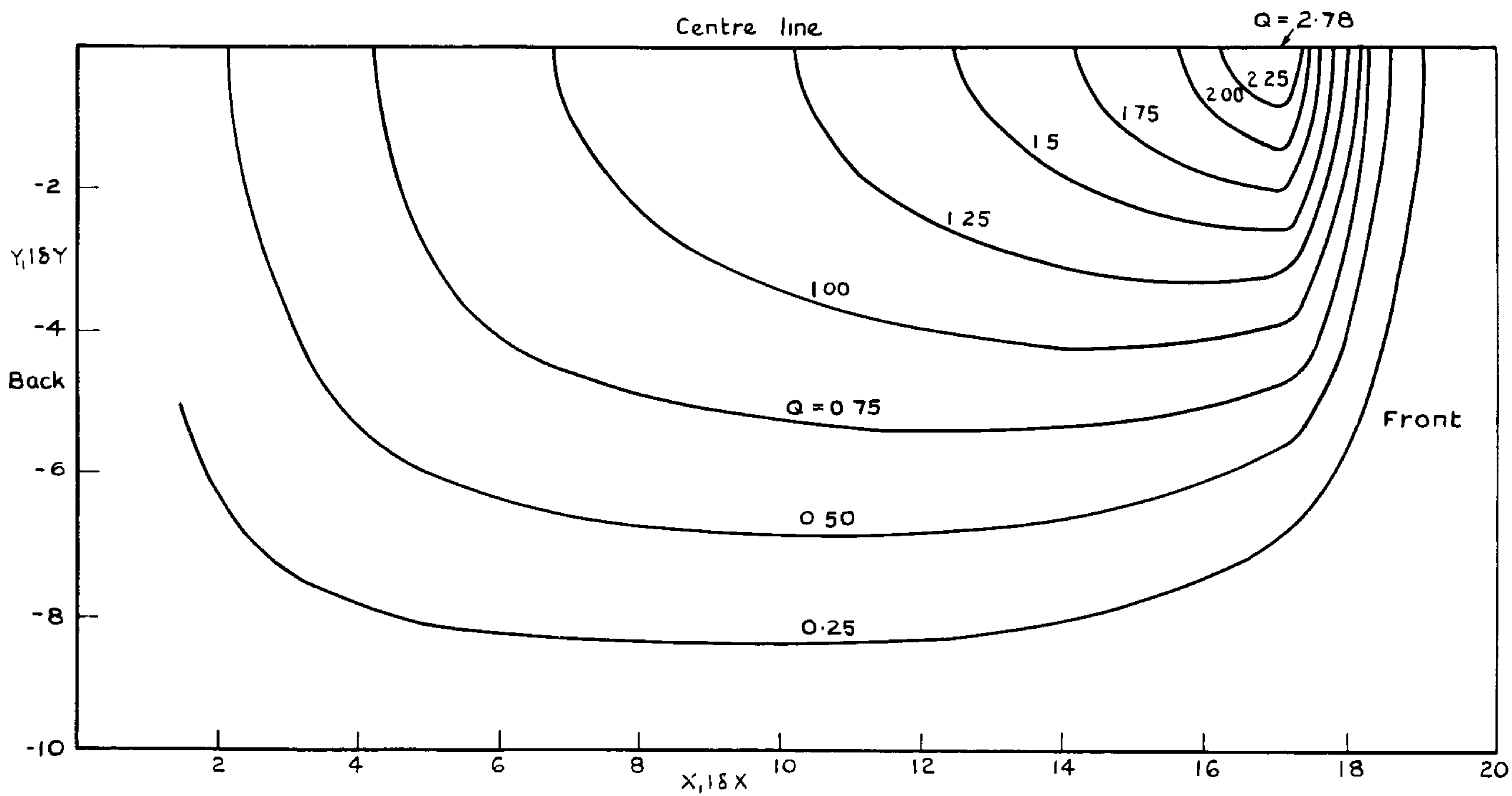
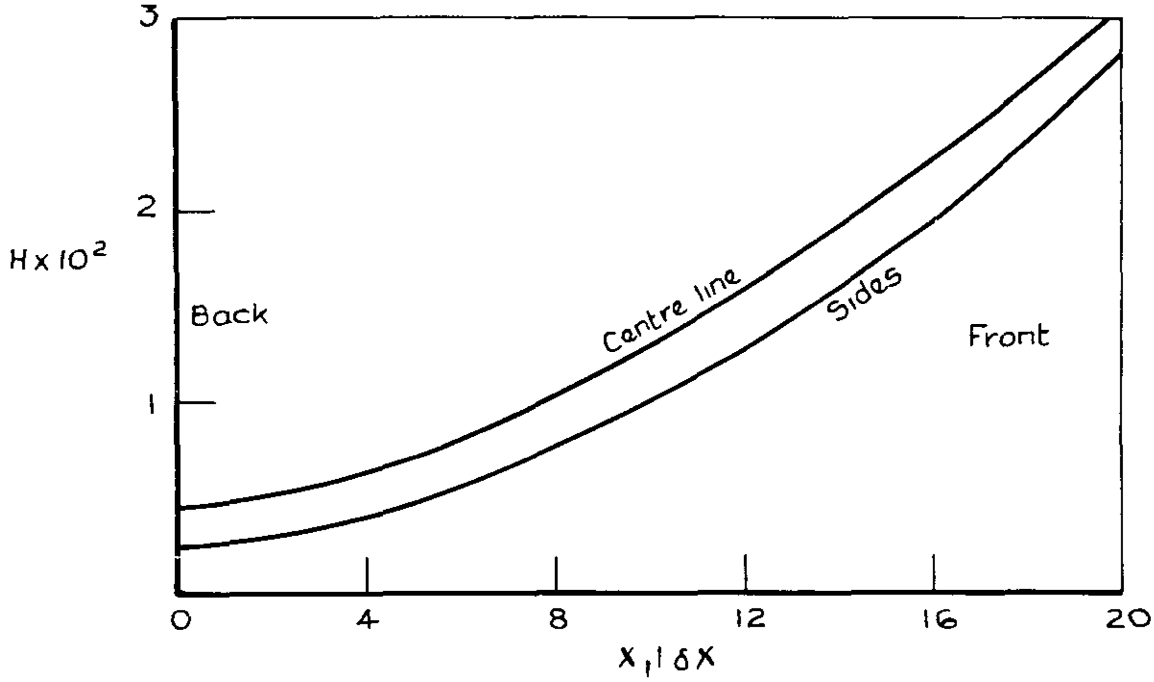
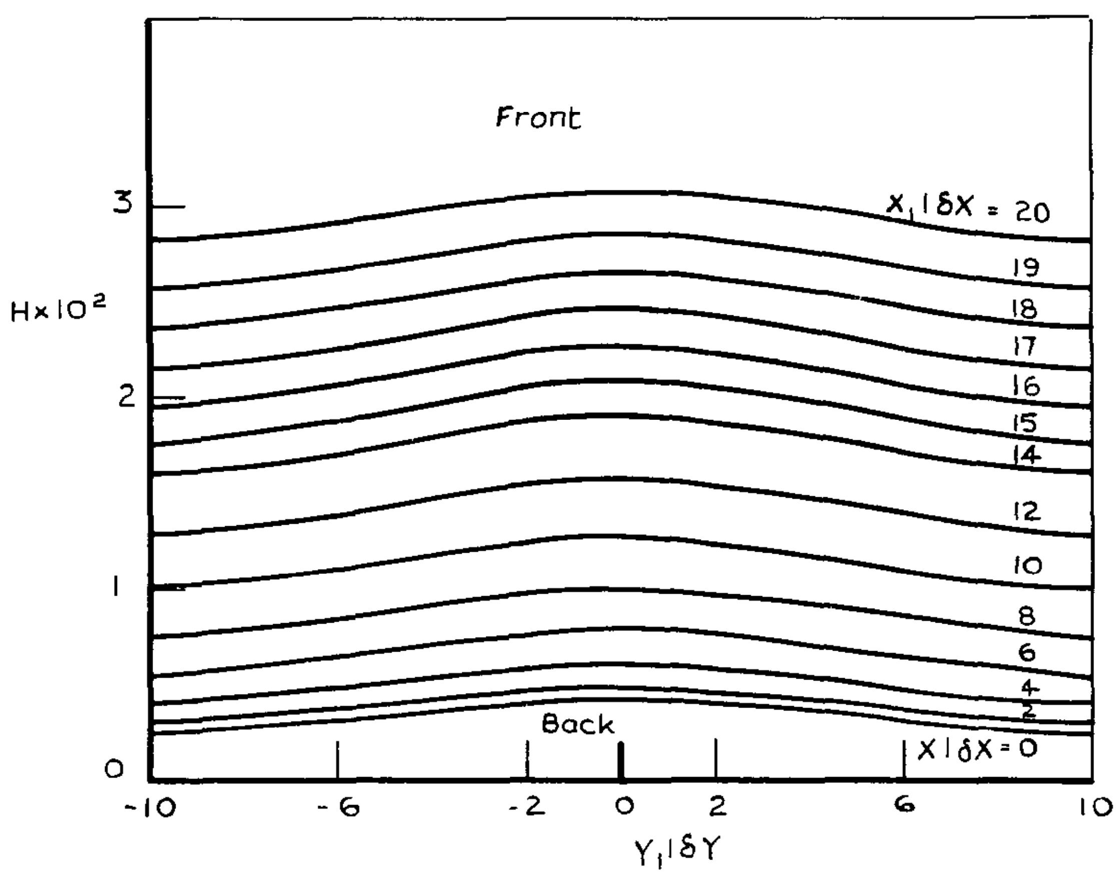


Fig. 7 Isobars. Wheel not rotating $H_0 = 0\ 0025$, $H_1 = 0\ 025$, $(u/u_1)^2 = 2{\cdot}78$



Shape at longitudinal sections $H_0 = 0\ 0025$



Height of tyre above the ground, $H_0 = 0\ 0025$

Fig. 8 Shape of the tyre

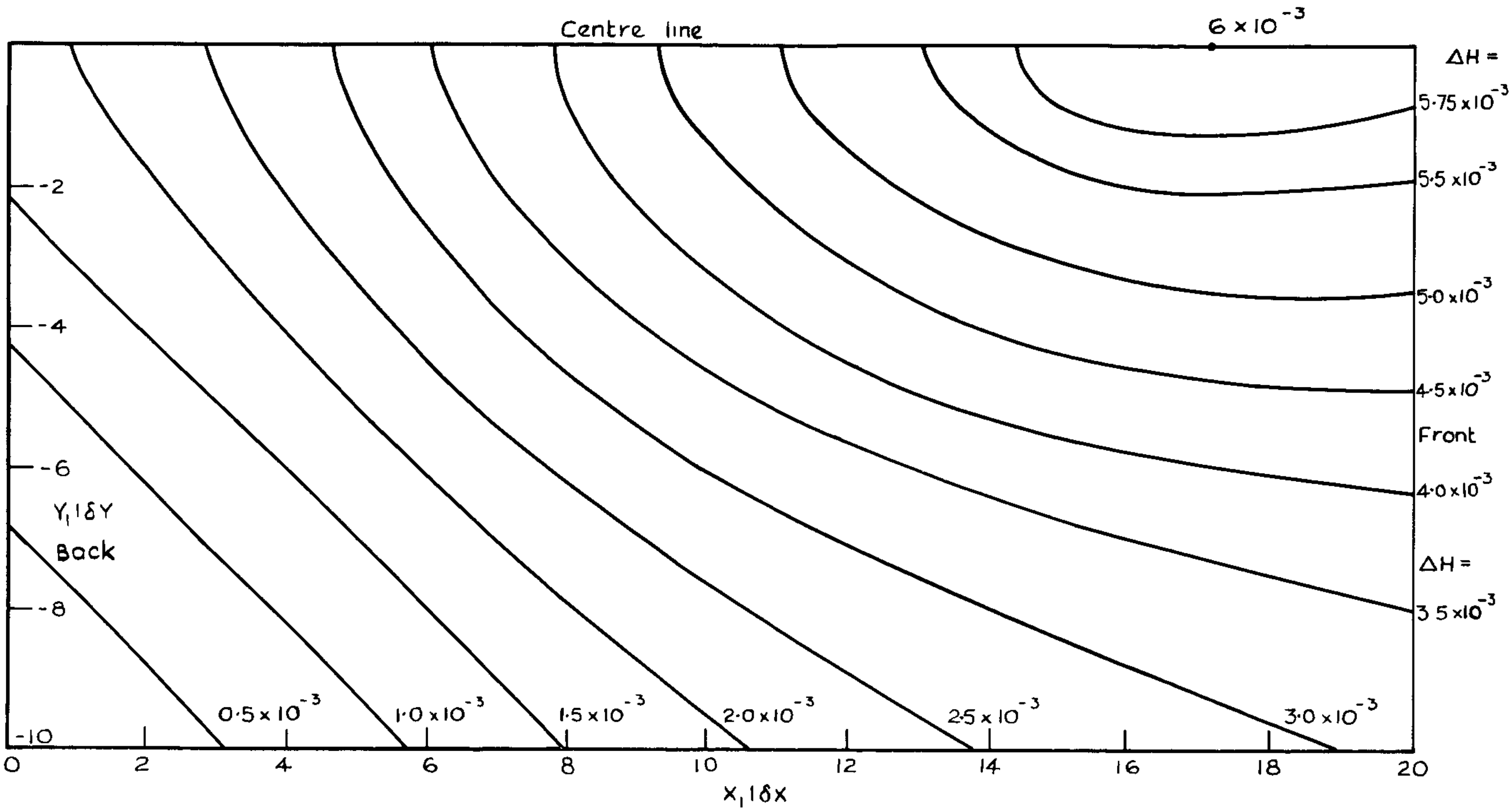


Fig 9 Lines of equal tyre deflection ΔH. Wheel not rotating. $H_o = 0\ 0025, H_1 = 0{\cdot}025$

Fig. 10 "Squeeze" films

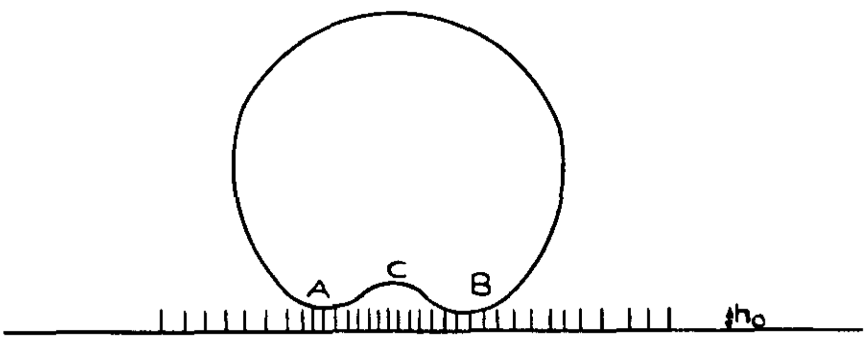


Fig 11 Situation in "partial" aquaplaning

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