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A TECHNIQUE FOR STUDYING HIGH-VELOCITY DRAWING IN POLYMERS

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SUMMARY

The principle of elastic relaxation is considered as a technique for drawing polymers at high velocities. The theory is worked out and some tests are described in which skeins of low density polyethelene filaments have been drawn using some heat-set terylene webbing as the elastic. These tests, in which polythene was drawn at a velocity of about 50 feet per second at a temperature of about 18°C, demonstrated that the technique is feasible. Comparison with the theory suggests that the force to draw at this speed is about twice that measured at very slow rates of drawing.

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* Replaces R.A.E. Technical Report 66262 - A.R.C. 30005

CONTENTS

1 INTRODUCTION

The specific work done in the cold drawing of polymers is substantial, being of the order of 10^4 - 10^5 foot-lb per lb of material, so that some interest has been shown in the use of these materials in energy absorbing mechanisms. The use of undrawn nylon has been established in emergency aircraft arresters[1] but other polymers, particularly low density polythene, show excellent drawing characteristics and are better to employ in certain mechanisms than nylon at normal temperatures.

It is well known that the ability to draw without brittle fracture depends upon temperature[2] and, to some extent, it is known that the velocity of drawing that can be attained depends also on temperature but our knowledge here is limited because of the lack of a method of drawing the polymers at a sufficiently high velocity. Ballistic test machines of the Avery type are limited to the order of 10 feet per second and centrifugal-type impact testers have been able to do some tests on energy to break when the specimen has been engaged at 40 feet per second.

The purpose of this Report is to describe a technique whereby velocities greater than mentioned above, probably greater than 100 feet per second, can be achieved by the method of elastic relaxation of certain materials. The theory of the method is given and supported by a description of a rig to do some experimental evaluation with polythene.

2 THE PRINCIPLE OF ELASTIC RELAXATION

When a length of elastic cord or wire is strained under tension and one end is released this end travels away with a finite velocity and the front of relaxed strain travels through the cord at a related faster velocity. If v and c are respectively the particle velocity in the relaxed cord and the velocity of propagation of the relaxation front then the strain e initially in the cord is related to v and c, for small strains at least, by

$$e = \frac{v}{c} . \tag{1}$$

Reference to work on straining by impact[3] will help to clarify this simple relation. The velocity, c, in a perfectly elastic cord is defined, in terms of quantities directly measurable on a chord or fibre, by

$$c^2 = \frac{\text{Stretch modulus}}{\text{Mass per unit length}} \tag{2}$$

rather than in terms of Young's modulus and density. The stretch modulus is defined by the ratio of the tension T, to the strain e. If the mass per unit length is denoted by m then equation (2) can be written

$$c^2 = \frac{T}{e\,m} = \frac{T\,c}{m\,v} \tag{3}$$

from which it follows that

$$T = m\,v\,c \tag{4}$$

and

$$\tfrac{1}{2}\,Te\ \text{(strain energy)} = \tfrac{1}{2}\,m\,v^2\ \text{(kinetic energy)} \tag{5}$$

The above equations relate to a complete relaxation when the tension in the cord behind the front falls to zero and all strain energy is converted into kinetic energy.

Provided thermodynamic effects and time dependent relaxations are negligible, equation (4) represents a condition which must always be satisfied across the relaxation front, namely, that the change of force must be equal to the rate of change of momentum. This mechanical concept, which has been discussed more fully elsewhere[4] can be extended to cord structures and imperfectly elastic material in which the wavefront has a finite width. It can also be extended to a partial relaxation,

$$T_1 - T_2 = m\,v\,c \tag{6}$$

where T_2 is the tension remaining on the cord when the end is released. The velocity of relaxation will now be less than the velocity of free relaxation but should be constant, for constant force, until the reflected relaxation front returns from the fixed end of the cord.

3 APPLICATION OF THE PRINCIPLE TO STUDY THE DRAWING OF LOW DENSITY POLYTHENE

Low density polythene will draw and extend up to 1500% before breakage at normal room temperatures. The actual load-extension characteristics vary with grades of material and are sensitive to temperature. Some measurements on spun filament polythene have been at low rates of strain[5] and some results are reproduced at Fig.1. Further measurements on the 0·040 inch diameter samples used in these tests are given in Table 1. The material appears to be substantially elastic up to approximately half the breaking tension at which tension the material begins to extend under constant force for many times the original length. The extension is accompanied with the formation of draw shoulders which travel through the material and thereby change the cross section. For example, the 0·026 inch diameter filament is substantially elastic up to a tension of 0·55 lb, will draw steadily at about 0·65 lb and will eventually break at about 1·10 lb. A filament of 0·040 inch diameter will commence to draw at about 1·7 lb.

From tests on groups of 0·026 inch filament[5] it appears that the initial stretch modulus works out to be about 5·5 lb per unit extension per filament operative over the first 7% of extension. This can be used to estimate the initial time lag before which the material will draw.

Since polythene will draw at a constant force over several times its original length then it is possible to visualise a condition of dynamic equilibrium of a tensioned elastic cord relaxing partially against the reaction of a plastic specimen such that the drawing takes place at near constant velocity. In the absence of forces due to acceleration the drawing force, F, for the specimen can be identified with T_2 of equation (6) and the initial tension, T_1, in the cord before release must be in excess of F for the equilibrium velocity. (To be general it will be assumed that F is not wholly independent of v - the drawing velocity.)

Practical problems are to attach the elastic cord to the plastic specimen, to provide a restraint to the end of the elastic cord so that it can be tensioned and to effect a quick release of this restraint to start the drawing. In providing the attachment and release a concentrated mass, M, is involved in the connection between the cord and the specimen. Whilst every effort in the design of a test apparatus will be made to keep this connection mass small, it must be taken account of in analysing the motion during initial relaxation.

4 MOTION DURING INITIAL RELAXATION

There are two factors that must be taken account of in studying the build-up of the velocity of relaxation to an equilibrium value. Firstly, there is the inertia forces due to a mass, M, between the cord and the specimen and secondly, the specimen substantially obeys Hooke's law at tensions below the drawing force. However, it is being quite general to write

$$T_2 = M\dot{v} + F(v) \qquad (7)$$

so that the equation of motion is

$$T_1 = m v c + F(v) + M \dot{v}. \qquad (8)$$

The simplest case to consider is when F, the force to draw the polymer sample, is constant. Then

$$v = \left(1 - \exp - \frac{m c t}{M}\right) v_e \quad \text{where } v_e = \frac{T_1 - F}{m c} \qquad (9)$$

v_e is the equilibrium velocity which is approached during the short time interval before the relaxation has been reflected back from the end of the elastic cord. Clearly the smaller the mass, M, in relation to the mass m c the sooner will equilibrium velocity be approached.

It is not believed that the force F to draw the polymer is wholly independent of the velocity of drawing; in fact there are reasons to suspect that it could increase slowly with velocity. The force can be expressed as a power series of v, thus

$$F(v) = F_0 + F_1 v + F_2 v^2 + \ldots \qquad (10)$$

If only the first two terms are included equation (9) is modified to

$$v = \left(1 - \exp - \left(\frac{m c + F_1}{M} t\right)\right) v_e \quad \text{where} \quad v_e = \frac{T_1 - F_0}{m c + F_1}. \qquad (11)$$

If the first three terms are retained then the equation is modified to

$$v \Big/ \left(1 + \frac{F_2}{m c + F_1} v\right) =$$

$$\frac{(T_1 - F_0)(m c + F_1)}{(m c + F_1)^2 + F_2(T_1 - F_0)} \left(1 - \exp - \left\{\frac{m c + F_1}{M} + \frac{2F_2(T_1 - F_0)}{M(m c + F_1)}\right\} t\right) \qquad (12)$$

when F_2 is a small quantity with respect to m c. The solution of equation (8) is a standard form when the first three terms are retained and the approximate form of equation (12) for small values of F_2 is developed in the Appendix. By performing the experiments with different weights of elastic cord and over a range of speeds it should be possible to estimate the first and second order coefficients F_1 and F_2.

In deriving the above equation it has been assumed that the drawing force is acting from the time of release. This is not so with a real material because, below the drawing force, the undrawn polymer is elastic. The force due to the extension of the polymer in this case is proportional to the displacement x that is $F = k x$. Thus for this initial stage before drawing equation (8), can be written

$$T_1 = m c \dot{x} + k x + M \dot{x} \qquad (13)$$

or

$$\ddot{x} + \frac{m c}{M} + \frac{k}{M}\left(x - \frac{T_1}{k}\right) = 0. \qquad (14)$$

The solution of the above equation is

$$x = \frac{T_1}{k}\left(1 + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t}\right), \qquad (15)$$

where $\lambda_1 = \frac{m c}{2M} + \sqrt{\frac{m^2 c^2}{4M^2} - \frac{k}{M}}$ and $\lambda_2 = \frac{m c}{2M} - \sqrt{\frac{m^2 - c^2}{4M^2} - \frac{k}{M}}$,

having regard to the boundary conditions that $x = 0$ and $\dot{x} = 0$ when $t = 0$.

In the above solution the roots can be real or complex according to whether $m^2 c^2$ is greater or less than $4Mk$. When the roots are real the solution is aperiodic and it is interesting to note that this is associated with reducing the coupling mass M. In the experimental system described later this mass was large enough to make the solution periodic.

The matter of optimising the mass, M, in relation to the cord constant $m c$ and the stiffness k could be studied in more detail but it is clear that the reduced reaction from the specimen in the initial stages would help the acceleration towards the equilibrium velocity so that v would be greater than the value given by equation (9).

5 MATERIALS FOR ELASTIC RELAXATION

5.1 Steel

Music wire is a useful material because it is nearly perfectly elastic and it is cheap. However, the strain is low so that the length of wire necessary to test the specimen could be inconveniently long.

The velocity of strain disturbances in music wire is about 16700 feet per second. The limiting velocity of snatch on the end which would induce the ultimate strain in music wire, is about 160 feet per second when the ultimate stress is 120 tons per square inch. In particular 20 swg piano wire has a breaking tension of about 330 lb and a mass of $1{\cdot}07 \times 10^{-4}$ slugs per foot. To have a velocity of relaxation of 50 feet per second with a residual tension of 50 lb, the initial tension in the wire would require to be 140 lb. The length of wire to give one foot relaxation travel would be about 200 feet and the test time would be 24 milliseconds. An example of a stress/strain curve for steel music wire is given at Fig.2.

5.2 Hot stretched terylene

Among the textile fibres hot stretched terylene is attractive because its relaxation of stress with strain is fairly linear[6]. Heat-set terylene, that is yarn which has been heated and held to constant length, is not quite so linear but, as some tubular woven braid was available from such yarns, it was tried. Terylene has a practical advantage over music wire in that it is more extensible and strain disturbances travel slower. Therefore, a shorter length of cord is required.

Some load-unload curves were obtained for some 400 lb tubular braid which are given in Fig.3. However since these measurements were done at relatively low speeds compared with those of the application, the recovery involves some time dependent relaxation. Two samples were available, one woven from 125 denier and the other from 250 denier yarns, but their differences were insignificant. The relaxation of strain is not quite linear. The assumption is made that all the strain energy released is converted into chordwise kinetic energy. The velocity so obtained is called the free relaxation velocity. From the curves of Fig.3, it is possible to obtain a relaxation between peak tension for a loading cycle and the strain energy released, which relation can be replotted as one between tension and free relaxation velocity as in Fig.4. In this figure equation (8) has been restated for the final non-accelerated state with a more practical interpretation. To use this curve to estimate a partial relaxation velocity a horizontal line representing the initial tension is drawn. As an example, if the partial relaxation velocity is 50 feet per second then the vertical residual between the initial tension and the curve at the 50 feet per second point on on the horizontal scale represents the residual tension. It may not be clear that one does not mark 50 feet per second to the left of the 200 feet per second mark. However, if the regain of energy as the tension falls in Fig.3 is equated to kinetic energy then the relation between the partial relaxation and the residual tension illustrated in Fig.5 is obtained for the interpolated value for an initial tension of 128 lb, as drawn by the dotted curve in Fig.3. This gives substantially the same answer as obtained from Fig.4.

It appears feasible to reach relaxation velocities of 300 feet per second with hot-stretched terylene. Whilst a higher velocity is theoretically possible, the non-linear region of the stress-strain relationship is encroached even more and the simple linear theory will become less valid. However, velocities greater than those possible with steel should be possible with terylene.

The effective velocity of propagation of strain disturbances can be calculated from the free relaxation velocity given in Fig.4 as the ratio T/mv. This propagation velocity is less than that of a single fibre but increases steadily towards such a value as the initial tension from which the relaxation takes place is raised (Fig.6).

To determine the length of cord required for a test, reference is made to the interpolated dotted curve on Fig.3. The velocity of propagation of the strain front for this tension is estimated from Fig.6 to be 6200 feet per second for 128 lb initial tension so that on a 100 foot length of cord a test time of about 30 milliseconds is available.

6 DESIGN OF TEST RIG

6.1 Principle of operation

The plastic specimen which is to be drawn is mounted on a carriage such that one end can be released from the main body of the carriage but is still held at the other end. This carriage initially slides freely on the main supporting frame. The releasable end is attached to a long length of cord, 100 feet of 400 lb terylene webbing in this case, whilst the other end of the carriage is connected through a cable over a pulley to a weight. This weight determines the tension applied to the cord when it is tensioned. As soon as the tension lifts the weight the releasable end is free and the carriage is locked to the test frame. The tensioned cord will relax and rapidly accelerate to the equilibrium relaxation velocity appropriate to the force which will draw the specimen.

6.2 Description of the experimental rig

The test rig is diagrammatically illustrated in Fig.7 and is described in relation to its initial plan for use. Photographs of overall and close-up views are given in Figs.8 and 9 respectively. The polythene test specimen (1) is wound into spools (2). The specimen with spools is mounted on the carriage (3) where one spool is held by the pin (4) and the other spool is mounted on a subsidiary member (5) which is restrained horizontally to the main carriage (3) by the pin (6). This pin passes unobstructedly through a cut-away portion of the main frame (7) upon which the carriage (3) rests. The catapult cord (8) is attached to the member (5) through the pin (9). To the other end of the carriage (3) a cable (10) passes over a pulley (11) to a weight (12) resting upon the ground.

In the original and principal method of operation the release at the correct tension is done automatically through a system of microswitches. The winch tensioning the catapult cord is wound slowly until the tension is sufficient to raise the weight off the ground. The actual tension in the cord will be in excess of the weight because there is a small frictional force between the carriage and the frame. The carriage (3) then slides slowly along the top of the frame until it reaches the first microswitch (13) which operates the indicator light (14). This signifies that the catapult is tensioned to the desired value and that the photographer can start the camera. A further movement of the carriage brings it into contact with a second microswitch (15) which operates a release (16). This release allows a small weight (17) to fall and to pull out the pin (6) thereby transferring the tension from the catapult cord to the cable (10) through the polythene specimen.

When the forward end of the carriage reaches the microswitch (15) the other end clears the pawl (18) which is moved by the spring (19) into a position to prevent the carriage being moved to the left due to the excess of the cable tension over the drawing force in the polythene.

When the member (5) is freed from the carriage (3) a slight movement will operate a third microswitch (21) and so actuate a flash (20) to record on the camera the initial movement of the member (5).

The main frame of the rig was built as a light braced frame using standard slotted angle sections, the frame being anchored to a wall by Rawlbolts. Ideally, it should be heavier because the application of a sudden load to the frame would cause it to vibrate in the audio frequency range. Associated with this vibration there could be some fluctuation in force on the specimen.

6.3 Methods of operation

The automatic method of operation which forms the basis of the description above raised some uncertainties with regard to sources of error in a rather unrefined experimental apparatus. For example, if the pawl failed to engage the velocity of drawing could be modified to some uncertain degree because of motion of both ends. In order to check the principle some tests were made with the carriage (3) fixed to the frame (7). In this case the tension was estimated from the extension of the catapult cord. This is a method which could be more reliable if steel wire were used but with terylene, even with hot-stretched material, there is some creep although it is not much at extensions below 5% as used in these experiments.

With the carriage fixed as described above the release of the pin (6) was under the control of the operator. Release in this method could also be done when the carriage was free of the main frame and was also adopted when the very high speed camera was used.

6.4 Inertia and rigidity limitations

Ideally it is desired to apply an instantaneous velocity to the specimen but the mass of the member (5) and the spool (9) will give the end a finite acceleration. It is necessary to make these connection pieces as light as possible; in this case the member (5) weighs 1·65 oz and the spool 0·5 oz giving a total weight of 2·15 oz. For the particular weight of terylene cord used the rate of build up of velocity is shown in Fig.6 showing that the equilibrium velocity is barely reached within the time the tension has relaxed.

Another factor which should not be overlooked is the limited rigidity of the frame. When the carriage (3) comes against the pawl a stress is applied to the frame and this can generate vibration at the natural frequency of the frame. This will cause oscillations of the spool (2) and these may show up as a ripple on the velocity at which the spool (9) is observed to move. Ideally, the frame should be heavy and rigid with plenty of structural damping to absorb the shock.

7 RESULTS

7.1 The test specimens

Although basically the material should be studied as single filaments there is a more practical need to examine the material in the form of an energy absorbing link consisting of a number of turns over spools. Thus a link was made with sufficient turns to be compatible with the 400 lb terylene webbing. From Fig.4 it could appear desirable to try a link which could draw at about 50 lb and to allow this to be drawn by the relaxing of a length of webbing tensioned to about 112 lb.

A number of turns were wound over the spools whose centres were set $5\frac{3}{4}$ inches apart. This enabled the specimen to be extended about $\frac{1}{4}$ inch when inserted into the test rig so that it was then under a small tension. On the basis of the slow speed drawing characteristics of the polythene the spools were wound to twenty five turns to give the estimated 50 lb drawing tension but it was soon found that the force was higher when drawn at speed. Later, specimens were only wound to 17-18 turns.

Because some early work[7] by Messrs. F. G. Miles Engineering Ltd on the use of undrawn nylon in energy absorption showed that it was essential to wind the yarn twice round each spool before passing to the other end this technique was first used in winding the polythene specimens. However, if was found that polythene could be simply wound over the spools, with half turn at each spool, without the material breaking prematurely and specimens were wound in this way for the tests described.

7.2 Preliminary tests

Some thirty were made which included some standard 2 inch dumb-bell test pieces cut from pressure moulded polythene sheet. These cut test pieces did not drawn as reliably as the 0·040 inch filament, there being some random fracture before the specimen was fully drawn so that work was concentrated on the filament which could draw without fracture. For these tests the movement of the specimen was photographed with an Eclair camera working at 240 frames per second but this camera speed did not prove fast enough.

7.3 Final measurements

Three tests were made in which the photography was taken with an Eastman Type III camera operating at 970 frames per second and it is these three tests which have been analysed and reported. The laboratory temperature was about 18°C.

For purposes of measurement a scale consisting of alternate black and white vertical bars of 1 inch width was placed 9 inches behind the specimen. Since the camera was 16 feet from the specimen measurements made with reference to this scale should be reduced by 4·5%. The distance moved by the end of the specimen between one frame and the next was taken as a measure of the velocity of movement.

Altogether in any test there were about forty significant frames and the accuracy with which each measurement could be made left a fair random component. Therefore, a three point smoothing of the measurements was made and these smoothed means are plotted in Fig.11. Corrections for scale parallax and camera speed have been made. The first test shows some erratic behaviour but the other two give results which are substantially as the theory predicts. All these tests show very definite indications of the moment the reflected relaxation front returned to the specimen.

The velocities reached in these tests were lower than predicted but were of the order of 50 feet per second, the value which is was desired to demonstrate was possible without premature fracture for the engineering application. In order to comply with the requirements of the theory the force exerted by the reaction to the drawing would require to be higher. Allowing for the friction of the carriage the initial tension reached is estimated at 128 lb. In the second test the carriage was locked and the tension estimated from the extension. However, the resulting velocity is greater than in the third test, where the operation of the rig was as primarily described. This suggests that the tension had been overestimated. The velocity reached at the end of the third test, the relevant case, is 45 feet per second and the velocity has been rising steadily toward an asymptotic value of 48 feet per second. Referring to Fig.4 it is seen that the free relaxation velocity at 128 lb tension is 197 feet per second and at an equilibrium relaxation velocity of 48 feet per second the drawing force is estimated at 105 lb. This is rather more than twice the value estimated from slow speed performance suggesting that the force to drawn a filament at these speeds is about 3·1 lb instead of the 1·7 lb quoted in Table 1. This dynamic drawing force is about 90% of the breaking strength indicating that the maximum speed for drawing must be only just above those recorded in these experiments.

With reference to the initial acceleration determined from the tests a comparison between Figs.10 and 11 shows that there is not a large discrepancy between experiment and theory; if anything the actual acceleration is less. A discrepancy in this direction is to be expected because any influence of the inertia of the polythene has been neglected. The accuracy of the individual points is, however, not good enough for any refined comparisons.

It appears possible, from Fig.11, to determine fairly precisely the time that the relaxation returns to the specimen since the intersection of the curves through the points before and after this time is fairly sharp. The intersections are not identical in time on each curve and are later the higher the relaxation velocity. This is consistent with other observations that strain disturbances travel faster the higher the tension[8]. The estimated velocities of propagation are 6800, 6000 and 6500 feet per second for the respective tests. The values related to the curve of Fig.6 are all slightly greater than the computed values.

8 DESIGN FOR MINIATURE TESTING

It is obviously desirable to be able to design a piece of equipment on which single filaments, or at least single loops can be tested. The problem of design is to keep the mass connecting the plastic test specimen to the catapult cord as low as possible. One proposal is to have a small ceramic thimble at the connection over which a loop of the plastic passes. This thimble has a small central hole which is sufficient to take a loop of piano wire to connect to the end of the catapult cord and a copper or other suitable wire which will restrain the cord before release. The copper wire is held, as a V, by two terminals on a frame and the release is effected by passing a sufficient electric charge through the wire to fuze it within the ceramic eye.

Another proposal by which specimens could be drawn initially at nearly constant velocity is to allow the catapult cord to begin to relax and to travel a short distance before it engages the end of the specimen. The catapult cord carries a mass, M, and this is allowed to accelerate to a velocity in excess of the equilibrium velocity of drawing. On impacting a mass, M_2, connected to the end of specimen, there is a sharing of momentum and the combined momentum will have a velocity consistent with the equilibrium velocity of drawing.

9 CONCLUSIONS

A technique for investigating the drawing of polymers at speed has been described by which it has been demonstrated that low density polythene filament can be drawn at velocities up to fifty feet per second. It has also been shown that the force generated when the material is drawn at these speeds is higher than, about double, that obtained at very slow rates of drawing. Deceleration tests of weights confirm this conclusion in the equation of work done to loss of potential energy.

The theory of the relaxation process shows that, by using relaxation cords of different weights, it should be possible to resolve the first order influence of speed on the drawing force.

The technique has potentialities for further development and refinement.

Appendix A

DETAILED EVALUATION OF INTEGRAL

The following standard form, quoted in mathematical tables,

$$\int \frac{dx}{a\,x^2 + b\,x + c} = \frac{1}{\sqrt{b^2 - 4a\,c}} \log \frac{2a\,x + b - \sqrt{b^2 - 4a\,c}}{2a\,x + b + \sqrt{b^2 - 4a\,c}} \qquad \text{(A-1)}$$

can be used to evaluate the velocity when the force is expressed to the second derivative. This standard form is to be compared with

$$\int - \frac{dt}{M} = \int \frac{dv}{-(T_1 - F_0) + (m\,c + F_1)\,v + F_2\,v^2} + \text{constant} \qquad \text{(A-2)}$$

so that

$$a = F_2, \quad b = m\,c + F_1, \quad c = -(T_1 - F_0)$$

and

$$\sqrt{b^2 - 4a\,c} = \sqrt{(m\,c + F_1)^2 + 4F_2(T_1 - F_0)}. \qquad \text{(A-3)}$$

When F_2 is small compared with m c

$$\sqrt{b^2 - 4a\,c} = (m\,c + F_1) + \frac{2F_2(T_1 - F_0)}{m\,c + F_1}. \qquad \text{(A-4)}$$

Evaluating the integral of equation (A-2)

$$-\frac{t}{M} =$$

$$\frac{1}{mc + F_1 + 2F_2(T_1 - F_0)/(mc + F_1)} \log \frac{2F_2 v - 2F_2(T_1 - F_0)/(mc + F_1)}{2F_2 v + 2(mc + F_1) + 2F_2(T_1 - F_0)/(mc + F_1)} + \text{const.}$$

or

$$-\left\{\frac{(mc + F_1)^2 + 2F_2(T_1 - F_0)}{M(mc + F_1)}\right\} t = \log \frac{v - (T_1 - F_0)/(mc + F_1)}{v + (mc + F_1)/F_2 + (T_1 - F_0)/(mc + F_1)} + \text{const.}$$

.... (A-5)

When t = 0, v = 0; so that the constant of integration is

$$-\log \frac{-(T_1 - F_0)/(m\,c + F_1)}{(m\,c + F_1)/F_2 + (T_1 - F_0)/(M\,c + F_1)}.$$

Now consider an equation of the form

$$e^{-\lambda t} = \frac{1 - Av}{1 + Bv} \tag{A-6}$$

and comparing this equation with (A-5) above

$$\lambda = \frac{(m\ c + F_1)^2 + 2F_2(T_1 - F_0)}{M(m\ c + F_1)}$$

$$A = \frac{m\ c + F_1}{T_1 - F_0}$$

$$B = 1 \Big/ \left(\frac{m\ c + F_1}{F_2} + \frac{T_1 - F_0}{m\ c + F_1} \right) = \frac{F_2(m\ c + F_1)}{(m\ c + F_1)^2 + F_2(T_1 - F_0)} .$$

Equation (A-6) can be transformed to

$$\frac{v}{1 + B\ v} = \frac{1}{A + B}\left(1 - e^{-\lambda t}\right) \tag{A-7}$$

and it follows, when F_2 is small compared with m c, that B is small compared with A. Therefore,

$$\frac{v}{1 + \frac{F_2\ v}{m\ c + F_1}} = \frac{T_1 - F_0}{m\ c + F_1}\left(1 - e^{-\left\{\frac{mc+F_1}{M} + \frac{2F_2(T_1-F_0)}{m(mc+F_1)}\right\}t}\right) \tag{A-8}$$

is the solution for v in terms of t for first order coefficients of F_2.

Table 1

Low density polyethylene filament diameter nominally 0·040 inch supplied by Messrs. Courtaulds Ltd.

Diameter	1·03 mm
Tex	797 gm/km
Tenacity	1·9 gm/tex
Drawing tension	1·7 lb
Breaking tension	3·4 lb
Breaking extension	1500%

SYMBOLS

c	velocity of propagation of strain
e	strain
k	stiffness constant
m	mass per unit length of cord
t	time
v	velocity of relaxation
x	displacement of end of specimen
F	force in specimen
F_0, F_1, F_2	force coefficients
M	mass of connection
T	cord tension

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6	J. E. Swallow	The shrinkage and extensibility of heated yarns Part III - Terylene. R.A.E. Technical Note No. Chem.1344, Jan.1959
7		General report on methods of absorbing energy by means of undrawn nylon, with particular respect to landing of parachuted equipment. F. G. Miles Engineering Ltd., Test Note No.12 1964
8	B. J. Brinkworth	The velocity of propagation of strain in textile cables. R.A.E. Technical Note No. ME.211 Nov.1955

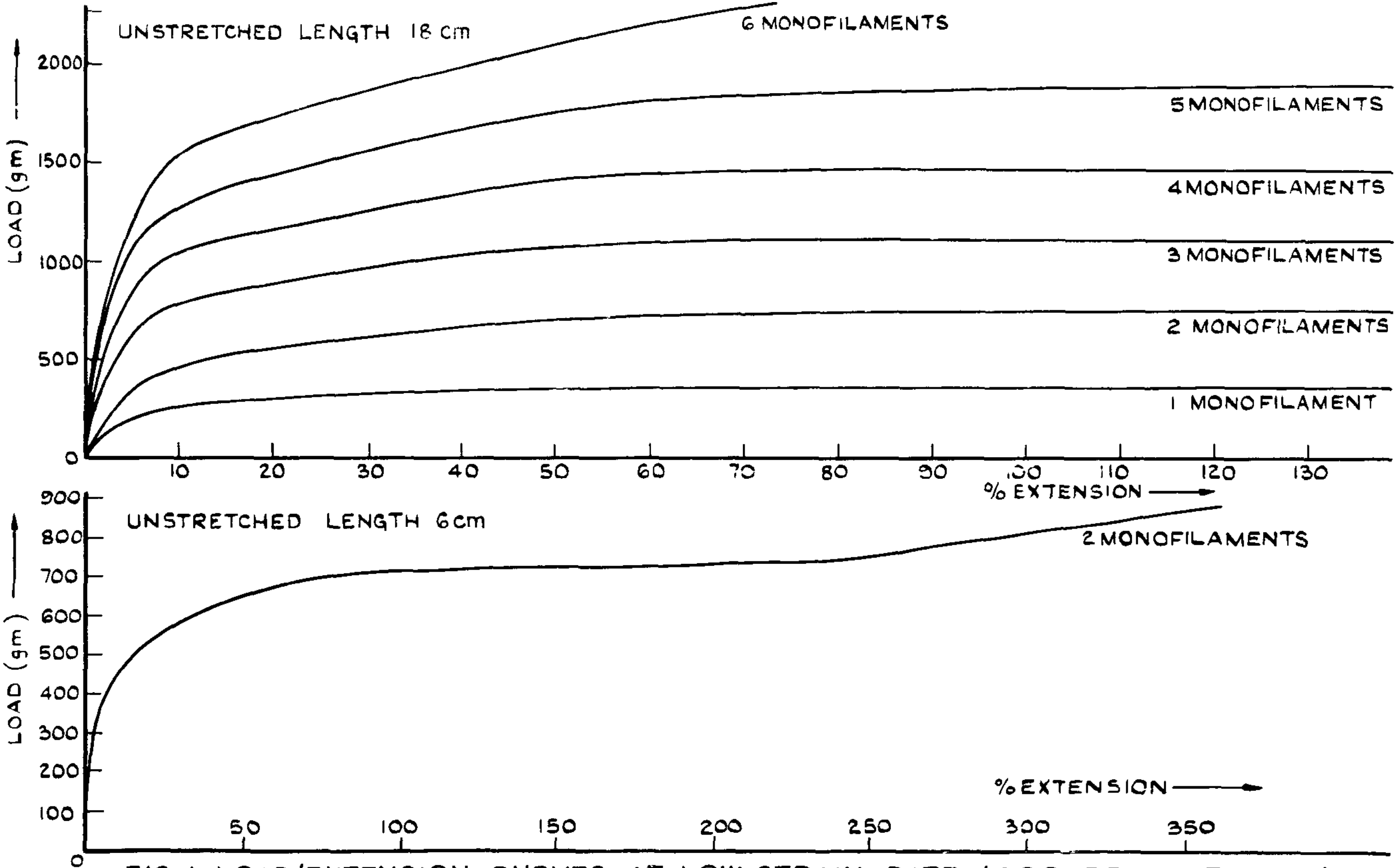


FIG. 1 LOAD/EXTENSION CURVES AT LOW STRAIN-RATE (GOODBRAND TESTED) UP TO 0·5 (cm/sec)

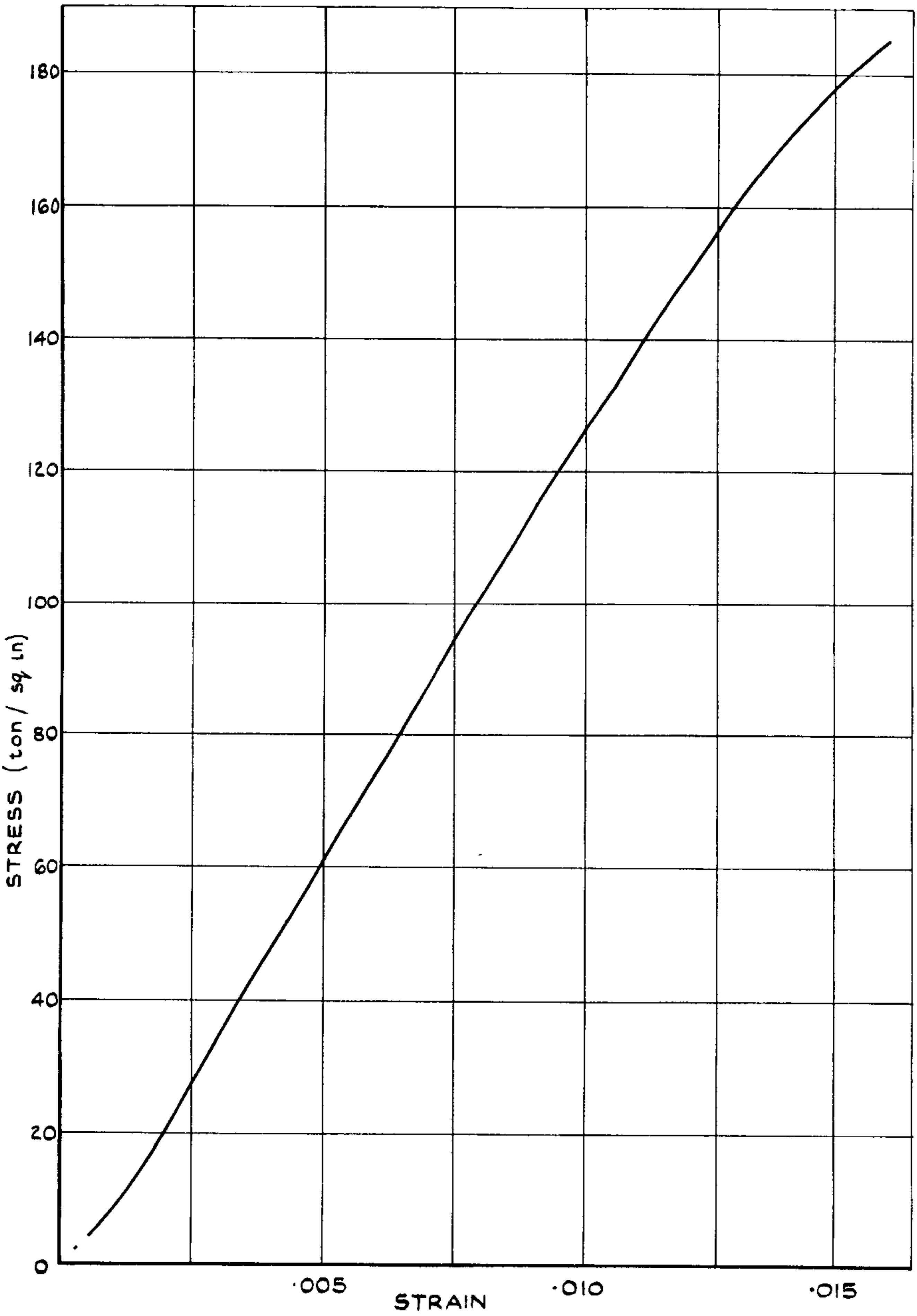


FIG. 2 LOAD EXTENSION CURVE FOR SAMPLE 26 SWG MUSIC WIRE
ABSTRACT FROM RAE REPORT D I 57 JUNE 1937

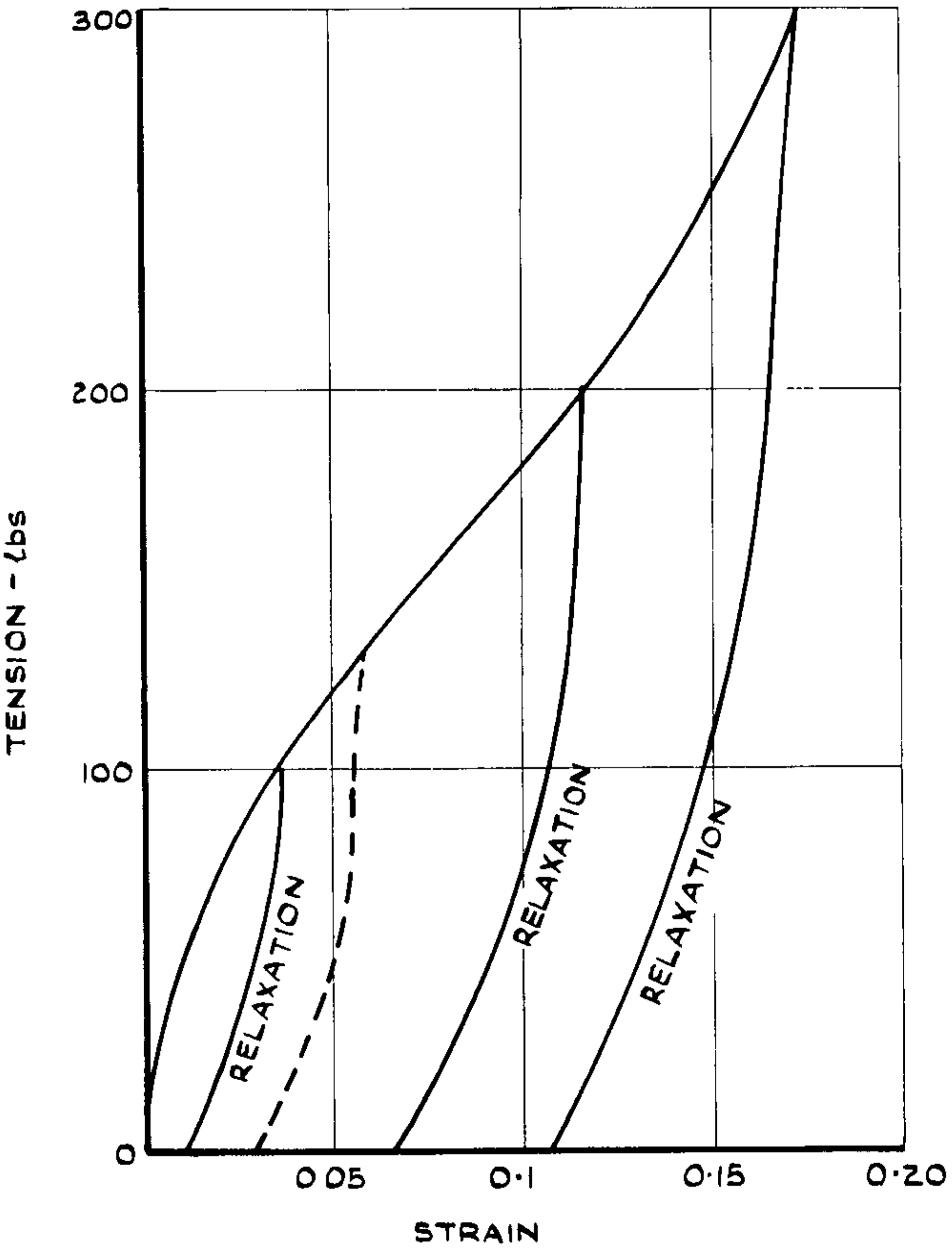
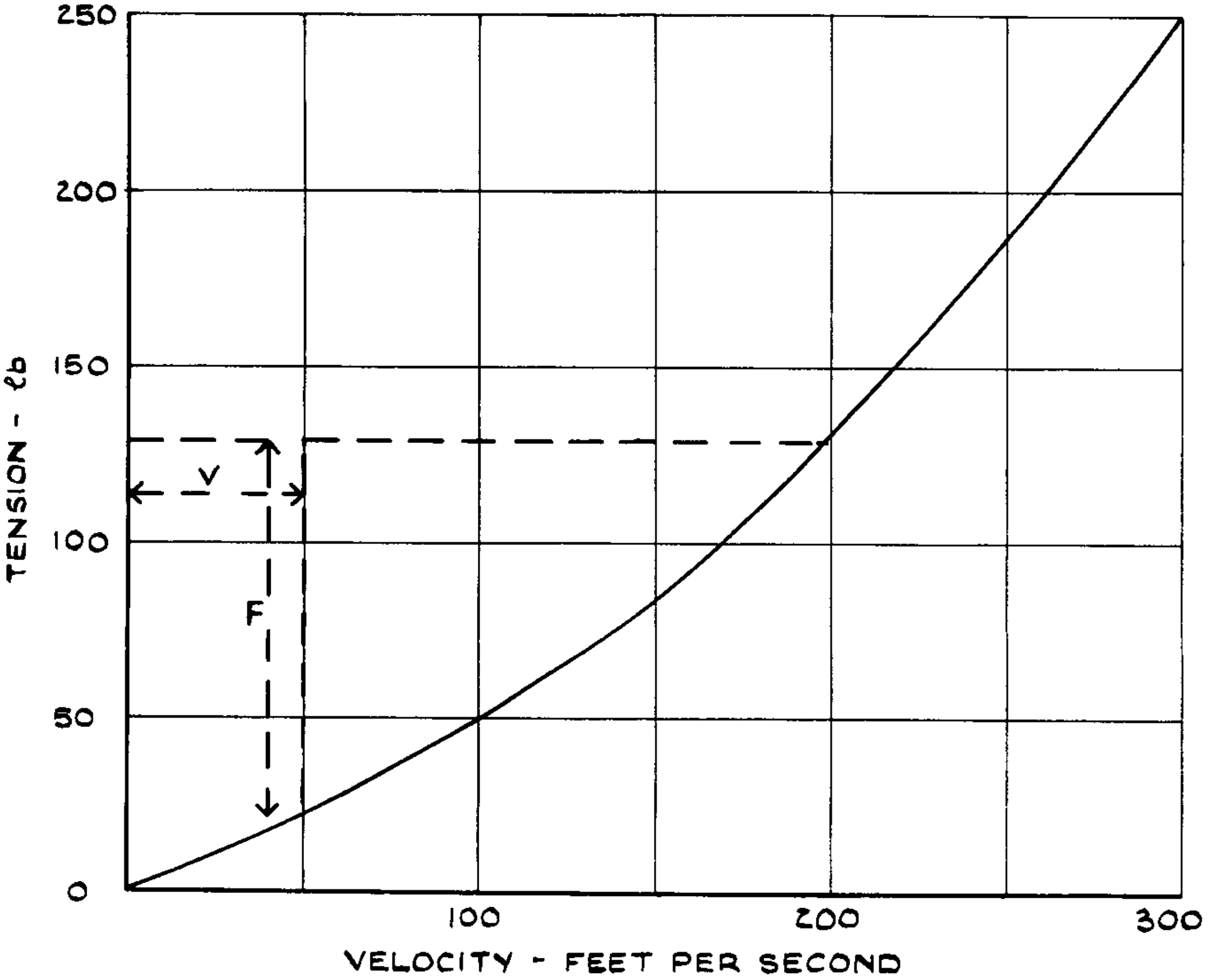


FIG 3 TERYLENE TUBULAR BRAID
TENSION - STRAIN . EXTENSION - RELAXATION CURVES .
250 DENIER YARN - 5003 GRMS PER KILOMETER
OR 1 045 x 10^{-4} SLUGS PER FOOT



$$\underset{\text{INITIAL TENSION}}{T} = \underset{\text{CORD 'CONSTANT'}}{(mc)}\ \underset{\text{RELAXATION VELOCITY}}{V} + \underset{\text{DRAWING FORCE}}{F}$$

FIG 4 TERYLENE TUBULAR WOVEN CORD
MASS $1{\cdot}045 \times 10^{-4}$ SLUGS PER FOOT
RELATION BETWEEN TENSION AND FREE RELAXATION VELOCITY

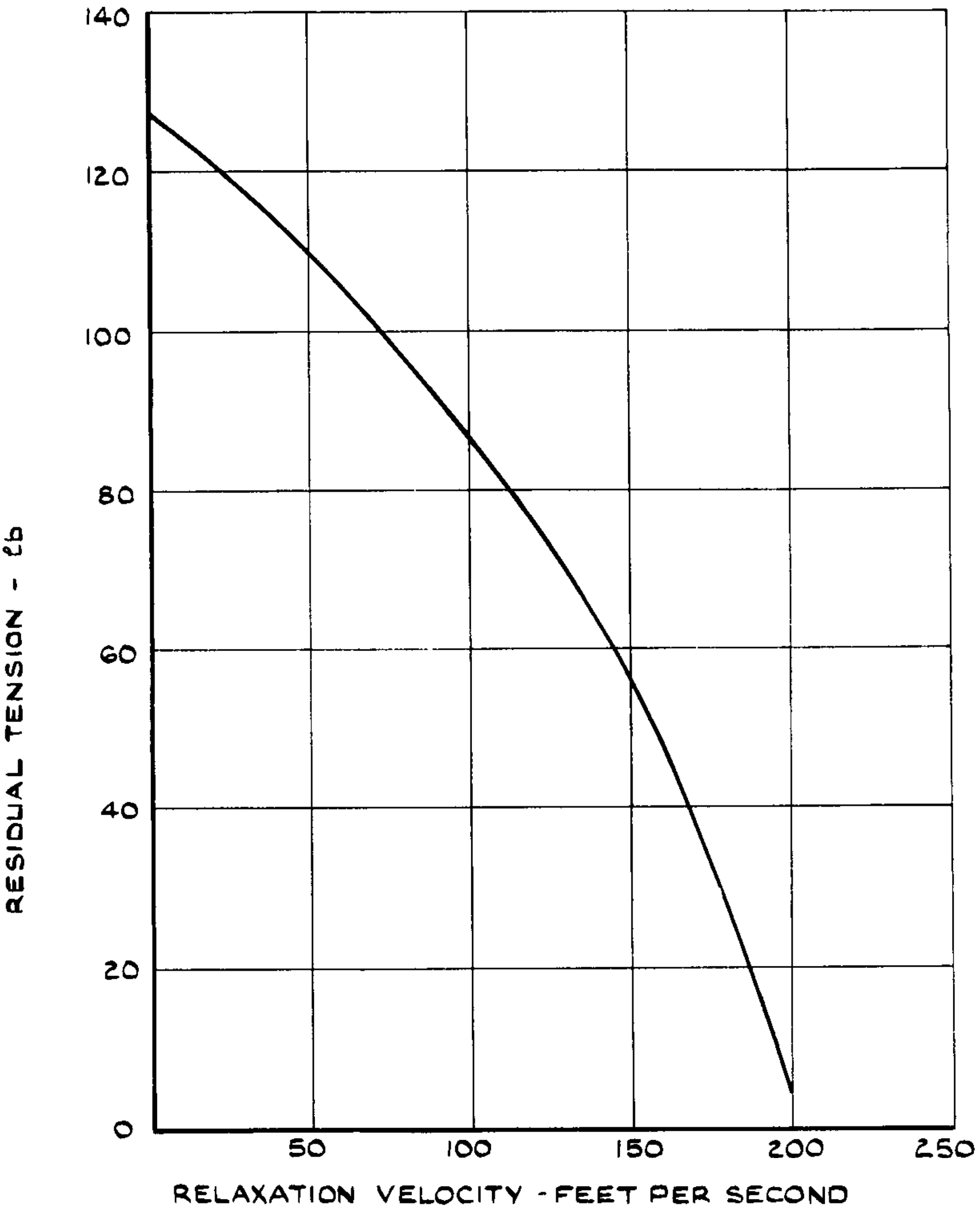


FIG 5 RELATION BETWEEN PARTIAL RELAXATION VELOCITY AND RESIDUAL TENSION FOR PARTICULAR CASE

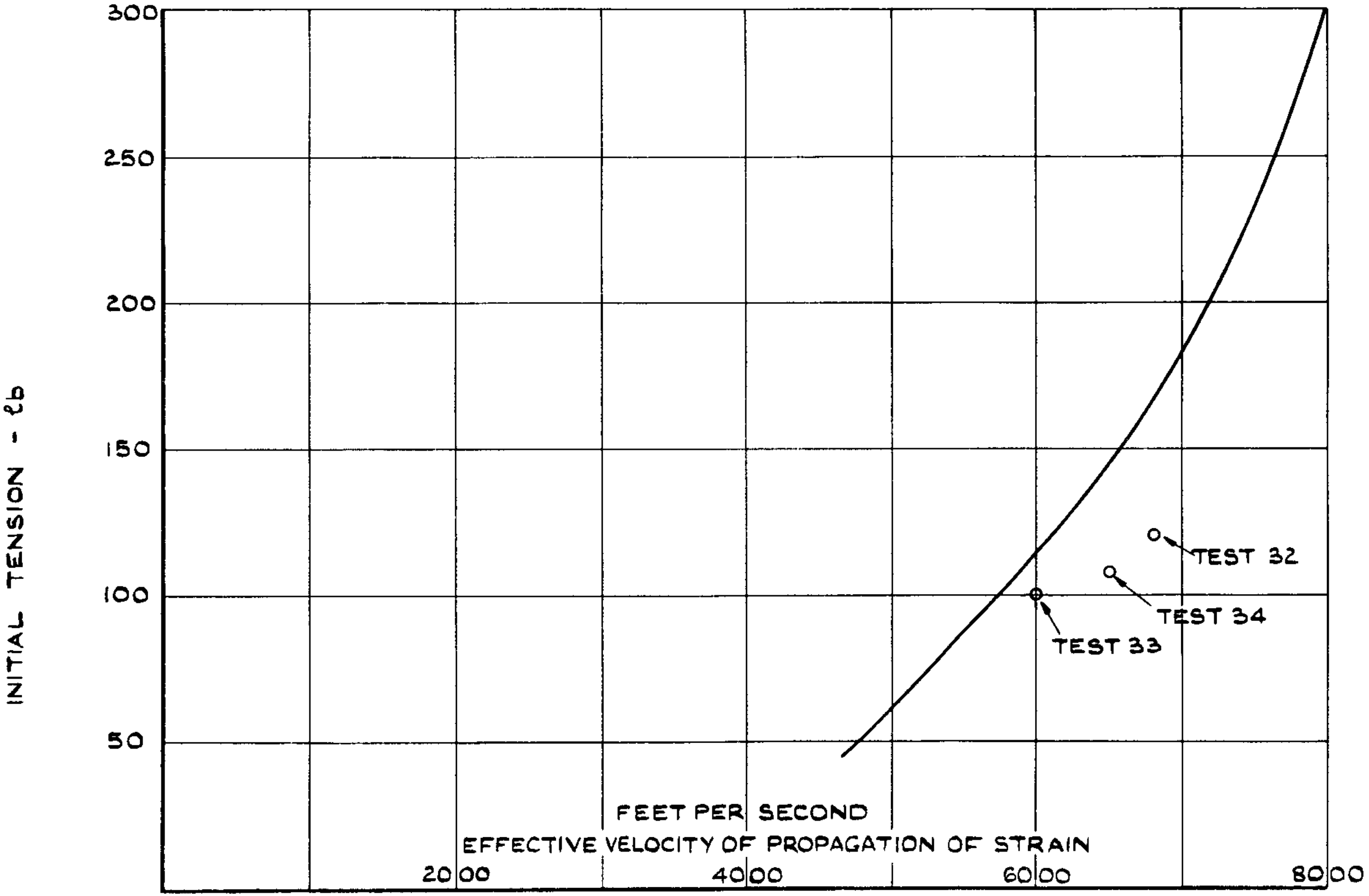


FIG. 6 TERYLENE TUBULAR WOVEN CORD
DEPENDENCE OF VELOCITY OF PROPOGATION OF STRAIN ON INITIAL TENSION

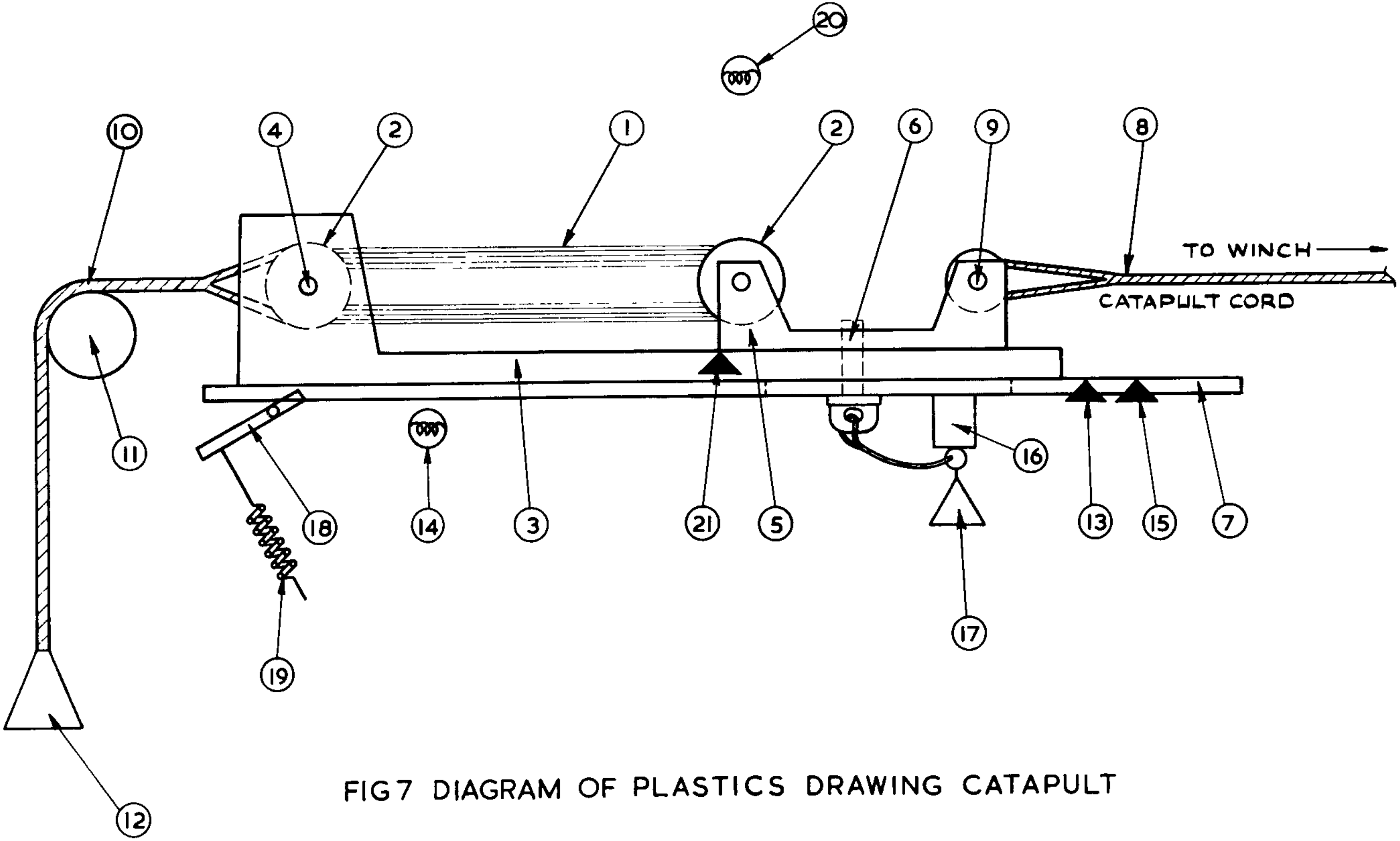


FIG 7 DIAGRAM OF PLASTICS DRAWING CATAPULT

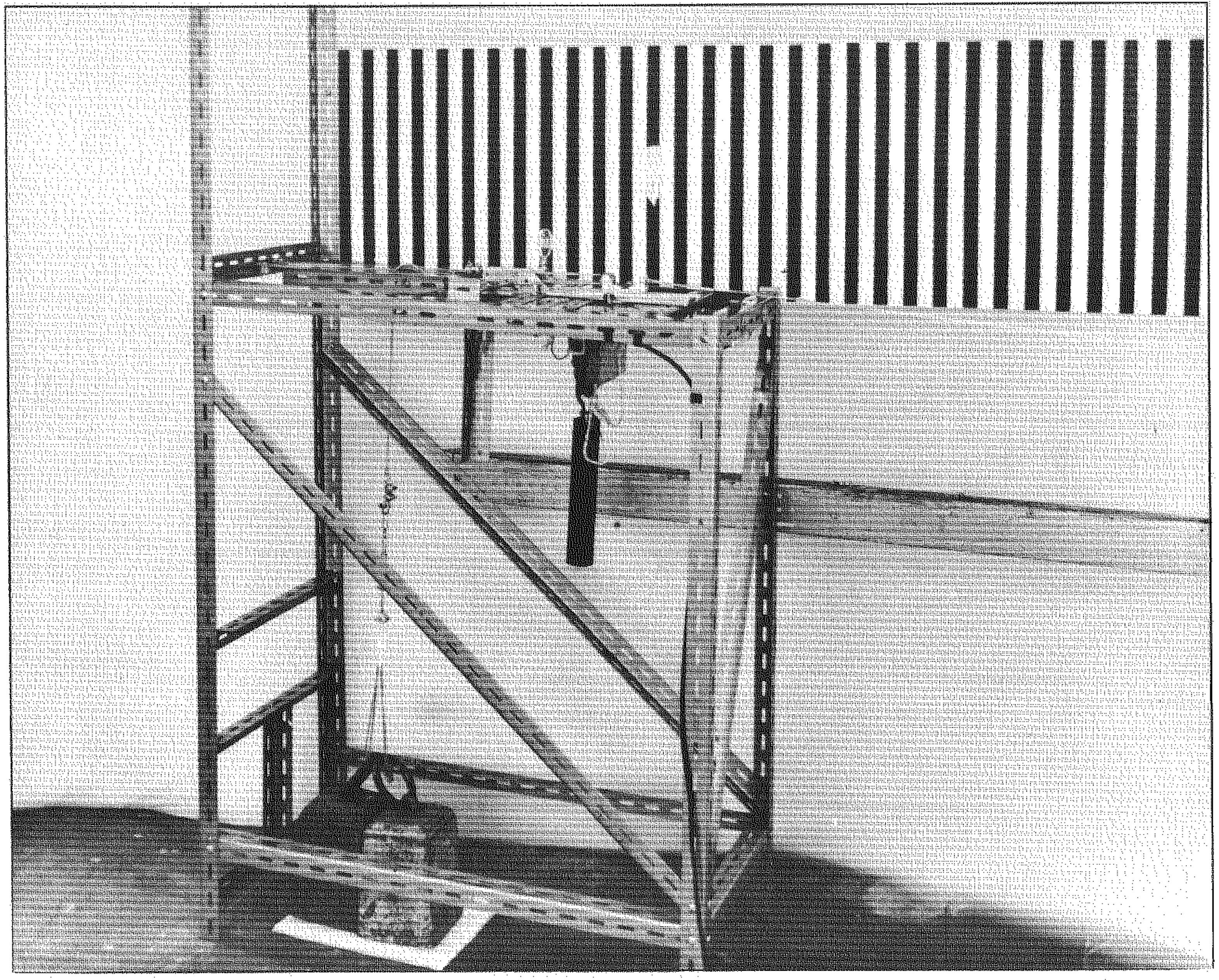


Fig. 8

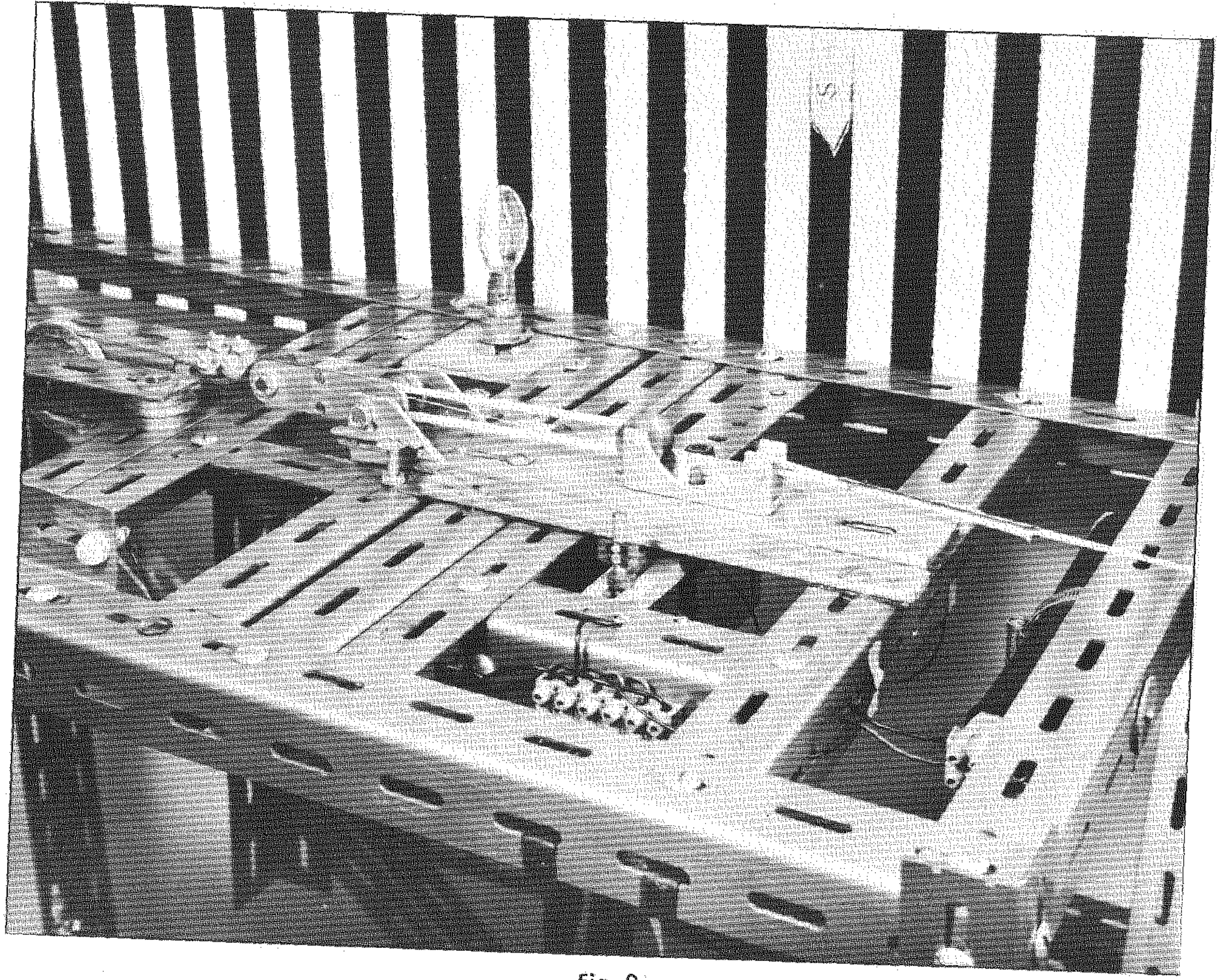


Fig. 9

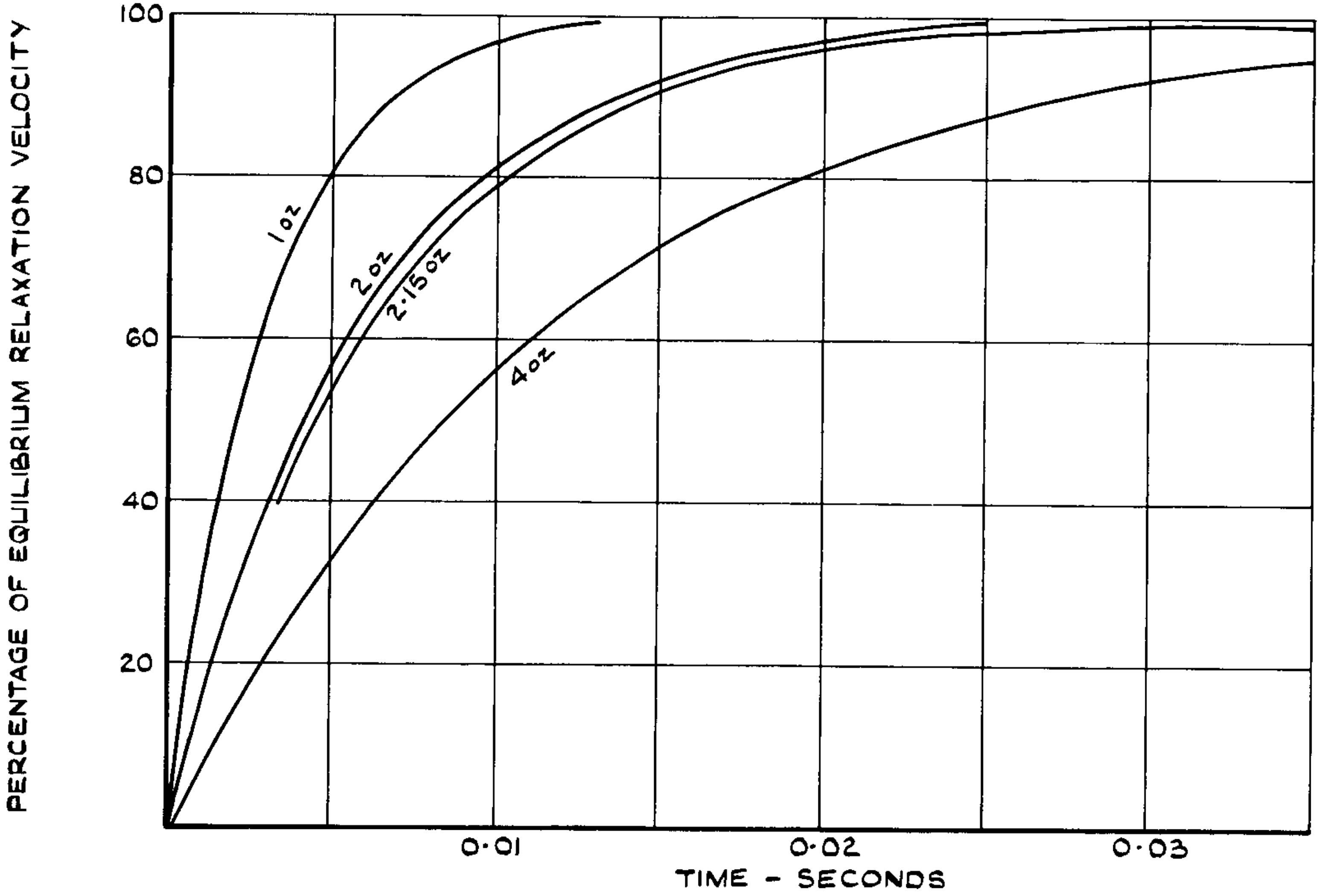


FIG. 10 INFLUENCE OF MASS OF CARRIAGE ON ACCELERATION

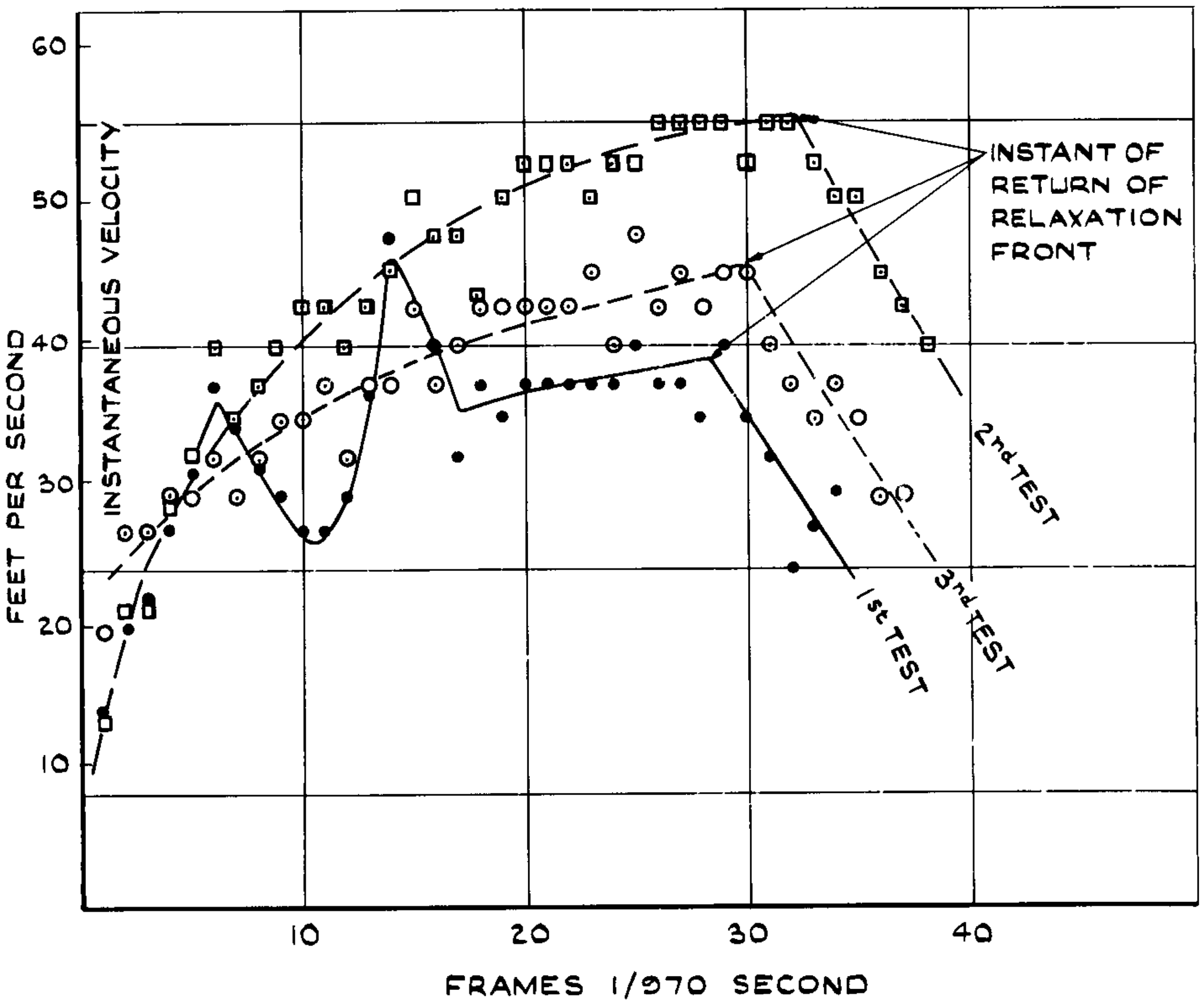


FIG 11 VARIATION OF INSTANTANEOUS DRAWING SPEED WITH TIME

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Stevens, G. W. H.
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