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An Experimental Investigation of Laminar Heat Transfer in a Uniformly Heated Tube Rotating about a Parallel Axis

by

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An Experimental Investigation of Laminar Heat Transfer in a Uniformly Heated Tube Rotating about a Parallel Axis

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SUMMARY

The results of an experimental investigation which studies the influence of rotationally induced buoyancy on heat transfer in a tube which rotates about an axis parallel to itself is presented. Data obtained with water and 100% glycerol appear to confirm the qualitative description of flow given by two theoretical analyses. A reliable qualitative method of prediction is still unavailable.

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1. Introduction

When a fluid flows through a tube which is rotating about some arbitrary axis, as shown in Fig. 1, the presence of centripetal and Coriolis acceleration components may cause secondary flow to occur in the planes perpendicular to the axis of the tube. Flow configurations of this type are often used in the design of cooling systems for certain rotating components, notably turbine rotor blades and the rotor conductors of large electrical machines. Under these circumstances density gradients in the fluid resulting from the temperature distribution give rise to buoyancy forces which further influence the flow field. When the angular velocity of the tube is large these buoyancy effects must be taken into account in the design calculations. However, little information is currently available concerning the influence of rotation on heat transfer and flow resistance. A case of particular interest is that where the axis of the tube is parallel to, but displaced from, the axis of rotation. This system is used for the forced cooling of electrical machine rotor conductors and is of current importance in the design of generator sets in the power range 500-1000 MW where water cooled rotor conductors are employed.

Morris[1,2] considered established laminar flow in this type of system both with and without heat transfer. For a uniformly heated tube it was shown, using a series expansion technique for the solution of the basic conservation

equations,/

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equations, that significant changes occurred in the heat transfer and pressure drop data. However, the solutions presented were restricted to low rotational speeds and heat flux owing to the form of the series used for solving the equations. The influence of the earth's gravitational field along the axis of the tube was also included in the analysis. This flow geometry comprised part of a thermosyphon circuit tested by Morris[3] and Davies and Morris [4] which had a projected application for cooling the rotor conductors of electrical prime movers. In practical applications using this type of rotating cooling geometry it is unlikely that conditions of established flow will occur and Humphreys, Morris and Barrow[5] presented the results of an experimental investigation using air as the working fluid in the entry region. Le Feuvre[6] has also presented the results of an experimental programme where this cooling system is used for the cooling of the rotor drums of electrical prime movers.

Mori and Nakayama[7] made a theoretical appraisal of this problem for established laminar flow and uniformly heated walls. These authors assumed a gross secondary flow consistent with high rotational speeds and subsequently derived heat transfer and pressure drop data for a variety of Prandtl numbers. They also showed that the influence of Coriolis forces was significant.

It is the purpose of the present communication to show that experimental data derived with water and glycerol for this rotating configuration shows qualitative agreement with the predictions of Mori and Nakayama. However, before the experimental details are presented a brief review of the two known attempts to obtain a theoretical solution will be made. This will also serve to illustrate the salient features of the flow regime obtained with this rotating system.

2. Review of Current Theoretical Work

The severity of the analysis of the flow and heat transfer behaviour in the rotating tube considered here may be illustrated by a brief consideration of the qualitative features of the flow involved. Fig. 2 shows the physical model and also indicates the co-ordinate system which will be used.

Under the influence of the radial component of acceleration the cooler and thus less dense particles of fluid tend to move towards the outer portion of the tube periphery. Conversely the warmer fluid near the heated walls moves towards the axis of rotation and thus sustains a buoyancy motivated secondary flow in the cross section of the tube. When one further considers the axial flow along the tube it is apparent that the migration of fluid in the cross stream direction and its associated re-distribution of momentum produces an increase in the resistance to flow. This is important as it affects the design of the pumping equipment in situations which utilize this cooling geometry. In fact it can be shown that, as a result of the secondary flow, the axial velocity profile deviates from its symmetrical form with a stationary tube so that the location of the point of maximum velocity moves towards the axis of rotation. The secondary flow also increases the heat transfer from the tube surface to the fluid.

We require a theoretical solution which permits evaluation of the velocity components in each of the three co-ordinate directions, the pressure distribution and the temperature distribution. When this has been achieved the calculation of heat transfer and flow resistance is but an easy step.

The theoretical attempts currently available have been restricted to the case where the tube is uniformly heated and the properties of the fluid, with the exception of density, are invariant. Also, distances along the tube sufficiently removed from the entrance to enable the assumption of a velocity

field/

field which is only weakly dependent on axial location have been considered.

Using these assumptions the present author[2] obtained the following laminar flow equations. (Symbols are as defined in the List of Symbols).

Conservation of Momentum

$$u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right] + \Omega^2\beta\left[T_w - T\right]\left[r + H\cos\theta\right]$$

$$+ 2\Omega\beta\left[T_w - T\right]v \qquad \ldots (1)$$

$$u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + \frac{uv}{r} = -\frac{1}{\rho r}\frac{\partial v}{\partial \theta} + \nu\left[\nabla^2 v - \frac{v}{r^2} + \frac{2}{r^2}\frac{\partial u}{\partial \theta}\right] - H\Omega^2\beta\left[T_w - T\right]\sin\theta$$

$$- 2\Omega\beta\left[T_w - T\right] \qquad \ldots (2)$$

$$u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \theta} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w \qquad \ldots (3)$$

Conservation of Mass

$$\frac{\partial (ru)}{\partial r} + \frac{\partial r}{\partial \theta} = 0 \qquad \ldots (4)$$

Conservation of Energy

$$u\frac{\partial T}{\partial r} + \frac{v}{r}\frac{\partial T}{\partial \theta} + w\frac{\partial T}{\partial z} = \alpha\nabla^2 T \qquad \ldots (5)$$

By assuming the existence of a weak secondary flow solutions for the velocity and temperature fields were subsequently derived by expanding the solution about the known result for a non-rotating tube. As a result the following expressions for a resistance coefficient, c_f, and Nusselt number, Nu_m, were calculated

$$c_f = \frac{16}{Re_p[1 - 0\cdot 2100(Re_p\, Ra_r/4608)^2]^2} \qquad \ldots (6)$$

where

$$c_f = -\frac{a}{w_m^2}\frac{\partial p}{\partial z} \qquad \text{(Fanning friction factor)} \qquad \ldots (7)$$

$$Re_p = -\frac{a^3}{4\nu^2\rho}\frac{\partial p}{\partial z} \qquad \text{(Pseudo Reynolds number)} \qquad \ldots (8)$$

Ra_r/

$$Ra_r = \frac{\beta H\Omega^2 r a^4}{\alpha\nu} \quad \text{(Rotational Rayleigh number)} \qquad \cdots (9)$$

The above definitions of the resistance coefficient and Reynolds number become those usually associated with pipe flows where the pipe is stationary.

$$Nu_m = \frac{[a' - (Re_p\, Ra_r/d')^2\,(e' + f'Pr + g'Pr^2)]}{[j' - (Re_p\, Ra_r/d')^2\,(n' + s'Pr + t'Pr^2)]} \qquad \cdots (10)$$

where

$a' = 0{\cdot}2500$	$d' = 4608$	$e' = 0{\cdot}0328$
$f' = 0{\cdot}0000$	$g' = 0{\cdot}0018$	$j' = 0{\cdot}0417$
$n' = 0{\cdot}0133$	$s' = 0{\cdot}0035$	$t' = 0{\cdot}0009$

and

$$Nu_m = \frac{q}{\pi k(T_w - T_m)} \qquad \cdots (11)$$

The definition of Nu_m given above uses the difference between the temperature of wall and the integrated mean fluid temperature (as obtained by simple integration of the temperature profile) as a representative temperature difference for the motivation of heat transfer. In practice one uses the mixed mean or bulk temperature, T_b, of the fluid. A Nusselt number involving T_b may be calculated but the algebraic labour involved is considerable.

Because of the requirement of a weak secondary flow for the applicability of equations (6) and (10) these solutions must be restricted to low rotational speeds and heating rates. Indeed, extrapolation of the results into regions where the product of Re and Ra is large indicates the "run away" nature of the solution technique. This point is illustrated in Fig. 3. However, the following interesting features emerge from the solutions.

Sample calculations indicate that increases in the resistance to flow and heat transfer result even with the weak secondary flow considered and that the proportional impairment in resistance can be greater than the improvement of heat transfer. This is an important feature when design assessment is being made. Also even though the influence of Coriolis force modifies the velocity and temperature fields it has no subsequent influence on the heat transfer and resistance to flow.

It is unlikely in practical situations that the rotational speeds and heat transfer rates are low enough to produce only weak secondary flow. Accordingly, Mori and Nakayama[7] considered the case where the secondary flow in the central region of the pipe was intense enough to enable gross assumptions concerning its nature to be made. Thus by assuming that the cross section of the pipe may be treated as a core flow where shear and heat transfer is dominated by secondary flow effects and a region near the wall of pseudo boundary layer nature, these authors proceed to develop expressions for relative increases of Nusselt number and resistance coefficient using a momentum integral technique. These increases are based again on known analytic expressions for non-rotating tubes with the same boundary conditions. The actual expressions derived were as given below

For Prandtl number $\geq$ unity

$$c_f = 0{\cdot}104\, c_{fo}\, (3F(Pr) - 1)^{1/5}\, (Re\, Ra_r)^{1/5} \qquad \cdots (12)$$

$$Nu_b = \frac{0{\cdot}191\, Nu_{bo}\, (3F(Pr) - 1)^{1/5}\, (Re\, Ra_r)^{1/5}}{F(Pr)[\,(1 + 1/10\, Pr\, F(Pr)\,)\,]} \qquad \cdots (13)$$

where

$$Re = \frac{2aw_m}{\nu} \quad \text{(Reynolds number)} \qquad \cdots (14)$$

$$F(Pr) = \frac{2}{11}\left[1 + \sqrt{\left(1 + \frac{77}{4Pr^2}\right)}\right] \qquad \cdots (15)$$

$$Nu_b = \frac{q}{\pi k(T_w - T_b)} \qquad \cdots (16)$$

For Prandtl number $\leq$ unity

$$c_f = 0{\cdot}149\, c_{fo}\, (F(Pr) - 1 + 1/3F(Pr)\,)^{1/5}\, (Re\, Ra_r)^{1/5} \qquad \cdots (17)$$

$$Nu_b = \frac{0{\cdot}273\, Nu_{bo}\, (F(Pr) - 1 + 1/3F(Pr)\,)^{1/5}\, (Re\, Ra_r)^{1/5}}{F(Pr)\, (1 + 1/10\, Pr\, F(Pr)\,)} \qquad \cdots (18)$$

where

$$F(Pr) = \frac{1}{5}\left[2 + \sqrt{\left(\frac{10}{Pr^2} - 1\right)}\right] \qquad \cdots (19)$$

In order to account for the effect of Coriolis force Mori and Nakayama recommend that the expressions given by equations (12), (13), (17) and (18) are multiplied by a correction factor, K , given by

$$K = \left[1 - \frac{0{\cdot}486\, (3F(Pr) - 1)^{2/5}\, J}{F(Pr)\, (PrF(Pr)\, 5 + 2)\, (Re\, Ra_r)^{3/5}}\right. \qquad \cdots (20)$$

for $Pr \geq 1$, and

$$K = \left[1 - \frac{0{\cdot}995\, (F(Pr) - 1 + 1/3F(Pr))^{2/5}\, J}{F(Pr)\, (PrF(Pr)\, 5 + 2)\, (Re\, Ra_r)^{3/5}}\right. \qquad \cdots (21)$$

for $Pr \leq 1$, where

$$J = \frac{2a^2\Omega}{\nu} \qquad \cdots (22)$$

3. Description of Apparatus

The experimental data to be reported in this paper was obtained using a rotating thermosyphon apparatus originally designed for the investigation of thermosyphon characteristics. A detailed description of the equipment has been presented by Morris[3]. However, for the sake of completeness, a brief description of the salient features will now be given.

A tubular closed loop in the form of a rectangle ABCD was constrained to rotate about the centre-line of the limb CD, as illustrated in Fig. 4. The limb CD formed part of a built up rotor which could be rotated at speeds in the range 0 - 300 rev/min. The circuit ABCD could be completely filled with various fluids.

The test section itself consisted of a copper rod 25.4mm external diameter with a 6·35mm diameter hole bored through the centre. Pyrotenax heating cable was embedded in a helical groove machined on the outside of the test section and silver soldered at a number of positions to ensure good thermal contact. The heating cable was composed of Nichrome resistance wire sheathed in a thin walled tube of stainless steel. Mutual electrical insulation between the resistance wire and the sheath was achieved with highly compressed magnesium oxide powder. The nominal length of the test section was 304·8mm, giving a length to diameter ratio of 48.

Nickel chrome/nickel aluminium thermocouples were soldered at both ends of the heated portion of the test section and also at a location 24 diameters downstream of the entry station. It was at this location that the heat transfer measurements were evaluated. Both ends of the test section were sealed with threaded end caps which were fitted with a copper/constantan thermocouple to permit measurement of the fluid temperature at the inlet and exit stations.

All thermocouple signals were taken to the stationary measuring equipment via a miniature instrumentation slip ring assembly located at one end of the rotor. External heat loss was reduced by covering the outside of the heater with a layer of refactory cement approximately 6mm thick. The heater assembly could easily be removed from the apparatus and the flow circuit was completed by fitting the short copper tubes seen at the inlet and exit stations into perspex radial limbs BC and DA.

During operation, the fluid in the limb CD was cooled using mains water flowing inside a coil fitted within this limb. Rotary seals at each end of the rotor permitted this secondary coolant to flow into and out of the apparatus.

The fluid under test is thus caused to circulate within the closed loop owing to the well known thermosyphon effect, the rate of flow being governed by the heat transfer from the test section for a given geometry of flow.

Direct measurement of the flow rate achieved could not be performed in this particular rotating test section. However this parameter could be calculated using a heat balance method after making measurements of heater power consumption, fluid temperature rise and external heat loss from the test section. The heat loss at any operating condition was determined from a series of heat loss calibration experiments performed with the interior of the test section filled with granulated cork.

Two/

Two experimental programmes were performed using water and 100% glycerol as the test fluids. Tests were conducted at rotational speeds of 50, 100, 200 and 300 rev/min which, for the apparatus used, gave centre-line centripetal accelerations in the range 0 - 15g.

4. Results and Discussion

The required dimensionless parameters derived from the test data obtained with water are shown plotted in Fig. 5 as Nu_b against $Re\ Ra_r$. It is seen that it is possible to discern separate lines for each of the four rotational speeds used and that the relative spacing is such that, for a given value of $Re\ Ra_r$, the heat transfer is reduced at the higher rotational speeds. Although for each individual data point it was not possible to control the value of Pr (because of variations in the properties of the fluid) observation of the range of Pr indicated that a mean value in the region of 6 was applicable. Consequently a reference line calculated from the results of Mori and Nakayama at $Pr = 6$ has been drawn for convenience of illustration.

It is seen that, although the test data is generally higher than that predicted, the data can be correlated by series of lines which are parallel to the prediction. Further, the parameter J, which relates the influence of Coriolis force to viscous force, also varies over each data point at a given rotational speed. At the maximum speed of 300 rev/min this parameter had a value in the region of 700. Consequently for $J = 700$ the correction factor suggested by Mori and Nakayama has been used and the result is shown in Fig. 5.

Theoretically it appears that the influence of J is not too significant in this case. However the test data shows a much greater sensitivity to J than suggested theoretically. It was found possible to derive an empirical equation which correlated the data for water and which allowed for the influence of J. The actual correlation was

$$Nu_b = 25{\cdot}7\ (Re\ Ra_r)^{1/5}\ J^{-0{\cdot}4} \qquad \ldots (23)$$

and is shown plotted, together with the test data, in Fig. 6.

It should be noted that the test data was taken at an axial location 24 diameters downstream from the entrance. It is likely with water that the influence of the entrance is still being felt and that any induced swirl on the fluid from the ducting upstream of the test section itself has had insufficient distance to decay. There remains as a result the unresolved point as to what extent J is reflecting the effect of Coriolis force and entrance effects. The entry effect will of course also be dependent on the speed of rotation. This effect of induced entry swirl has also arisen in the work of Humphreys, Barrow and Morris [5] where tests with air in the lower turbulent range were conducted.

One would expect the effect of J to be less pronounced when a fluid of high Pr was used in view of the associated large viscosity whether J reflected the effect of Coriolis force or entrance conditions. Accordingly, data for large values of Pr should be more likely to correlate on a single line when plotted as Nu_b against $Re\ Ra_r$. The data obtained with 100% glycerol as the test fluid is shown plotted in this form in Fig. 7 and indeed it was found impossible to discern separate lines at each of the rotational speeds. Although once more the Pr varied throughout the experimental programme, a mean value of

about/

about 1000 was taken for the reference line of Mori and Nakayama. As before the theoretical line underestimates the test data but has a similar gradient.

The fact that with both fluids the theoretical predictions fall significantly below the test data is easy to explain. Both attempts to predict the effect of rotation on heat transfer base their calculations on theoretical conditions for a non-rotating tube. That is on the classical work of Nusselt for a uniformly heated tube. This work which is reported by Goldstein[8] ignores the temperature dependence of the properties and as a consequence a constant value for Nu_b or Nu_m results. This independence of heat transfer on Re and Pr is well known to be contrary to experimental evidence. Consequently as the present analyses reduce to the Nusselt result when the rotational speed is zero the entire range of numerical predictions must underestimate the true state of affairs.

5. Concluding Remarks

In conclusion, the following general remarks pertain to the general state of knowledge for this particular rotating system.

For the case of laminar flow both the currently available theoretical attempts fail to permit reliable prediction of the heat transfer and pressure drop and serve only to highlight some of the salient physical features of the flow. Both analyses appear to become invalid when, in the case of Morris, extrapolated towards higher values of $Re\,Ra_r$ and, in the case of Mori and Nakayama, towards the lower $Re\,Ra_r$ range. There is need for a more detailed analysis which ideally should account for entrance effects and also a temperature dependent viscosity.

There is a definite lack of reliable experimental data particularly in the case of pressure drop measurements. A research facility capable of operating at 1000g is currently being designed at the School of Applied Science, University of Sussex. This apparatus will use air as the working fluid and may be fitted with heated test sections with length/diameter ratios up to 100. It is envisaged that pressure drop data will also be taken.

List/

List of Symbols

a	radius of test section
a'	constant
c_f	friction factor in rotating tube
c_{fo}	friction factor in stationary tube
d	constant
e'	constant
f'	constant
F	function dependent on Prandtl number
g'	constant
j'	constant
k	thermal conductivity
K	correction for Coriolis force
n'	constant
p	pseudo pressure
q	heat flux
r	radial co-ordinate
s'	constant
t'	constant
T_b	bulk temperature
T_m	mean temperature
T_w	wall temperature
u	radial velocity
v	tangential velocity
w	axial velocity
w_m	mean axial velocity
z	axial co-ordinate

α/

α	thermal diffusivity
β	expansion coefficient
ρ	density
ν	kinematic viscosity
Ω	angular velocity
θ	angular co-ordinate
Ψ	inclination of tube relative to axis of rotation
ψ	gradient of wall temperature

Dimensionless Groups

J	rotational Reynolds number
Re	Reynolds number
Re_p	pseudo Reynolds number
Ra_r	rotational Rayleigh number
Pr	Prandtl number
Nu_b	Nusselt number based on bulk temperature
Nu_m	Nusselt number based on mean temperature
Nu_o	Nusselt number for stationary tube

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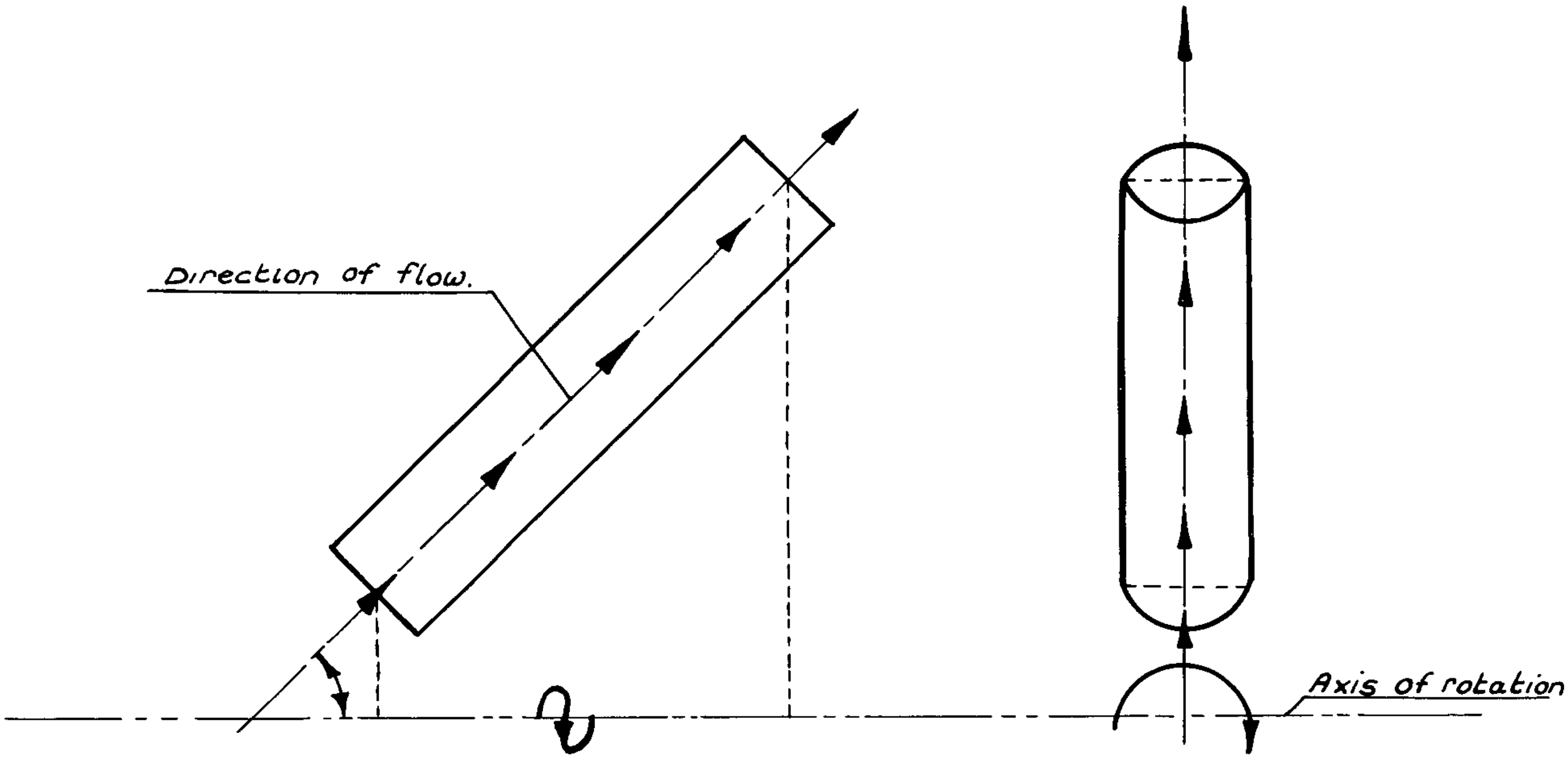
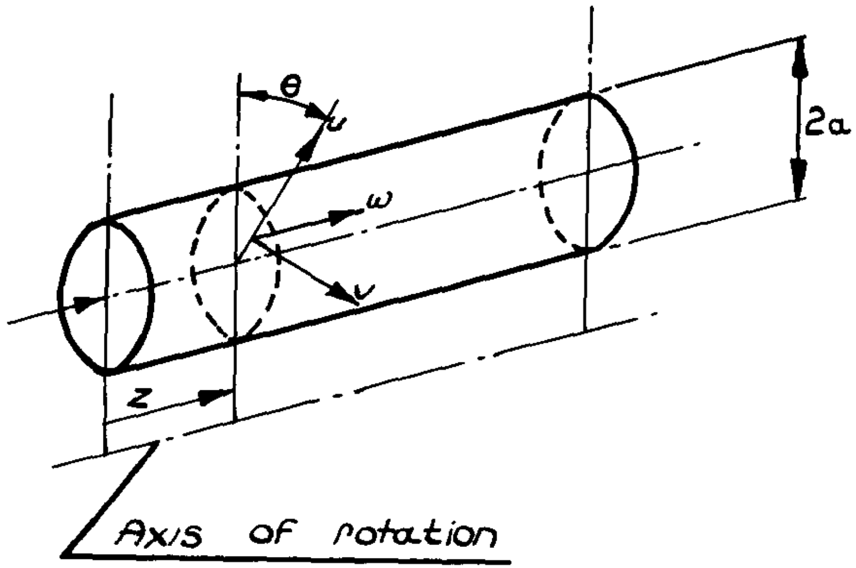
BMG



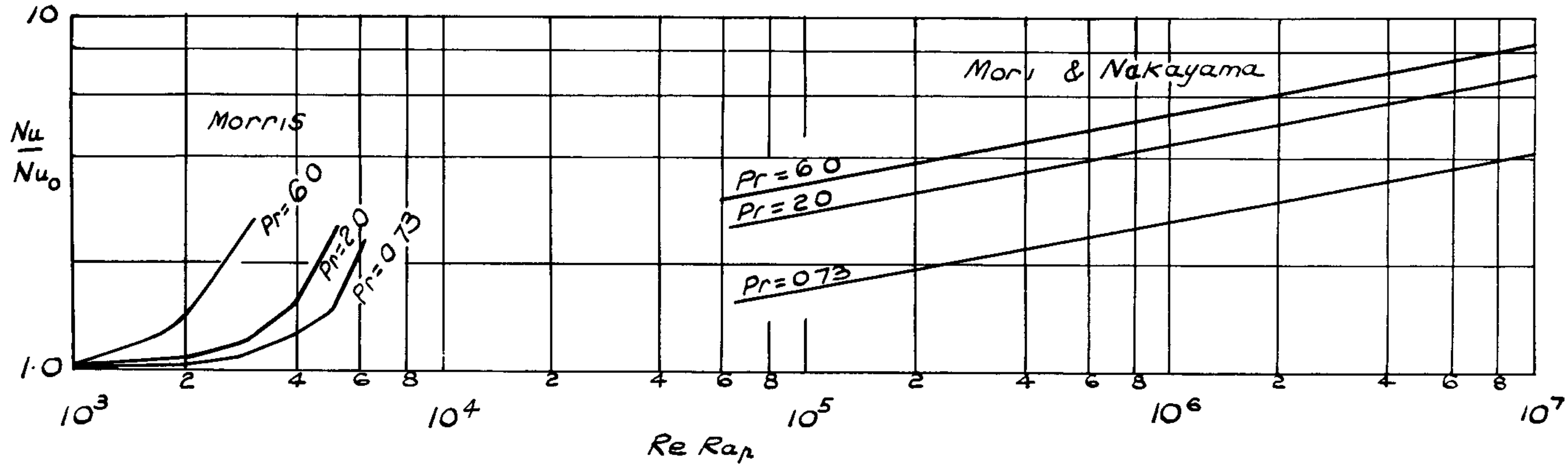
Typical Location of Tube Relative to the Axis of Rotation.

Figure 1



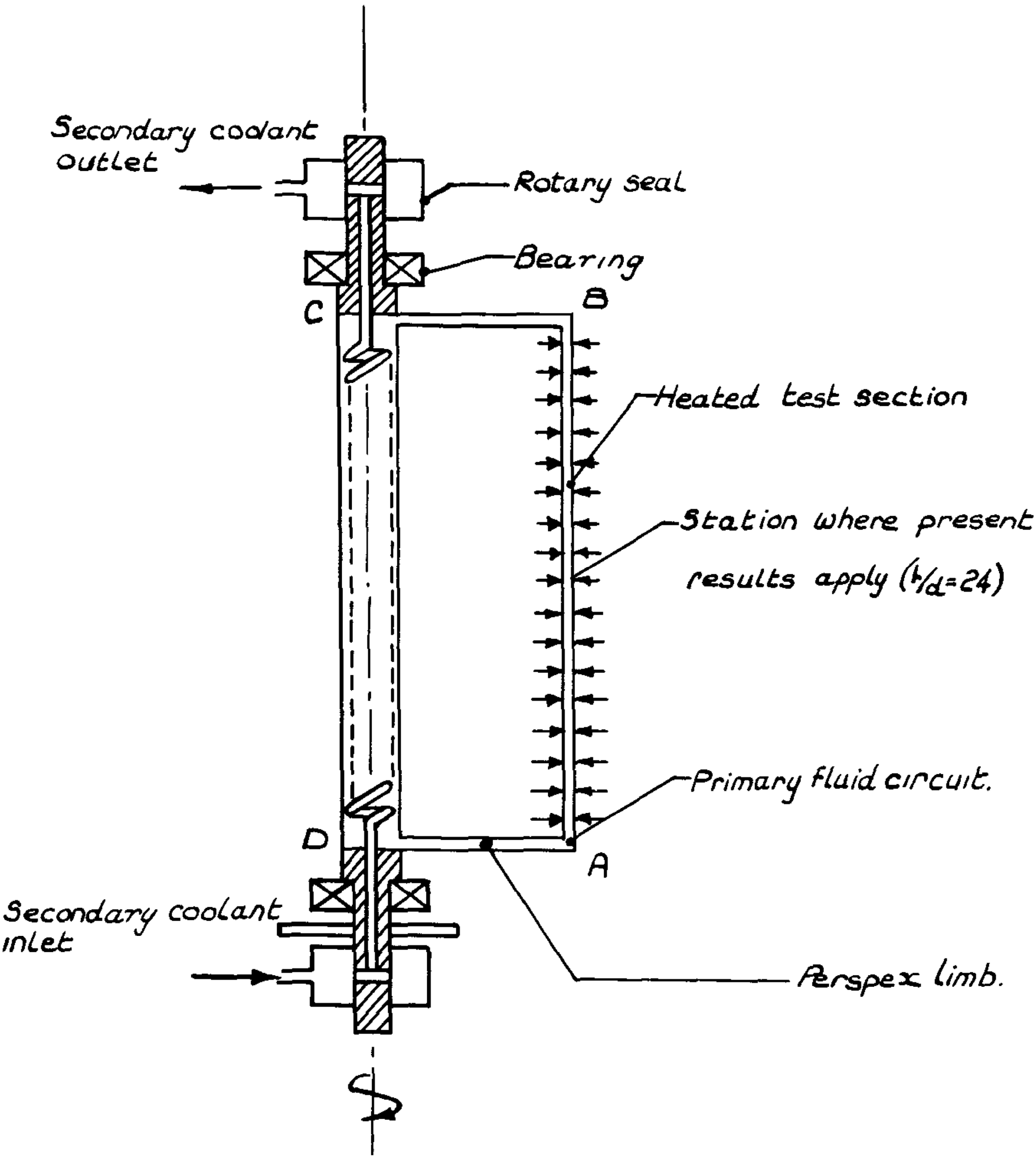
Physical Model and Coordinate System

Figure 2



Theoretical Variation of Heat Transfer with Rotation

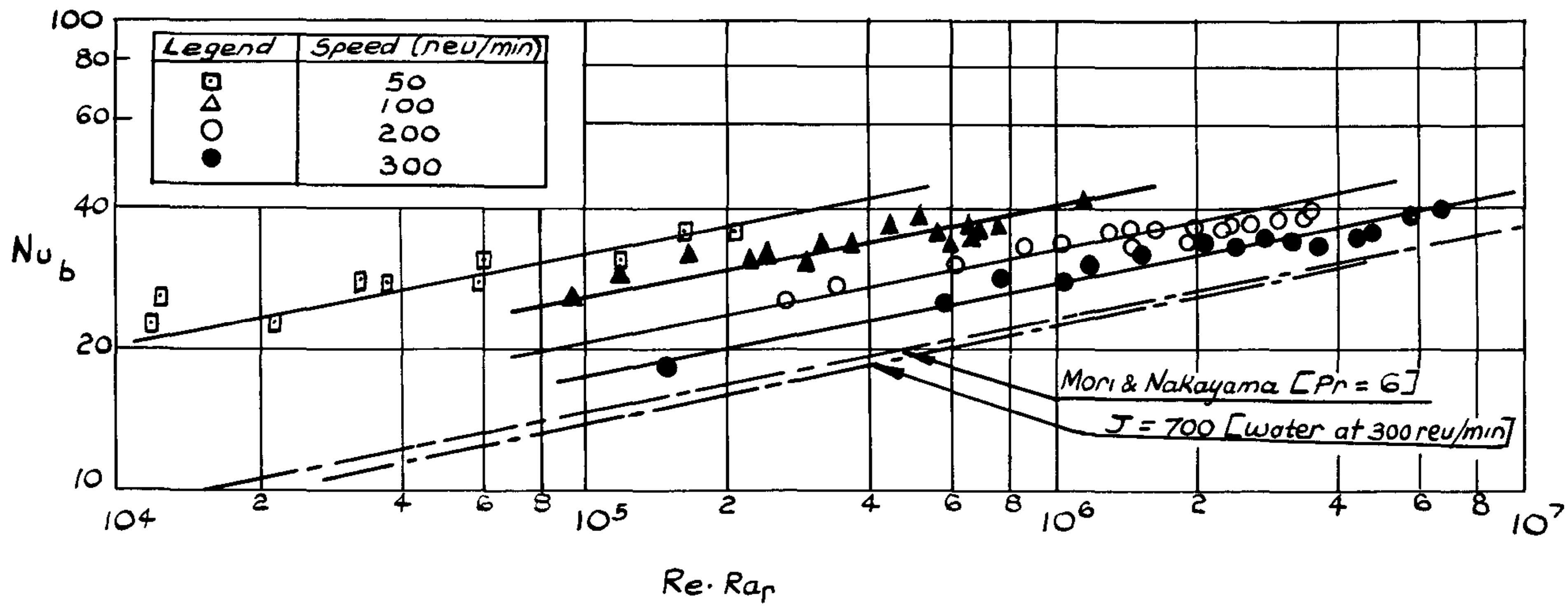
Figure 3



Schematic Layout of Rotating Thermosyphon Test Loop.

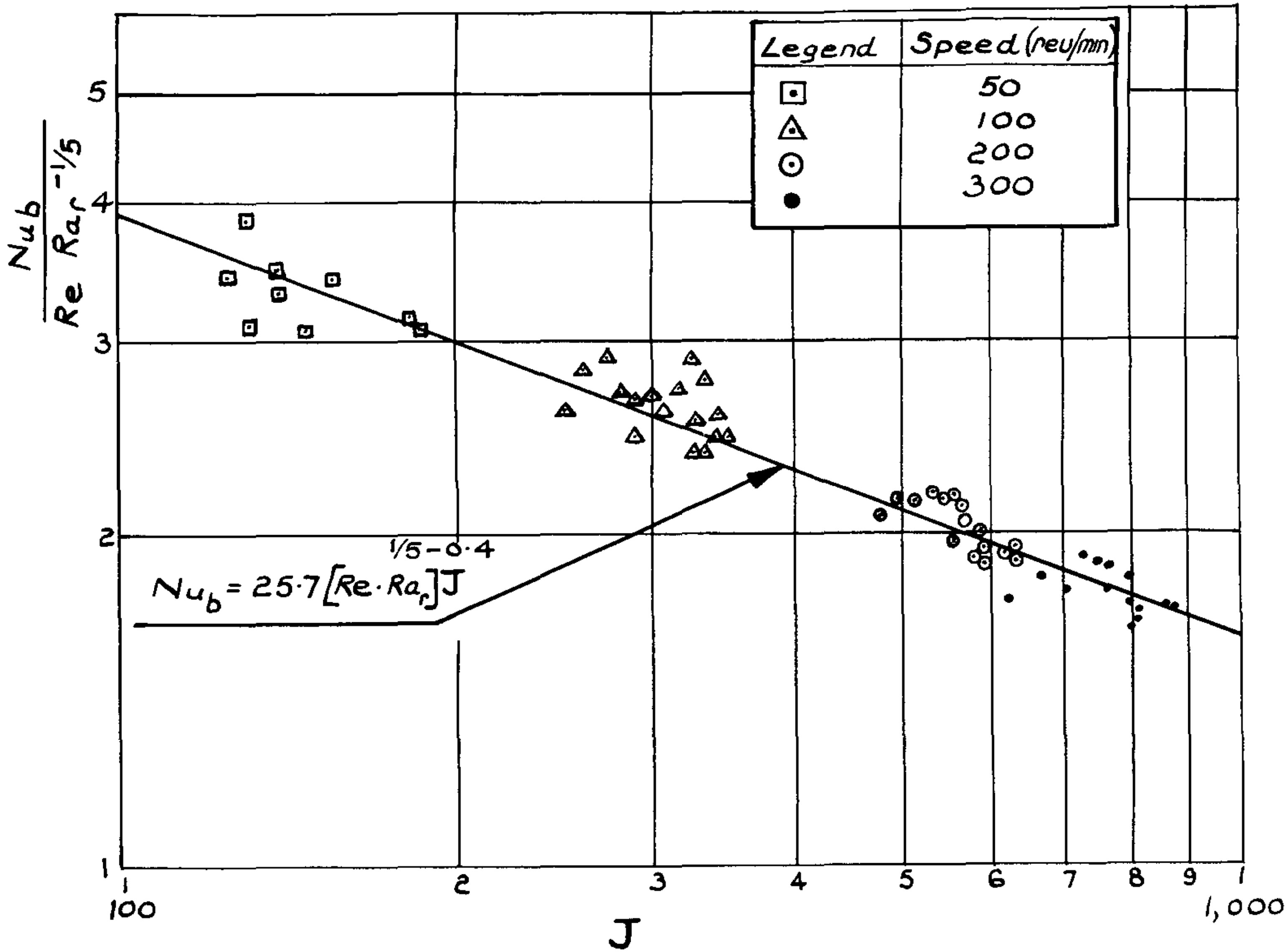
Figure 4.

Re·Ra$_r$



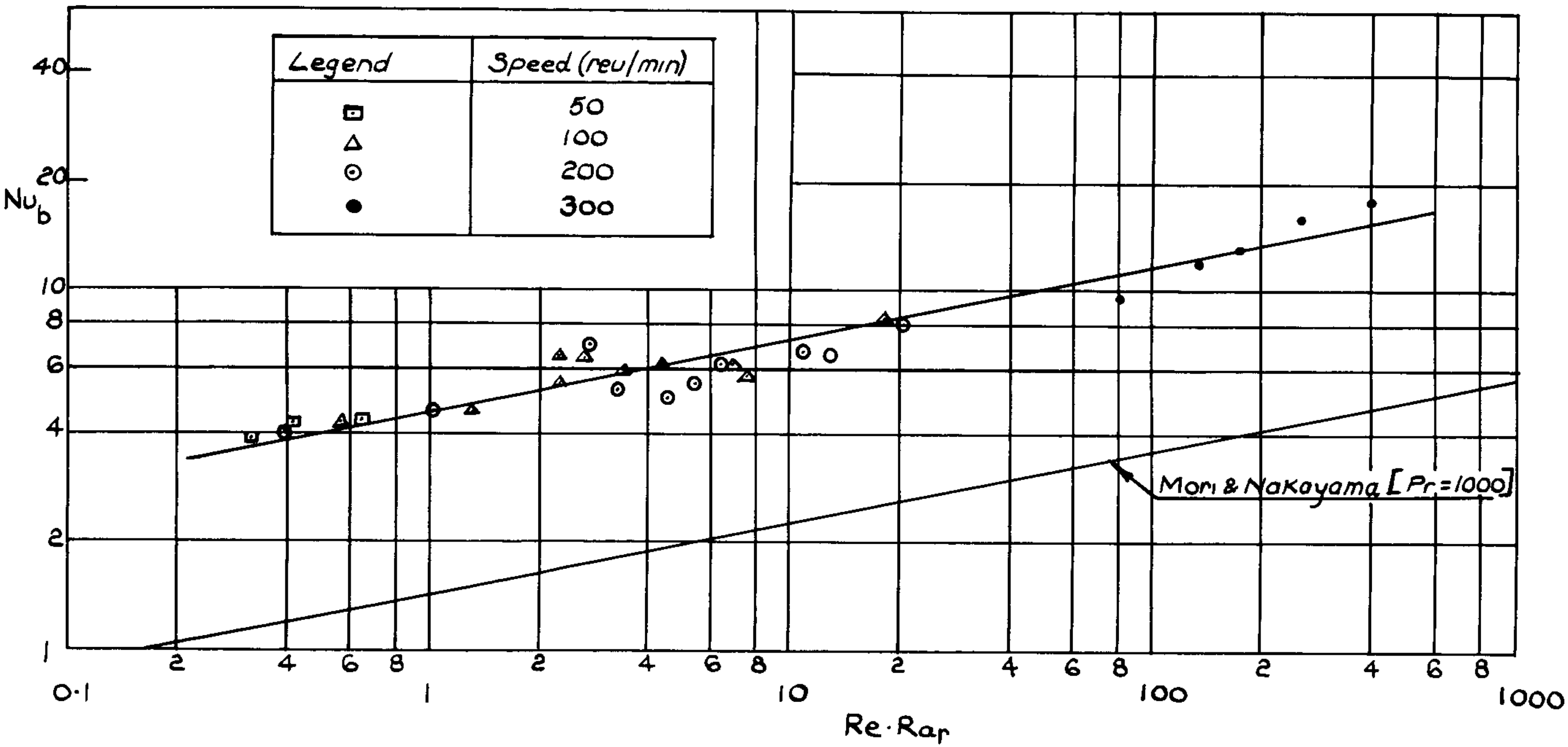
Variation of Nu$_b$ with Re Ra$_r$ for a Number of Rotational Speeds: Fluid-Water.

Figure 5.



Suggested Correlation of Data for Water Over the Rotational Speed Range Tested.

Fig 6.



Variation of Nu_b with $Re \cdot Ra_r$ for a Number of Rotational Speeds. Fluid – 100% glycerol.

Figure 7

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AN EXPERIMENTAL INVESTIGATION OF LAMINAR HEAT TRANSFER IN A UNIFORMLY HEATED TUBE ROTATING ABOUT A PARALLEL AXIS

The results of an experimental investigation which studies the influence of rotationally induced buoyancy on heat transfer in a tube which rotates about an axis parallel to itself is presented. Data obtained with water and 100% glycerol appear to confirm the qualitative method of prediction is still unavailable.

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