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An EMA Program for the Analysis of Plane Stress Problems

by

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AN EMA PROGRAM FOR THE ANALYSIS OF PLANE STRESS PROBLEMS

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SUMMARY

A computer program in Extended Mercury Autocode (EMA) is described for the finite element analysis of plane stress problems in regions of arbitrary geometry, using constant strain triangular elements.

*Replaces R.A.E. Technical Report 67275 - A.R.C. 30433.

CONTENTS

1 INTRODUCTION

This Report describes a program in Extended Mercury Autocode (EMA) for the finite element analysis of plane stress problems in regions of arbitrary geometry, in media with uniform orthotropic or isotropic properties; the adaptation of the program to include variable thickness and variable elastic constants 13 trivial. The displacement[1] method is employed, and the region to be analysed is divided into triangular elements[2] between the vertices of which the displacements are assumed to vary linearly. Instruction3 are given for the use of the program, the choice of grid pattern is discussed and the method of analysis is summarized. The program is primarily intended for use on Atlas, but it may be used on any other computer with an EMA compiler provided that details such as Job heading are modified appropriately.

2 NOTATION

Underlined symbols are used to denote matrices.

N_ℓ, h	defined after equation (1)
x_i, y_i	coordinates of a typical element vertex
D_i	elastic constants, defined after equation (1)
M	bandwidth parameter of $\underline{K}_{pp}$ matrix, defined in section 4
W	number of displacement components
D	number of prescribed zero displacements
U	number of prescribed non-zero displacements
R	number of known non-zero applied forces which do not correspond to displacement3 that are prescribed zero
C	= W-D-U, number of unprescribed displacements
$\underline{r}$	displacement matrix
$\underline{R}$	force matrix
$\underline{k}_\ell$, $\underline{T}_\ell$, $\underline{M}$	defined in equation (1)
$\underline{\sigma}_\ell$, $\underline{V}_\ell$	defined in equation (3)
$\underline{K}$	stiffness matrix
{ }	denote3 a column matrix

Superscript

t	denote3 a matrix transpose

Subscript

ℓ	typical triangular element
p,q,r	defined in section 3.

3 OUTLINE OF METHOD

This section gives a brief account of the method of analysis for those users who wish to understand the working of the program. The reader with no experience of matrix notation is referred to standard texts [3,4] for an explanation of the simple matrix operations employed.

The deformation of the finite element idealization is defined by a column matrix, $\underline{r}$, of the displacement components at the nodes of the grid formed by the triangular elements; a column matrix $\underline{R}$ denotes the corresponding force components. These matrices are used to define the loading, and are divided into submatrices as follows:

$\underline{r}_p$ unprescribed displacements
$\underline{r}_q$ prescribed non-zero displacements
$\underline{r}_r$ prescribed zero displacements
$\underline{R}_p$ prescribed forces corresponding to $\underline{r}_p$
$\underline{R}_q$ unprescribed forces corresponding to $\underline{r}_q$

The force components applied at the vertices of a typical element ℓ, of thickness h, are related to the corresponding displacement components by the following 6 x 6 stiffness matrix:

$$\underline{k}_\ell = \frac{N_\ell h}{2} \underline{T}_\ell \underline{M} \underline{T}_\ell^t \qquad (1)$$

where $N_\ell = \dfrac{1}{x_1y_2 + x_2y_3 + x_3y_1 - (y_1x_2 + y_2x_3 + y_3x_1)}$,

$$\underline{T}_\ell = \begin{vmatrix} y_{23} & 0 & -x_{23} \\ 0 & -x_{23} & y_{23} \\ y_{31} & 0 & -x_{31} \\ 0 & -x_{31} & y_{31} \\ y_{12} & 0 & -x_{12} \\ 0 & -x_{12} & y_{12} \end{vmatrix}, \qquad \underline{M} = \begin{bmatrix} D_1 & D_2 & 0 \\ D_2 & D_3 & 0 \\ 0 & 0 & D_4 \end{bmatrix}$$

$$x_{ij} = x_i - x_j,$$

and where the coordinates of the vertices and the force and displacement components are numbered as shown in Fig.1; D_1, D_2, D_3 and D_4 represent the appropriate elastic constants. The superscript t denotes a matrix transpose.

The stiffness matrix $\underline{K}$ for the assembled idealization is formed by adding the ooeffioients of the element stiffness matrices $\underline{k}_{\ell}$ into the appropriate rows and columns, this matrix is divided into the submatrices,

$$\underline{K} = \begin{bmatrix} \underline{K}_{pp} & \underline{K}_{pq} & \underline{K}_{pr} \\ \underline{K}_{qp} & \underline{K}_{qq} & \underline{K}_{qr} \\ \underline{K}_{rp} & \underline{K}_{rq} & \underline{K}_{rr} \end{bmatrix},$$

so the displacement components are related to the corresponding applied forces by the equations'

$$\begin{bmatrix} \underline{R}_p \\ \underline{R}_q \end{bmatrix} = \begin{bmatrix} \underline{K}_{pp} & \underline{K}_{pq} \\ \underline{K}_{qp} & \underline{K}_{qq} \end{bmatrix} \begin{bmatrix} \underline{r}_p \\ \underline{r}_q \end{bmatrix}.$$

The unknown displacements are thus given by

$$\underline{r}_p = \underline{K}_{pp}^{-1} \left[\underline{R}_p - \underline{K}_{pq}\, \underline{r}_q \right]. \tag{2}$$

The $\underline{K}_{pp}$ matrix is banded and symmetric, and has its largest coefficients on the leading diagonal, it is inverted, in the program, by the Choleski decomposition method, using a translation of the algorithm by Martin and Wilkinson[5].

The stresses $\underline{\sigma}_{\ell}$ in a typical element are given by

$$\underline{\sigma}_{\ell} = \{\sigma_x\ \sigma_y\ \tau_{xy}\} = N_{\ell}\, \underline{M}\, \underline{T}_{\ell}^{t}\, \underline{V}_{\ell} \tag{3}$$

where $\underline{V}_{\ell}$ is the column matrix of the displacements of the element vertices, in the order shown in Fig.1.

4 GRID PATTERN

The triangular elements used in practice are chosen on the basis of past experience in similar problems and, when necessary, by comparing results obtained using elements of different sizes. Care should be taken in choosing a suitable grid to avoid elongated elements, as these give relatively inaccurate results. Smaller elements are needed in areas where the stress gradients are expected to be high, but it is worthwhile choosing a pattern which is sufficiently regular for the results to be plotted and interpreted without too much difficulty.

In problems where distributed loadings are applied, the equivalent concentrated forces at the nodes of the grid formed by the elements should preferably be obtained on a rigorous virtual work basis[6], but simpler techniques based directly on considerations of equilibrium are often adequate.

When the idealization has been selected, the nodal displacement components are numbered in the following order:

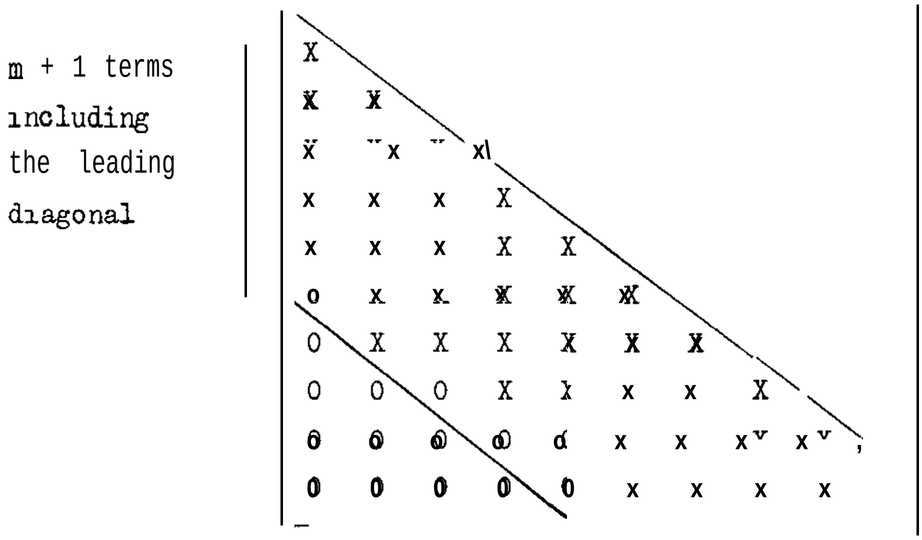
$$\underline{r} = \begin{bmatrix} \underline{r}_p \\ \underline{r}_q \\ \underline{r}_r \end{bmatrix}$$

where the submatrices are defined in section 3.

A simple example is now given to illustrate the numbering system. Consider the square isotropic plate ABCD, loaded as shown in Fig.2 and idealized as shown in Fig.3. Reference numbers 1-10, in Fig.3, correspond to unprescribed displacements, 11 and 12 to prescribed non-zero displacements and 13-18 to prescribed zero displacements.

The maximum size of problem that can be analysed in a single computation is discussed in section 6 and depends, among other things, on the bandwidth of the $\underline{K}_{pp}$ matrix, which is defined here by a parameter M; the value of this parameter is calculated by taking the maximum difference between the reference numbers of the unprescribed displacement components $\underline{r}_p$ at adjacent nodes. This parameter is of significance because terms in the stiffness matrix $\underline{K}$ coupling non-adjacent nodes are zero.

In large problems where the bandwidth is of importance, it is sometimes worthwhile to make a diagram of the $\underline{K}$ matrix showing the position of the non-zero terms. For instance, the lower half of the $\underline{K}_{pp}$ matrix for the above example, in which the bandwidth parameter is 5, may be shown diagrammatically as



where 0 denotes a zero and X a non-zero term. Such a diagram sometimes reveals a more efficient numbering system for the displacement components.

5 INPUT DATA

The input data is specified below in the order in which it must be read into the computer. The symbols in the left-hand column opposite certain items are used in the calculation of the directives at the beginning of the program (see section 6)and are not punched on the tape. The data may be presented in either fixed- or floating-point form. Typical data for input is illustrated in Appendix B.

Data for Chapter 0:

	Problem number, to help the user identify the different oases computed.
D_1, D_2 D_3, D_4	Components of the elastic matrix $\underline{M}$, defined after equation (1).
	Thickness of the plate.
W	Total number of displacement components (2 x number of nodes).
D	Number of prescribed zero displacements.

M	Bandwidth parameter of K_{pp}, defined in section 4.
	Number of different basic 'types' of elements, where each basic 'type' of element has a different size, shape or orientation, two different orientations of identical elements give two different basic 'types'.
U	Number of prescribed non-zero displacements.
R	Number of prescribed non-zero forces.
a) X_1, Y_1 X_2, Y_2 X_3, Y_3	Coordinates of the vertices of a typical element of a basic 'type', numbered as shown in Fig.?.
	Number of elements which are identical in size, shape and orientation to the above reference element (including the reference element).
	Reference numbers of the displacement components at the vertices of each of these elements in turn, in the order in which the coordinates are specified.
	The data sequence from (a) above is repeated for all the basic types of element,

Data for Chapter 1:

	When R is zero, jump to (b) below. Prescribed non-zero forces, preceded by their reference numbers.
b)	When U is zero, jump to (0). Prescribed non-zero displacements in the numerical order given by their reference numbers.
c)	The data for Chapter 0, from (a), are now repeated for reasons given below.
	The data are ended by the symbols * * * z.

The basic element data are read in a second time for the calculation of the stresses since this information has been overwritten in the execution of the earlier part of the program. The input is left in this somewhat cumbersome form so that the program can be used on any machine with an EMA compiler irrespective of the peripheral facilities.

6 JOB HEADING AND PROGRAM SIZE

The job heading required for an Atlas tape is illustrated in Appendix B and discussed more fully in the appropriate manual[7]. The method of calculating the relevent parameters is discussed in this section.

The coding which precedes the title in the second line of the Job heading depends on the Atlas installation employed, and the identity of the user.

The number of blocks of store needed for the execution of the program is given approximately by

$$23 + N/512,$$

where N is the total number of main variables which is calculated as indicated below. In normal running on the Manchester Atlas, 140 blocks cannot be exceeded, although more may be used by special request.

The costing on Atlas does not depend on the number of instructions requested, so a generous allowance can be made without affecting the cost, a more accurate estimate may then be made if the computations are repeated for any reason.

The number of lines of output is given by the smallest integer greater than (or equal to)

$$20 + W/5 + \text{total number of triangles.}$$

The number of main variables used is given by the total of the numerical values of the directives listed below, plus eight.

The directives are specified in numerical form at the beginning of Chapter 1, which immediately follows the title, as shown in Appendix A. These directives take the numerical values calculated from the following expressions:

$A \rightarrow 2$
$B \rightarrow 2$ or U, whichever is larger
$D \rightarrow 4$
$F \rightarrow (M + 1)C$

$G \rightarrow UC$

$H \rightarrow (M + 1)C$

$X \rightarrow 21$ or C, whichever is larger

$Y \rightarrow 2$

where $C = W - D - U$, and where the other symbols after the arrows are defined in section 5.

It is difficult to estimate the size of the largest problem that can be tackled with a single idealization because of the large number of parameters involved. If we consider, for example, problems in which no non-zero displacements are specified, i.e. $U = 0$, and a storage capacity of 140 blocks, then the limiting size as far as storage is concerned, is given approximately by

$$(1 + 2M)(W - D) = 60000 .$$

Hence, by reducing the bandwidth parameter M of the $\underline{K}_{pp}$ matrix, the number of nodes in the grid may be increased.

If a problem is too large to be analysed in a single computation, a coarse grid of elements can be employed in a preliminary analysis, and then smaller areas of particular interest can be reanalysed using the coarse grid results as boundary conditions. An alternative and more accurate method is to subdivide the idealization and analyse each subregion separately. A relatively simple additional program is then required, however, to complete the analysis by reassembling the deformed subregions.

7 OUTPUT

The output is printed in floating-point form in the following order:

(a) Problem number.

(b) Non-zero displacements.

The displacements $\underline{r}_p$ and $\underline{r}_q$ are printed in tabular form, 5 terms to a row. The first displacement of each row is preceded by its corresponding reference number.

(c) Stresses

The results for triangles of the same basic type are presented on consecutive lines and a space is left between the results for different types of element. Each line begins with the triangle number, followed by the stresses σ_x, σ_y and τ_{xy} (the columns are headed Sx, Sy and Sxy in the printout). The form of the output is illustrated in Appendix B.

Consequences of data faults

Data faults can obviously give rise to nonsensical results. They can moreover cause the program to stop prematurely with the caption 'FAIL IN INVERSION'. This implies that a data error has made the $\underline{K}_{pp}$ matrix singular. The captions 'EXCESS BLOCKS' or 'S V OPERAND' can also be produced by this kind of fault, although they may also be due to a wrong estimation of the size of the problem.

8 ADAPTATIONS TO INCLUDE VARIABLE THICKNESS AND VARIABLE ELASTIC CONSTANTS

The program may easily be modified to include variable thickness provided that the thickness of each element may be assumed. uniform. The program is altered in such a way that the element thicknesses are read in with the element reference numbers, so that each term of the element stiffness matrix can be multiplied by the appropriate thickness before being added into the submatrices of $\underline{K}$. The expression at the beginning of Chapter 1 containing the thickness h is then omitted.

The elastic constants may similarly be varied by reading in the appropriate values with the reference number of the individual elements. If elements of the same type have different elastic properties, then it is necessary to calculate the stiffness matrices of the elements individually. The elastic constants are, of course, required again when the stresses are calculated.

Appendix A

PROGRAM DETAILS

Figs.5 and 6 give flow diagrams illustrating the organisation of the program, which is divided into the two chapters described below; Chapter 0 forms the two submatrices of the stiffness matrix $\underline{K}$ that are required in the calculations, and Chapter 1 calculates the displacements and stresses.

Chapter 0

The basic data is first read in, followed by the coordinates of a typical triangle of the first 'type'. The lower half of the element stiffness matrix is calculated and the required matrix elements are added into the appropriate positions in the $\underline{K}_{pp}$ and $\underline{K}_{pq}$ submatrices, for each triangle of this type; the procedure is then repeated for all other element types. As can be seen in section 3, $\underline{K}_{pp}$ and $\underline{K}_{pq}$ are the only submatrices of $\underline{K}$ required in the analysis.

$\underline{K}_{pp}$ is stored in the variables H_1 in the form required for the Martin and Wilkinson[5] program for matrix inversion by the Choleski method. The diagonal terms of the matrix are stored in the end column of a C by (M + 1) matrix, which has the following form when C is 5 and M is 2:

Stored array			Lower triangle of conventional array				
X	X	a_{11}	a_{11}				
X	a_{21}	a_{22}	a_{21}	a_{22}			
a_{31}	a_{32}	a_{33}	a_{31}	a_{32}	a_{33}		
a_{42}	a_{43}	a_{44}	0	a_{42}	a_{43}	a_{44}	
a_{53}	a_{54}	a_{55}	0	0	a_{53}	a_{54}	a_{55} .

The $\underline{K}_{pq}$ matrix is stored in the variables G_i, in the transposed form $\underline{K}_{qp}$. The form of the $\underline{K}$ matrix is shown in Fig.7, the shaded areas being the only regions of the matrix which are stored.

Chapter 1

The displacements are calculated using the matrix equation (1) and all the non-zero displacements are pnnted cut. The stresses in each triangle are then calculated using equation (2) and printed cut.

The printout of the program commences on the opposite page.

```
TITLE
PLANE   STRESS   ANALYSIS

CHAPTER1

A→2
B→2
D→4
F→60
G→20
H→60
X→21
Y→2

1)J=ψINT PT(C+0.1)
I=1(1)J
XI=0
REPEAT

JUMP17,R=0
I=1(1)R
READ(K)
READ(XK)
XK=XK/H
REPEAT

JUMP18,U=0
17)R=ψINT-PT(U+0.1)
I=1(1)R
READ(BI)
REPEAT


I=1(1)J
FI=0
REPEAT

I=1(1)J
L=1(1)R
N=ψINT PT(LC-C+I+0.1)
FI=FI+BLGN
REPEAT
XI=XI-FI
REPEAT

18)CAPTION
NON-ZERO   DISPLACEMENTS
NEWLINE

I=1(1)J
JUMP2,I>M
P=M-I+1
JUMP3
2)P=0
3)R=I-M+P
```

App.A Cont.

```
N=P(1)M
S'=N-1
Q=M-N+P
T'=MI+N+I-M
Y=HT'
JUMP5,P>S'

K=P(1)S'
O'=MI+K+I-M
T'=MR+Q+R-M
Y=Y-FO'FT'
Q=Q+1
REPEAT

5) JUNP6 ,N≠M
JUMP24,0>Y
T'=MI+N+I-PI
A=ψSQ RT(Y)
FT'=1/A
JUMP7
6)T'=MI+N+I-M
O'=MR+R
FT'=YFO'
R=R+1
7)REPEAT
REPEAT

S'=M-1
I=1(1)J
JUMP9,I>M
P=M-I+1
JUMP10
9)P=0
10)Q=I
Y=XI
JUMP13,P>S'

K=S'(-1)P
O=O-I
T'=MI+K+I-M
Y=Y-FT'HQ
REPEAT

13)O'=MI+I
HI=YFO'
REPEAT

I=J(-1)1
Q=J-I
JUMP11,Q>M
P=M-J+I
JUMP1a
11)P=0
12)Y=HI
Q=I
JUMP14,P>S'
```

```
K=S'(-1)P
Q=Q+1
T'=MQ+K+Q-M
Y=Y-FT'HQ
REPEAT

14)O'=MI+I
HI=YFO'
REPEAT

R=ψINT PT(U+0.1)
JUMP60,R=0

J=ψINT PT(C+0.1)

I=1(1)R
K=I+J
HK=BI
REPEAT

60)K=ψINT PT(D/5+0.1)
21)JUMP15,K=0
I=1(1)K
B=5I-4
PRINT(B)2,0
L=1(1)5
P=5I+L-5
PRINT(HP)0,3
REPEAT
NEWLINE
REPEAT

15)L=ψINT PT(D-5K+0.1)
JUMP16,L=0
B=5K+1
PRINT(B) a.0
I=1(1)L
P=5K+I
PRINT(HP)0,3
REPEAT
NEWLINE

16)J=ψINT PT(D+1.1)
L=ψINT PT(W+0.1)
I=J(1)L
HI=0
REPEAT

CAPTION
STRESSES
NEWLINE
```

App.A Cont.

```
CAPT ION
TRIANGLE  SX                SY                SXY
NEWL INE
R=1(1)S
I=0(1)2
READ(XI)
READ(YI)
REPEAT

A0=X2-X1
A1 =X0-X2
A2=X1-X0

B0=Y1-Y2
B1=Y2-Y0
B2=Y0-Y1

C=X2Y0+X0Y1+X1Y2-Y2X0-Y0X1-Y1X2
A=1/C

READ(N)
I=1(1)N
PRINT(I)2,0
J=1(1)6
READ(L)
FJ=HL
REPEAT

B=D1B0F1+D2A0F2+D1B1F3+D2A1F4+D1B2F5+D2A2F6
B=AB
PRINT(B)0,5
B=D2B0F1+D3A0F2+D2B1F3+D3A1F4+D2B2F5+D3A2F6
B=AB
PRINT(B)0,5
B=D4A0F1+D4B0F2+D4A1F3+D4B1F4+D4A2F5+D4B2F6
B=AB
PRINT(B)0,5
NEWL INE
REPEAT
NEWL 1 NE
REPEAT

JUMP 31
24)CAPTION
FAIL IN INVERSION
31)ACROSS1/0
CLOSE
```

```
CHAPTER0
VARIABLES1

1)READ(A)
CAPTION
PROBLEM NUMBER
SPACE
PRINT(A)4,0
NEWLINE
I=1(1)4
READ(DI)
REPEAT
READ(H)
READ(W)
READ(D)
READ(M)
READ(S)
READ(U)
READ(R)
D=W-D
C=D-U


J=ψINT PT(MC+C+0.1)
I=0(1)J
HI=0·
REPEAT


J=ψINT PT(UC+0.1)
I=0(1)J
GI=0·
REPEAT
P=1(1)S
I=0(1)2
READ(XI)
READ(YI)
REPEAT

A0=X2-X1
A1=X0-X2
A2=X1-X0

B0=Y1-Y2
B1=Y2-Y0
B2=Y0-Y1

C'=X2Y0+X0Y1+X1Y2-Y2X0-Y0X1-Y1X2
A=0.5/C'
```

```
X1 =D1 BoBo+D4AoAo
X2=D2AoBo+D4AoBo
X3=D3AoAo+D4BoBo
X4=D1BoB1 +D4AoA1
X5=D2AoB1+D4BoA1
X6=D1B1B1 +D4A1A1
X7 =D2A1Bo+D4AoB1
X8=D3A1Ao+D4BoB1
X9 =D2A1B1 +D4A1B1
X10=D3A1A1 +D4B1B1
X11=D1BoB2+D4AoA2
X12=D2AoB2+D4BoA2
X13=D1B1B2+D4A1A2
X14=D2A1B2+D4B1A2
X15=D1B2B2+D4A2A2
X16=D2A2Bo+D4AoB2
X17=D3AoA2+D4BoB2
X18=D2A2B1 +D4A1B2
X19=D3A1A2+D4B1B2
X20=D2A2B2+D4A2B2
X21=D3A2A2+D4B2B2

I=1(1)21
XI =AXI
REPEAT

READ(O)
L=1(1)O
J=1(1)6
READ(FJ)
REPEAT

N-o
I=1(1)6
J=1(1)I
N=N+1
JUMP29,FJ>FI
A=FI
B=FJ
JUMP30
29)A=FJ
B=FI
30)JUMP4,A>D
JUMP25,A>C
T=ψINT-PT(MA+B+o.1)
HT=HT+XN
JUMP4
25) JUMP4, B>C
T=ψINT PT(CA-CC-C+B+o.1)
GT=GT+XN

4)REPEAT
REPEAT
REPEAT
REPEAT
ACROSS: /1
CLOSE
```

Appendix B

SIMPLE EXAMPLE OF JOB HEADING, INPUT DATA AND OUTPUT

This appendix illustrates the form of the Job heading, input data and output for the simple example described in section 4. The triangular elements are numbered in the order shown in Fig.4, and the element data are read into the computer in the following order; A_1 to A_4, B_1 to B_4. The printout commences on the following page.

JOB HEADING

```
JOB
F1001/2406/1 J.SINFIELD R.A.E. PLANE STRESS ANALYSIS
STORE 24 BLOCKS
COMPUTING 8000 INSTRUCTIONS
OUTPUT
0 LINE PRINTER 32 LINES
COMPILER EMA
AUXILIARY(0,0)
MAIN→179
```

VARIABLE DIRECTIVES

```
A→2
B→2
D→4
F→60
G→20
H→60
X→21
Y→2
```

DATA

```
1
1 0.3 1 0.35
0.1 18 6 5 2 2 2

0 0 0.5 0.5 0 0.5
4
2 3 4 5 1 11
6 7 12 8 4 5
13 14 6 7 2 3
15 16 9 10 6 7

0 0 0.5 0 0.5 0.5
4
2 3 6 7 4 5
6 7 9 10 12 8
13 14 15 16 6 7
15 16 17 18 9 10

1 1
8 1
1 1

0 0 0.5 0.5 0 0.5
4
2 3 4 5 1 11
6 7 12 8 4 5
13 14 6 7 2 3
15 16 9 10 6 7

0 0 0.5 0 0.5 0.5
4
2 3 6 7 4 5
6 7 9 10 12 8
13 14 15 16 6 7
15 16 17 18 9 10

***Z
```

OUTPUT
- - - - -

PLANE STRESS ANALYSIS

5. START OF CHAPTER 1
185. START OF CHAPTER 0
PROGRAMME ENTERED

PROBLEM NUMBER 1
NON-ZERO DISPLACEMENTS

1	2.321,	1	9.592	6.495,	-1	8.749	3.511
6	a. 246		1.650	2.198,	1	5.257	7.718
11	1.000		1.000				

STRESSES

TRIANGLE	SX		SY		SXY	
1	-2.87101,	1	-7.97512		1.12899,	1
2	-1.43808,	1	-9. 27382,	-1	1.32771,	1
3	-2.30102		4.91785,	-1	7.41470	.
4	-4.98859		1.50693	·	1.00197,	1
1	-1.57417		1.91461		1.05221	
4	a.576 20		2.67229,	1	x.16746	
	9.90167, -	1	3.30056		5.77 241	
3	4.63071		1.54357,	1	3.67984	

Appendix C

COMPARISON WITH AN EXACT SOLUTION

A comparison is now made between the results obtained using this program and the exact results obtained by Morley[8] for an isotropic square plate, encastré at one and, and loaded as shown in Fig.8. The plate and the loading are symmetric about the OX axis, so the upper half only is analysed, using the grid shown in Fig.9. The stresses along the edges of the plate are shown in Figs.10 and 11. As this finite element idealization prescribes uniform stresses within each element, the results are made up of lines of constant stress , Joined at the nodes to give a step formation. These stresses are, in effect, an average of the stresses over the area of each element, so they cannot be expected to agree completely with the exact stresses along the edge of the plate. Figs.10 and 11 demonstrate, however, that the finite element results follow the exact curve, differing most, of course, in the immediate vicinity of the point where the exact results have an infinite value.

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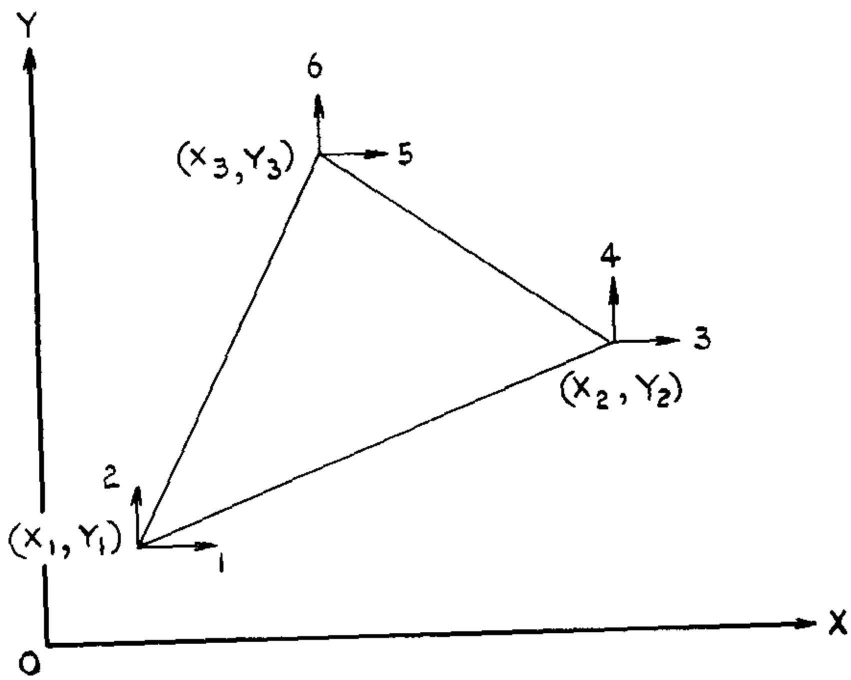

Fig 1 Typical triangle

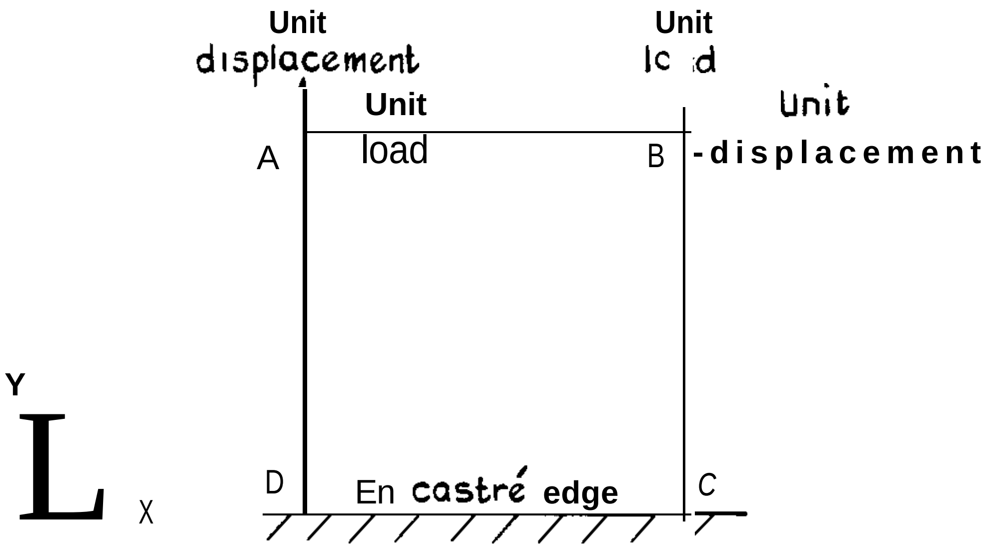

Fig .2 Square plate with encastré edge

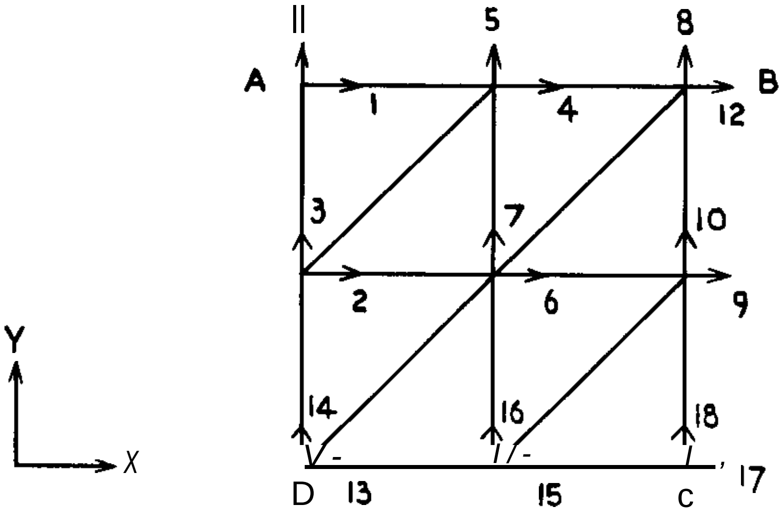

Fig.3 Grid for the square plate

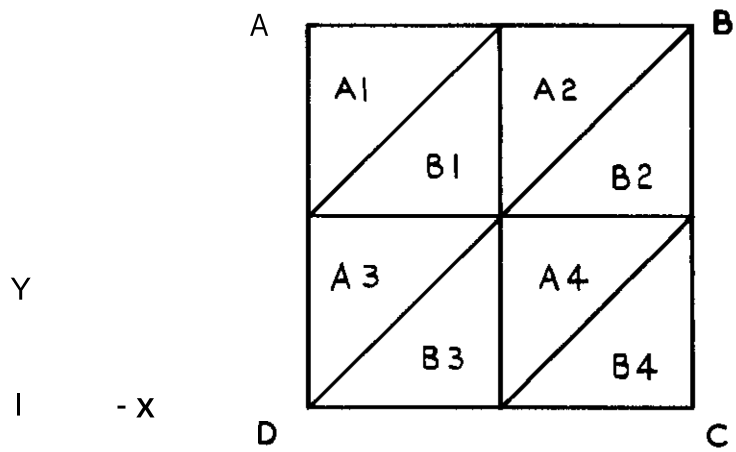

Fig.4 Triangle numbering system for the squore plate

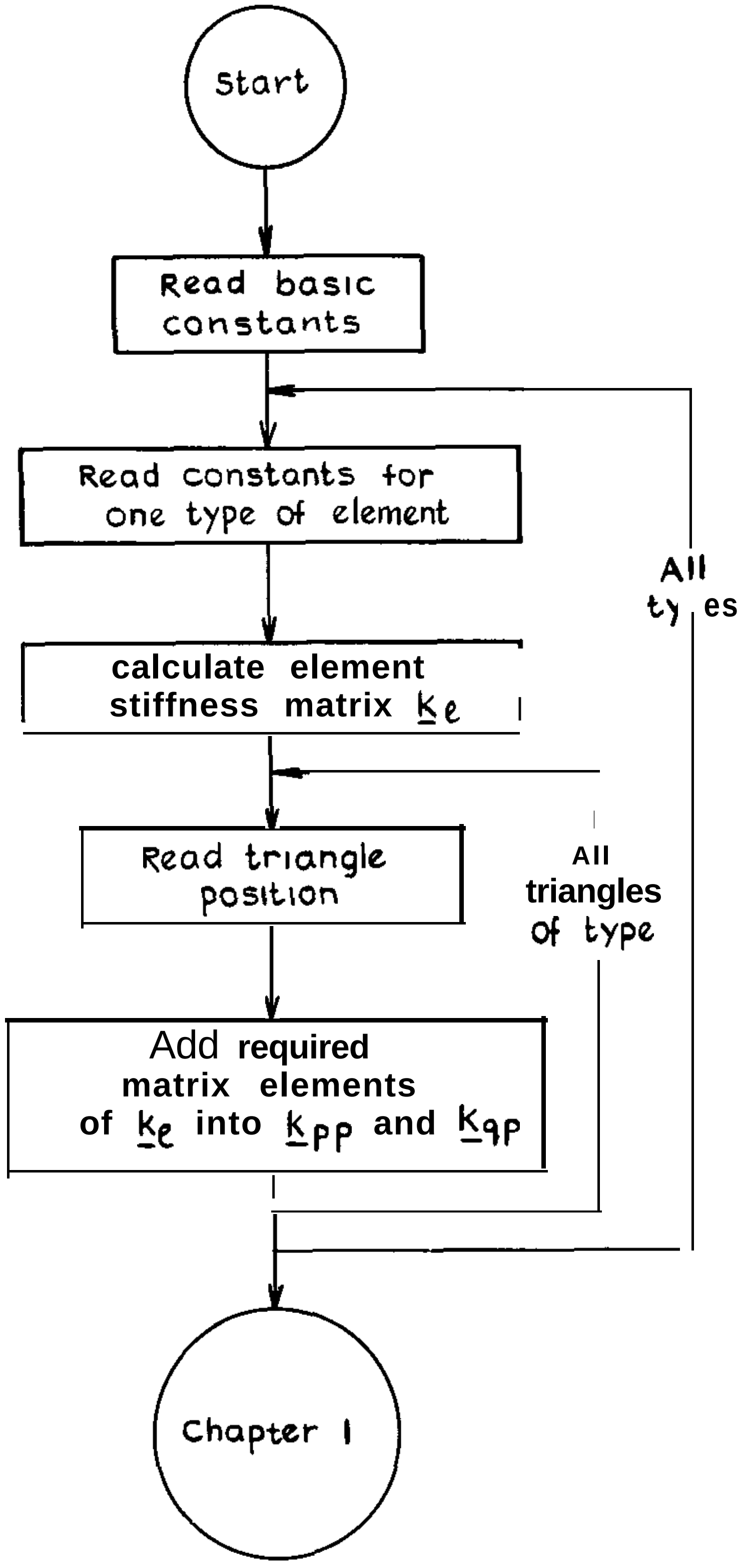

Fig.5 Flow diagram for Chapter 0

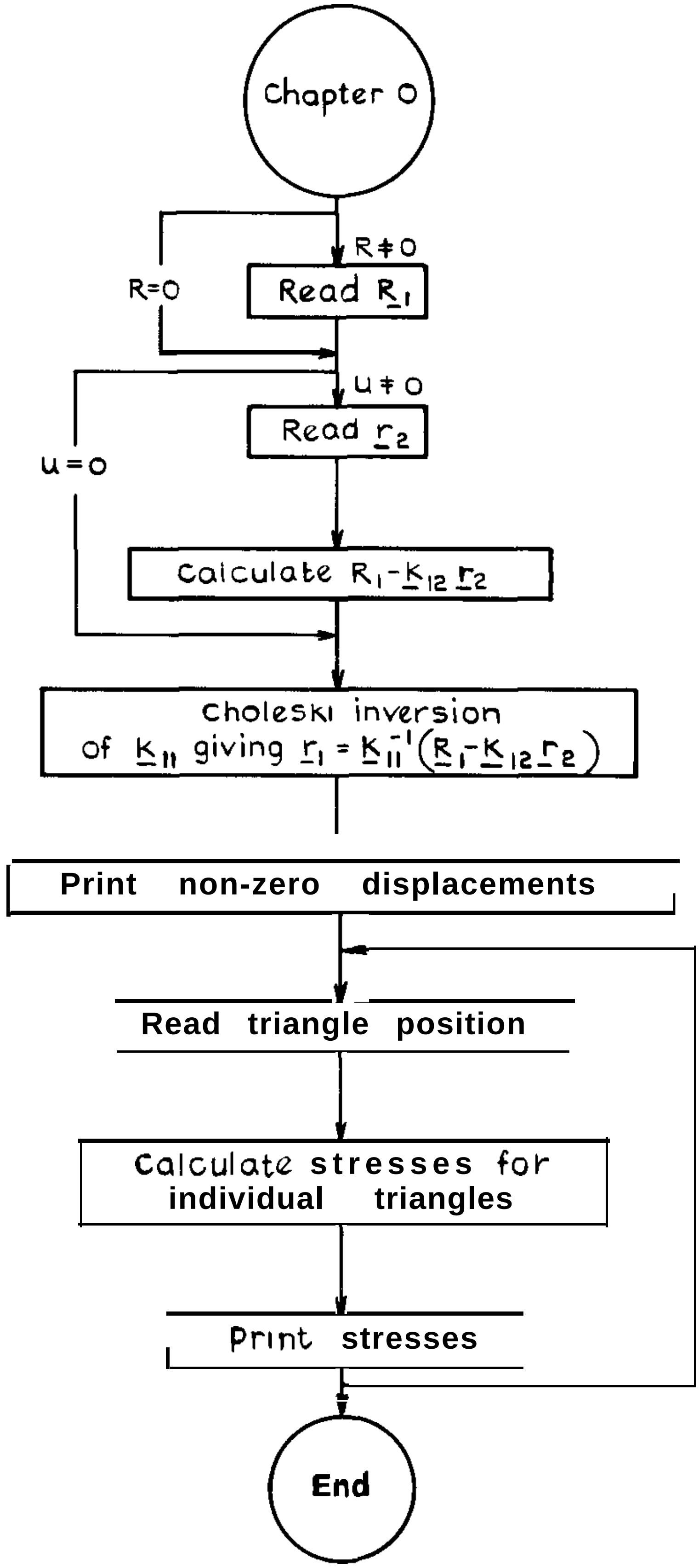

Fig 6 Flow diogrom for Chopter I

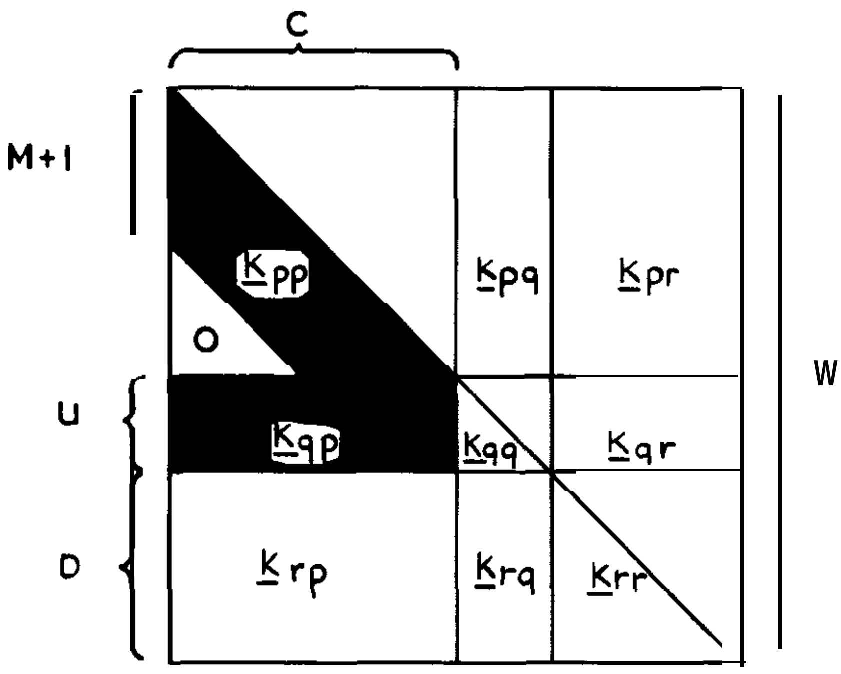

Fig.7 K matrix diagram

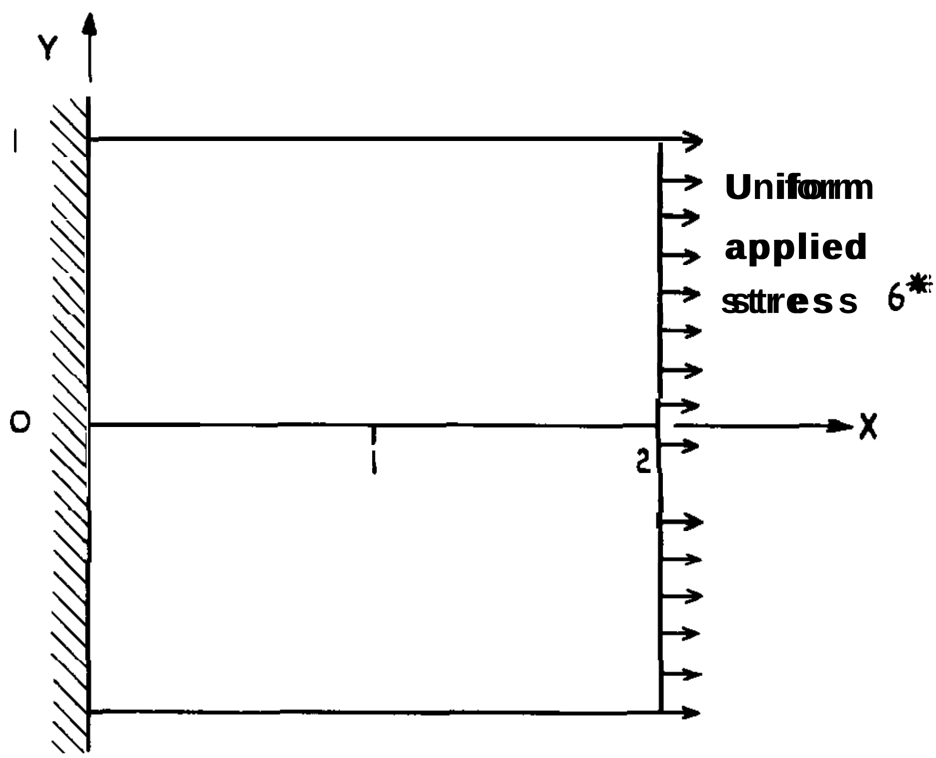

Fig.8 Plane stress problem analysed exactly by Morley

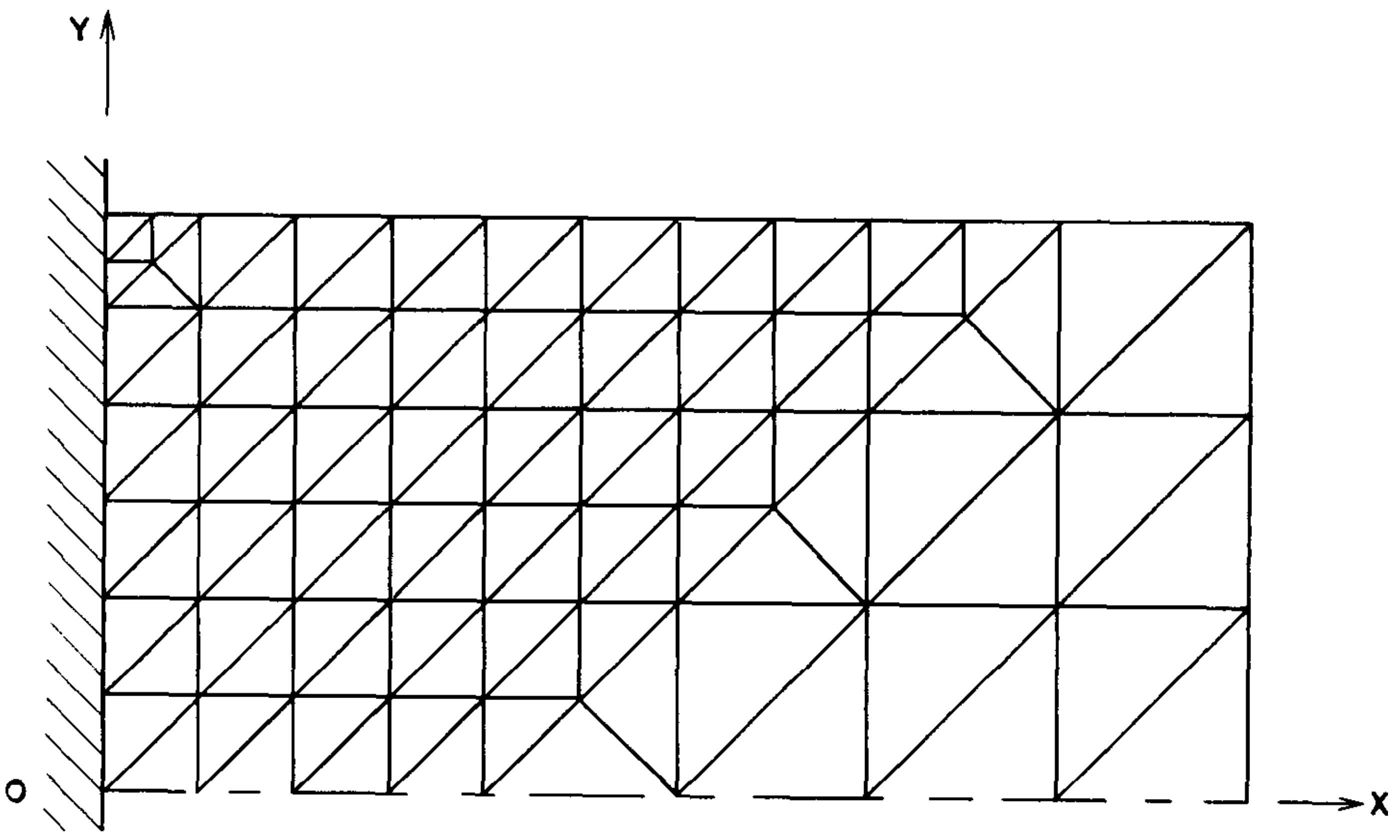

Fig 9 Idealisation of the upper holf of the plate shown in Fig 8

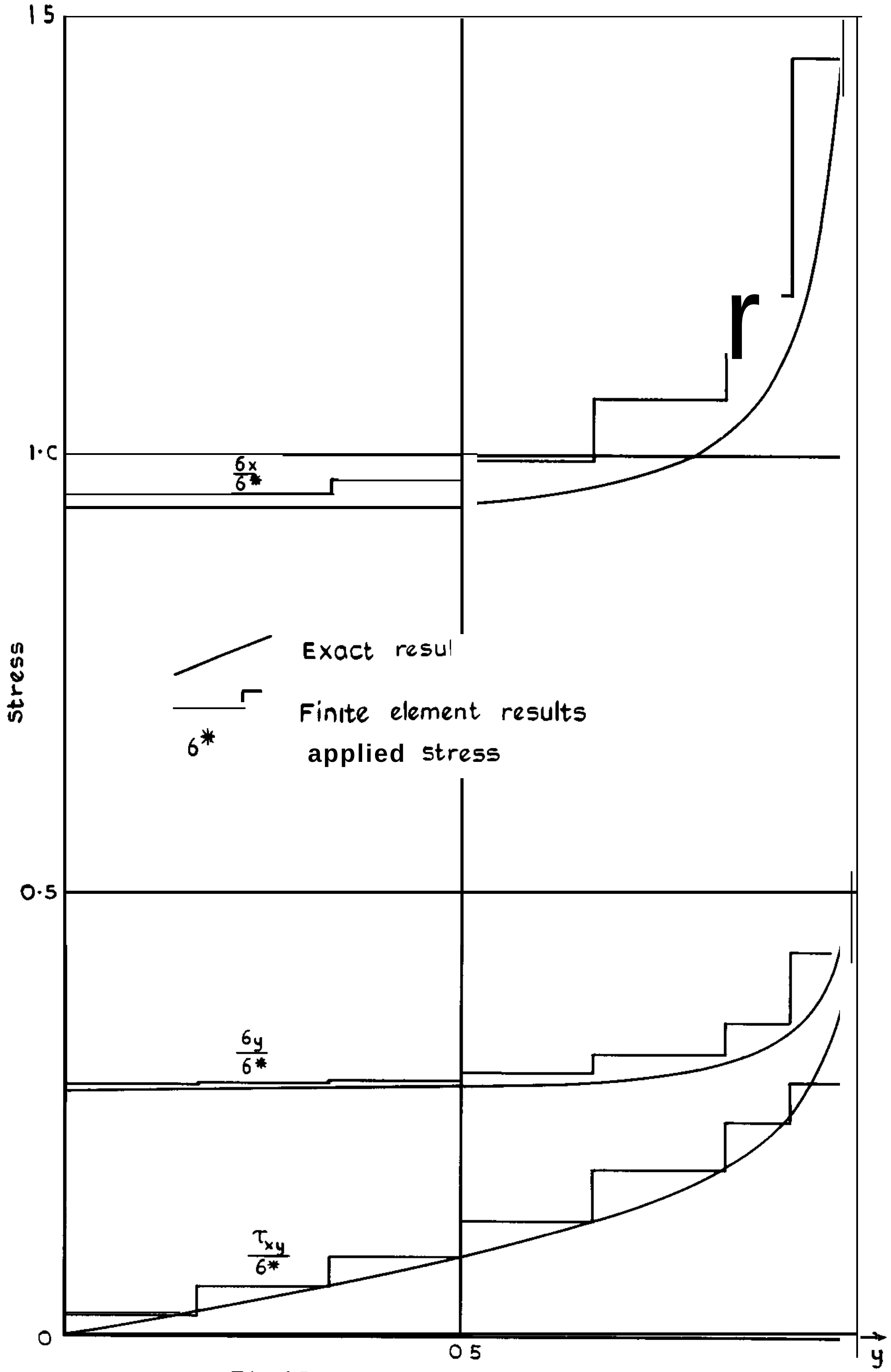

Fig.10 Stresses along x=0

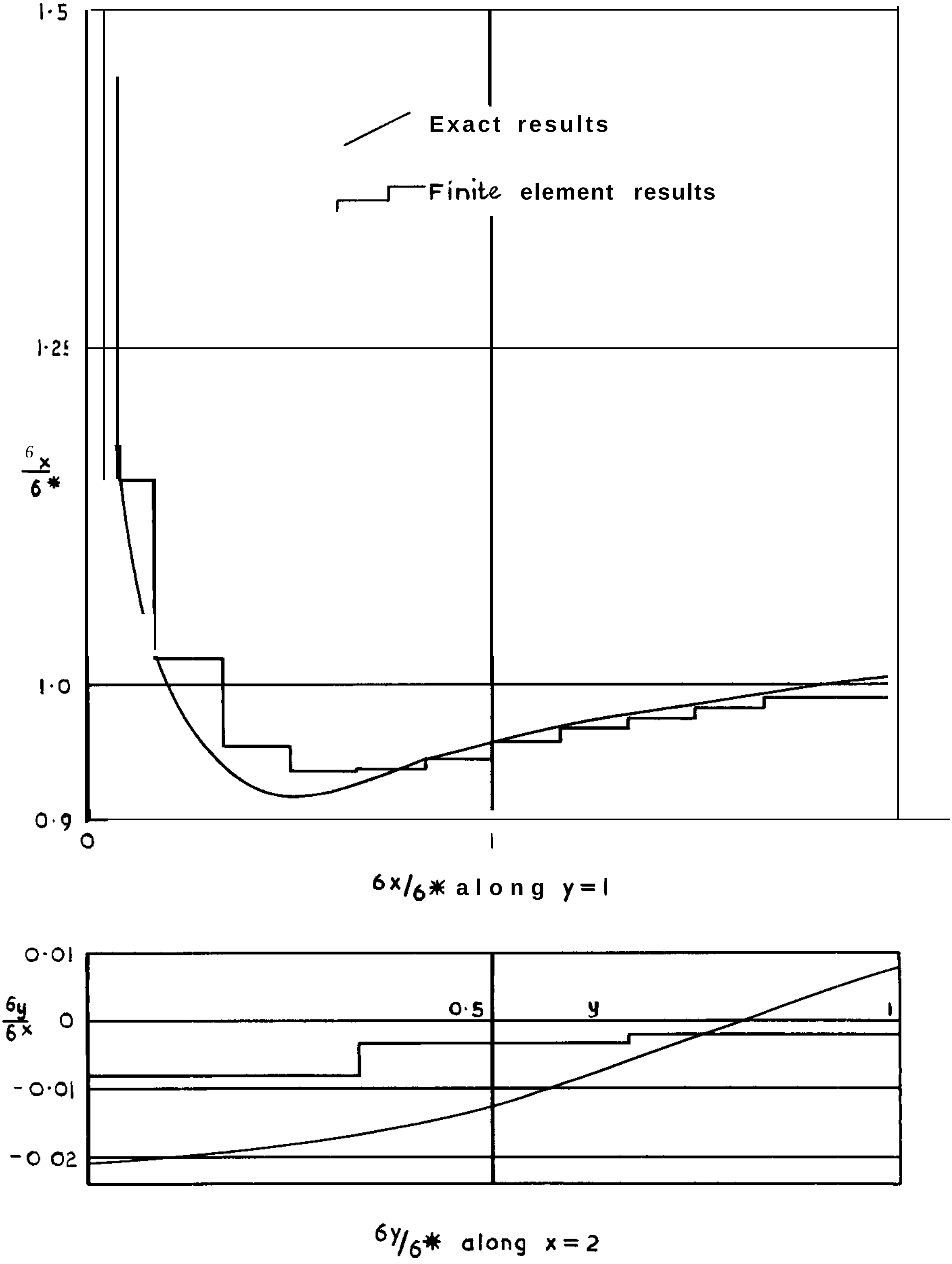

Fig.11 Stresses along the free edges

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