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An Analysis of some major Factors Involved in Normal Take-off Performance

by

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AN ANALYSIS OF SOME MAJOR FACTORS INVOLVED IN NORMAL TAKE-OFF PERFORMANCE

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SUMMARY

An analytical study has been made of the effect of such parameters as wing loading, aspect ratio, thrust-weight ratio, and number of engines, on the takeoff performance of fixed wing aircraft. Expressions are derived for the take-off lift coefficients which give the shortest take-off distance, the highest take-off wing loading, and climb conditions Just meeting the airworthiness requirements. Examples of the analysis applied to two designs of current Interest are given.

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1 INTRODUCTION

The aim of this paper is to provide a simple framework within which the various factors involved in improving normal take-off performance may he considered. The term 'normal take-off' is here used, in the sense suggested by Sutcliffe[1], to imply that lift is generated aerodynamically by fixed wings, and that take-off distances of more than 2000 ft are contemplated. The exact method of generating the lift is not, in fact, important in the present analysis, but such high-lift devices as conventional mechanical flaps, b.l.c. flaps, end some jet-flap schemes come within its scope. In preparing the paper the field of civil transport aircraft was mainly in mind, but parts of the analysis may be applied equally readily to military requirements.

Nowadays, routine calculations of take-off performance during a" aircraft's design are usually made on digital computers, using programs which often embody the experience gained from previous designs, and take account of the many individual features of the aircraft which cannot be neglected when accurate numerical estimates of take-off performance are required. Such computer programs may also be used for more systematic studies into the effect of various parameters on take-off performance, but, although this approach may yield a wealth of numerical detail, there is some danger that a genuine understanding may not be achieved. What appears to be needed to support such studies is a" analysis which is simple enough to allow the dominant features to be kept in sight, and this is the aim of the present work.

Such a" analysis does not attempt to vie with the detailed computer calculation when it comes to predicting the exact performance of a given aircraft in a particular situation, but it does aim to show the broad trends in performance as different parameters are varied. Obviously, an eye must be kept on the simplifying assumptions that have been made, and there may well be cases where the particular features of a design result in eve" the broad trends being contrary to those predicted by the analysis. Eve" then, such anomalies may be better understood when they are recognized as intelligible departures from the usual pattern.

The body of the paper is divided into six main sections. The first (section 2) deals with the equations which have been chosen to represent the take-off manoeuvre, and gives numerical examples of the effect of the principal parameters on take-off performance. Then follow four sections in which expressions are dewed for the lift coefficients at which various significant limiting features in take-off performance occur. These are the lift coefficients giving the shortest take-off distance at a given wing loading (section 3); the largest wing loading from a" airfield of specified length (section 4), airworthiness climb gradient limits (section 5), and, zero rate of climb (section 6). I" each case account is taken of an engine failing as the aircraft becomes airborne.

In section 7 the results of the analysis are applied to two designs of current Interest the swept-wing subsonic transport, and the allwing short range aerobus.

One particular feature of the analysis is that it deals with limitations in useable lift coefficients arising only from performance considerations, rather than from purely aerodynamic factors, such as stalling. It may therefore be regarded as setting up targets for the aerodynamicist, in terms of the take-off lift and drag coefficients to be achieved.

2 THE TAKE-OFF MANOEUVRE

The design of civil aircraft from the viewpoint of take-off performance is largely dictated by airworthiness requirements[2,3]. For twin-engined aircraft, in particular, the manoeuvre which usually provides the critical design case is that in which a" engine falls at the least favourable moment. Performance analysis of this situation involves a detailed study into the merits of continuing the take-off, or of attempting to stop within the runway length remaining. The latter obviously involves a knowledge of the aircraft's braking performance, which may, in turn, depend on such details as whether airbrakes and lift spoilers are fitted. The layout of the engines may also have an Important effect on the ability to use reverse thrust. Therefore, quite detailed aspects of the aircraft design are involved, even in a preliminary assessment of the take-off performance.

Other factors which increase the difficulty of estimating take-off performance accurately are the uncertainty surrounding the aerodynamic characteristics when close to the ground, and the non-linearities introduced into the equations of motion because of changing ground effect with height, and because of changing aircraft configuration. Finally, the airborne portion of the take-off is partially dependent on the control technique used by the pilot.

Any attempt to represent all of these features in a general analysis would result in it becoming so cumbersome that even the major trends would tend to be lost in a profusion of detail. The manoeuvre has therefore been simplified as far as possible, consistent with revealing these general trends.

2.1 The ground roll

A combination of elementary dynamics and aerodynamics gives the following equatmn of motion for an aircraft accelerating along the runway -

$$\frac{W}{g} v \frac{dv}{ds} = T - \tfrac{1}{2}\rho v^2 S C_{D_G} - \mu\left(W - \tfrac{1}{2}\rho v^2 S C_{L_G}\right) \qquad (1)$$

where C_{L_G} and C_{D_G} are the lrft and drag coefficients appropriate to the ground attitude.

The thrust T, in practice, varies slightly with speed, but if it is represented by some equivalent constant value*, equatmn (1) may be Integrated to give the &stance needed to accelerate up to any take-off speed.

It will be convenient to replace the takeoff speed in the resulting expression by the lift coefficient needed to sustain level flight at that speed. This substitution is given by the equation:-

$$v = \sqrt{\frac{2\omega}{\rho C_L}} \qquad (2)$$

which stems directly fmm the definition of lift coefficient, and tbe balance of lift and weight in level flight (neglectmg engine thrust components). The relationship between speed, wing loading and lift coefficient given by equation (2) is shown graphically in Fig.1, for relevant ranges of the variables, as an aid in interpreting the numerical results obtained later in the paper.

With this substitution the equatmn for the ground roll distance, obtained by integration of (1), is -

---

*The value of the thrust at 0 7 of the take-off speed is generally used for jet aircraft[4,12]. It will be more convenient in this analysis, however, to take T as the thrust at the take-off speed, and this will generally involve only a small error Where greater accuracy is required the method suggested in Appendix B (1) may be adopted. Equation (1) may also be integrated if T is a function of v^2.

$$s_G = \frac{\omega}{\rho g\,(C_{D_G} - \mu C_{L_G})} \log_e \left[\frac{T/W - \mu}{T/W - \mu - \dfrac{(C_{D_G} - \mu C_{L_G})}{C_L}} \right] \tag{3}$$

which may be expanded into the series expression -

$$s_G = \frac{\omega}{\rho g\, C_L\,(T/W - \mu)} \left\{ 1 + \tfrac{1}{2}\,\frac{(C_{D_G} - \mu C_{L_G})}{C_L\,(T/W - \mu)} + \tfrac{1}{3}\,\frac{(C_{D_G} - \mu C_{L_G})^2}{C_L^2\,(T/W - \mu)^2} + \ldots \right\} \tag{4}$$

The factor outside the brackets in this expression derives from the simple unresisted motion of the aircraft, (i.e. from elementary dynamics), while the terms inside the brackets take into account aerodynamic lift and drag effects during the ground roll For typical values of these parameters the third and higher terms in the series may be ignored, with negligible loss in accuracy, while the second term will usually be small (<0.1) compared with unity. Also, for concrete runways, the rolling coefficient of friction, μ, will be as low as 0.02–0.03, and it may be permissible to neglect it, in comparison with T/W, in some parts of the analysis.

A further simplification adopted for much of the subsequent analysis is to assume that the factor $(C_{D_G} - \mu C_{L_G})$ remains constant for the range of take-off lift coefficient investigated.

In cases where future designs of high lift device might produce excessive profile or induced drag during the ground roll, it is probable that means would be sought for delaying their operation until just before the lift-off speed was reached. in order to avoid excessive ground runs. Under these circumstances the present analysis would still remain valid.

It is seen, from the above discussion, that the ground run distance is largely dominated by the simple mechanics of accelerating the aircraft mass up to a speed at which it can become airborne, using the specified take-off lift coefficient. Equation (4) shows that this distance will increase in direct proportion to the wing loading, and in inverse proportion to the thrust-weight ratio, take-off lift coefficient, and atmospheric density.

To give some idea of the numerical values Involved, Fig.2 shows values of the take-off ground run, calculated from equation (4), for lift coefficients ranging between 1 and 4, thrust-weight ratios from 0.2 to 0.5, and wing loadings of 50, 100 and 150 lb/ft^2. (The value of μ was taken to be 0.02, and of $C_{D_G} - \mu C_{L_G}$ to be 0.05, for these calculations.) For this range of variables the largest ground run is 12,000 ft and the smallest is about 400 ft.

The curves of Fig.2 show clearly the law of diminishing returns, between ground roll distance and increasing take-off lift coefficient, arising from their inverse relationship. For instance, the reduction in ground run due to increasing the take-off C_L from 1 to 2 is double that due to increasing the C_L from 2 to 4. The same situation applies to increasing the thrust-weight ratio for conventional engine installations. Thrust deflection may be employed to make better use of the thrust available[1], but this is beyond the scope of the present paper.

2.2 The airborne distance from lift-off to the screen height

The estimation of the ground run distance given in the previous section was comparatively straightforward and, even with the simplifying assumptions made for this analysis, could be expected to give results which do not depart far from what would be measured in practice. Reliable estimation of the airborne distance to the screen height is more difficult. Some of the reasons for this uncertainty were mentioned at the beginning of this section. They Include, varying ground effect, changing aircraft configuration, e.g. under carnage retraction, and the influence of different piloting techniques.

The various methods of estimating the airborne &stance which have been published[5,6,7,8,9,10] all depend, at some stage, on parameters which can be evalusted only from experimentally derived data. This data may appear either as an increase in speed during the transition to steady climbing fight, or as an excess in normal acceleration, or as en increment in lift coefficient, depending on the theoretical model of the take-off assumed in the different methods. Whichever it may be, such parameters have not usually been measured directly during the experimental work. The values quoted are such es to yield the measured take-off performance when substituted into the theoretical model of the take-off. There can be little doubt that such experimental data contain concealed empirical factors which allow for the differences between the real take-off and the assumed theoretical model.

When it comes to choosing a model for the present analysis, we are faced with the further difficulty of not knowing how piloting techniques may change between conditions of high and low thrust-weight ratio, or with different take-off speeds. Fortunately, we shall find that the optimum takeoff lift coefficients are defined fairly clearly, irrespective of the take-off technique assumed. The uncertainties in the methods of estimating the airborne part of the take-off distance should, however, be recognized.

For the purposes of the present analysis, the method which gives the airborne distance in the most convenient form is that quoted by Ewans and Hufton[10].-

$$s_a = \frac{k_2 V^2}{g} + \frac{h}{\tan \gamma_c} \qquad (5)$$

Since a derivation of this equation does not seem to be generally available, the analysis is given in Appendix A. Tbe two assumptions are •

(1) That the lift coefficient remains constant, at the value for level flight at the lift-off speed, throughout the transition.

(2) That (Thrust-Drag) remains constant during the transition.

With these assumptions the transition to climbing flight is effected by a phugoid type of motion, the curvature of the flight path being produced by the excess lift generated, as the speed rises above the lift-off speed. In practice, it is assumed that this phugoid motion is not allowed to develop fully, action being taken by the pilot to stabilise the aircraft in a steady climb when the flight path gradient γ first attains its equilibrium value γ_c.

Equation (5) is only strictly accurate when the aircraft has attained the steady climbing condition by the time it reaches the screen height, h. Appendix A shows that this condition should be satisfied for the cases of interest in this Report, although it may affect the calculated take-off distances at low C_L.

The theoretical analysis yields a value for k_2 in equation (5) of $1/\sqrt{2}$, but the authors state[10] that a value $k_2 = 1$ gives estimates of airborne distance which are in better agreement with the values measured during actual take-offs.

The two components of equation (5) have been termed the 'transition distance', s_T, and the 'climb distance', s_c. These are illustrated in Fig.3 It should be noted that the 'transition distance', as defined here, is not the distance in which the steady climb path is achieved. It may best be regarded as the difference between the actual airborne distance and the distance which would have been achieved had the aircraft climbed straight up to the screen at its final steady climbing angle.

If we adopt the suggested value[10] of k_2 = 1, and again use equation (2) to substitute the take-off C_L, in place of the take-off speed, the expression for the 'transition distance' becomes -

$$s_T = \frac{2w}{\rho g \, C_L} \tag{6}$$

The transition distance is seen to be independent of thrust-weight ratio, but its variation with wing loading, take-off lift coefficient, and density, is similar to that of the ground-roll distance. Numerical values for the transition distance are shown in Fig.4, for the same range of variables used before, i.e. C_L from 1 to 4, and wing loadings of 50, 100 and 150 lb/ft² The transition distances range between about 4000 ft and 300 ft

The second component of the airborne distance, termed here the 'climb distance', is given by the ratio of the screen height h, (usually 35 ft for civil requirements, and 50 ft for military requirements) to the steady climb gradient The latter is determined by the balance of thrust, drag, and weight components, acting along the flight path

It was mentioned earlier that the critical design case for multi-engined aircraft (particularly twins) is frequently that in which an engine fails during the take-off The general case of an engine failing anywhere during the take-off is beyond the scope of the present analysis, but the effect of an engine failure at lift-off may be included, quite simply, by applying a factor f, (= $1-1/n$, where n is the total number of engines), to the thrust for the airborne part of the take-off Engine failure at lift-off is included in all the calculations and graphs of this Report When one engine fails, on twin, three and four engined aircraft, f thus has the values 0.5, 0 67 and 0.75, respectively. Where a very large number of engines are fitted, $f \to 1$, and $f = 1$ also applies, of course, to a take-off with all engines operating. (The values of thrust-weight ratio, T/W, given in this paper will always refer to the total installed thrust, measured at the take-off speed - see footnote on Page 4. Thrust losses due to engine failure are always accounted for by the factor f, as described above.)

It should be noted that the engine failure at lift-off, represented here, does not, in general, constitute such a severe case as that considered in establishing the, so-called, 'balanced field length',* stipulated by the airworthiness requirements. Nonetheless, it is felt that the analysis should give a worthwhile indication of the effect of engine failure on take-off performance.

The balance of forces acting along the flight path during the steady climb is given by -

$$fT - D = W \sin \gamma_c$$

---

*The balanced field length is the take-off distance required when an engine fails at such a point that the distances for continuing the take-off, and for successfully abandoning it, are equal This is, generally speaking, the worst point at which an engine can fail. If an engine fails earlier, a shorter distance is required to stop, if it falls later, a shorter distance results from continuing the take-off.

For small angles of climb, and equating the lift equal to the weight, this may be written -

$$\gamma_c = fT/W - D/L \qquad (7)$$

If, for simplicity, the aircraft drag coefficient* is expressed in the classical form -

$$C_D = C'_{D_0} + \frac{C_L^2}{\pi A_e} \quad ,$$

where C'_{D_0} is the zero-lift drag coefficient with the flaps down, and $A_e(=A/k)$ is a" 'effectwe' aspect ratio[11], equation (7) becomes -

$$\gamma_c = fT/W - \frac{C'_{D_0}}{C_{L_c}} - \frac{C_{L_c}}{\pi A_e} \qquad (8)$$

and hence the climb &stance is given by -

$$s_c = \frac{h}{fT/W - \frac{C_{D_0}}{C_{L_c}} - \frac{C_{L_c}}{\pi A_e}} \qquad (9)$$

It may be noted that, unlike the expressions for s_G and s_T, s_c is '"dependent of the aircraft wing loading.

The method of calculating tbe transition distance, discussed earlier in this section, implies that the speed rises during the transition, attaining a value -

$$v_c = v(1+2k_2\ \gamma_c)^{1/2} \ ,$$

at the point where the steady climb angle is reached (see Appendix A). Strictly speaking, the climb gradient given by equation (8) should be evaluated at a climb lift coefficient, C_{L_c}, lower than tbe take-off C_L, by the factor $\frac{1}{1+2k_2\ \gamma_c}$. This however could only be evaluated iteratively. I" practice the climb gradients corresponding to the optimum take-off lift coefficients are found to be close to tbe minima specified by the airworthiness requirements, i.e. of the order of a few per cent. The difference between the take-off and climb lift coefficients has therefore bee" ignored in this analysis. Admittedly, this implies a slight philosophical inconsistency, since the method of calculating the transition depends on an increase in speed, which is then ignored in evaluating the climb path. However, the practical effect on the evaluation of optimum lift coefficients is small. The effect of the various parameters occurring in

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*There must be some doubt as to the value of C'_{D_0} which is appropriate, since undercarriage retraction will have started during this phase. While each case must be treated on its merits, it is suggested that the profile drag due to the undercarriage may be found to be roughly compensated by the reduction in lift-dependent drag, due to ground effect, and that the value for the aircraft with undercarriage retracted should therefore be used.

equation (9) is illustrated in Fig.5. The variation of climb distance with take-off lift coefficient is plotted for a twin-engined aircraft of $A_e = 5$ and T/W = 0.3, and then the effect of changing thrust-weight ratio (Fig.5a), aspect-ratio (Fig.5b), and number of engines (Fig.5c) from this datum case are shown. In each case an engine failure is assumed to occur at lift-off.

The curves are all seen to be of the same general form, with a fairly constant value of the climb distance at the lower lift coefficients, but steepening rapidly as some critical value of the lift coefficient is approached, the climb gradient then tending to infinity asymptotically. The nature of this variation is best discussed with the aid of a diagram showing the aircraft's drag characteristics as a plot of D/L against C_L (Fig.6). On this diagram the component of D/L due to the lift-dependent drag is a straight line through the origin of slope $1/\pi A_e$, while the component due to the zero-lift drag is a rectangular hyperbola. The sum of these, forming the total D/L curve, has the well known properties[11] of a minimum value

$$\left(\frac{D}{L}\right)_{min} = \sqrt{\frac{4C'_{D_o}}{\pi A_e}}$$

occurring at a lift coefficient, $C_L = \sqrt{\pi A_e C'_{D_o}}$. Also, the components due to zero-lift and lift dependent drag are equal at this point.

The climb angle γ, given by equation (7), is represented on this diagram by the vertical intercept between the total D/L curve and a horizontal line drawn through the ordinate fT/W. At the point marked A on the diagram, where these two lines intersect, the climb gradient naturally has the value zero, and this determines the C_L at which the climb distance tends to infinity. A more detailed analysis of this condition will be given later (section 3.1).

The level of the relatively flat portion of the curves of climb distance against C_L (Fig.5) may be found, approximately, by substituting the value of $(D/L)_{min}$ in equation (9). i.e

$$(s_c)_{min} = \frac{h}{fT/W - \sqrt{\frac{4C'_{D_o}}{\pi A_e}}} \tag{10}$$

2.3 The total take-off distance

The ground roll, transition distance and climb distance, given by equations (4), (6) and (9), are added to give an expression for the total take-off distance :

$$s = \frac{\omega}{\rho g\, C_L\,(T/W - \mu)}\left\{1 + \tfrac{1}{2}\frac{(C_{D_G} - \mu C_{L_G})}{C_L\,(T/W - \mu)}\right\} + \frac{2\omega}{\rho g\, C_L} + \frac{h}{fT/W - \frac{C'_{D_o}}{C_L} - \frac{C_L}{\pi A_e}} \tag{11}$$

From the previous discussion of the way in which the components of this expression vary with C_L, it is evident that there exists an optimum lift coefficient which will give the shortest take-off distance for a given wing loading, aspect ratio, etc. Increasing C_L results, initially, in a reduction in overall take-off distance, because it represents s lowering in take-off speed, and therefore s shorter ground run. As take-off C_L is increased still further, this improvement in ground run is progressively outweighed by an increase in climb distance, due to the shallower climb gradient resulting from higher lift dependent drag. The D/L diagram (Fig.6), discussed earlier, is again useful for illustrating this point. At A the ground run (and transition) sre at their shortest (because V is lowest), but the climb gradient is zero, so that the total take off dlstsnce is infinite. As C_L is reduced (and take-off speed increased) the ground run lengthens, but the climb gradient increases, and so reduces the climb dlstsnce. At A' tbe climb gradient is greatest further reduction in C_L will obviously increase both airborne and ground distances. Clearly, between A' and A there exists an optimum C_L for minimum take-off distance.

As an illustration, Flg.7 shows the variation in total take-off dlstsnce with C_L for the example aircraft considered earlier, i.e. one having two engines, an effective aspect ratio of 5, C'_{D_0} = 0.03, and s wing loading of 100 lb/ft². Both optimum C_L, and the corresponding take-off dlstsnce are seen to depend markedly on thrust-weight ratio, ranging in value from $C_{L_{opt}}$ = 1.3, with s take-off distance of 8500 ft, for $T/W = 0.25$, to $C_{L_{opt}}$ = 2.9, with s distance of 2000 ft, for T/W = 0.50. A more detailed examination of the factors which determine $C_{L_{opt}}$ IS given in section 3.

2.4 A note on semi-empirical take-off dlstsnce charts

It is common practice, in aircraft project work, to make use of semi-empirical take-off distance charts for preliminary performance estimates. Examples msy be found in the sppropnste text books[12,13,14], and, ss an illustration, such s chart based on data given by Brooks[15] is shown in Flg.8. The parameter on which these charts are usually based is •

$$\frac{\omega}{T/W\,\sigma C_L} ,$$

and it may be seen that (neglecting μ) this is proportional to the factor outside the bracket in equation (4), which was shown earlier to derive from the simple unresisted motion of the aircraft during the ground roll. It is evident that the basic assumption underlying the use of such charts is that the total take-off dlstsnce varies roughly in proportion to the ground rol l distance. These charts are found to be reasonably accurate in practice, and are particularly useful for assessing the effect of variation in the design parameters about s well established datum condition. The present analysis shows, however, that there would be large errors at lift coefficients approaching the critical, and the charts given later in this paper may be used to establish whether this condition applies.

3 THE OPTIMUM LIFT COEFFICIENT GIVING MINIMUM TAKE-OFF DISTANCE

3.1 Derivation of s theoretical expression for the optimum C_L

The reasons for the occurrence of an optimum lift coeffmient, giving minimum take-off distance, have already been briefly outlined in section 2.3. We shall now derive an expression for this optimum, in terms of the aircraft thrust-weight ratio, aspect ratio, number of engines, and other relevant parameters. In theory this could be done by differentiating equstmn (11), as it stands, but the resulting expression is found to be too unwieldy to be of value in the present analysts. Two simplifying steps have therefore been taken to make the expression more manageable.

(1) The function $\left\{1 + \frac{1}{2}\frac{(C_{D_G} - \mu C_{L_G})}{(T/W - \mu)\,C_L}\right\}$ in equstlon (11) has been replaced by a constant λ, typically of value 1.1. Although this function does itself contain the Independent variable C_L, the contribution of the term in which C_L sppesrs is small. This simplification is not considered to detract seriously from the value of the analysis.

(2) The term $\frac{C_{D_0}'}{C_L}$ in the 'climb distance' component of equation (11) has, for the moment, been neglected. Potentially, this is a more serious omission, for the effect of the zero-lift drag is quite significant at low thrust-weight ratios, although lift dependent drag forms the major proportion of the drag at the higher lift coefficients However, an approximate method of taking account of the zero-lift drag has been developed, and will be discussed subsequently

With the two simplifications described above, the expression for the take-off distance, given by equation (11), is reduced to •

$$s' = \frac{\omega}{\rho g\, C_L}\left(2 + \frac{\lambda}{T/W}\right) + \frac{h}{fT/W - \dfrac{C_L}{\pi A_e}} \tag{12}$$

(results obtained from the simplified analysis, neglecting the zero-lift drag, will be denoted by primed symbols, e.g. s', to distinguish them from results obtained when taking the zero-lift drag into account).

Differentiating equstlon (12) w.r.t. C_L, and setting equal to zero •

$$\frac{ds'}{dC_L} = -\frac{\omega}{\rho g\, C_L^2}\left(2 + \frac{\lambda}{T/W}\right) + \frac{h/\pi A_e}{\left(fT/W - \dfrac{C_L}{\pi A_e}\right)^2} = 0$$

yields the value of the C_L for minimum distance •

$$[C_L]'_{MD} = \frac{\pi A_e\, fT/W}{1 + \sqrt{\dfrac{\rho g h}{\omega}\,\pi A_e\,\dfrac{T/W}{(2T/W + \lambda)}}} \tag{13}$$

The expression below the root sign is less than unity, for practical values of the parameters, (ranging typically from about 0.1 to 0.6).

The function which forms the numerator of equstlon (13), i.e. $\pi A_e\, fT/W$, will occur repeatedly during this analysis, and it is worthwhile studying its significance with the aid of the drag diagram previously used (Flg.6). The lift dependent drag component of D/L, given by $C_L/\pi A_e$, Intersects the line representing the available thrust-weight ratio at the pant marked B, *so that,* at this point, $C_L/\pi A_e = fT/W$. It is therefore evident that the numerator in equstlon (13) is equivalent to the *lift coefficient at which the lift dependent drag equals the available thrust. We* shall henceforward denote this lift coefficient by the special symbol $[C_L]_\nu$, so that •

$$[C_L]_\nu = \pi A_e \; fT/W \qquad (14)$$

As $[C_L]_\nu$ plays such a prominent part in this analysis, it is worth being able to evaluate it rapidly, and Figs.9 and 10 provide charts for this purpose. Fig.9 is the more general chart of $[C_L]_\nu/f$, allowing for any number of engines, while Fig.10 gives individual graphs, allowing rapid evaluation for the most common cases of two, three and four engined aircraft.

Equation (13) may now be usefully regarded as relating the C_L for minimum take-off distance to $[C_L]_\nu$, by means of the factor -

$$\frac{[C_L]'_{MD}}{[C_L]_\nu} = \frac{1}{1 + \sqrt{\frac{\rho g h}{\omega} \pi A_e \frac{T/W}{(2T/W + \lambda)}}} \; . \qquad (15)$$

Numerical values of this factor are plotted in Fig.11, for a range of aspect ratio, wing loading and thrust-weight ratio, (assuming I.S.A. sea level conditions, and a screen height of 35 ft). The factor is seen to be relatively insensitive to variation of the parameters, which would themselves produce large changes in $[C_L]_\nu$ (and thus, in $[C_L]'_{MD}$ itself). For instance, for $A_e = 1$ and $\omega = 50$ lb/ft², the value of $[C_L]'_{MD}/[C_L]_\nu$ is about 0.85, while for $A_e = 10$ and $\omega = 100$ lb/ft², the value is about 0.75. For the same range of aspect ratio, the value of $[C_L]_\nu$ will have increased by a factor of 10. We may thus deduce that (ignoring, for the moment, the effects of zero lift drag) the C_L for minimum take-off distance is a roughly constant proportion (say, about 0.75) of $[C_L]_\nu$. Thus $[C_L]'_{MD}$ may he taken to increase roughly in proportion to the effective aspect ratio, the thrust-weight ratio, and f, the factor accounting for the thrust loss due to engine failure. Changes in wing loading have only a minor effect.

We now turn to the method of taking some account of the zero-lift drag in this analysis. The important part played by the climb gradient in determing the minimum take-off distance has been noted in the previous discussion (section 2.3). In the analysis given above (i.e. neglecting zero-lift drag) a relationship has been derived between the optimum C_L and $[C_L]_\nu$, but, if zero-lift drag is neglected, $[C_L]_\nu$ is the lift coefficient giving zero rate of climb. We now make the assumption that the same relationship holds, at least to a first order, between the true optimum C_L and the true C_L for zero rate of climb when taking account of zero-lift drag in both cases. This may be expressed as -

$$[C_L]_{MD} \approx \frac{[C_L]'_{MD}}{[C_L]_\nu} [C_L]_{ZRC} \; . \qquad (16)$$

This assumption is difficult to test analytically, but it seems reasonable, and has been found, by comparison with exact calculations from equation (11), to give better predictions of $[C_L]_{MD}$ than those from the simpler analysis.

In order to use equation (16), it is now necessary to derive an expression for the zero-rate-of-climb lift coefficient, $[C_L]_{ZRC}$, which includes the zero-lift drag term. Setting $\gamma = 0$ in equation (8), and taking the appropriate root of the resulting quadratic, gives the solution:

$$[C_L]_{ZRC} = [C_L]_\nu \left\{ \tfrac{1}{2}\left[1 + \sqrt{1 - \frac{4C'_{D_o}}{\pi A_e (fT/W)^2}}\right]\right\} \qquad (17)$$

When combined with equations (15) and (16), this gives the following expression for $[C_L]_{MD}$:

$$[C_L]_{MD} = \frac{\left\{ \tfrac{1}{2}\left[1 + \sqrt{1 - \frac{4C'_{D_o}}{\pi A_e (fT/W)^2}}\right]\right\}}{1 + \sqrt{\frac{\rho g h}{\omega} \pi A_e \frac{T/W}{(2T/W + \lambda)}}} [C_L]_\nu \, . \qquad (18)$$

This expression differs from that given by the simpler analysis (equation (15)) only by the term within brackets in the numerator, which mey be regarded as a correction factor in the relationship between $[C_L]_{MD}$ and $[C_L]_\nu$, to allow for the effects of zero-lift drag. Values of this factor

$$\left\{ \tfrac{1}{2}\left[1 + \sqrt{1 - \frac{4C'_{D_o}}{\pi A_e (fT/W)^2}}\right]\right\} = \frac{[C_L]_{ZRC}}{[C_L]_\nu}$$

are given graphically in Fig.12, for thrust-we&it ratios between 0.1 and 0.5, values of the parameter C'_{D_o}/A, ranging between 0.003 and 0.012, and for twin, three and four engined aircraft.

At high values of T/W, Fig.12 shows that the factor $[C_L]_{ZRC}/[C_L]_\nu$ is close to unity, indicating that the zero-lift drag term is relatively unimportant, and that the simplified analysis given earlier is therefore adequate. With reducing T/W, however, the factor diminishes, tending towards a limiting value of ½, which occurs at a different critical value of T/W, depending on the number of engines and the value of C_{D_o}/A_e. The drag diagram of Fig.6 is again helpful in understanding this variation. The ratio $[C_L]_{ZRC}/[C_L]_\nu$ is represented on this diagram by the ratio of the lift coefficients at the points marked A and B. This is seen to diminish progressively as the value of fT/W is lowered, until the minimum drag point is reached at A'. This occurs when

$$T/W = \frac{1}{f}\sqrt{\frac{4C_{D_o}}{\pi A_e}} \, , \qquad (19)$$

and level flight at any value of C_L is not possible for thrust-weight ratios below this. The limiting value of $[C_L]_{ZRC}/[C_L]_\nu = ½$, at this point, is also seen to stem directly from the characteristics of the D/L v C_L curve discussed previously (section 2.2).

The pmposed quick method of finding the C_L for minimum take-off distance may now be summarized as follows.

For the given values of T/W, A, and number of engines, read off the value of $[C_L]_\nu$ from Fig.10 (or Flg.9). Find tbe appropriate value of $[C_L]'_{MD}/[C_L]_\nu$ from Fig.11 and then apply tbe correction factor for profile drag, $[C_L]_{ZRC}/[C_L]_\nu$, from Flg.12. The C_L for minimum take-off distance is then given by•

$$[C_L]_{MD} = \frac{[C_L]'_{MD}}{[C_L]_\nu} \frac{[C_L]_{ZRC}}{[C_L]_\nu} [C_L]_\nu$$

3.2 Comparison of optimum C_L's from published performance estimates with results from the present analysis

The values of C_L for minimum take-off distance given in previously published papers may be usefully compared with those for similar conditions deduced from the present analysis, to provide further justification for the simplifying assumptions adopted.

Futcher and Wedderspoon[16] give the results of a computer study into the effect of fitting high-lift devices to a twin jet transport aircraft, of aspect-ratio 8, and 25° sweepback. Estimates of the take-off performance, for different values of lift coefficient, wing loading, and thrust-weight ratio, were made, using a digital computer program which was known to give reliable results for this class of aircraft.

The take-off distances given are for a 'balanced field length', i.e. with an engine failing at the least favourable moment, rather than at the instant of lift-off. The computer program also gave a much more detailed mathematical representation of the take-off manoeuvre than was possible in the present analysis. Nevertheless, the values of the optrmum C_L for minimum take-off distance, found in the computer study, agree remarkably well, as Illustrated in Flg.13, with the values calculated from the present analysis, using the same values of the relevant parameters.

The conclusions of the authors of this study[16] may usefully be quoted "for a *given thrust/weight ratio there is an optimum value of* $C_{L_{max}}$ *above which take-off distance increases, although this optrmum value may not be usable due to airworthiness limitations*". The latter point will be examined later in this paper (sections 5 and 6).

Other calculations given in a paper by Johnston[17] are really more relevant to the STOL regime, and the effect of an engine failure was not included, (i.e. $f = 1$ in the nomenclature of the present analysis). However, a wide range of aspect-ratio (6 to 14), wing loading (20 to 60 lb/ft²), and thrust/weight ratio (0.38 to 0.6) was covered. Again, the agreement shown with the results of the present analysis in Flg.13 is satisfactory, although there is a tendency for the optimum lift coefficients given by this analysis to be slightly lower than those calculated by Johnston, at the higher values of CL. Johnston's conclusions from his numerical investigation[17] are well supported by tbe present theoretical work, *e.g.* "*for a given power loading, thrust loading and wing aspect ratio there always exist an optimum* $C_{L_{max}}$ *for which the take-off distance to 50 ft is minimized. Further, this optrmum* $C_{L_{max}}$ *is very nearly independent of wing loading but increases quite quickly to beyond seven for thrust loadings (static) less than two and for aspect ratios greater than ten*".

The work of Johnston" has been extended and carned into greater detail I" a paper by Mair and Edwards[18]. The results given there are not in a form which allows exact comparison with the results of tbe present analysis, but tbe 'maximum useful lift coefficients' quoted by Mair are compared with the optimum lift coefficients of the present analysis in Fxg.13. It may be noted that the 'maximum useful lift coefficients' were arbitrarily defined[17] as those giving a take-off drstance 15% greater than the minimum possible. The fact that the lift coefficients given by the present analysis are slightly larger than the 'maximum useful' values, as shown in Fig.13, is therefore entirely reasonable. The general conclusions arrived at by Mair and Edwards are in agreement with those of the other references quoted above.

4 THE OPTIMUM LIFT COEFFICIENT GIVING MAXIMUM TAKE-OFF WING LOADING

The same methods of analysis as those used in section 3 may now be used to tackle a slightly different aspect of the take-off performance problem*. Assume that a fixed take-off distance has been specified and we wish to find the take-off CL which allows the maximum wing loading to be used.

Rearranging equation (11)

$$\omega = \frac{\rho g\, C_L (T/W - \mu)}{1 + 2(T/W - \mu) + ?/, \dfrac{(C_{D_G} - \mu C_{L_G})}{(T/W - \mu)\, C_L}} \left[s - \frac{h}{fT/W - \dfrac{C'_{D_o}}{CL} - \dfrac{C_L}{\pi A_e}} \right] \tag{20}$$

With the same two simplifications as those discussed at the beginning of section 3.1, this reduces to :

$$\omega' = \frac{\rho g\, C_L\, T/W}{(2T/W + \lambda)} \left[s - \frac{h}{fT/W - \dfrac{C_L}{\pi A_e}} \right] \tag{21}$$

where the primed symbol, ω', once again denotes that the zero-lift drag has been neglected in the analysis. Differentiating equation (21), and setting equal to zero :

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*Dr. M.H.L Waters has drawn my attention to the fact that the relationships for optimum C_L dewed in this, and in the previous section, should, theoretically, be identical. This may be shown in the following general manner

Let $F(C_L, \omega, s) = 0$ be any relation between CL, ω and s, and consider increments δC_L, $\delta\omega$ and δs such that

$$F(C_L + \delta C_L,\ \omega + \delta\omega,\ s + \delta s) = 0$$

then

$$\frac{\partial F}{\partial C_L}\delta C_L + \frac{\partial F}{\partial \omega}\delta\omega + \frac{\partial F}{\partial s}\delta s = 0$$

To find a minimum of s for fixed ω, we set $\delta\omega = 0$ and seek solutions of $\delta s/\delta C_L = 0$, i.e. $\partial F/\partial C_L = 0$. Similarly, to find a maximum of ω for fixed s, we set $\delta s = 0$ and seek solutions of $\delta\omega/\delta C_L = 0$, i.e. again, $\partial F/\partial C_L = 0$.

Equations (15) and (22) of the text may, Indeed, be derived from each other by substituting for s' or ω from equation (12). However, the introduction of the correction factor for zero-lift drag into the recommended expressions for $[C_L]_{MD}$ and $[C_L]_{M\omega}$ (equations (18) and (24) respectively) has led to a slight inconsistency, so that the theoretical identity demonstrated above does not hold for these approximate expressions.

$$\frac{d\sigma'}{dC_L} = \frac{\rho g\, T/W}{(2T/W + \lambda)} \left[s - \frac{h}{fT/W - \frac{C_L}{\pi A_e}} - \frac{hC_L}{\pi A_e \left(fT/W - \frac{C_L}{\pi A_e} \right)^2} \right] = 0$$

yields tbe value of C_L, for maximum wing loading :

$$[C_L]'_{M\omega} = fT/W\, \pi A_e \left(1 - \sqrt{\frac{h}{sfT/W}} \right)$$

or, using the notation adopted in section 3.1 :

$$\frac{[C_L]'_{M\omega}}{[C_L]_\nu} = \left(1 - \sqrt{\frac{h}{sfT/W}} \right) \qquad (22)$$

Values of this expression are plotted in Frg.14 for twin and four engined aircraft, covering a range of take-off distances from 2000 ft to 10000 ft, end of thrust-weight ratios from 0.2 to 0.4. (The screen height assumed is 35 ft.)

Once again, tbe variation of $[C_L]'_{M\omega}/[C_L]_\nu$ is relatively small, for the range of parameters of practical interest, compared with the variation of $[C_L]_\nu$ Itself. As with $[C_L]'_{MD}$, $[C_L]'_{M\omega}$ may be taken to increase roughly in proportion to the aspect ratio, the thrust-weight ratio, and tbe factor *f*.

The effect of zero-lift drag may now be taken into account, using the same argument as in section 3.1. It is assumed that the relationship between $[C_L]'_{M\omega}$ and $[C_L]_\nu$ (which is the C_L for zero rate of climb when zero-lift drag is neglected) also holds true. at least to the first order, for the relationship between $[C_L]_{M\omega}$ and $[C_L]_{ZRC}$, when both the latter include the effects of zero-lift drag. This may be expressed as :

$$[C_L]_{M\omega} \approx \frac{[C_L]'_{M\omega}}{[C_L]_\nu} [C_L]_{ZRC} \qquad (23)$$

Then, substituting from equations (22) and (17) gives :

$$[C_L]_{M\omega} = \left(1 - \sqrt{\frac{h}{sfT/W}} \right) \left\{ \tfrac{1}{2} \left[1 + \sqrt{1 - \frac{4C'_{D_o}}{\pi A_e (fT/W)^2}} \right] \right\} [C_L]_\nu \qquad (24)$$

The method of using the charts given in this paper, to find the optimum C_L for maximum wing loadmg, may now be summarized as follows.-

For the given values of *T/W, A,* and number of engines, read off the value of $[C_L]_\nu$ from Fig.10 (or Flg.9). Find the appropriate value of $[C_L]'_{M\omega}/[C_L]_\nu$, for the specified take-off distance, from Flg.14, and the correction factor for profile drag, $[C_L]_{ZRC}/[C_L]_\nu$, from Flg.12. Then the C_L for maximum wing loadmg is given by -

$$[C_L]_{M\omega} = \frac{[C_L]'_{M\omega}}{[C_L]_\nu} \frac{[C_L]_{ZRC}}{[C_L]_\nu} [C_L]_\nu$$

5 CLIMB GRADIENT REQUIREMENTS

Another Important feature of take-off performance is the need for an adequate climb gradient, once the aircraft is airborne. This may provide another critical design case, particularly for an engme failure cm a twin engined aircraft. B.C.A.R.[2] lay down that the minimum climb gradients, following an engme failure, on twin, three and four engined aircraft, shall be 0.024, 0.027 and 0.030 respectively, during the second segment climb. (i.e. with the flaps in the take-off position, but with undercarriage retracted, and no ground effect.)

An equatmn for the climb gradient, γ, at any lift coefficient, has already been derived (equatmn (8)). If R is the minimum climb gradient, specified in the airworthiness requirements, the lift coefficient at which the requirement can just be met, denoted by $[C_L]_{CGL}$, *(CCL* = climb gradient limited) can be found from -

$$fT/W - \frac{C'_{D_o}}{[C_L]_{CGL}} - \frac{[C_L]_{CGL}}{\pi A_e} = R \qquad (25)$$

It may be noted that the aircraft wing loadmg does not occur in this expressmn. The relative magnitudes of the terms which contribute to the climb gradient equatmn are illustrated diagramatically in Flg.15. This shows the breakdown of the thrust-weight ratio which must be Installed to meet the airworthiness requirements, into the contributions needed to overcome zero-lift drag, lift dependent drag, to provide the specified climb gradient, and to guard against an engme failure. The disadvantage of the twin engined aircraft, relative to the four-engined, when the need to guard against engme failure is considered, is very evident.

The quadratic equatmn (25) may be solved to give the expression for climb gradient limited lift coefficient -

$$[C_L]_{CGL} = \frac{\pi A_e (fT/W - R)}{2} \left[1 + \sqrt{1 - \frac{4C'_{D_o}}{\pi A_e (fT/W - R)^2}} \right]$$

or, with the notation adopted in the previous sections -

$$[C_L]_{CGL} = \tfrac{1}{2} \left(1 - \frac{R}{fT/W} \right) \left[1 + \sqrt{1 - \frac{4C'_{D_o}}{\pi A_e (fT/W - R)^2}} \right] [C_L]_\nu \qquad (26)$$

It may be noted that the expression previously dewed for $[C_L]_{ZRC}$, (equation (17)), is the special case of equation (26) when $R = 0$.

Values of the factor $[C_L]_{CGL}/[C_L]_\nu$ are given graphically in Fig.16, for thrust-weight ratios between 0.1 and 0.5, values of the parameter C_D'/A, ranging between 0.003 and 0.012, and for twin, three and four engined aircraft. The curves bear: general resemblance to those for $[C_L]_{ZRC}/[C_L]_\nu$, given in Fig.10, and many of the comments made in section 3.1 about that factor apply here also. However, at the higher values of T/W the curves tend to a value $\left(1 - \frac{R}{f T/W}\right)$, rather than to unity, while the thrust-weight ratio at which the critical value, $[C_L]_{CGL}/[C_L]_\nu = ½$, occurs is given by •

$$T/W = \frac{1}{f}\left[\sqrt{\frac{4C_{D_o}'}{\pi A_e}} + R\right] \tag{27}$$

compared with the value given by equation (19)

We may now summarize the method of finding the C_L at which the aircraft becomes climb gradient limited, as follows •

For the given values of T/W, A, and number of engines, read off the value of $[C_L]_\nu$ from Fig.10 (or Fig.9). Find the appropriate value of $[C_L]_{CGL}/[C_L]_\nu$ from Fig.16. Then

$$[C_L]_{CGL} = \frac{[C_L]_{CGL}}{[C_L]_\nu}[C_L]_\nu$$

6 SPEED MARGINS OVER ZERO RATE OF CLIMB SPEED

One of the principal conditions which has to be satisfied in current airworthiness requirements[2,3] is that of maintaining an adequate safety margin above the aircraft's stalling speed, at all times. It is now recongnized, however, that a new generation of aircraft is about to appear, in which the traditional concepts of stalling have little relevance. This is the family of aircraft having very highly swept, or slender wings, which may be taken to angles of incidence well outside the range of practical use before any breakdown in the flow, analogous to the conventional stall, occurs. It has been suggested[19] that, in this situation, the 'zero rate of climb speed' may assume a new significance, as being the lower limit to the range of safe operational speeds. The proposed airworthiness regulations for Concorde[20] reflect this, by specifying that the initial climb out speed with one engine inoperative shall be not less than 1.15 times the zero rate of climb speed. Such a speed margin is equivalent to a margin of 1.32 on lift coefficient – that is to say, that the maximum lift coefficient which may be used, without infringing the above requirement, may be found as 1/1.32, i.e. 0.76, of $[C_L]_{ZRC}$, as given by equation (17).

7 EXAMPLES OF THE APPLICATION OF THE ANALYSIS

The way in which the lift coefficients discussed in this paper are affected by an aircraft's design parameters, such as T/W, A_e, ω, etc., has already been briefly considered in the foregoing sections, following the derivation of the theoretical expressions for each of them. The method of using Figs.9, 10, 11, 12, 14 and 16, to obtain numerical values for $[C_L]_{MD}$, $[C_L]_{M\omega}$, $[C_L]_{CGL}$ and $[C_L]_{ZRC}$, has also been summarized at the end of each section. By way of illustration, we shall now consider the values of these lift coefficients for two cases of current interest[20,21], the subsonic swept wing transport aircraft, and the short range all wing aerobus.

7.1 Subsonic swept wing transport aircraft

The example used earlier in this paper will be considered first. This was a twin engined aircraft of effective aspect ratio 5, with a wing loading of 100 lb/ft^2. Fig.17 shows again the curves of take-off distance against take-off lift coefficient, for different values of thrust-weight ratio, (first shown in Fig.7), but with the boundaries of optimum C_L, and the C_L for climb gradient limit, from the present analysis, superimposed. The boundary to give a margin in C_L of 1.32 over $C_{L_{ZRC}}$ is also shown, out of Interest, although it is not currently an airworthiness requirement for this class of aircraft. Incidently, it will be seen that the boundary of optimum C_L given by the analysis is in close agreement with the actual optima of the takeoff distance curves (which were obtained by exact calculation from equation (11)), giving some further confidence that the approximations used for overcoming the difficulties in the analysis are valid.

The curves given in Fig.17 are all for the same value of zero-lift drag coefficient, $C'_{D_0} = 0.03$, this being a typical value for current transport aircraft at take-off flap setting. Its use, at higher values of C_L than are currently used, say > 1.5, implies the use of high lift devices which do not incur significant increases in zero-lift drag, e.g. flaps with b.l.c., rather than ordinary flaps at larger deflections.

Fig.17 shows that there is little to choose between the three C_L boundaries at currently used values of thrust-weight ratio end take-off lift coefficient (i.e. $T/W = 0.25$ and $C_L = 1.5$). Bearing in mind the limitations of the analysis, it is certainly not possible to assert that any one of them is dominant. At the higher values of thrust-weight ratio, however, the analysis shows that the climb gradient limit would not occur until a lift coefficient appreciably above the optimum for take-off distance, so the latter would tend to be the dominant feature. These lift coefficients are larger than those which can currently be produced sufficiently economically for use during take-off.

We may now broaden the discussion by examining the effect of varying the number of engines on this design. Fig.18 shows the variation of $[C_L]_{MD}$ and $[C_L]_{CGL}$ with thrust-weight ratio for twin, three and four engined aircraft. In all cases the tendency for the climb gradient limit to dominate at low T/W, and the C_L for optimum distance to dominate at higher T/W, is present. The significant feature of the three and four engined aircraft is, however, that the numerical values of the C_L's involved lie well above what is achievable, under take-off conditions, in the current state of aerodynamic practice. It follows that improved flap designs, giving higher take-off lift coefficients must be accompanied, on the twin, by increases in thrust-weight ratio before the potential gains in takeoff performance can be realised. For three and four engined aircraft, on the other hand, a straightforward increase in take-off lift coefficient may, by itself, result in improved performance.

Fig.19 shows some numerical results for the alternative problem discussed in section 4, that of finding the C_L which allows the maximum wing loading to be flown from a specified take-off distance (in this case 6000 ft). It is found, in fact, that the lift coefficient for maximum wing loading $[C_L]_{M\omega}$ is generally close to that for minimum distance, $[C_L]_{MD}$ and consequently the remarks made earlier in this section about the relationship between $[C_L]_{MD}$ and $[C_L]_{CGL}$ also apply to $[C_L]_{M\omega}$.

7.2 The allwing aerobus

We now apply the analysis to a radically different type of aircraft, the slender allwing aerobus of the type discussed by Küchemann and Weber[21] and by Lee[22]. For this we may take the values $A_e = 1$, ω = SO lb/ft^2 and $C'_{D_0} = 0.012$.

Fig.20 shows the variation of the lift coefficients previously discussed; $[C_L]_{MD}$, $[C_L]_{CGL}$ and $0.76\,[C_L]_{ZRC}$, with thrust-weight ratio, for twin and three engined versions of this aircraft.

The first, end most obvious feature, is that the whole order of the lift coefficients being considered is reduced by a factor of about five, compared with those in the previous example. Since, to a first order, the take-off distance is proportional to wing loading, and inversely proportmnal to lift coefficient and thrust-weight ratio, it is evident that thrust-weight ratios considerably above those used for more conventional aircraft will be essential. This has the twofold effect of both shortening the take-off distance directly, and of allowing higher take-off lift coefficients to be used, before one or other of the limiting features Intervenes. It is also evident from Fig.18, that this limiting feature is likely to be an airworthiness margin over zero rate of climb conditions, (section 6), rather than the more usual climb gradient limitation. In fact, this is a case where the take-off performance seems likely to depend more on the factors discussed in this paper, than on the magnitude of the lift coefficients which can be generated aerodynamically.

8 CONCLUSIONS

By simplifying the equations of motion used to represent the take-off manoeuvre, it has been possible to derive closed-form theoretical expressions for some of the lift coefficients of particular interest in evaluating take-off performance. These are the lift coefficients for shortest take-off distance, for maximum wing loading (at a specified take-off distance), for just meeting the airworthiness climb gradient requirements, and for providing an adequate speed margin over the zero rate of climb speed. Charts are provided which enable the values of these lift coefficients to be found rapidly.

For those cases where the zero-lift drag is small, relative to the lift-dependent drag, it is found that all four of these lift coefficients depend primarily on a function which is directly proportional to thrust-weight ratio, effective aspect ratio and the engine failure thrust loss factor f (since the analysis took account of en engine failing at the moment of lift-off). For cases where the zero-lift drag cannot be Ignored, a method of correcting the simpler analysis is suggested.

When applied to a typical subsonic swept wing transport aircraft configuration, the analysis shows that, at present day values of thrust-weight ratio, the boundaries of optimum lift coefficient for shortest take-off distance, and for climb gradient limit, are fairly close together. At higher values of thrust-weight ratio, the optimum C_L for shortest distance appears to become dominant. Numerically, these lrft coefficients for *twin* engined aircraft are close to what can currently be achieved from aerodynamic considerations. Any developments leading to higher take-off lift coefficients may, therefore, have to be accompanied by increases in installed thrust-we&t ratio, with possible economic implications in cruising flight.

For aircraft with more than two engines, the lift coefficients for optimum take-off distance, and for climb gradient limit, are somewhat higher than those currently achieved, and there is therefore greater scope for the efficient utilization of aerodynamic developments towards higher take-off lift coefficients.

When applied to a slender allwing aerobus the analysis shows that the optimum, and climb gradient limited lift coefficients, are only about one-fifth es large as those for the conventional swept wing aircraft. While this ties in well with the lift producing capabilities of slender wings, end suggests that there may not be much scope for the use of high lift devices on such aircraft, it does mean that comparable take-off performance must be achieved by the use of low wing loadings and high thrust-weight ratios.

In this paper only one facet of en aircraft's performance envelope, namely that of take-off, has been considered. For a proper appreciation of the overall design problem it is important that similar analyses for the climb, cruise, descent and landing should be made, and that the structural and economic aspects should be considered.

Appendix A

A DERIVATION OF THE EXPRESSION FOR THE AIRBORNE DISTANCE USED IN SECTION 2.2. TOGETHER WITH SOME COMMENTS ON THE MANOEUVRE IT REPRESENTS

Ewans and Hufton quote[10] the expressmn for the airborne part of the take-off distance (given as equation (5) of this paper) and cite, as the derivation, a paper then in preparation. This later paper did not become available, but the expression may be obtained as follows.

Notation

D = drag (lb)

g = acceleration due to gravity (ft/sec²)

h = height above ground (ft)

L = lrft (lb)

s = horizontal distance (ft)

T = thrust (lb)

V = aircraft speed during the transition (ft/sec) T.A.S

V_{LO} = aircraft speed at lift-off (ft/sec) T A S

V_C = aircraft steady climb speed (ft/sec) T.A.S.

W = weight (lb)

γ = aircraft climb gradient during the transition (rad)

γ_c = equilibrium climb gradient (rad)

The suffix 1 applied to the variables s and h denotes their values when the climb gradient first attains its final steady value.

Assumptions

It is assumed that both the lift coefficient, C_L, and the difference between thrust and drag, $(T-D)$, remain constant at their lift-off values throughout the transition. Small climb gradients are only considered to allow a linearised treatment.

Analysis

With the above assumptions the equation of motion normal to the flight path becomes •

$$\frac{L}{W} = 1 + \frac{V^2}{g}\frac{d\gamma}{ds} \qquad \text{(A-1)}$$

and since C_L is assumed constant,

$$\frac{L}{W} = \left(\frac{V}{V_{LO}}\right)^2 . \qquad \text{(A-2)}$$

If the speed rise during the transition is small we may, furthermore, substitute V_{LO} for V in equation (A-l) with only small loss in accuracy. Then (A-l) and (A-2) together give:-

$$\frac{(V^2)\,g}{V_{LO}^4} - \frac{d\gamma}{ds} = \frac{g}{V_{LO}^2} \quad . \tag{A-3}$$

The equation of motion along the flight path is.-

$$\frac{d(V^2)}{ds} + 2g\gamma = 2g\,\frac{(T-D)}{W} \tag{A-4}$$

and since $(T-D)/W$ is assumed constant, and $= \gamma_C$ (A-4) gives -

$$\frac{d(V^2)}{ds} + 2g\gamma = 2g\,\gamma_C \; . \tag{A-5}$$

Eliminating (V^2) between equations (A-3) and (A-S) leads to the differential equation:

$$\frac{d^2\gamma}{ds^2} + \frac{2g^2\gamma}{V_{LO}^4} = \frac{2g^2}{V_{LO}^4}\,\gamma_C \tag{A-6}$$

which has the solution (for initial conditions y = 0 when $s = 0$)

$$y = \gamma_C\left(1 - \cos\frac{\sqrt{2}g}{V_{LO}^2}\,s\right) \; . \tag{A-7}$$

This expression represents a phugoid type of motion of constant amplitude about a mean climb path of gradient γ_C. It Is assumed that in practice this phugoid motion is not allowed to develop fully, action being taken by the pilot to stabilise the aircraft in a steady climb when the flight path gradient y first attains its equilibrium value γ_C. From equation (A-7) this is seen to occur when'

$$s = s_1 = \frac{\pi V_{LO}^2}{2\sqrt{2}g} \; . \tag{A-8}$$

Now, since y = dh/ds,

$$dh = \gamma_C\left(1 - \cos\frac{\sqrt{2}g}{V_{LO}^2}\,s\right) ds \tag{A-9}$$

which gives upon integration -

$$h = \gamma_C\left(s - \frac{V_{LO}^2}{\sqrt{2}g}\sin\frac{\sqrt{2}g}{V_{LO}^2}\,s\right) \tag{A-10}$$

and the height when the equilibrium climb gradient is first attained is •

$$h = h_1 = \gamma_C\left(s_1 - \frac{V^2_{LO}}{\sqrt{2}g}\right) \qquad \text{(A-11)}$$

The flight path above h_1 is assumed to be a steady climb at gradient γ_C. The height equation for $h > h_1$ may therefore be written'-

$$h = h_1 + \gamma_C(s - s_1) = \gamma_C\left(s - \frac{V^2_{LO}}{\sqrt{2}g}\right) \qquad \text{(A-12)}$$

As illustrated in Fig.3 of the main text, this may be considered to represent a steady climb path of gradient γ_C, stemming from a point $s_T = \frac{k_2 V^2_{LO}}{g}$ beyond the actual lift-off point. Although the analysis gives a value of $1/\sqrt{2}$ for the factor k_2 in this transition distance, Ewans and Hufton state[10] that a value of unity gives airborne distances which agree more closely with measured takeoffs.

It is evident from the above that the equation used for the airborne distance is only strictly valid for $h > h_1$, i.e. when tbe aircraft has attained the steady climb condition before reaching the screen height. [If this condition is not met equation (A-10) applies.]

From equation (A-9) it is seen that the above condition* will be satisfied if'•

$$s_a = s_T + s_C > s_1 \equiv \frac{\pi V^2_{LO}}{2\sqrt{2}g}$$

i.e. If

$$s_C/s_T > \left(\frac{\pi}{2} - 1\right)$$

i.e.

$$s_C/s_T > 0.57 \ .$$

Comparison of the curves of Figs.4 and 5 show that this condition is most likely to be contravened at low C_L, combined with high T/W and ω. The boundary where the condition is just satisfied has bee,, shown on Fig.7, and it is seen to he at lift coefficients below the optimum in all cases. In any event the errors involved are quite small. Even with s_C/s_T as low as 0.1, the airborne distance given by equation (A-12) is only about 15% greater than that given by the exact equation (A-10).

The variation of speed during the transition may be found by differentiating (A-7) to give $d\gamma/ds$, and substituting in (A-3):.

$$\frac{d\gamma}{ds} = \frac{\gamma_C\sqrt{2}g}{V^2_{LO}}\sin\frac{\sqrt{2}g}{V^2_{LO}}s \qquad \text{(A-13)}$$

and from (A-3):-

$$V^2 = V^2_{LO}\left(1 + \sqrt{2}\,\gamma_C\sin\frac{\sqrt{2}g}{V^2_{LO}}s\right) \qquad \textbf{(A-14)}$$

---

*This condition is examined in slightly greater detail in Ref.24.

and thus the speed at $s = s_1$, when the steady climb gradient is first attained is:-

$$V_C = V_{LO}\left(1+\sqrt{2}\,\gamma_C\right)^{1/2} \qquad \text{(A-15)}$$

The manoeuvre represented by the expression derived above

A method which is sometimes used for estimating the airborne distance during take-off assumes that a constant normal acceleration of $(1 + n)g$ is applied, from lift-off until the point at which the steady climb gradient is achieved. It is off interest to find the value of n which would allow the height h, to be reached in the same distance, s_1, as that given by the method used in this Report.

For the manoeuvre with constant normal acceleration

$$\frac{d\gamma}{ds} = \frac{d^2h}{ds^2} = \frac{ng}{V_{LO}^2} \qquad \text{(A-16)}$$

and hence:

$$h = \frac{ngs^2}{2V_{LO}^2} \qquad \text{(A-17)}$$

substituting the value of s_1 from (A-8) into this equation gives a height:

$$h = \frac{n\pi^2 V_{LO}^2}{16g} \; . \qquad \text{(A-18)}$$

For the method used in the present report, the same value of s_1 substituted in equation (A-11) gives -

$$h_1 = \frac{\gamma_C V_{LO}^2}{\sqrt{2}g}\left(\frac{\pi}{2}-1\right) \; . \qquad \text{(A-19)}$$

The value of n which will make h in (A-18) and (A-19) equal is therefore.-

$$n = \frac{16\gamma_C}{\sqrt{2}\pi^2}\left(\frac{\pi}{2}-1\right) \equiv 0.654\;\gamma_C \; . \qquad \text{(A-20)}$$

This analysis uses the theoretical value of $k_2 = 1/\sqrt{2}$. If the recommended value of $k_2 = 1$ is applied consistently it is found that the values of n given by (A-20) are reduced by the factor $1/\sqrt{2}$.

The numerical values of n resulting from (A-20) for reasonable values of the climb gradient γ_C, are certainly smaller then the value 0.1 which is sometimes assumed when using the constant normal acceleration method of estimation, and it is probably true that the method used in this Report represents a relatively gentle manoeuvre. (This conclusion was also reached in Ref.25 on the basis of a somewhat similar analysis, but using a constant increment in lift coefficient, rather than normal acceleration, in tbe alternative method of analysis.) However, this may not be so very unrealistic in the difficult piloting situation which exists following an engine failure during take-off. And in any case, the numerical comparison should not be pursued too far, since, although the distances to reach a particular height have been matched in the above analysis, the climb gradient achieved at that point in the constant acceleration manoeuvre is only about three-quarters of rts final value. The important feature revealed by equation (A-20) is that the method used in the main analysis implies a variation in the 'gentleness' of the take-off which is proportional to the excess of thrust over drag. There is some evidence that this reflects whet occurs in practice. For instance, in a recent flight simulator study [23] it was found that the equivalent mean normal acceleration used during takwffs with all engines operating was 1.1 g, but that this fell to 1.04 g for take-offs with one engine failed.

Appendix B

SOME POSSIBLE REFINEMENTS TO THE ANALYSIS

There are one or two refinements to the analysis which have been omitted in the main text for fear of overcomplicating the discussion. In general they are concerned with second order effects, but they are summarized here in case they should be useful in Isolated cases.

(a) Large variation of engine thrust with speed

It was stated in section 2.1 that the value of engine thrust used for evaluating the take-off ground roll should strictly be that at about 0.7 of the take-off speed, but that the value at take-off would be used in the analysis. This was done so that only one value of thrust-weight ratio would appear in the analysis for both the gmund roll and the airborne distance. In cases where the variation of engine thrust with speed is very large, a better approximation would be obtained by using the thrust-weight ratio at 0.7 of the take-off speed, wherever *T/W* appears in the analysis, and accounting for the lower thrust in the climb by a suitable adjustment of the factor *f*, which was introduced in section 2.2 to cater for engine failure at lift-off.

(b) Large variation in lift coefficient between lift-off and steady climb

If it is known that a significant variation occurs between the lift coefficient at lift-off and that for steady climb, and if their ratio can be roughly evaluated, the methods given in the text may be used to give a crude approximation as follows. It is observed that the wing loading ω and the take-off CL always occur in the same combination in the expressions for ground roll and transition distance, while ω does not occur in the expression for climb distance. If the value of the lift coefficient in the climb is used throughout the analysis the errors which would have been Introduced into the calculation of the ground roll may be compensated by a simple modification of the value of ω used, in the ratio $\omega_{used} = \omega_{true} \times \frac{C_{L_C}}{C_{L_{TO}}}$. This method obviously needs to be used with care, since it takes no account of the extra distance needed for the acceleration implied by the change in lift coefficient. It may, however, be useful in the case of low aspect ratio aircraft, where a relatively small change in CL , in terms of speed, may correspond to a large change in drag, and, therefore, a significant change in climb gradient.

SYMBOLS

Symbol	Definition, etc.	Unit
A	Geometric aspect ratio	–
A_e	Effective aspect ratio (sectmn 2.2)	–
C_D	Drag coefficient	–
C_{D_G}	Drag coefficient during ground run	–
C'_{D_o}	Zero-lift drag coefficient, flaps down	–
C_L	Lift coefficient	–
C_{L_C}	Lift coefficient during the climb	–
C_{L_G}	Lift coefficient during ground run	–
$[C_L]_{CGL}$	Lift coefficient at which aircraft becomes climb gradient limited (section 5)	–
$[C_L]_{MD}$	Lift coefficient giving minimum take-off distance (sectmn 3.1)	–
$[C_L]_{M\omega}$	Lift coefficient giving maximum take-off wing loading (sectmn 4)	–
$[C_L]_{ZRC}$	Lift coefficient at which rate of climb is zero (section 3.1)	–
$[C_L]_\nu$	Lift coefficient at which the induced drag equals the available thrust (section 3.1)	
D	Aircraft drag	lb wt
f	Factor accounting for thrust loss following engine failure	
g	Acceleration due to gravity	ft/sec^2
h	Take-off screen height	
k	Induced drag factor	
k_2	Factor in equation (5) for airborne distance	
L	Aircraft lift	lb wt
R	Climb gradient required by airworthiness regulations (sectmn 5)	
S	wing area	ft^2
s	Total take-off distance	ft
s_a	Airborne distance (section 2.2)	ft
s_c	Climb distance (section 2.2)	ft
s_G	Ground roll distance (section 2.1)	ft
s_T	Transition distance (section 2.2)	ft
T	Engine thrust	lb wt
v	Aircraft speed	ft/sec
V_2	Climb speed after engine failure	kt
W	Aircraft weight	lb wt

SYMBOLS (Contd)

Symbol	Definition, etc.	Unit
γ	Climb gradient	tad
λ	Constant used in section 3.1	
μ	Coefficient of rolling friction	
ρ	Atmosphenc density	slug/cu. ft
σ	Relative density	
ω	Aircraft wing loading	lb/ft^2

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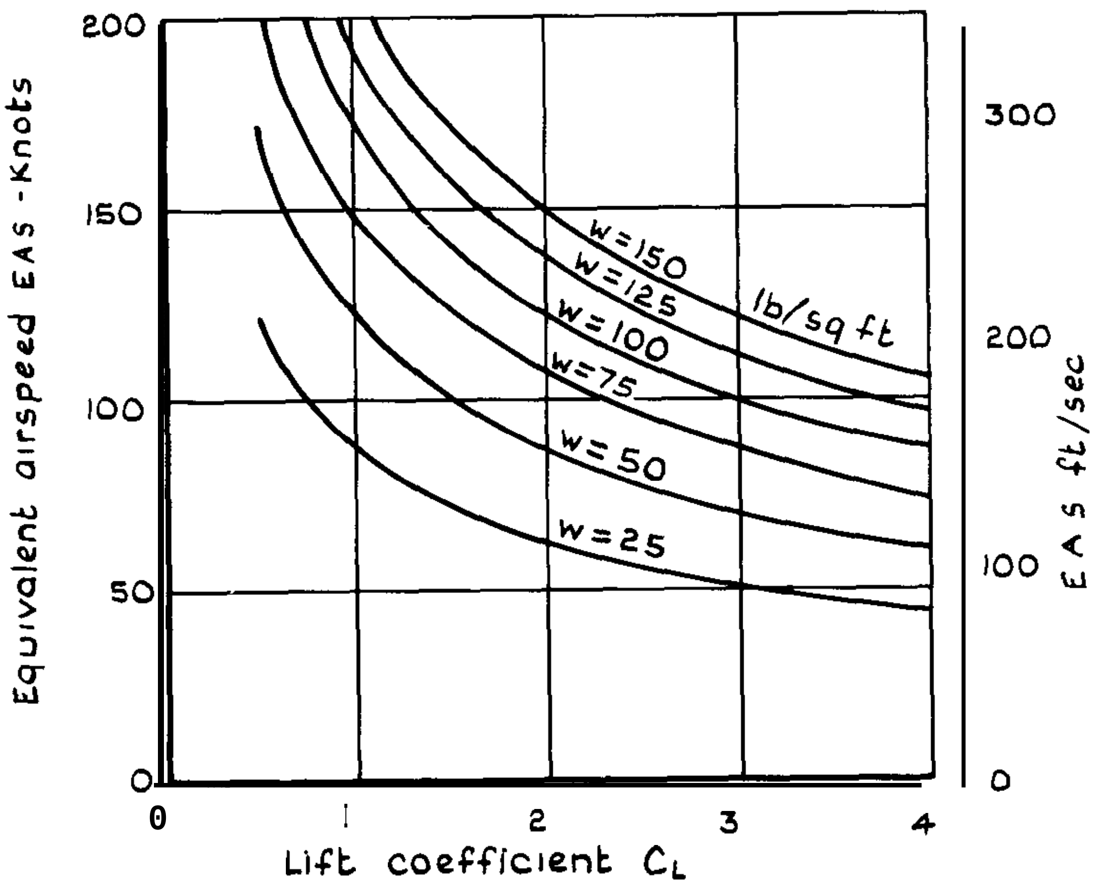


Fig.1 Relationship between wing loading, equivalent airspeed and lift coefficient for level flight

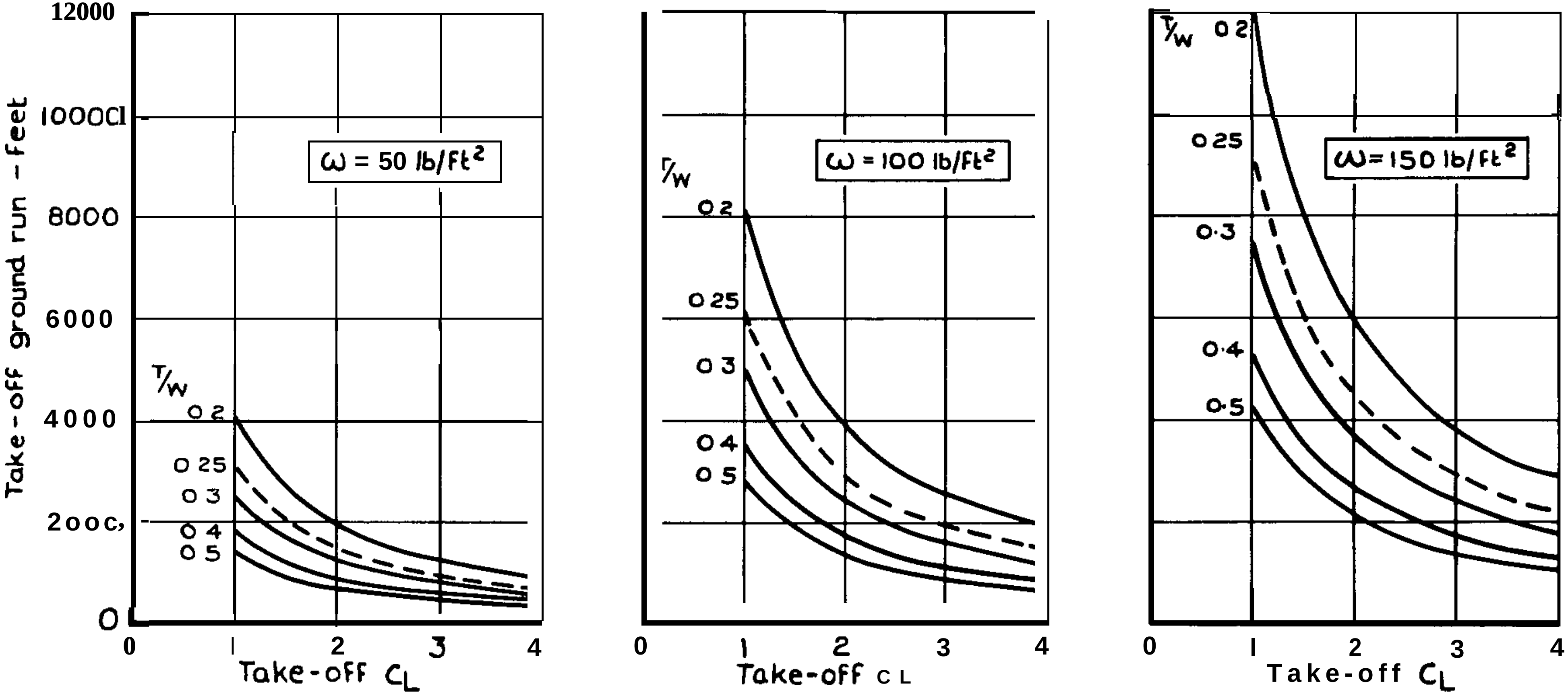
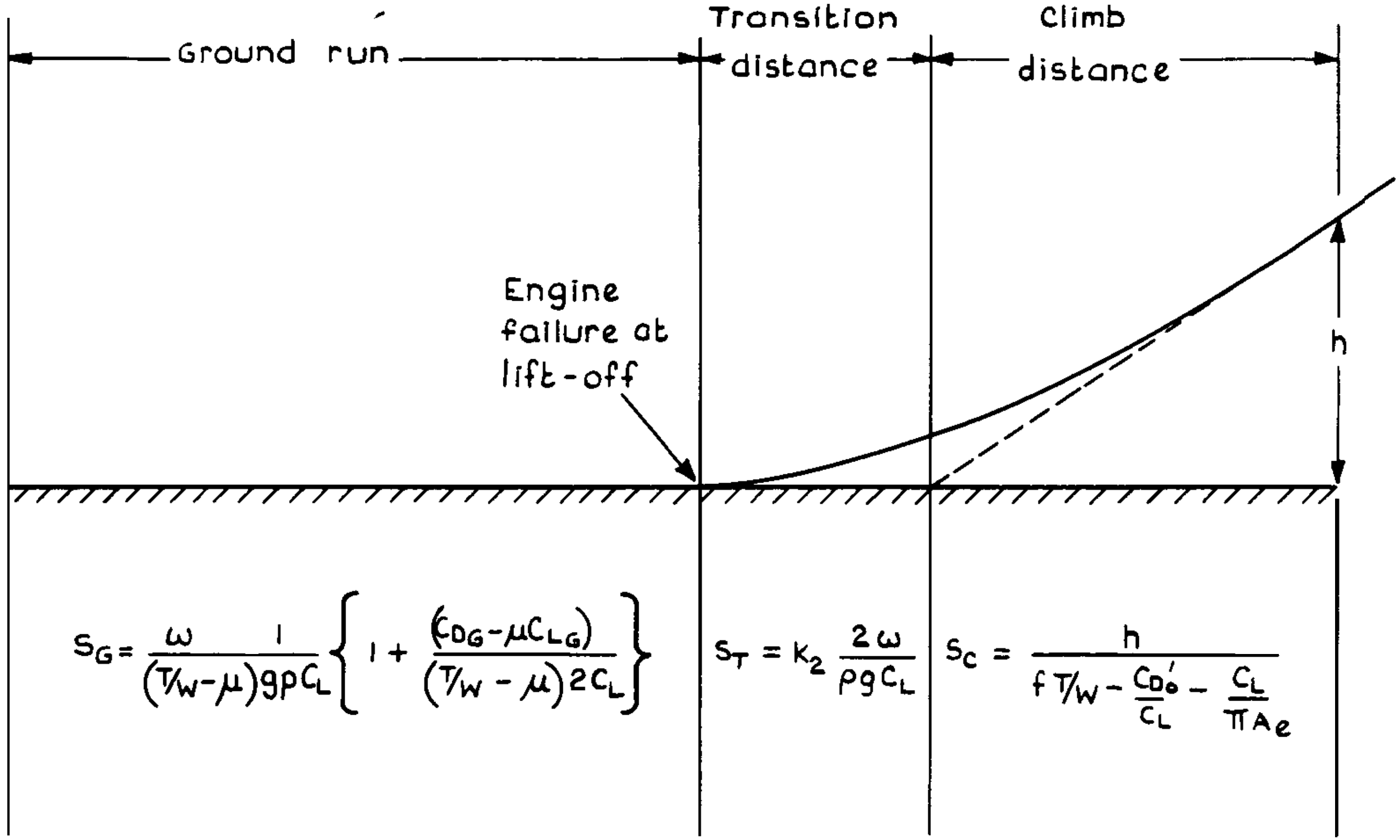


Fig. 2 Ground run to take-off. All engines operating.
$\left[\left(C_{D_G} - \mu C_{L_G}\right) = 0.05, \mu = 0.02\right]$ Sea level I S A



$$S_G = \frac{\omega}{(T/W-\mu)g\rho C_L}\left\{1 + \frac{(C_{DG}-\mu C_{LG})}{(T/W-\mu)2C_L}\right\}$$

$$S_T = k_2 \frac{2\omega}{\rho g C_L}$$

$$S_C = \frac{h}{f\,T/W - \frac{C_{D0}'}{C_L} - \frac{C_L}{\pi A e}}$$

Fig. 3 Notation and equations used for the take-off manoeuvre

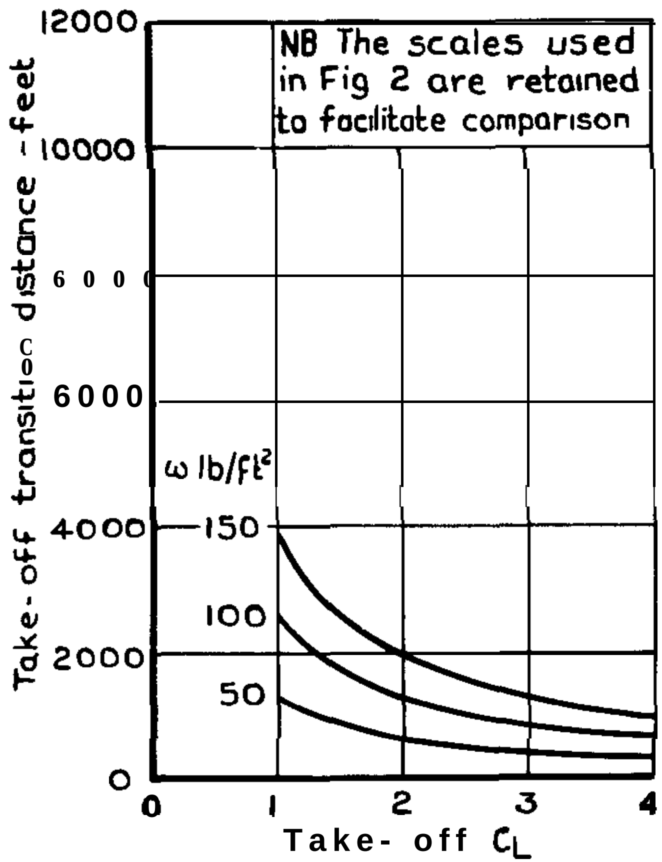
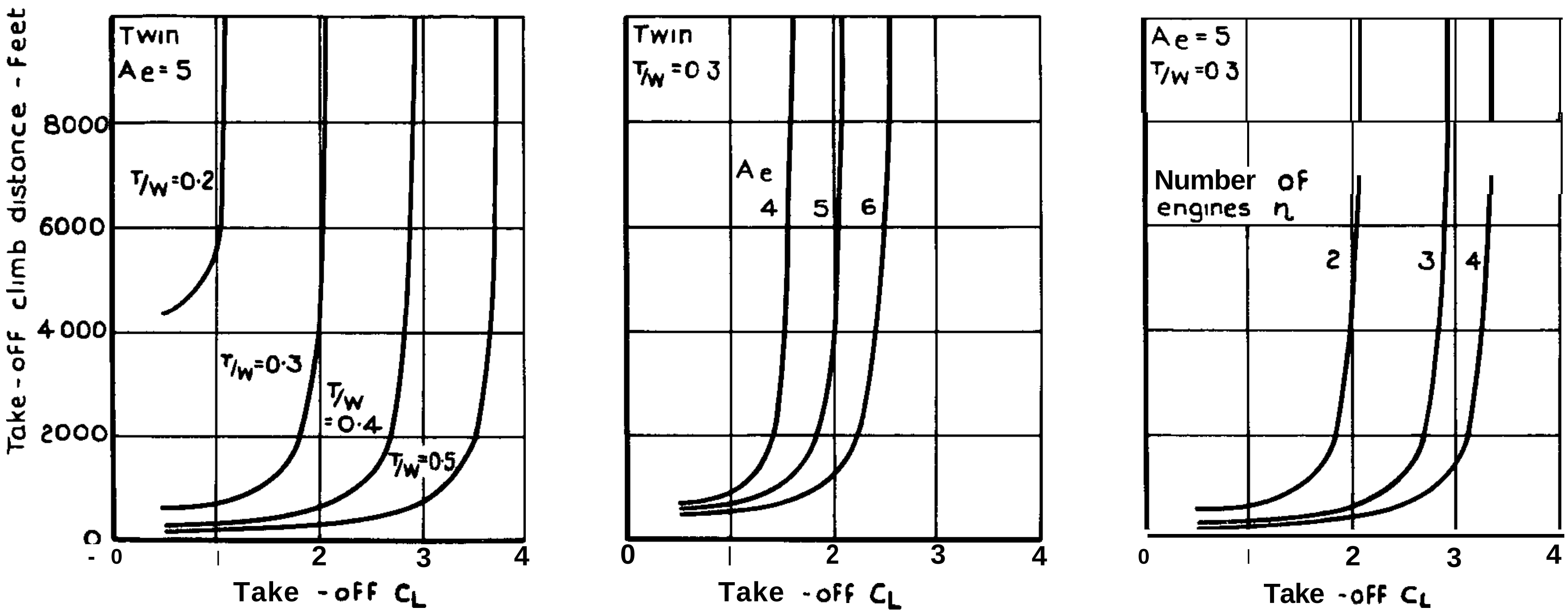


Fig. 4 Take-off transition distance (defined in Fig. 3). Variation with take-off C_L and wing loading



a Effect of varying thrust /weight ratio

b Effect of varying aspect ratio

c Effect of varying the number of engines

Fig. 5a-c Effect of various parameters on take-off climb distance. One engine failed. $C_{D'_O} = 0 \cdot 03$

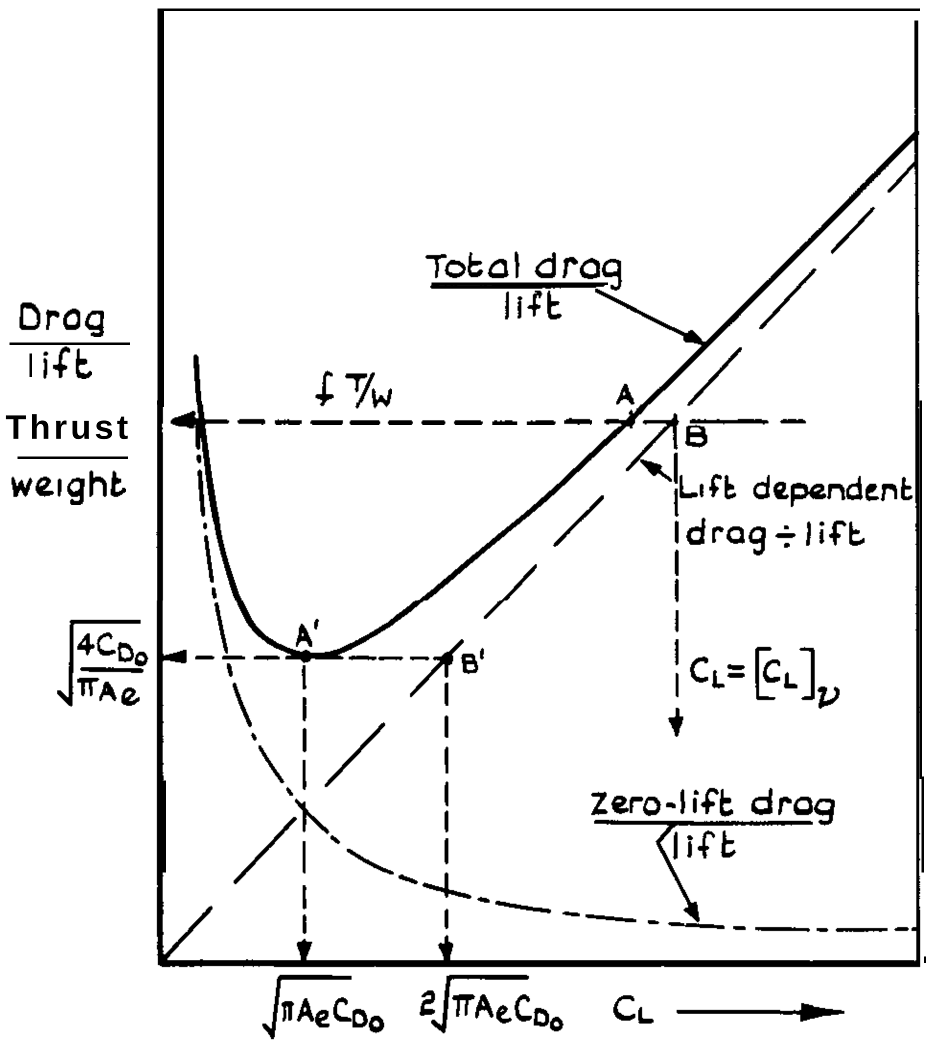


Fig. 6 Features of the ai rcrafts drag characteristics involved in the analysis

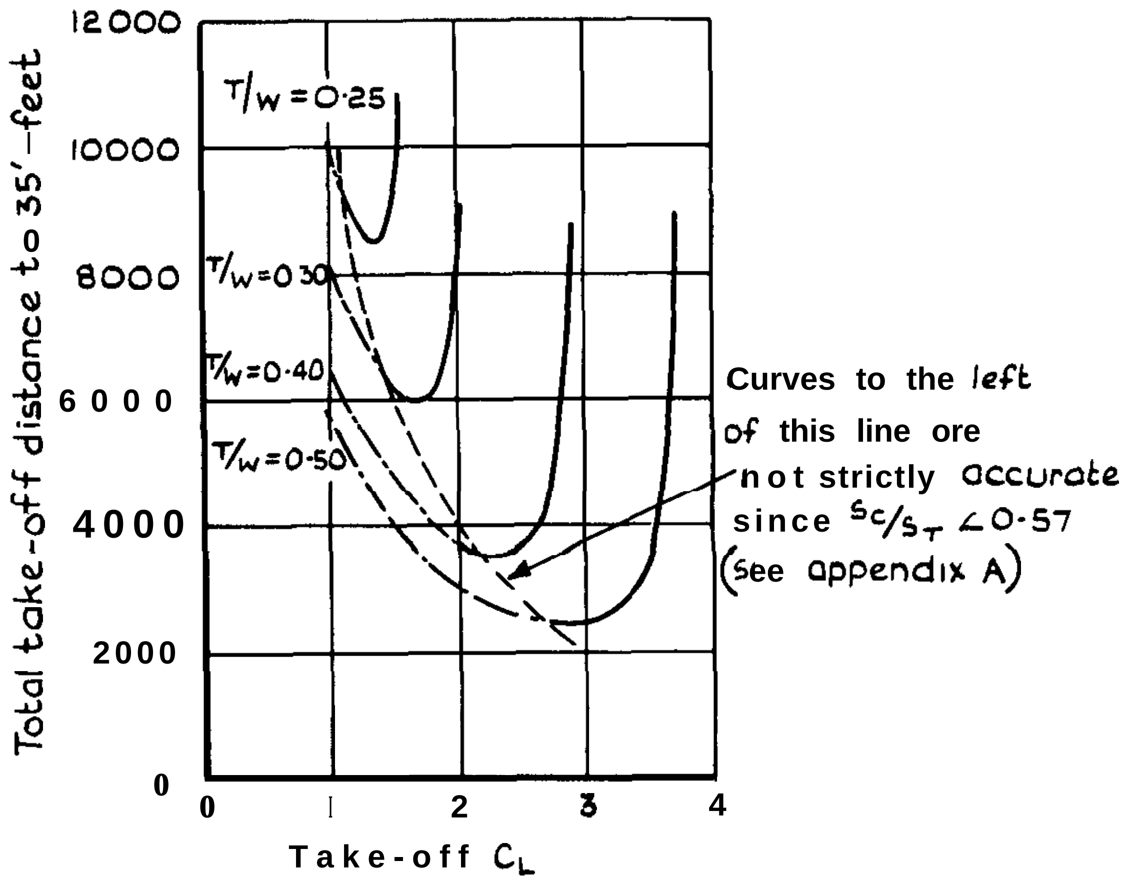


Fig.7 Variation in total take-off distance with C_L and T/W for a twin engined aircraft

$A_e = 5$ $W = 100$ lb/ft^2 $C_{D_0}' = 0{\cdot}03$

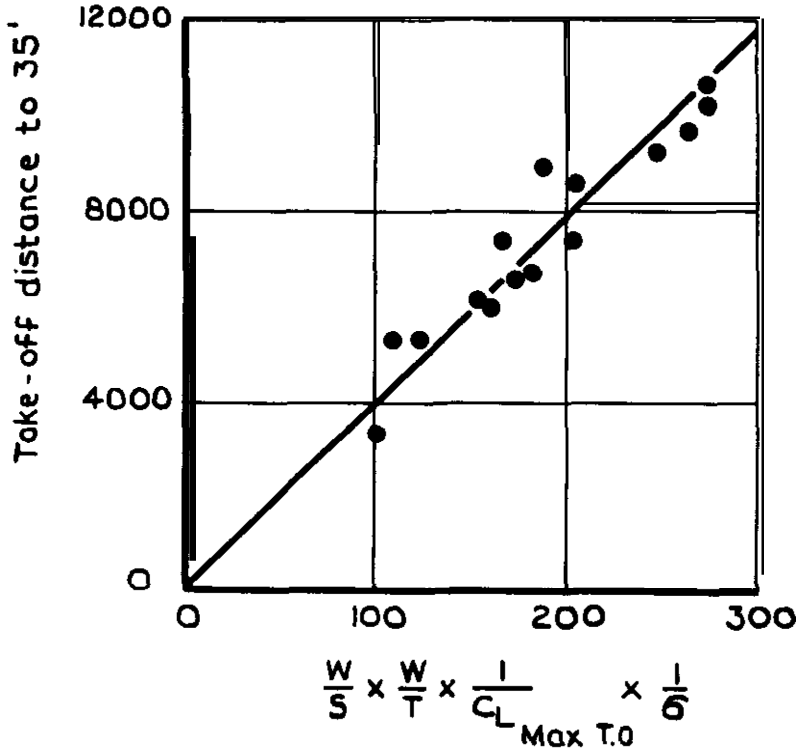


Fig. 8 Example of semi - empirical take- off design chart based on the performance of current transport aircraft (Data from ret 15)

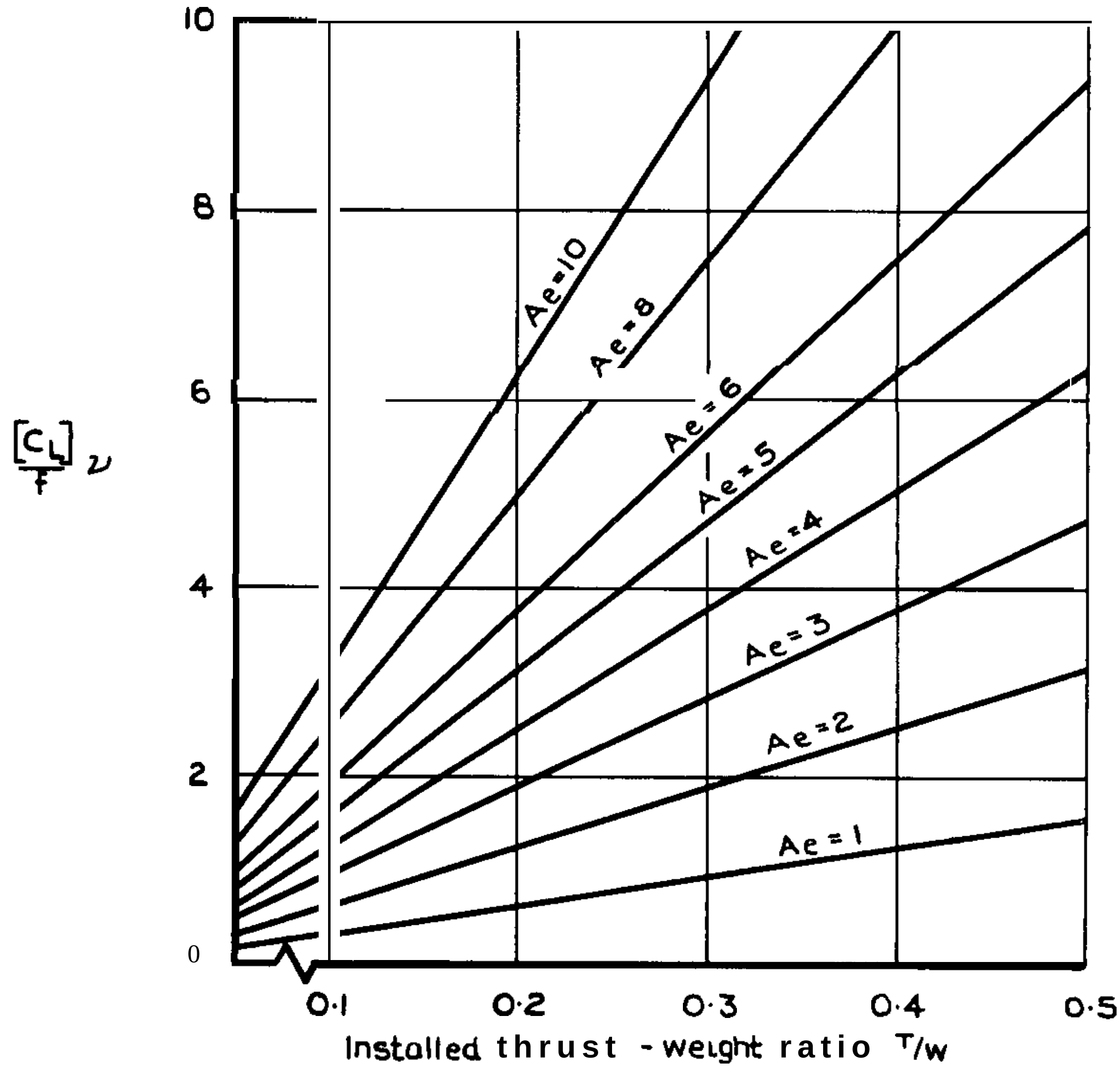
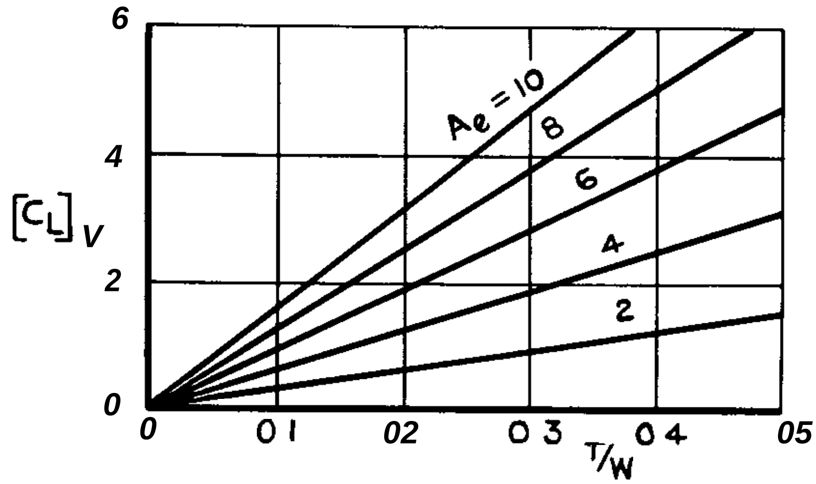
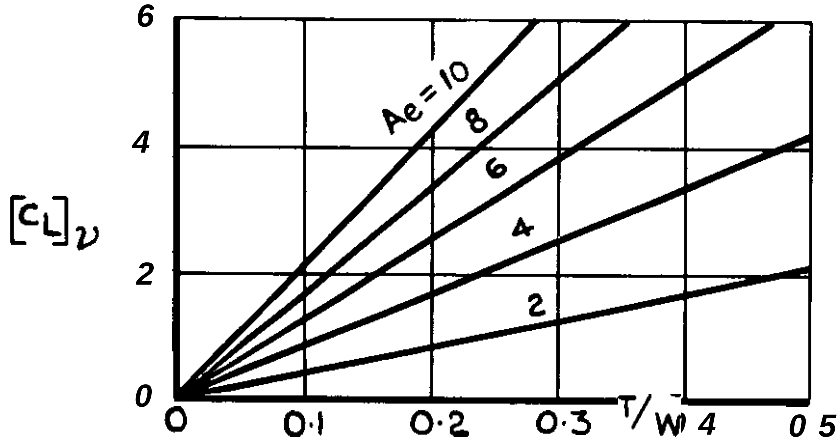


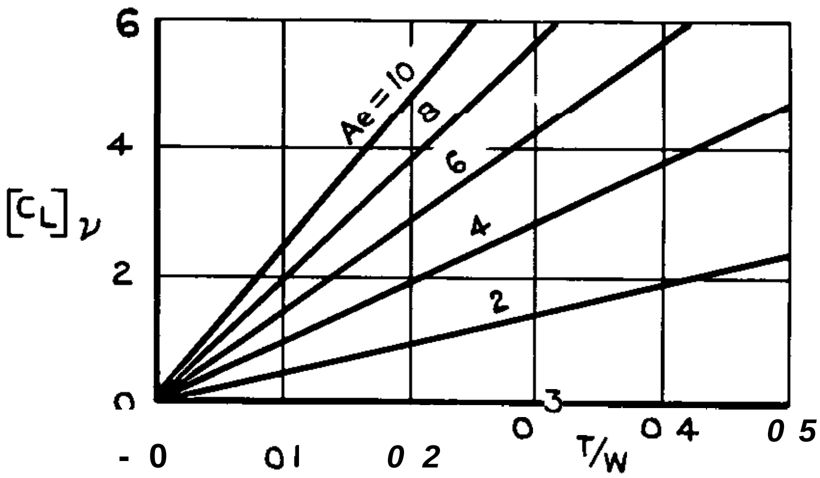
Fig. 9 General parameter $\frac{C_L}{f}$ for the lift coefficient at which the lift dependent drag equals the thrust with one engine failed



a Twin engined



b Three engined



c Four engined

Fig. 10a-c Lift coefficient at which the lift dependent drag equals the thrust with one engine failed

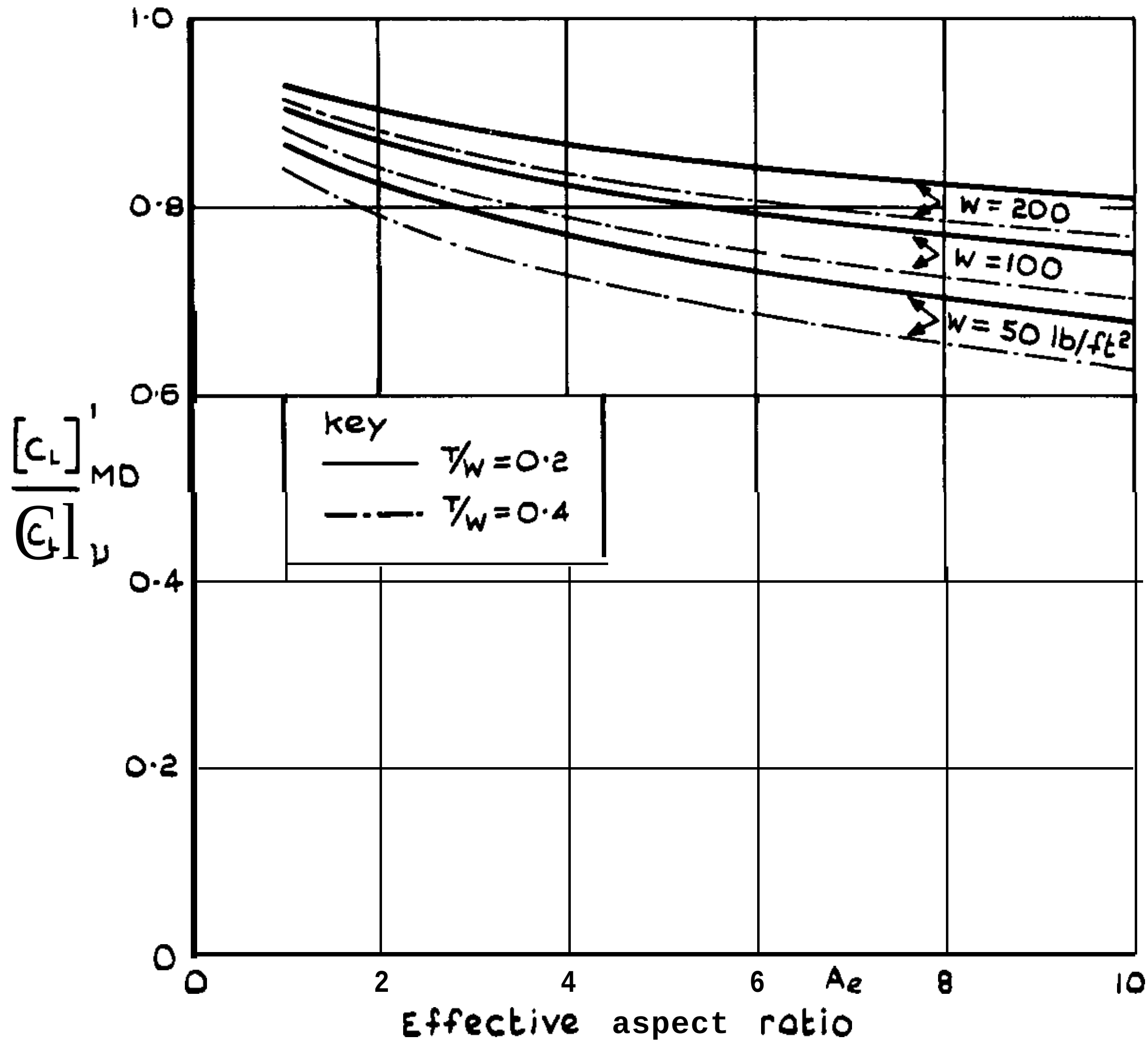
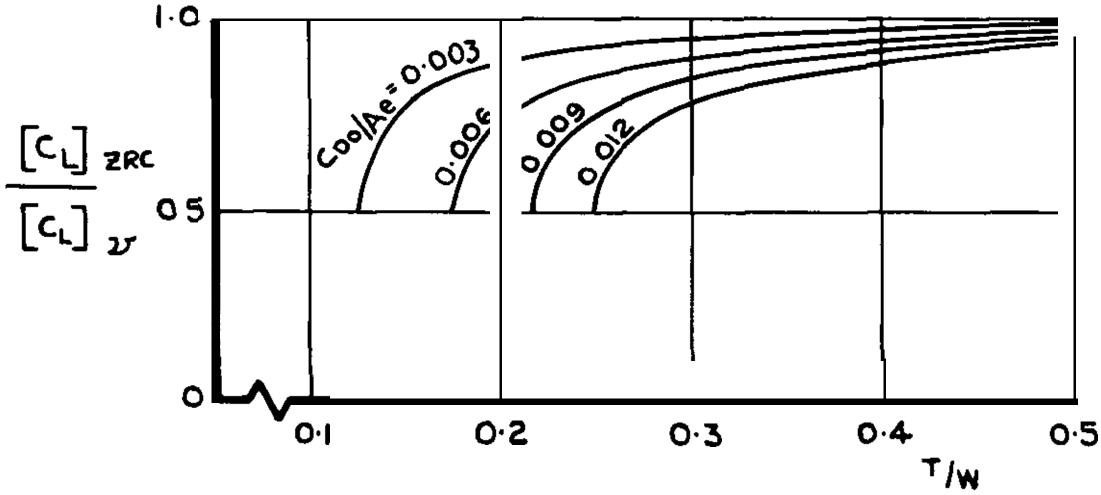
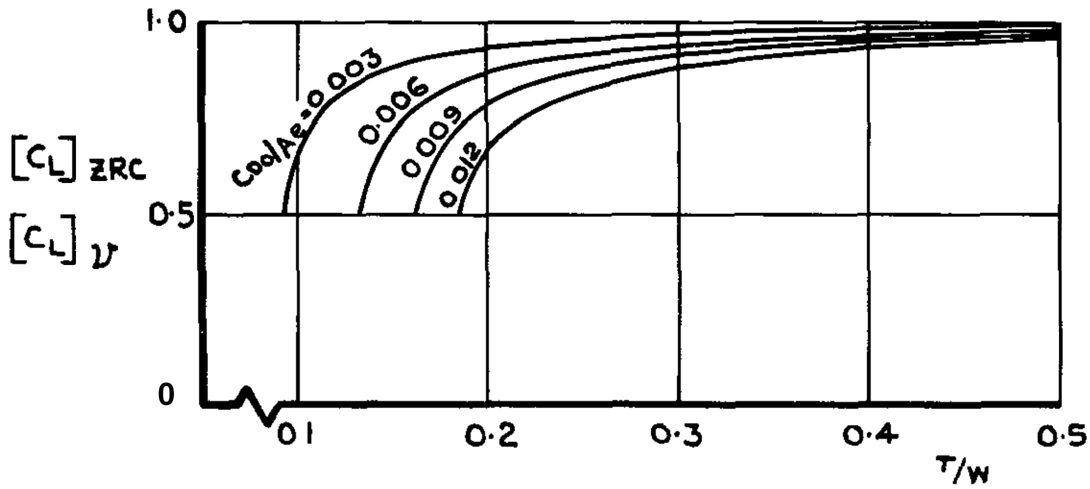


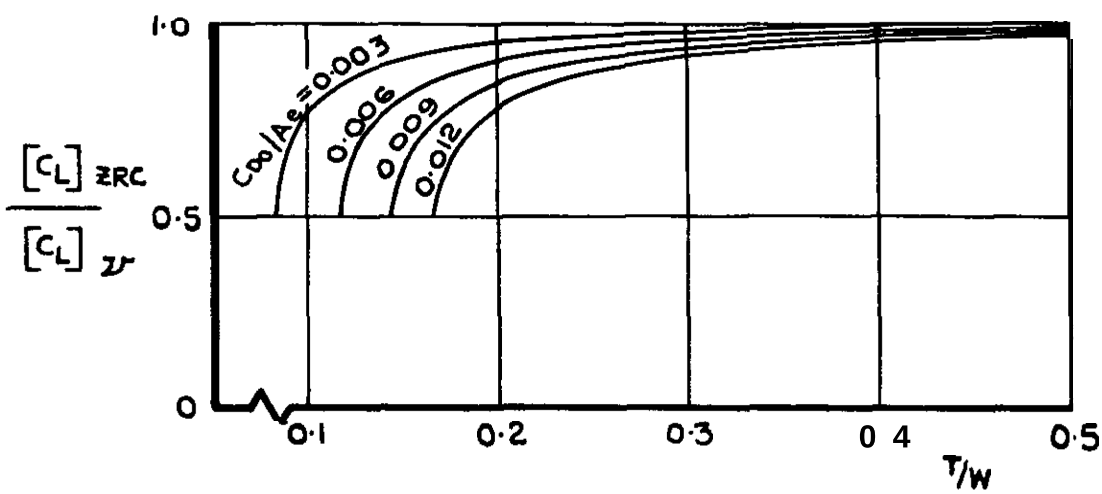
Fig II Ratio of the C_L for minimum take- off distance to the C_L at which the lift dependent drag equals the thrust . Sea level I S A. Screen height 35'



a Twin engined



b Three engined



c Four engined

Fig. 12a-c Ratio of the C_L for zero rate of climb to the C_L at which the lift dependant drag equals the thrust

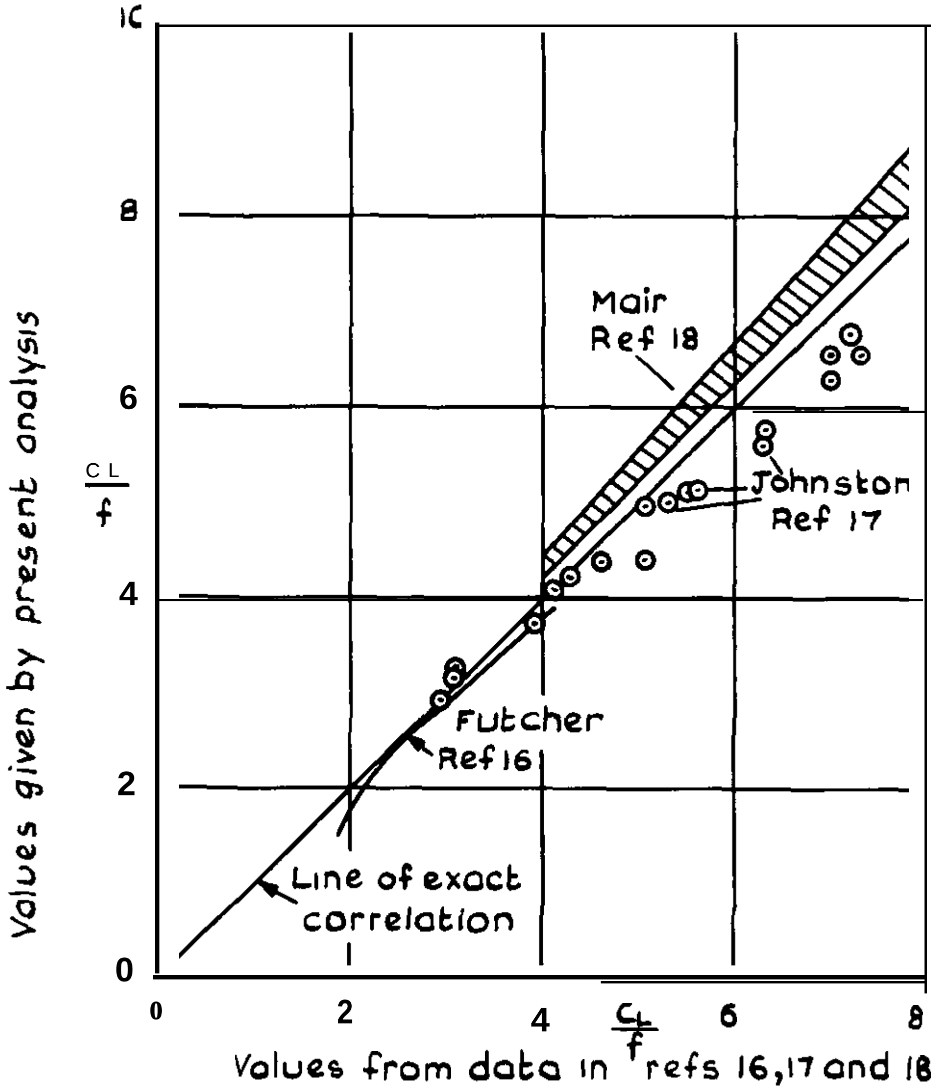
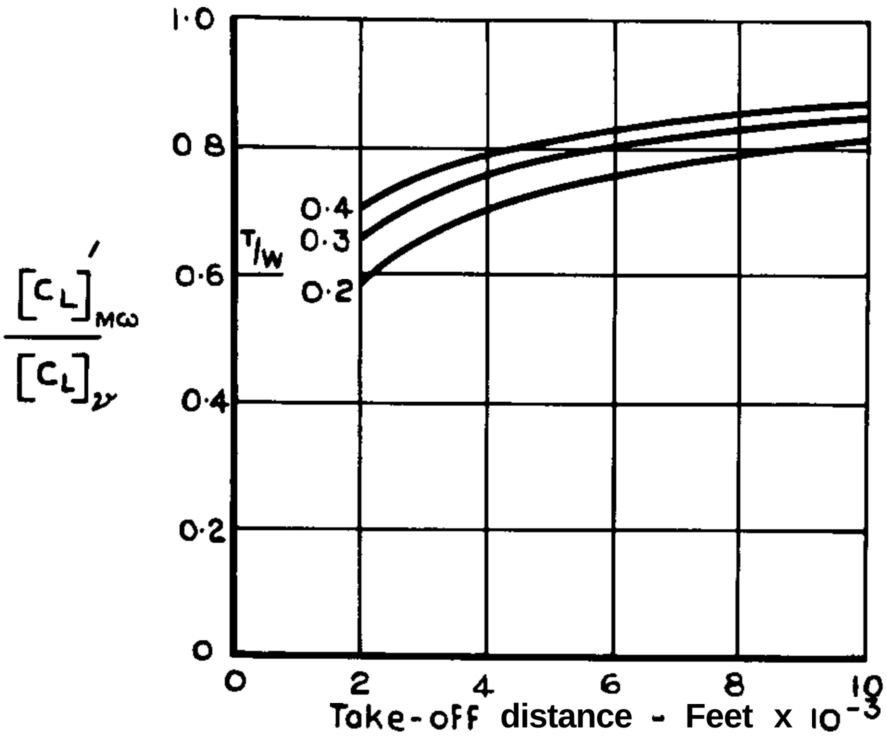
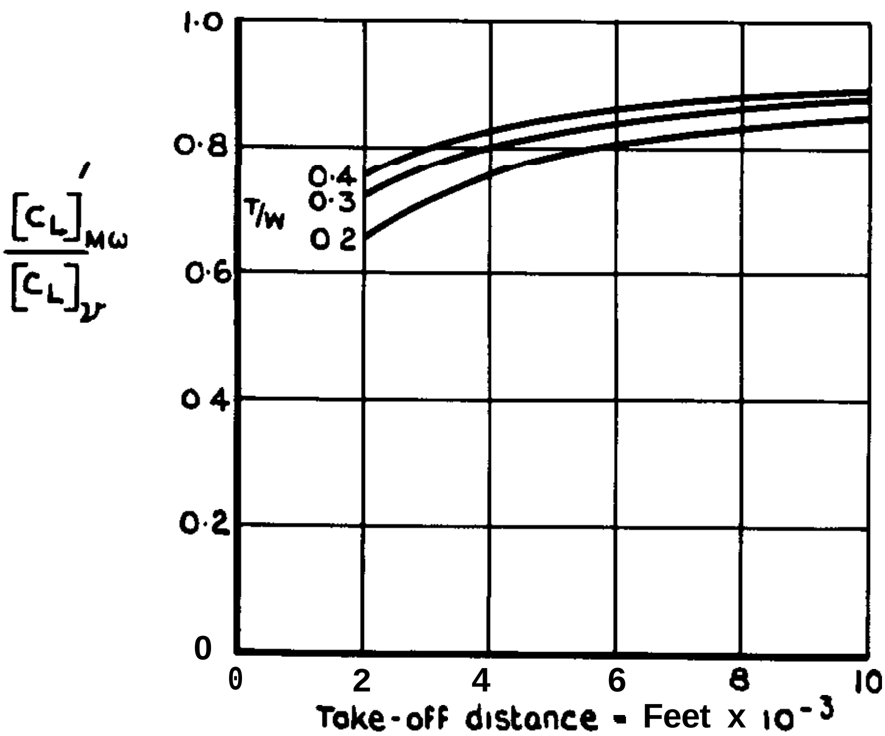


Fig.13 Comparison of take-off C_L's for minimum distance given by present theory with data from refs 16.17 and 18



a Twin engined



b Four engined

Fig. 14a & b Ratio of the C_L for maximum take-off wing loading to the C_L at which the lift dependant drag equals the thrust. Screen height 35'

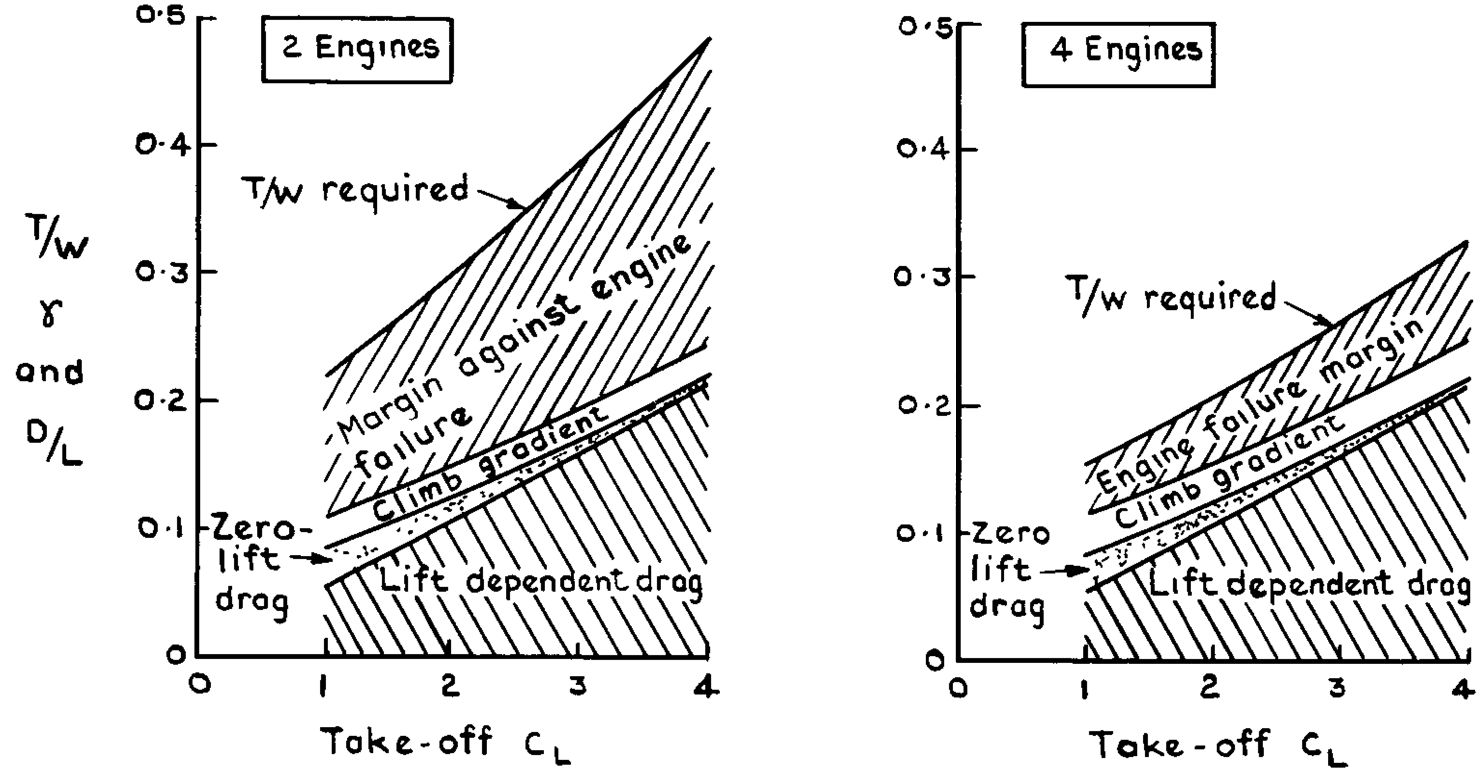
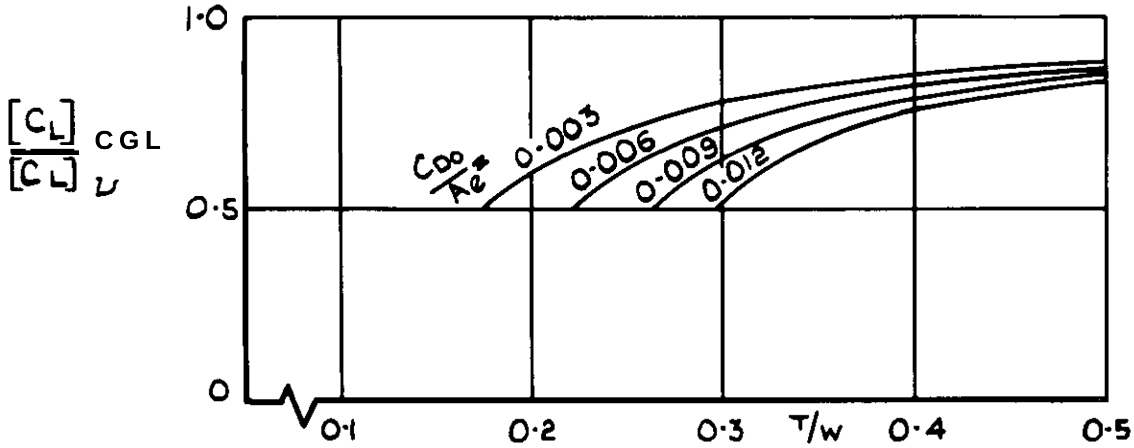
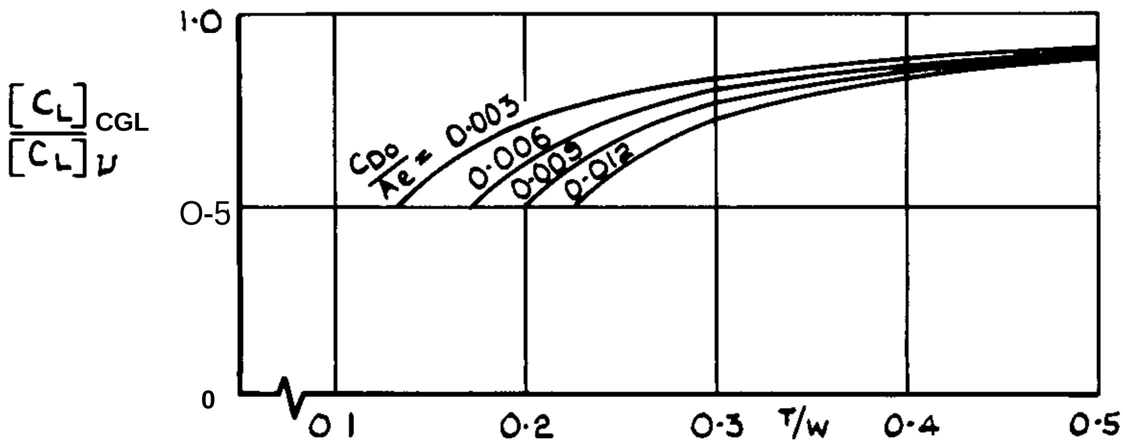


Fig.15 Breakdown of thrust-weight ratio required to meet airworthiness climb gradient requirements for twin and four engined aircraft

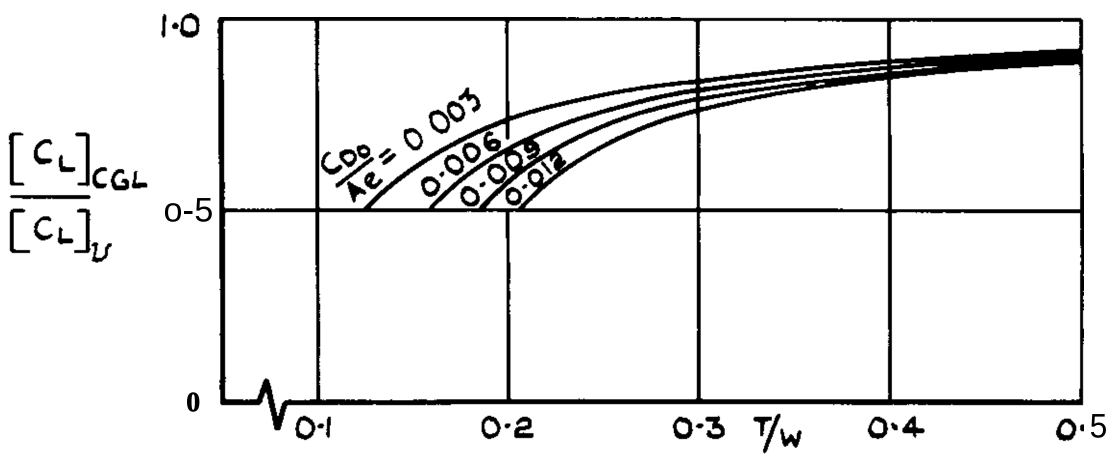
$C_{D_0} = 0{\cdot}03,\ A_e = 6$



a Twin engmed



b Three engined



c Four engined

Fig.16 a-c **Ratio of the C_L for BCAR climb gradient to the C_L at which the lift dependent drag equals the thrust**

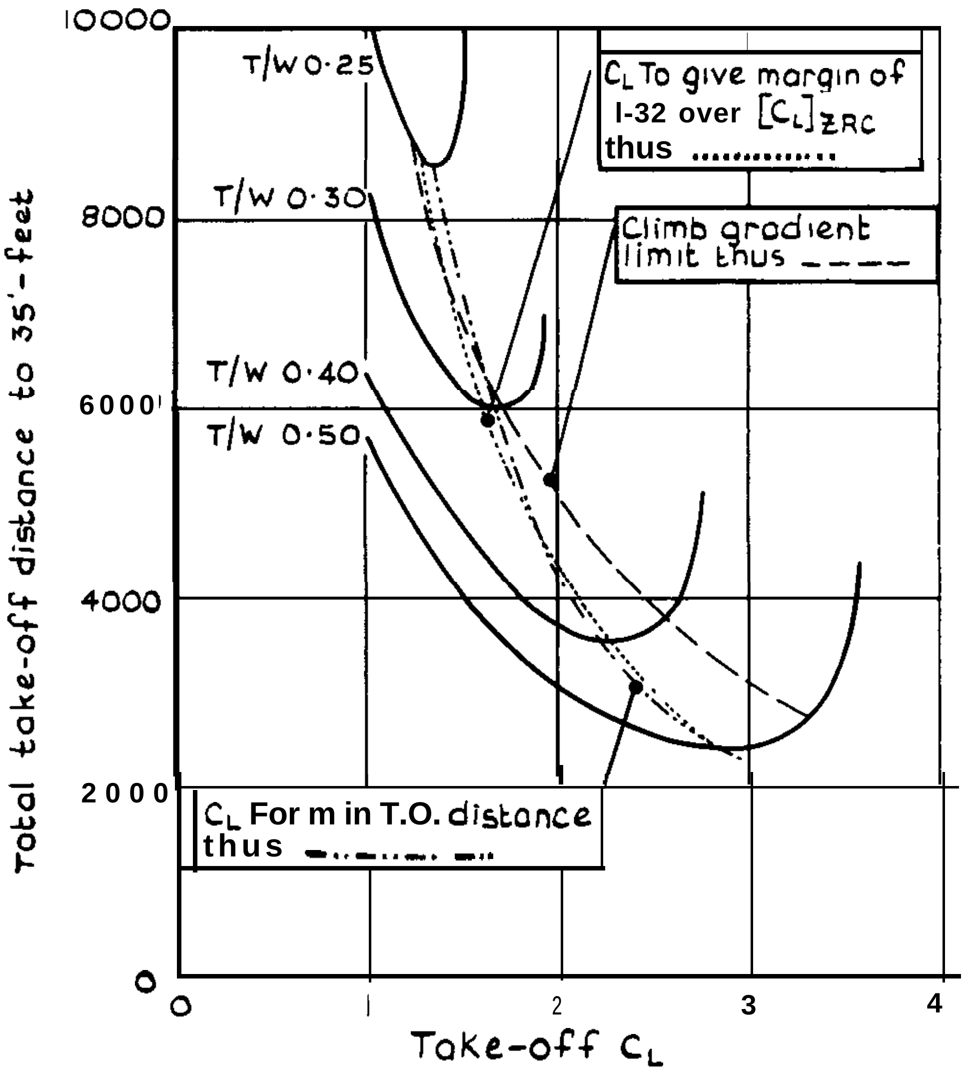


Fig.17 Limits in useable take-off C_L derived from the present analysis for the example given in Fig. 7

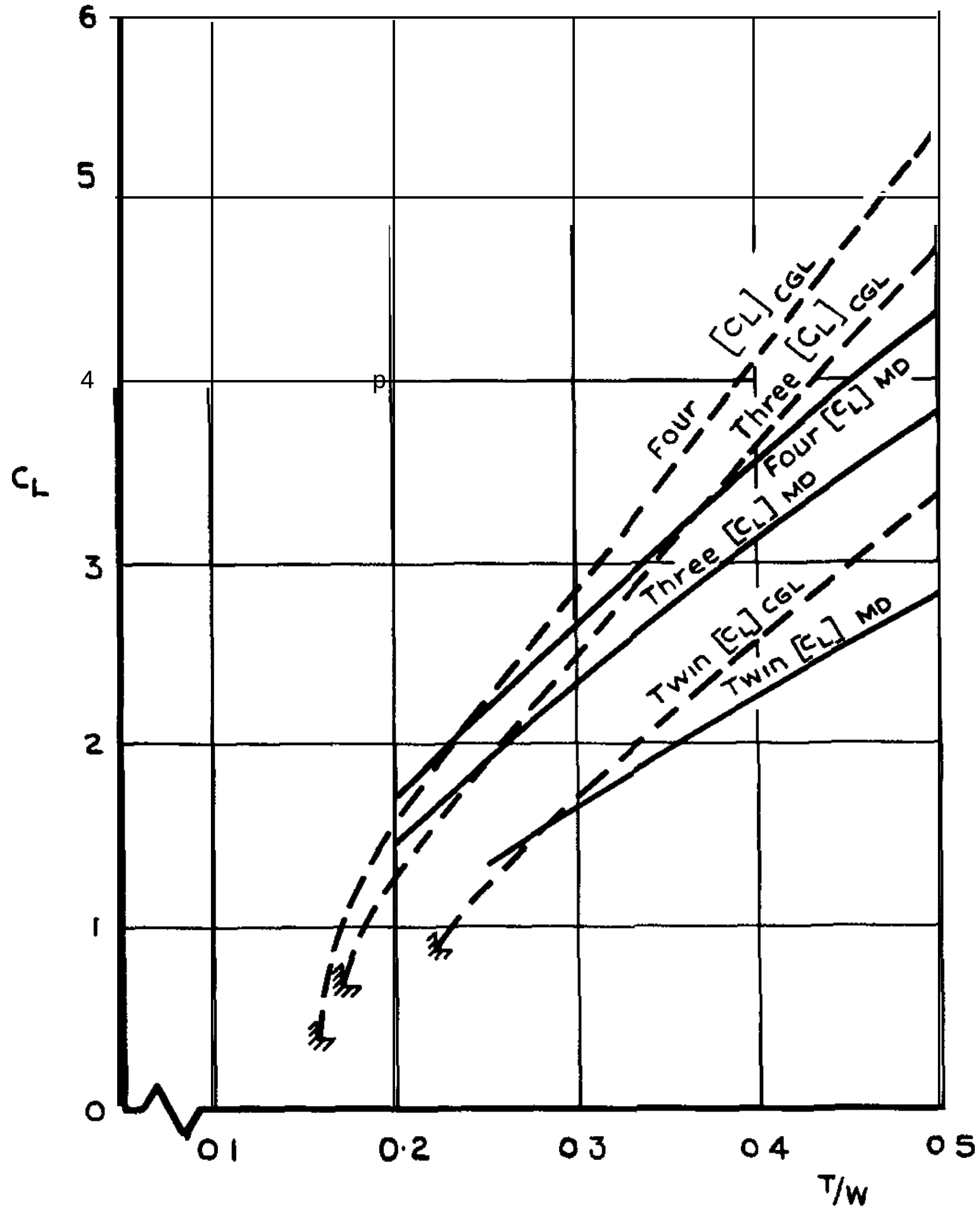


Fig. 18 Effect of varying the number of engines on the lift boundaries for subsonic transport aircraft

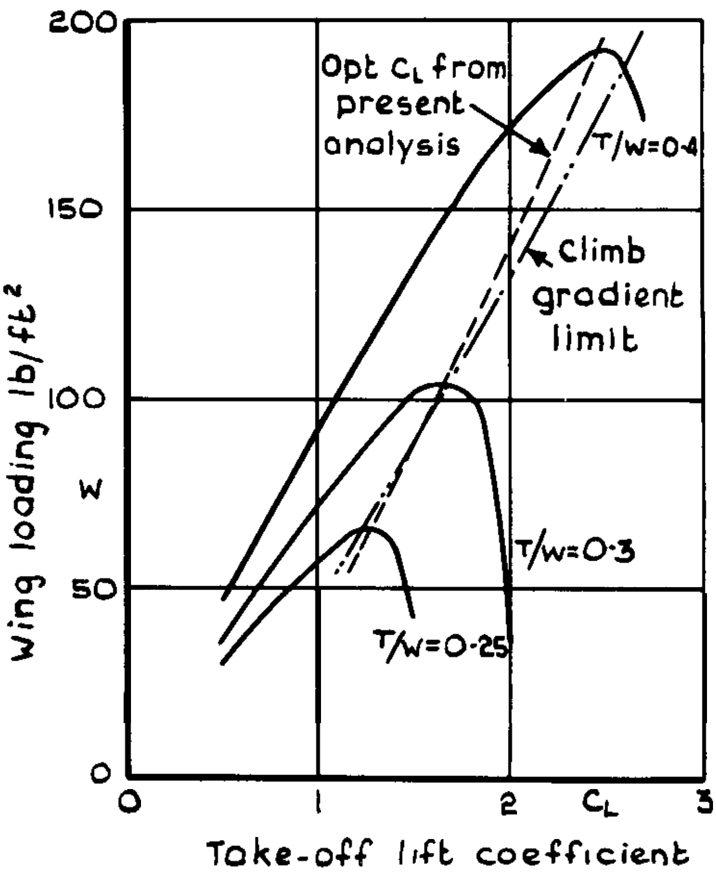


Fig. 19 Variation in wing loading with C_L and T/W to give a take-off distance of 6000 ft. Twin engined aircraft $A_e = 5$

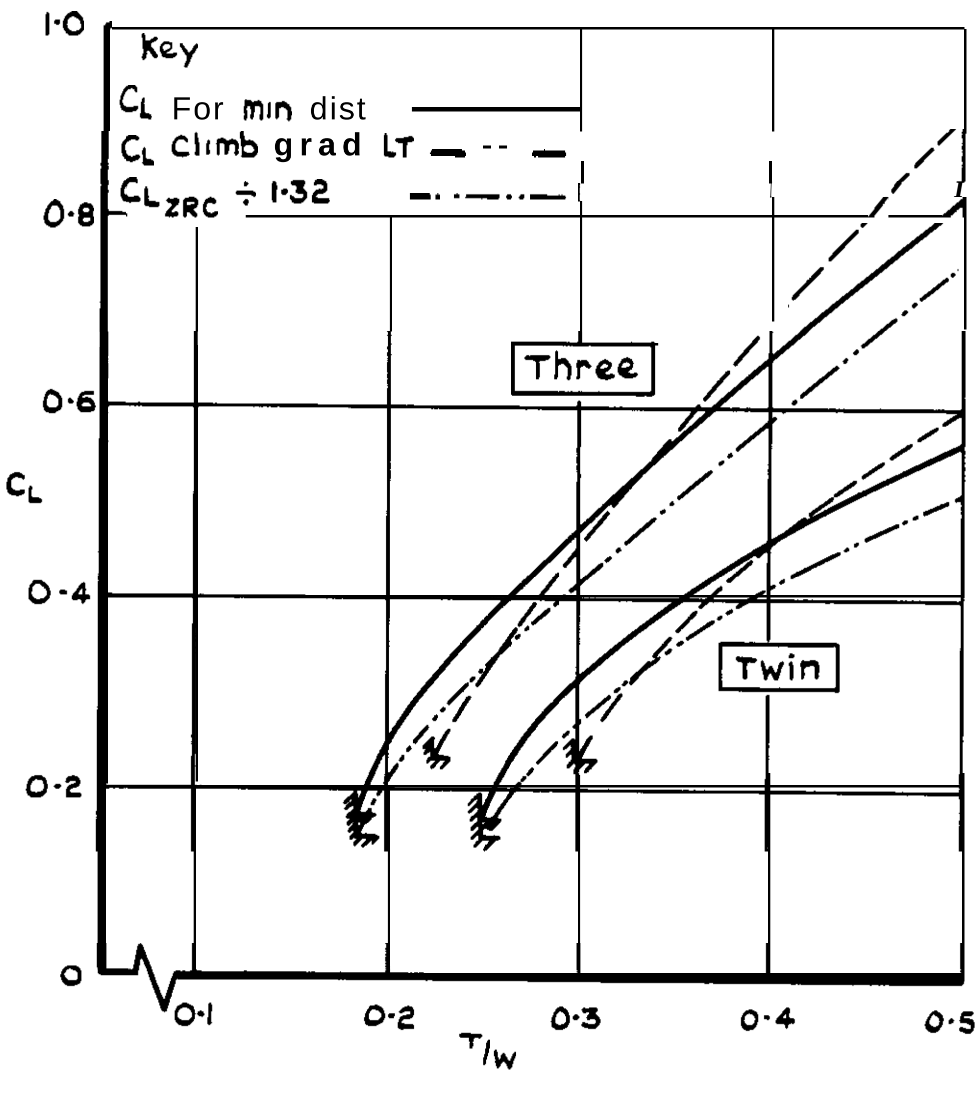


Fig.20 Lift coefficient limitations for an all wing aerobus

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A.R.C. C.P. No.1034
December 1967

533.6.015.1
533.6.013.13
533.693.048.1
533.6.013.644

Perry, D.H.

AN ANALYSIS OF SOME MAJOR FACTORS INVOLVED
IN NORMAL TAKE-OFF PERFORMANCE

An analytical study has been made of the effect of such parameters as wing loading, aspect ratio, thrust-weight ratio, and number of engines, on the take-off performance of fixed wing aircraft. Expressions are derived for the take-off lift coefficients which give the shortest take-off distance, the highest take-off wing loading, and climb conditions just meeting the airworthiness requirements. Examples of the analysis applied to two designs of current interest are given.

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An analytical study has been made of the effect of such parameters as wing loading, aspect ratio, thrust-weight ratio, and number of engines, on the take-off performance of fixed wing aircraft. Expressions are derived for the take-off lift coefficients which give the shortest take-off distance, the highest take-off wing loading, and climb conditions just meeting the airworthiness requirements. Examples of the analysis applied to two designs of current interest are given.

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