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Notes and Graphs for Boundary Layer Calculations in Compressible Flow

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Notes and Graphs for Boundary Layer Calculations in Compressible Flow

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24th August, 1951

SUMMARY

Formulae are given for calculating the skin friction coefficients and the various parameters associated with the boundary layer up to Mach numbers of 4. The calculations have been made for a laminar boundary layer with a sinusoidal velocity distribution; for a turbulent boundary layer with a "power law" velocity distribution, the distribution following "one fifth", "one seventh", "one ninth" and "one eleventh" power laws; and for a turbulent boundary layer with a "log law" velocity distribution.

Introduction

In 7634[1] some calculations are made on the effect of compressibility on a turbulent boundary layer. At the time (1942-43) this work was done no measurements of the velocity distribution existed and the analysis had to be based on the assumption that it was not appreciably affected by the mainstream being supersonic. This assumption could be rendered plausible, but it was a pure speculation and accordingly the analysis was only carried far enough to establish trends.

Since that time analytical reasons supporting this assumption have been given[2], and measurements have been made at the N.P.L.[3], R.A.E.[4] and in America[5] covering between them Mach numbers up to $2\frac{1}{2}$ and Reynolds numbers up to 20 million. These measurements have established the general truth of the assumptions made in 7634 and accordingly it seems worth while to complete the calculations and collate the information in one document in the hope it may be of use to designers.

The general method of calculation is as follows:-

(1) It is assumed that the velocity distribution can be written:-

$$u/U_1 \equiv f(\eta) \quad \text{with} \quad \eta \equiv y/\delta .$$

(2) A quantity α is defined by:-

$$(1 - \alpha)\left(1 + \frac{\gamma-1}{2} M_1^2\right) = 1 \qquad 0 \leqslant M_1 \leqslant \infty$$
$$0 \leqslant \alpha \leqslant 1.$$

(3)/

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(3) It is assumed that throughout the layer[*]:-

$$u^2 + 2\,C_p\,T = \text{Constant.}$$

In particular

$$T_W = T_1\left(1 + \frac{U_1^2}{2\,C_p\,T_1}\right)$$

$$= T_1\left(1 + \frac{\gamma-1}{2}M_1^2\right)$$

$$= (1-\alpha)^{-1}\,T_1$$

and

$$\rho_W = (1-\alpha)\,\rho_1$$

(since p is constant through the layer). Where $_1$ and $_W$ denote, as usual, free stream and wall values.

(4) It is assumed that:-

$$\mu \propto T^{\omega}$$

ω is a fraction whose value ranges from $\frac{1}{2}$ at very high temperatures to $8/9$ in wind tunnel work with atmospheric stagnation temperature. It seems unlikely that any error of practical importance to the aircraft designer will be introduced by putting:- $\omega = \frac{3}{4}$.

<u>Calculation of Displacement and Momentum Thicknesses</u>

The standard definitions are:-

$$\left.\begin{aligned} \delta^{\times} &\equiv \int_0^{\delta}\left(1 - \frac{\rho u}{\rho_1 U_1}\right)dy \\ \vartheta &\equiv \int_0^{\delta}\frac{\rho u}{\rho_1 U_1}\left(1 - \frac{u}{U_1}\right)dy \\ H &\equiv \delta^{\times}/\vartheta \end{aligned}\right\} \qquad \ldots(1)$$

[*]That is that the Prandtl number is unity. It is also assumed that there is no heat transfer between the layer and the wall.

On the assumptions just given these become:-

$$\left.\begin{aligned} \delta^{\times}/\delta &= \int_0^1 \left(1 - \frac{(1-\alpha)\,f}{1-\alpha f^2}\right) d\eta \\ \vartheta/\delta &= \frac{1-\alpha}{\alpha}\int_0^1 \left(1 - \frac{1-\alpha f}{1-\alpha f^2}\right) d\eta \\ H &= \delta^{\times}/\vartheta \end{aligned}\right\} \qquad \ldots(2)$$

The analytical and numerical work is subdivided under three heads:-

Laminar Boundary Layer

Turbulent Boundary Layer, Power Law

Turbulent Boundary Layer, Log Law.

Laminar Boundary Layer

It is assumed that*

$$f = \sin\left(\frac{\pi}{2}\eta\right).$$

Then the equations (2) yield:-

$$\delta^{\times}/\delta = 1 - \frac{2}{\pi}\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{2}} \text{arc tan} \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}$$

$$\vartheta/\delta = \frac{1-\alpha}{\alpha}\left\{\frac{2}{\pi}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2}} \text{arc tan} \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2}} - \frac{1-(1-\alpha)^{\frac{1}{2}}}{(1-\alpha)^{\frac{1}{2}}}\right\}.$$

The values of $\delta^{\times}/\delta$, ϑ/δ and H are plotted as functions of M over the range $0 \leqslant M \leqslant 5$ in figures 2, 3 and 4 respectively.

Turbulent Boundary Layer, Power Law

It is assumed that:-

$$f = \eta^{1/m}$$

where m is an integer and often $m = 7$.

Then/

*Accurate solutions exist[6], but are laborious to obtain. The above assumption is sufficiently accurate for the purpose in view and has the double advantage of being analytically simple and easily computable.

Then the equations (2) yield:-

$$\delta^{\times}/\delta = 1 - \frac{m}{m+1}(1-\alpha)\left\{1 + \frac{m+1}{m+3}\alpha + \ldots + \frac{m+1}{m+2n+1}\alpha^n + \ldots\right\}$$

$$= \frac{1}{m+1} + 2m \sum_{n=1} \frac{\alpha^n}{(m+2n-1)(m+2n+1)}$$

$$\vartheta/\delta = \frac{m}{(m+1)(m+2)}(1-\alpha)\left\{1 + \frac{(m+1)(m+2)}{(m+3)(m+4)}\alpha + \ldots + \frac{(m+1)(m+2)}{(m+2n+1)(m+2n+2)}\alpha^n + \ldots\right\}$$

$$= \frac{m}{(m+1)(m+2)}\left\{1 - 2\sum_{n=1}\frac{(2m+4n+1)\,\alpha^n}{(m+2n-1)(m+2n)(m+2n+1)(m+2n+2)}\right\}.$$

These series become more and more slowly convergent as α increases; this difficulty can be overcome when it is seen that equations (2) can also be written:-

$$\delta^{\times}/\delta = 1 - (1-\alpha)\, m \int_0^1 \frac{f^m}{1 - \alpha f^2}\, df$$

$$\vartheta/\delta = \frac{(1-\alpha)}{\alpha} - \frac{(1-\alpha)\, m}{\alpha}\int_0^1 \frac{(1 - \alpha f)\, f^{m-1}}{1 - \alpha f^2}\, df$$

both of which can be integrated by elementary methods in the form of a polynomial plus a log term. This polynomial, however, will contain several terms and so the method is only of practical use when the series expansion converges slowly.

The values of $\delta^{\times}/\delta$, ϑ/δ and H for $m = 5, 7, 9$ and 11 are plotted as functions of M over the range $0 \leqslant M \leqslant 4$ in figures 5, 6 and 7 respectively.

Turbulent Boundary Layer, Log Law

It is assumed that:-

$$f = 1 + A\,\ln\eta$$

where $$A = \frac{1}{K}\left(\frac{U_1}{U_{\tau w}}\right). \qquad \frac{k\,U_{\tau w}}{U_1}$$

The/

The integrals in equations (2) can be expanded in powers of A with coefficients functions of α to give:-

$$\delta^{x}/\delta = \frac{1+\alpha}{1-\alpha} A \left\{ 1 - \frac{2\alpha(3+\alpha)}{1-\alpha^2} A + \frac{6\alpha(1+6\alpha+\alpha^2)}{(1-\alpha^2)(1-\alpha)} A^2 - \frac{24\alpha^2(5+10\alpha+\alpha^2)}{(1-\alpha^2)(1-\alpha)^2} A^3 + \dots \right\}$$

$$\vartheta/\delta = A \left\{ 1 - \frac{2(1+\alpha)}{1-\alpha} A + \frac{6\alpha(3+\alpha)}{(1-\alpha)^2} A^2 - \frac{24\alpha(1+6\alpha+\alpha^2)}{(1-\alpha)^3} A^3 + \dots \right\}$$

but these series are quite useless for Mach numbers much greater than unity.

Now equations (2) can also be written:-

$$\delta^{x}/\delta = \int_0^1 \left\{ 1 - \frac{1-\alpha}{2\sqrt{\alpha}} \left(\frac{1}{1-\sqrt{\alpha f}} - \frac{1}{1+\sqrt{\alpha f}} \right) \right\} d\eta$$

$$\vartheta/\delta = \frac{1-\alpha}{\alpha} \int_0^1 \left\{ 1 - \tfrac{1}{2} \left(\frac{1-\sqrt{\alpha}}{1-\sqrt{\alpha f}} + \frac{1+\sqrt{\alpha}}{1+\sqrt{\alpha f}} \right) \right\} d\eta$$

and thus depend on:-

$$\left. \begin{aligned} I_1 &\equiv \frac{1}{\sqrt{\alpha A}} \int_0^1 \frac{d\eta}{\frac{1-\sqrt{\alpha}}{\sqrt{\alpha A}} - \ln \eta} \\ I_2 &\equiv \frac{1}{\sqrt{\alpha A}} \int_0^1 \frac{d\eta}{\frac{1+\sqrt{\alpha}}{\sqrt{\alpha A}} + \ln \eta} \end{aligned} \right\}$$

that is to say on the exponential-integral function.

In fact:-

$$\delta^{x}/\delta = 1 + \frac{1-\alpha}{2\alpha A} \left\{ \exp \frac{1-\sqrt{\alpha}}{\sqrt{\alpha A}} \,\mathrm{Ei} \left(- \frac{1-\sqrt{\alpha}}{\sqrt{\alpha A}} \right) + \exp \left(- \frac{1+\sqrt{\alpha}}{\sqrt{\alpha A}} \right) \mathrm{Ei} \left(\frac{1+\sqrt{\alpha}}{\sqrt{\alpha A}} \right) \right\}$$

$$\vartheta/\delta = \frac{1-\alpha}{\alpha} \left[1 + \tfrac{1}{2} \left\{ \frac{1-\sqrt{\alpha}}{\sqrt{\alpha A}} \exp \frac{1-\sqrt{\alpha}}{\sqrt{\alpha A}} \,\mathrm{Ei} \left(- \frac{1-\sqrt{\alpha}}{\sqrt{\alpha A}} \right) - \frac{1+\sqrt{\alpha}}{\sqrt{\alpha A}} \exp \left(- \frac{1+\sqrt{\alpha}}{\sqrt{\alpha A}} \right) \mathrm{Ei} \left(\frac{1+\sqrt{\alpha}}{\sqrt{\alpha A}} \right) \right\} \right].$$

The/

The values of $(1-\sqrt{\alpha})/\sqrt{\alpha}$ and $(1+\sqrt{\alpha})/\sqrt{\alpha}$ are given below in the table and as figure 8.

M	α	$(1-\sqrt{\alpha})/\sqrt{\alpha}$	$(1+\sqrt{\alpha})/\sqrt{\alpha}$
0	0	∞	∞
0.5	0.0476	3.583	5.583
1.0	0.1667	1.450	3.450
1.5	0.3103	0.7953	2.795
2.0	0.4444	0.5001	2.500
2.5	0.5555	0.3417	2.342
3.0	0.6429	0.2472	2.247
3.5	0.7101	0.1867	2.187
4.0	0.7619	0.1456	2.146
4.5	0.8020	0.1167	2.117
5.0	0.8333	0.0954	2.095

A typical value for A is 0.1.

So $(1-\sqrt{\alpha})/\sqrt{\alpha A}$ varies from 1 to 10 } omitting the low

and $(1+\sqrt{\alpha})/\sqrt{\alpha A}$ varies from 21 upwards } Mach number region.

The exponential-integral function is tabulated up to values of 10 for the independent variable and has the asymptotic expansion

$$\mathrm{Ei}\ (x) \sim \frac{\exp\ (x)}{x}\left\{1 + \frac{1}{x} + \frac{2}{x^2} + \frac{6}{x^3} + \frac{24}{x^4} + \ldots\right\}$$

which is rapidly convergent for $x > 10$ if only 3- or 4-figure accuracy is needed.

Thus we can write

$$\delta^{\times}/\delta = 1 + \frac{1-\alpha}{2\alpha A}\left[\exp\left(\frac{1-\sqrt{\alpha}}{\sqrt{\alpha A}}\right)\mathrm{Ei}\left(-\frac{1-\sqrt{\alpha}}{\sqrt{\alpha A}}\right) + \frac{\sqrt{\alpha A}}{1+\sqrt{\alpha}}\left\{1 + \frac{\sqrt{\alpha A}}{1+\sqrt{\alpha}} + 2\left(\frac{\sqrt{\alpha A}}{1+\sqrt{\alpha}}\right)^2 + \ldots\right\}\right]$$

$$\vartheta/\delta = \frac{1-\alpha}{\alpha}\left[1 + \tfrac{1}{2}\left[\frac{1-\sqrt{\alpha}}{\sqrt{\alpha A}}\exp\left(\frac{1-\sqrt{\alpha}}{\sqrt{\alpha A}}\right)\mathrm{Ei}\left(-\frac{1-\sqrt{\alpha}}{\sqrt{\alpha A}}\right) - \left\{1 + \frac{\sqrt{\alpha A}}{1+\sqrt{\alpha}} + 2\left(\frac{\sqrt{\alpha A}}{1+\sqrt{\alpha}}\right)^2 + \ldots\right\}\right]\right]$$

The values of $\delta^{\times}/\delta$, ϑ/δ and H over the range $0 \leq M \leq 4$ for Reynolds numbers of 10^5, 10^6, 10^7, 10^8, and 10^9 are plotted as figures 9, 10 and 11.

Skin Friction Coefficients

Because of the temperature difference between the wall and the mainstream, the several coefficients require careful definition. The standard definition is in terms of mainstream values as follows:-

$$C_f = \tau_0/\tfrac{1}{2}\rho_1 U_1^2$$

$$C_F = \frac{1}{x}\int_0^x C_f\, dx$$

$$R_x = \rho_1 U_1 x/\mu_1$$

but/

but it is sometimes convenient to define them in terms of "wall values", thus:-

$$C_{fW} = \tau_0 / \tfrac{1}{2}\rho_W U_1^2 = (1 - \alpha)^{-1} C_f$$

$$C_{FW} = \frac{1}{x}\int_0^x C_{fW}\,dx = (1 - \alpha)^{-1} C_F$$

$$R_{xW} = \rho_W U_1 x / \mu_W = (1 - \alpha)^{1+\omega} R_x.$$

Once a velocity distribution has been fixed, then the skin friction coefficients can be determined from the relationships:-

$$2d\theta/dx = C_f \qquad \frac{2d\theta}{dx} = C_f$$

$$2\theta/x = C_F \qquad \frac{2\theta}{x} = C_F.$$

the disposable constants being adjusted so that the formulae reduce to the incompressible form on putting $\alpha = 0$. Figures 12, 13 and 14 show curves of C_f and C_F calculated in this way. In this case the constants have been adjusted so that the formulae reduce for $\alpha = 0$ to the von Kármán-Kempf-Schoenherr type.

The same curves enable the thickness of the layer to be calculated since the relationship just given shows that figures 13 and 14 are a plot of $2\theta/x$ against R. Thus θ can be obtained when x is known and, since H is also known, δ and δ^x also by using the appropriate curve.

It seems likely that expressions in terms of free stream values are most likely to be of use to designers since the free stream values of the wind tunnel correspond to the atmospheric conditions of real life. But it is worth pointing out that C_{fW} and C_{FW} attain their incompressible values for a Reynolds number equal to

$$R_W \frac{T_1}{T_W}$$

and that therefore the values of C_{fW} and C_{FW} can be read off the M small curve of figures 12 to 14 by interpreting R in this sense.

BIBLIOGRAPHY/

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AA.

α is defined by $(1-\alpha)\left(1+\frac{\gamma-1}{2}M^2\right)=1.$ $\therefore\ \alpha=\frac{M^2}{M^2+5}$, $M^2=\frac{5\alpha}{1-\alpha}$ taking $\gamma = 1{\cdot}40$

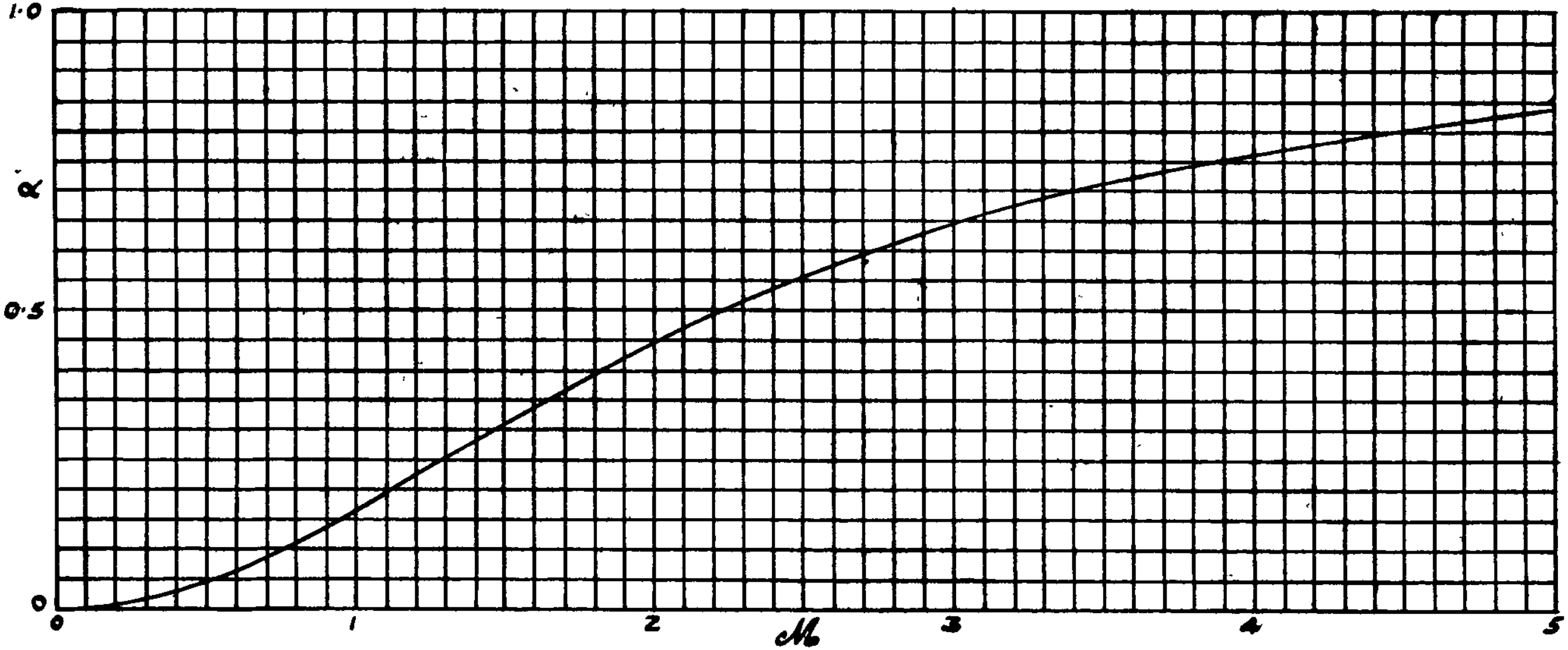
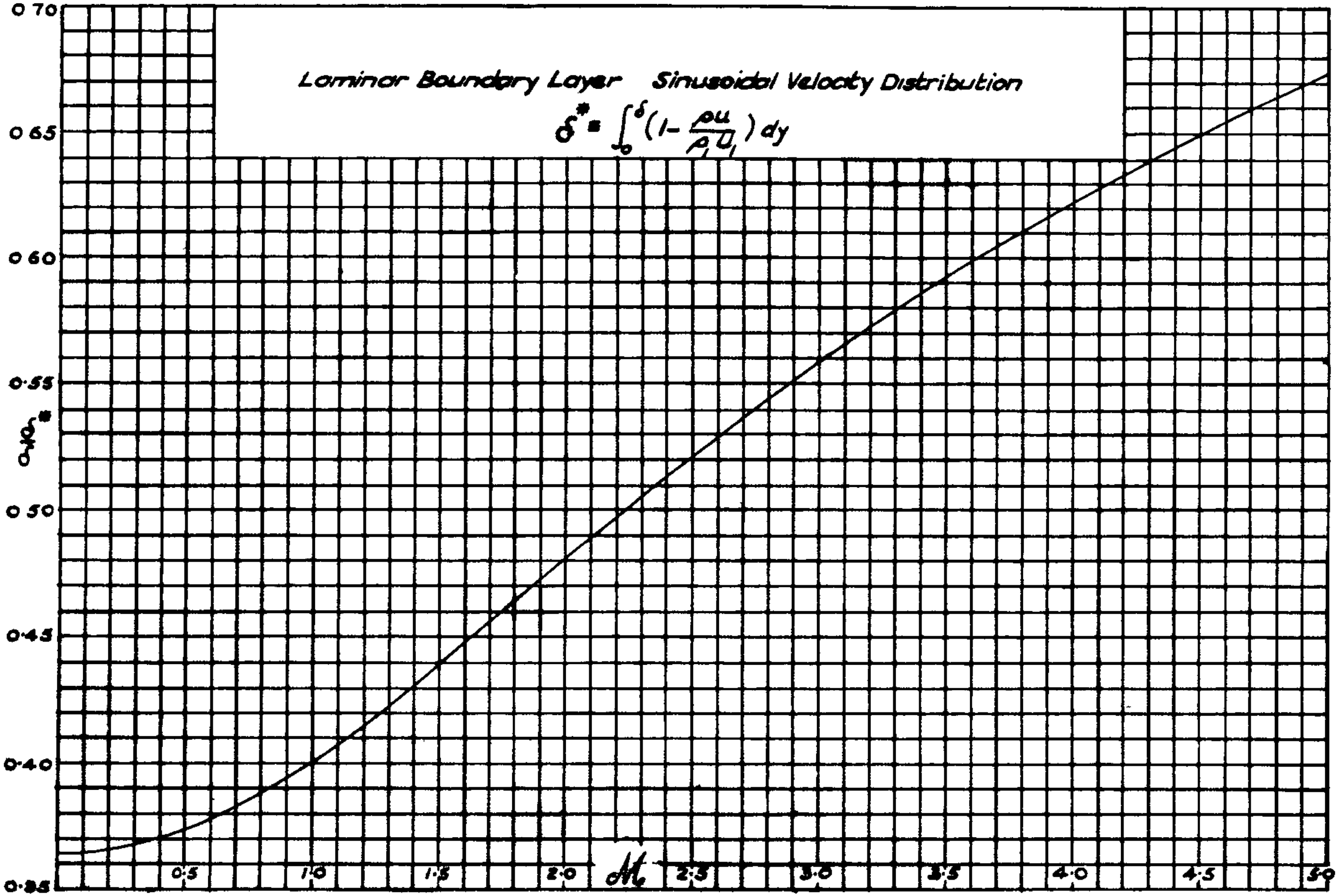
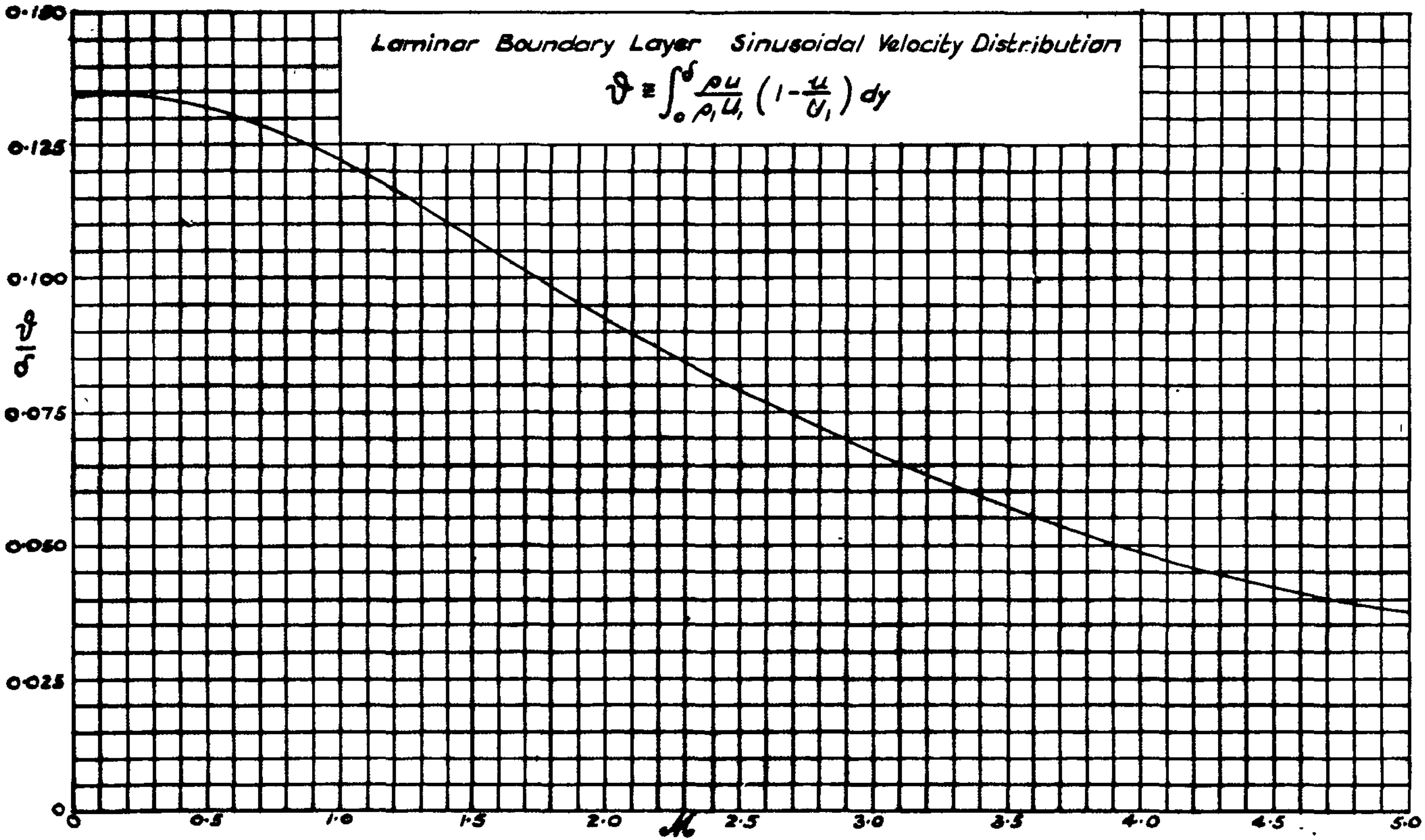


FIG. 1.



Laminar Boundary Layer Sinusoidal Velocity Distribution
$\delta^* = \int_0^\delta \left(1 - \frac{\rho u}{\rho_1 U_1}\right) dy$
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
M_1
$\frac{\delta^*}{\delta}$

FIG 2.



Laminar Boundary Layer Sinusoidal Velocity Distribution
$\vartheta = \int_0^\delta \frac{\rho u}{\rho_1 U_1}\left(1-\frac{u}{U_1}\right) dy$
0·150
0·125
0·100
0·075
0·050
0·025
0
$\frac{\vartheta}{\delta}$
0
0·5
1·0
1·5
2·0
2·5
3·0
3·5
4·0
4·5
5·0
M

FIG. 3.

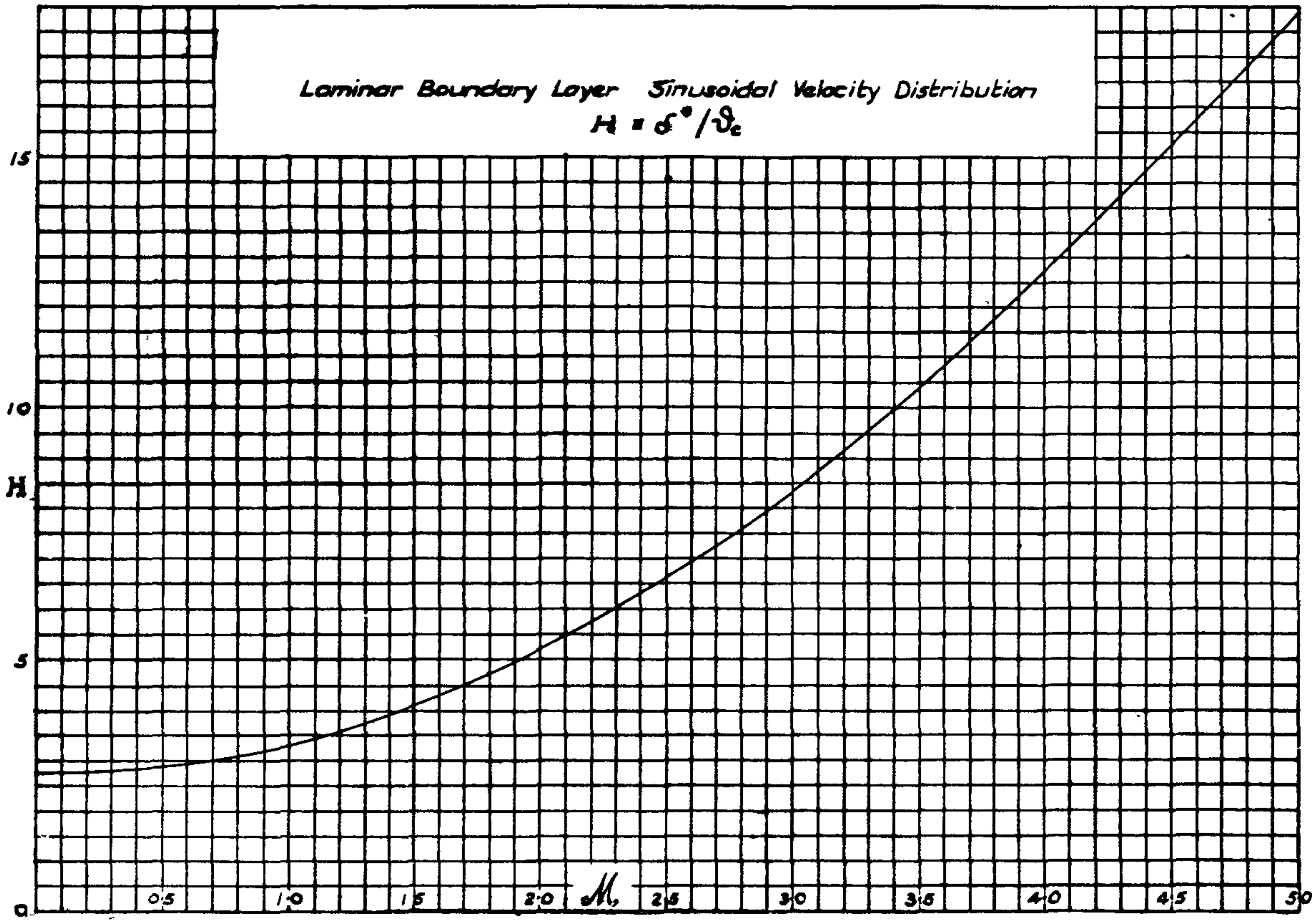
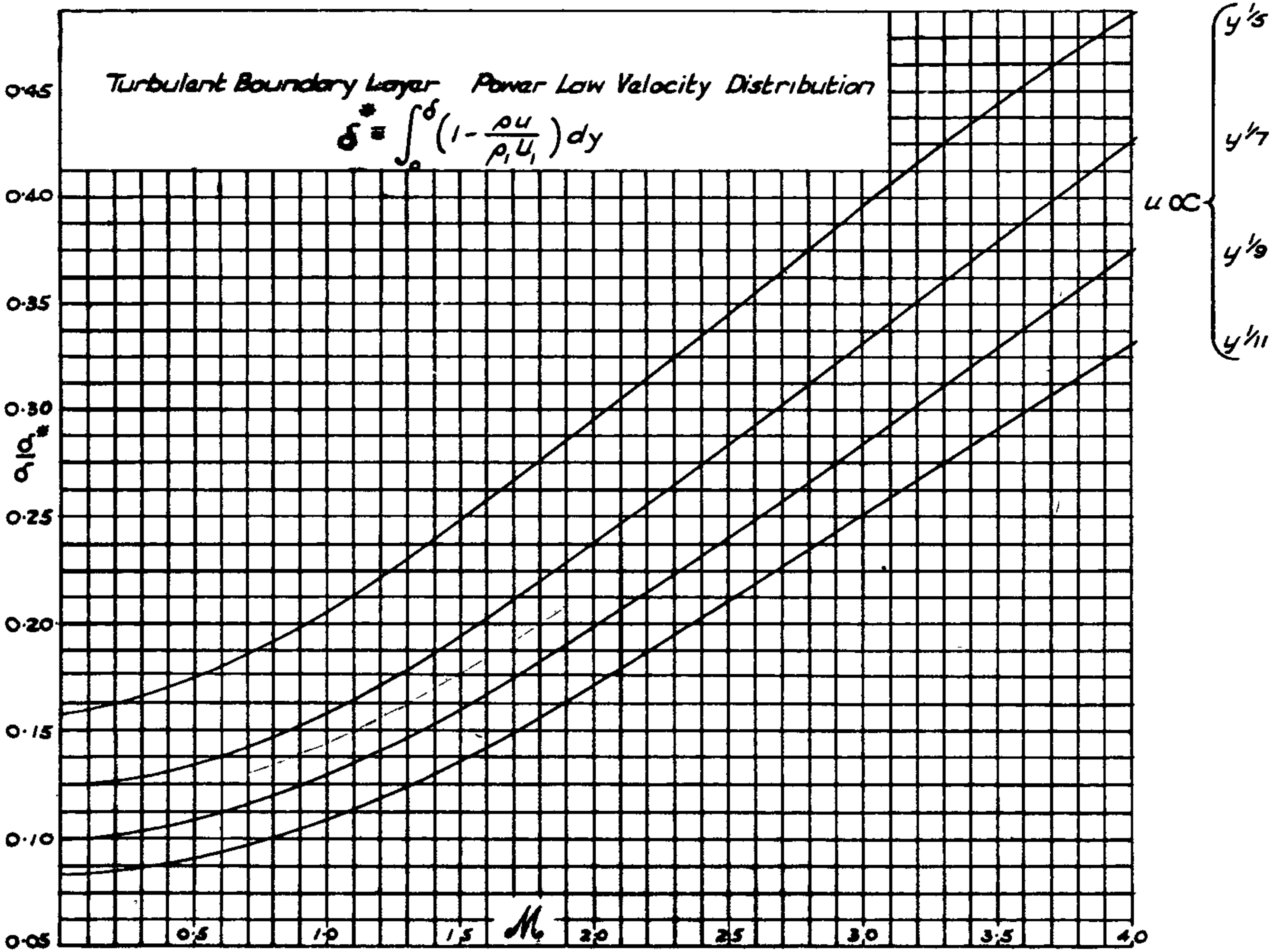

Laminar Boundary Layer Sinusoidal Velocity Distribution
H = δ*/ϑ_c
15
10
H
5
0
0·5
1·0
1·5
2·0
M₁
2·5
3·0
3·5
4·0
4·5
5·0

FIG. 4



Turbulent Boundary Layer Power Law Velocity Distribution
$\delta^* = \int_0^\delta \left(1 - \frac{\rho u}{\rho_1 U_1}\right) dy$
0·45
0·40
0·35
0·30
$\frac{\delta^*}{\delta}$
0·25
0·20
0·15
0·10
0·05
0·5
1·0
1·5
M_1
2·0
2·5
3·0
3·5
4·0
$u \propto$
$y^{1/5}$
$y^{1/7}$
$y^{1/9}$
$y^{1/11}$

FIG. 5.

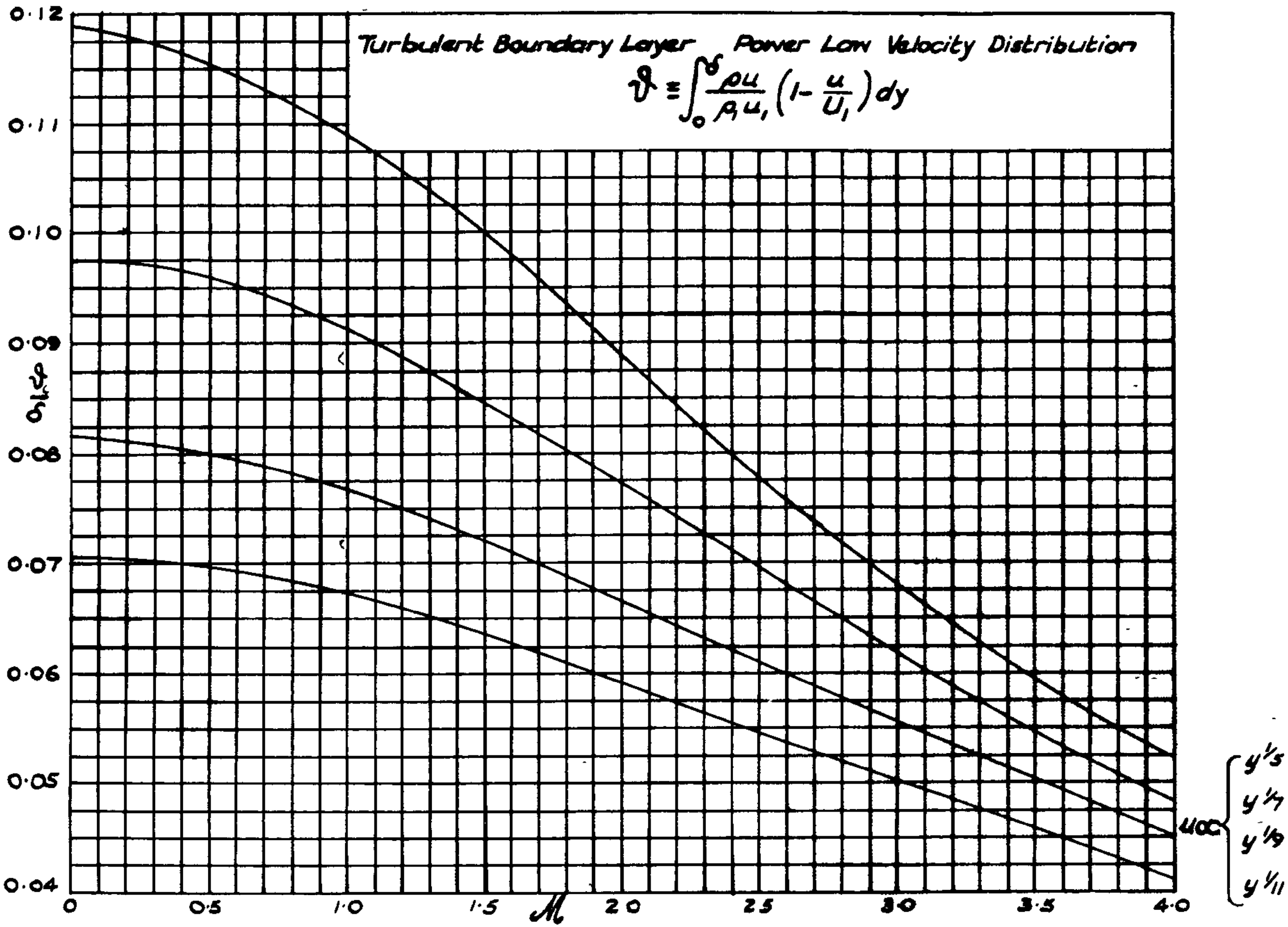

Turbulent Boundary Layer Power Law Velocity Distribution
$\vartheta \equiv \int_0^\delta \frac{\rho u}{\rho_1 u_1}\left(1-\frac{u}{U_1}\right)dy$
$\frac{\vartheta}{\delta}$
M
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
$u \propto$
$y^{1/5}$
$y^{1/7}$
$y^{1/9}$
$y^{1/11}$

FIG. 6.

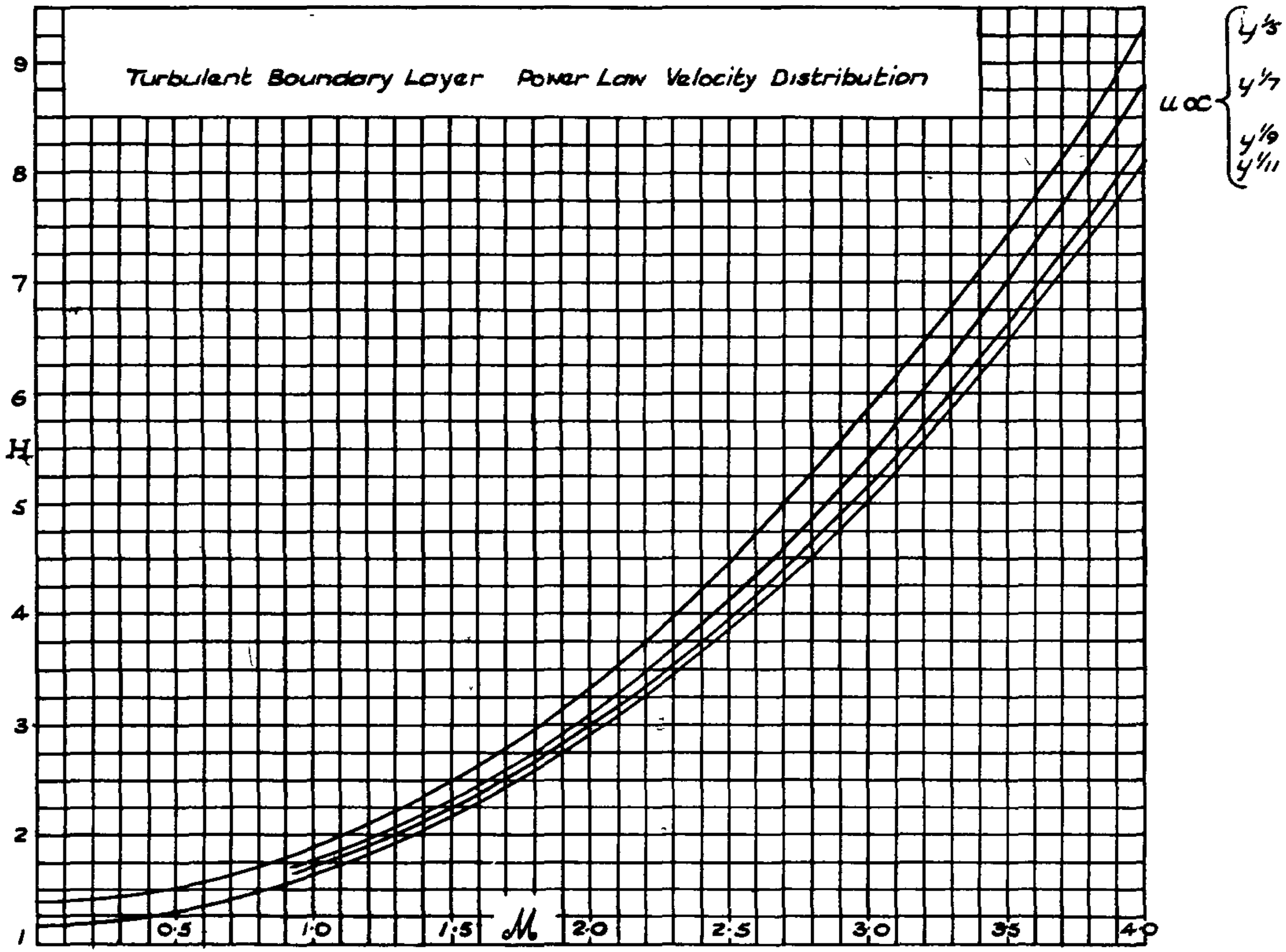

Turbulent Boundary Layer Power Law Velocity Distribution
9
8
7
6
H
5
4
3
2
1
0·5
1·0
1·5
M
2·0
2·5
3·0
3·5
4·0
u ∝ y^1/5
y^1/7
y^1/9
y^1/11

FIG. 7.

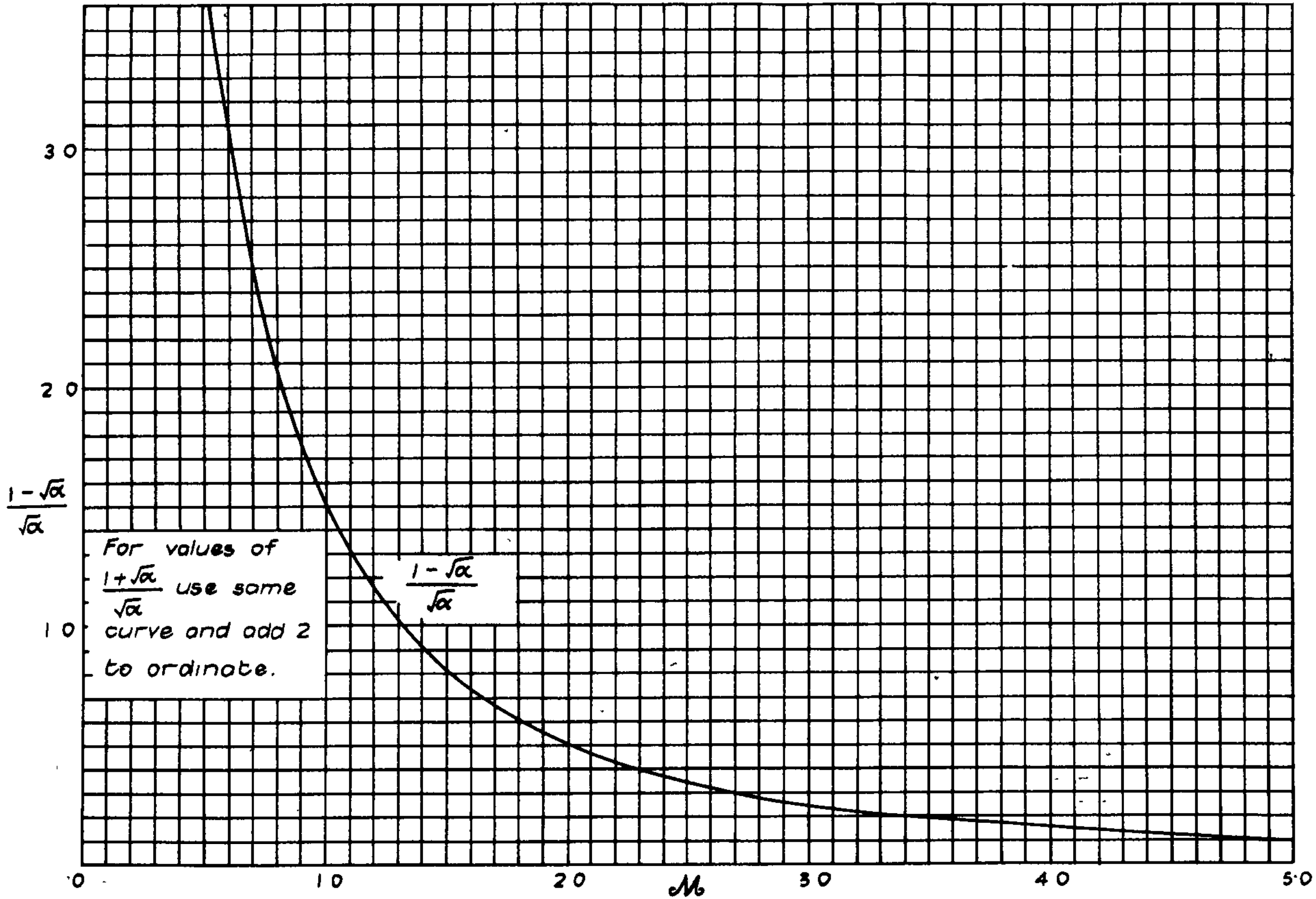
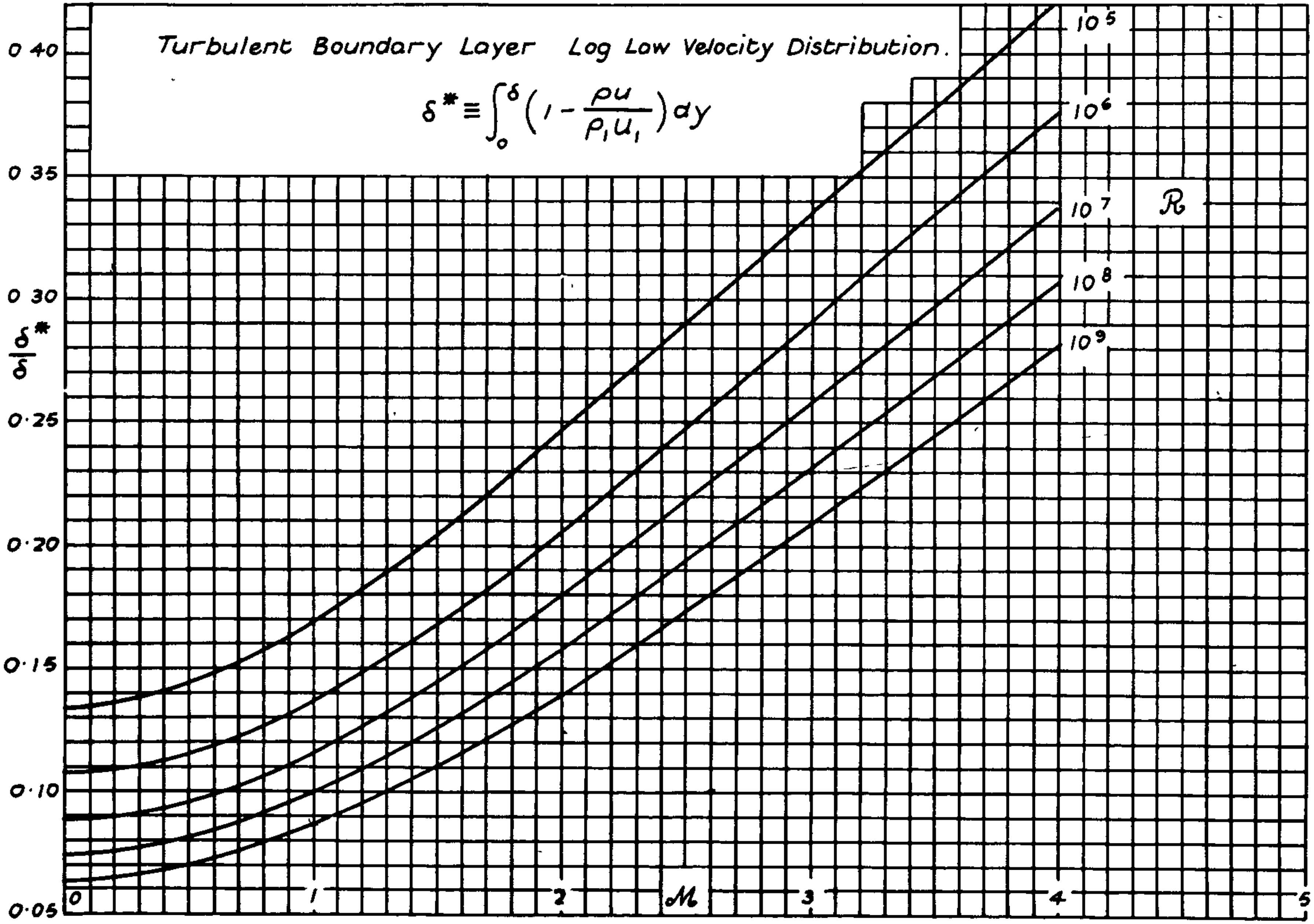

3·0
2·0
1·0
·0
1·0
2·0
3·0
4·0
5·0
$\mathcal{M}$
$\frac{1-\sqrt{\alpha}}{\sqrt{\alpha}}$
For values of $\frac{1+\sqrt{\alpha}}{\sqrt{\alpha}}$ use same curve and add 2 to ordinate.
$\frac{1-\sqrt{\alpha}}{\sqrt{\alpha}}$

FIG. 8.



Turbulent Boundary Layer Log Low Velocity Distribution.
$\delta^* \equiv \int_0^\delta \left(1 - \frac{\rho u}{\rho_1 u_1}\right) dy$
$\frac{\delta^*}{\delta}$
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
1
2
3
4
5
$\mathcal{M}$
10^5
10^6
10^7
10^8
10^9
$\mathcal{R}$

Fig 9.

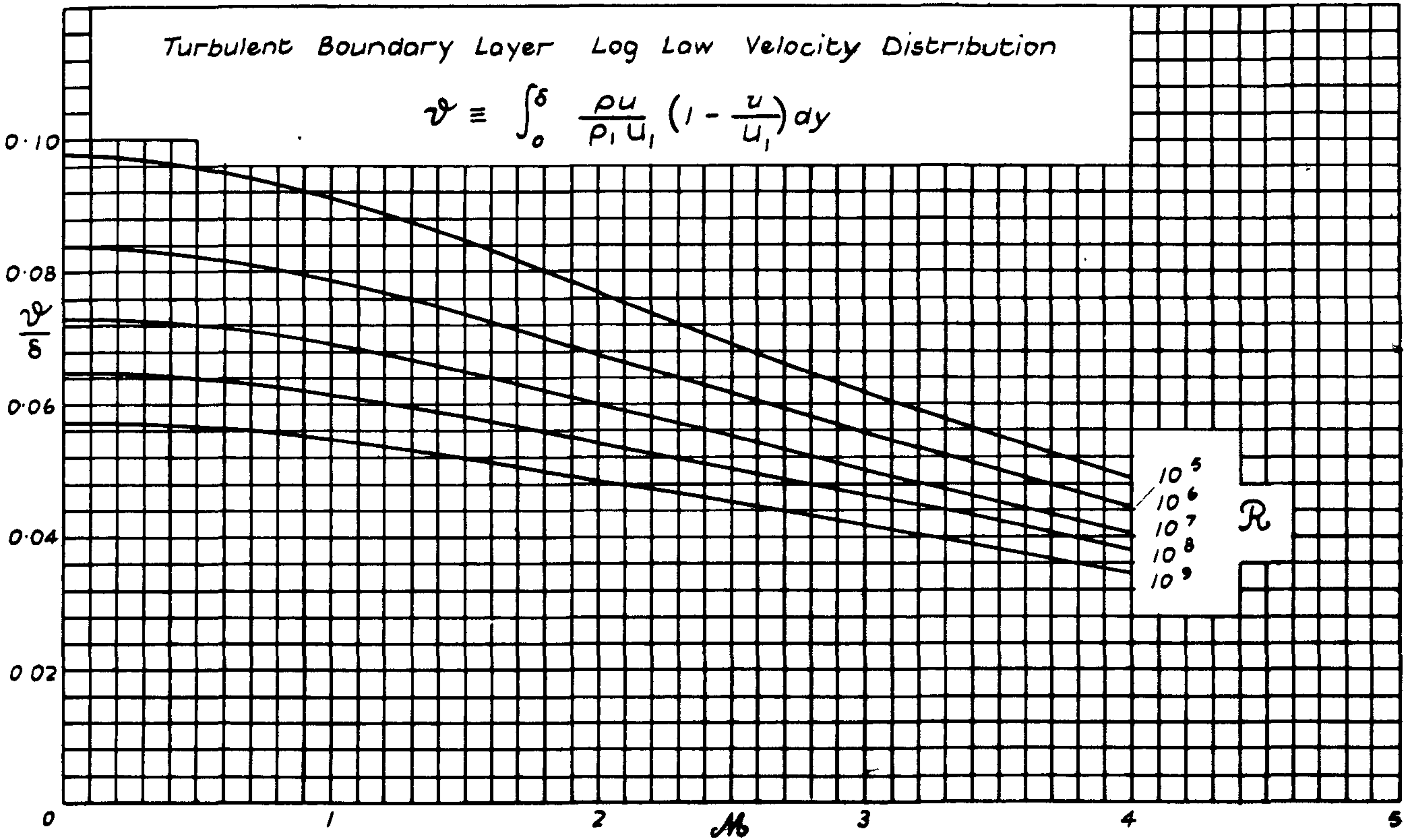
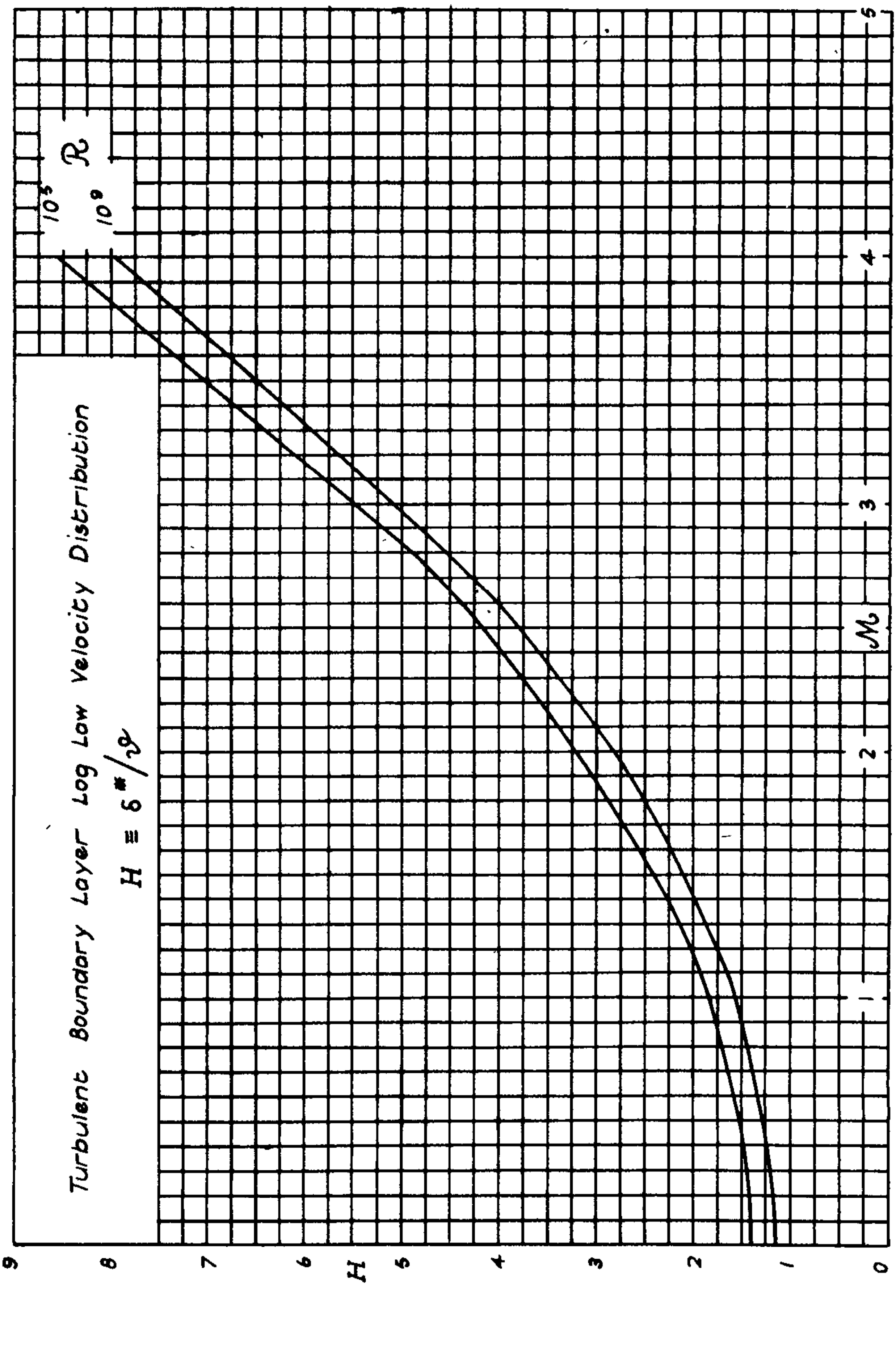
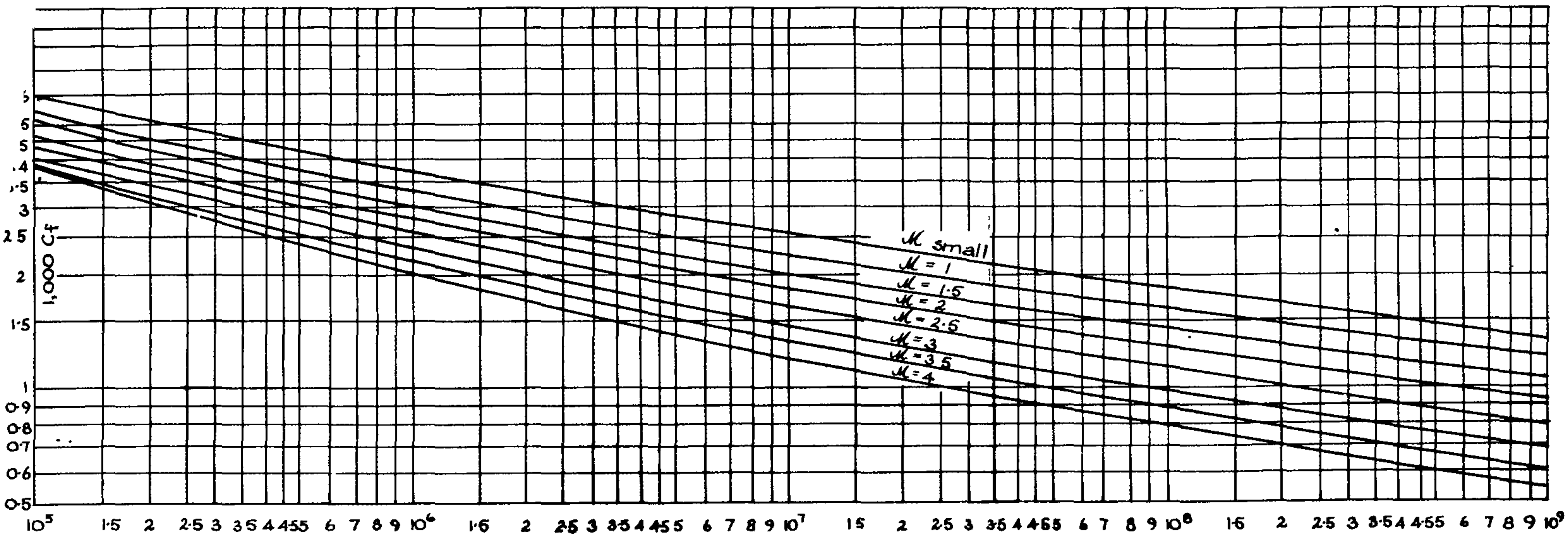

Turbulent Boundary Layer Log Law Velocity Distribution
$\vartheta \equiv \int_0^\delta \frac{\rho u}{\rho_1 u_1}\left(1 - \frac{u}{u_1}\right) dy$
0·10
0·08
0·06
0·04
0 02
$\frac{\vartheta}{\delta}$
0
1
2
3
4
5
M
10^5
10^6
10^7
10^8
10^9
R

FIG. 10.

FIG 11



Turbulent Boundary Layer Log Law Velocity Distribution
H ≡ δ*/θ
R
10^5
10^9
H
M
0
1
2
3
4
5
6
7
8
9



$$\sqrt{C_f}\ \log (RC_f) = 0{\cdot}242 \left\{1+\frac{\gamma-1}{2}M^2\right\}^{-\frac{1}{2}} + \left\{1{\cdot}75\ \log\left\{1+\frac{\gamma-1}{2}M^2\right\} - 0{\cdot}41\right\}\sqrt{C_f}.$$

FIG 12.

Skin Friction of Flat Plates, Compressible Flow.

Fig. 13

Flat Plate (Laminar) - Blasius

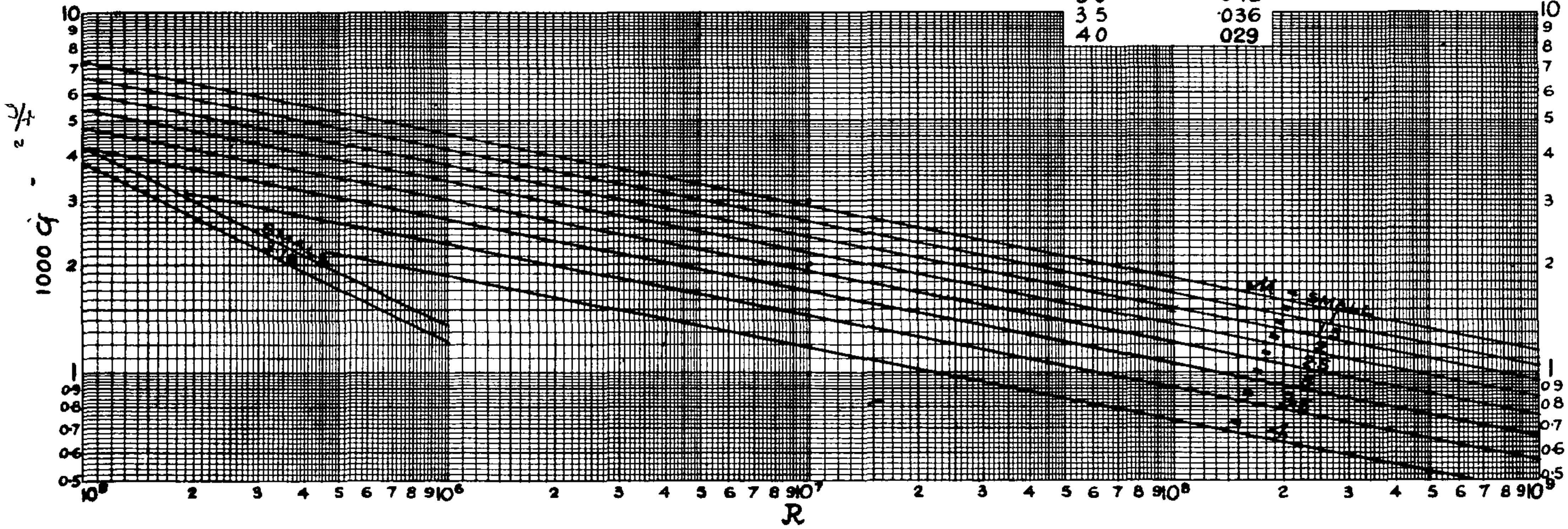
$C_f \sqrt{R} = f(M)$

M	$f(M)$
SMALL	1·328
1	1·308
2	1·269
3·16	1·209

Flat Plate (Turbulent) 1/7th Power Law.

$C_f \sqrt[5]{R} = f(M)$

M	$f(M)$
SMALL	0·073
1	066
1·5	·060
2·0	·054
2·5	·048
3·0	042
3·5	·036
4·0	029



Skin Friction of Flat Plates, Compressible Flow.

Flat Plate (Laminar), Blasius.

$C_f\sqrt{R} = f(M)$

M	$f(M)$
SMALL	1·328
1	1·308
2	1·269
3·16	1·203

Flat Plate (Turbulent), Log Law.

$$\sqrt{C_f}\ \mathrm{Log}(RC_f) = 0{\cdot}242\left\{1+\frac{\gamma-1}{2}M^2\right\}^{-\frac{1}{2}} + \frac{7}{4}\sqrt{C_f}\ \mathrm{Log}\left\{1+\frac{\gamma-1}{2}M^2\right\}$$

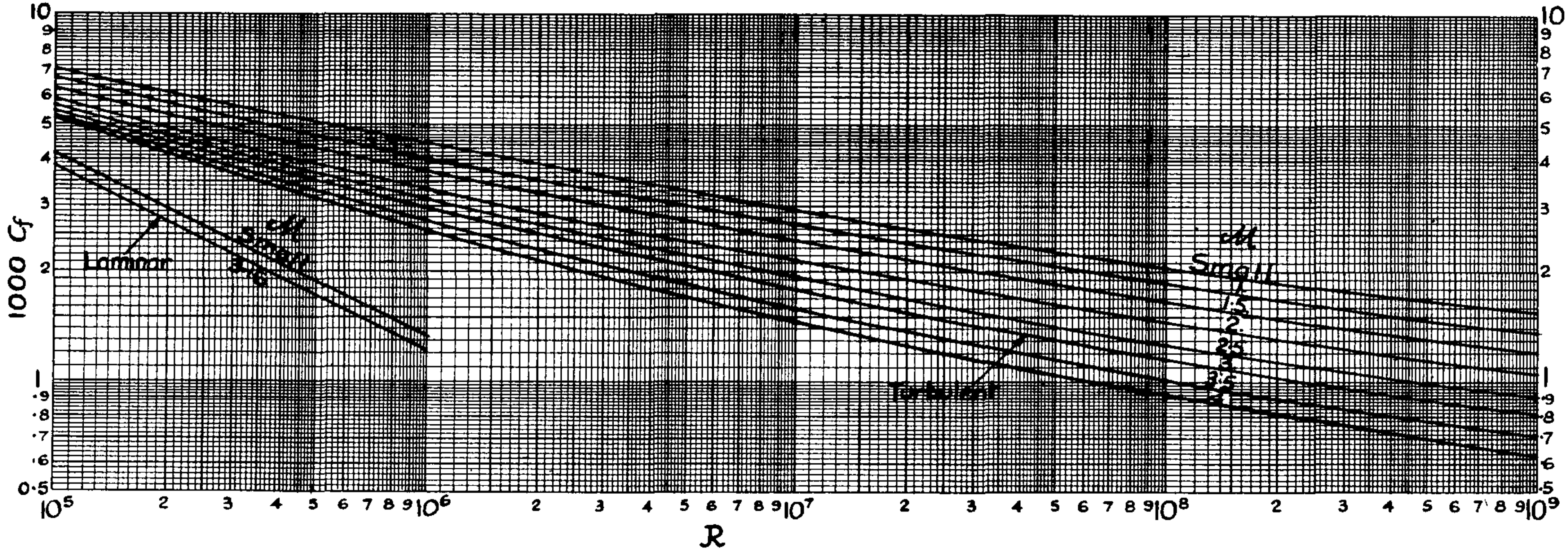


FIG. 14

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