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The Pressure Distribution at Zero Lift on a Slender Delta Wing at Transonic Speeds

by

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THE PRESSURE DISTRIBUTION AT ZERO-LIFT ON A SLENDER DELTA WING AT TRANSONIC SPEEDS

by

M. C. P. Firmin

SUMMARY

The pressure distribution has been measured on the rear of a slender delta wing with rhombic cross-sections as an extension to the programme of work on zero-lift drag at supersonic speeds. The thickness distribution was extreme in that it was designed to give rise to a marked adverse pressure gradient over the central part of the wing and a relatively large suction near the trailing edge at supersonic speeds.

The measurements have been compared with thin-wing theory and slender-thin-wing theory throughout the Mach number range of 0·8 to 1·3, except at sonic speed where approximate solutions are given for the sonic-thin-wing theory. The results for supersonic speeds have also been compared with a calculation method by the author reported previously.

* Replaces R.A.E. Technical Report 66172 - A.R.C. 28492

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1 INTRODUCTION

In some previous work, reported in Ref.1, on the pressure distribution over the rear of a slender delta wing, it was found that the terminal shock wave had moved forward from the trailing edge of the wing over the central region at the lowest supersonic Mach number at which tests are possible in the R.A.E. No.19 (18" × 18") wind tunnel. It was decided, therefore, to extend this work to lower Mach numbers in a different tunnel in order to confirm the result and also obtain some evidence on the transonic and subsonic behaviour of the flow over such a wing. These measurements were made during 1963.

2 DETAILS OF THE MODEL

The main details of the model are given in Fig.1 for completeness, but for its design features and a comparison of its shape with other wings tested previously the reader is referred to Refs.1 and 2.

The wing is of delta planform, unit aspect ratio, and has rhombic cross-sections. The centre-line distribution of thickness (Fig.2) is given by

$$\frac{Z(\xi,0)}{c_o} = \frac{v}{2\,s\,c_o^2}\;[\xi\;(1-\xi)\;(14{\cdot}0 + 52{\cdot}67\,\xi - 167{\cdot}33\,\xi^2 + 116{\cdot}67\,\xi^3)]$$

where ξ ($\equiv x/c_o$) is the chordwise station normalised with reference to the centre-line chord (c_o = 12·00 inches) and measured from the apex. All the planform edges had a nominal 0·002 inch radius and the thickness distribution was modified slightly to include the extra thickness due to the radiused planform edges.

3 DETAILS OF THE WIND TUNNEL AND ITS FLOW

The model was mounted in the 24" × 18" transonic tunnel, some design features of which are given in Ref.3. Details of the support system and the location of the model with respect to the sidewalls are given in Fig.3. Glass sidewalls were used in order to obtain Schlieren and Shadowgraph photographs even though the slotted sidewalls, as used for the roof and floor, would have reduced the tunnel interference. The Mach number in the working section was controlled primarily by suction through the slotted roof and floor for supersonic speeds, the suction being provided from the diffuser. At subsonic speeds the tunnel flow is controlled by a by-pass system so arranged that the total volume flow through the compressor is constant*. Mach numbers up to 1·3

* More recently the flow has been controlled at subsonic speeds (M > 0·7) by choking the flow in the diffuser.

are available and the measured Mach number uniformity on the centre-line in the empty tunnel over the length occupied by the model is better than ±0·002 up to M = 1·15, ±0·005 up to M = 1·20 and ±0·007 up to M = 1·30.

4 MEASUREMENTS

In transonic wind tunnels it is not possible to obtain results which are completely free from interference from the walls at all Mach numbers. In the present case the model was designed for use in an 18" × 18" supersonic wind tunnel at Mach numbers above 1·4 and its support and size were chosen accordingly. However, at a Mach number of 1·25 the shockwaves reflected from the roof and floor just miss the wing and the disturbance from the support should not influence the flow on the upper surface of the wing where pressures are measured. It may be concluded, therefore, that measurements at Mach numbers above 1·25 may be used as an extension of the results obtained previously[1]. The disturbances reflected from the roof and floor of the tunnel will be attenuated by the slotted walls and hence, since the disturbances reflected from the side-walls will miss the trailing edge of the model, the tunnel interference should be small in the Mach number range 1·11 to 1·25. At lower Mach numbers the measurements may be marred by the influence of the support and tunnel walls but they should be sufficiently reliable to give a qualitative impression of the flow over the wing. At subsonic speeds the slotted walls are designed to alleviate blockage effects and so, since the blockage ratio for the model in the tunnel is about 0·3%, any interference effects should not be large[4], since all the measurements were made with the model as zero incidence.

Since the 24" × 18" transonic tunnel is limited to relatively low Reynolds numbers (2×10^6 per ft) boundary layer transition was fixed by the use of distributed roughness in the region of the leading edges. It had been shown, in the previous tests on this model at supersonic speeds, that measurements of pressure coefficient with fixed transition at low Reynolds numbers were consistent with those with free transition at fairly high Reynolds numbers (10×10^6 per ft).

5 DISCUSSION OF RESULTS

The results of the measurements are given in Figs.4(a) - (o) in order of descending Mach number. The results at supersonic speeds follow the trend found previously at Mach numbers of 1·4 and above. As the Mach number is reduced the pressure near the centre of the trailing edge tends to rise and the region in which this compression takes place extends forward and laterally to the other chordwise station ($y/s = 0{\cdot}5$) at which measurements were made.

At other than sonic speed the results have, for completeness, been compared with the same theories as enumerated at the higher Mach numbers. Since at near sonic speeds the slender approximation ($|\beta^2 \phi_{xx}| << |\phi_{yy}| + |\phi_{zz}|$) holds, both the first order theories (viz thin-wing theory and slender-thin-wing theory) tend to the same result as the Mach number approaches unity; but they are both totally inadequate when compared with the measurements near the trailing edge for low supersonic speeds in failing to predict the recompression. This work (including the results in Ref.1) confirms, therefore, that slender-thin-wing theory should not be relied upon for the calculation of pressure coefficients in the region of a trailing edge at any supersonic Mach number. On the other hand, within the range of Mach numbers considered, it is only at very low supersonic speeds that thin-wing theory fails completely. The method of Ref.1, which takes at least some account of the second order corrections, is extremely successful at moderate supersonic speeds where the corrections are not very large. At very low supersonic speeds where the corrections are larger the method cannot be said to do more than indicate one possible reason why thin-wing theory fails. This method does, however, predict a compression near the trailing edge at low supersonic speeds similar to that found experimentally but this is probably fortuitous since the assumptions on which the method is based are then violated.

It is well known that at sonic speed (Fig.4(i)) the small-disturbance approximations are not sufficient to linearise the isentropic flow equation so general solutions are lacking. This has led several investigators to make further approximations in order to obtain approximate solutions which may be of general use. The differential equation applicable for small disturbances of a sonic stream is

$$\phi_{yy} + \phi_{zz} = (\gamma + 1)\,\phi_x\,\phi_{xx} \quad .$$

Since the present wing is geometrically slender, the equivalence theorem of Oswatitsch and Keune[5] may be employed to relate the flow past it to the flow past a body of revolution of the same cross-sectional area distribution. There are at least two different, but related, approaches to solving the equation above for a body of revolution. Randall[6] regarded $(\gamma + 1)\,\phi_{xx}$ as constant (λ) and he gave a method of obtaining an appropriate value for λ. An approximate general solution for bodies of revolution has also been obtained by Coles and Royce[7] making a different assumption in order to solve the differential equation. They assume that $\phi_x \, \phi_{xx}$ may be replaced by $\mu(x - \bar{x}) \, \phi_{xx}$ where μ and $\bar{x}$ are constants which may be determined for a particular body shape. If the assumptions made were strictly justified, the methods would be equivalent. Estimates from both the above methods have been included in Fig.4(i). It appears that differences in the assumptions made have a large influence on the estimates. As a further check on the methods estimates were compared for a Lord V wing, which is a less 'extreme' shape, but the same conclusion was reached. However, the two estimates do establish a common trend, which conform broadly with the experimental results except at points near the trailing edge, where the marked compression exists.

At subsonic speeds, Figs.4(j) - (o), the marked compression still exists near the trailing edge and at the lowest speed at which measurements were made (Fig.4(o)) thin-wing theory appears to work quite well although, clearly, it must fail at the trailing edge where the solution is singular. At higher subsonic speeds the experimental results are not as dependent on Mach number as predicted by the theories. This is not consistent with the findings on two-dimensional wing sections at near zero lift where, in general, the experimental results for the peak suction are more Mach number dependent at high sub critical speeds than suggested by the Prandtl-Glauert rule. The flow was super-critical ahead of the measuring stations for the results obtained at Mach numbers above 0·9 and so at these higher Mach numbers the measurements may be influenced by a weak upstream shock. Slender-thin-wing theory which has been included for completeness again deviates from thin-wing theory near the trailing edge, but this could partly be overcome by the technique suggested by Randall[6].

So far the discussion has concentrated on comparing the experimental results with idealized theories which do not allow for the existence of a boundary layer. The wing tested, as mentioned previously, had rather an extreme chordwise section, designed originally to explore the validity of the small-disturbance theory at supersonic speeds. This resulted in a shape having a marked adverse pressure gradient over about 30% of the root chord, starting between 40 and 50% of the root chord, throughout the Mach number range considered. In addition, for subsonic speeds it is well known that where a large adverse pressure gradient exists over the rear of a wing the boundary layer is important in determining the pressure distribution[8]. The differences between the experimental results and thin-wing theory are, in general, consistent with the kind of changes that may be expected from the influence of a boundary layer. Although boundary layer measurements have not been made in these tests the Schlieren evidence of Ref.1 suggested that the boundary layer thickens considerably on the rear part of the wing. The flow at the rear of the wing at supersonic speeds is similar to that in a compression corner with the pressure rise causing some upstream influence through the boundary layer.

Applying the conditions listed by Cooke[9], we should not expect a separated boundary layer; but some of the shadowgraph photographs (Fig.5) do appear to show a bifurcated shock at the trailing edge. Surface oil flow, however, confirmed that the flow was not separated; but a thicker film of oil remained near the trailing edge, which is consistent with a rapid reduction in skin friction and a thickening of the boundary layer. Another possible reason why the compression at the trailing edge does not occur through a single oblique shockwave attached to the edge is that the turning angle is too large for the local Mach number conditions. For this particular wing the local Mach number on the centre-line is close to the above 'detachment' condition at a free stream Mach number of 1·3, which is only slightly below the higher speed at which a compression was observed near the trailing edge. It is possible, therefore, that the rather large turning angle together with the influence of the boundary layer resulted in the compression being spread forward of the trailing edge.

Another consequence of the extreme section shape was that at transonic Mach numbers two separate regions of supersonic flow existed, one on the forward part of the wing, which was terminated by a shockwave in the region of 50-60% of the root chord at Mach numbers near unity, and another region nearer the trailing edge. The flow did not become fully supersonic up to the region of the trailing edge until a Mach number of about 1·05.

6 CONCLUSIONS

The transition from subsonic to supersonic flow proceeded smoothly as the Mach number was increased and the shockwaves were not sufficiently strong for the boundary layer to separate. The measurements presented confirm the results indicated in the previous tests that the terminal shockwave was not attached to the trailing edge until a Mach number above the range of these tests.

This work, including the results of previous tests, confirms that slender-thin-wing theory is in error for wings as 'extreme' in shape as the present and so it should not be relied upon in general. For the present wing it does provide better estimates at subsonic speeds than at supersonic speeds. Thin-wing theory on the other hand works fairly well near the trailing edge except at low supersonic speeds where the assumption of small velocity perturbations is violated and again very close to the trailing edge at subsonic speeds where the solution is singular. The author's method, which worked well at moderate supersonic speeds, gives the correct trend at low supersonic speeds but, as with thin-wing theory, the velocity perturbations cannot be considered small and therefore the agreement is probably fortuitous. At sonic speed the approximate methods of Randall and of Coles and Royce are not satisfactory since the results are inconsistent with the approximations used to obtain them. A satisfactory theoretical method is still required.

Boundary layer measurements have not been made in this experiment and no account has been taken of the boundary layer in any of the theoretical methods. If more accurate estimates are required for the pressure distribution on the rear of wings such as this at subsonic and transonic speeds then corrections will have to be made for its influence.

SYMBOLS

c_o	wing chord at centre-line
C_p	pressure coefficient $\left(\frac{P - P_\infty}{q}\right)$
M	mean Mach number of undisturbed stream
p	pressure
P_∞	static pressure in undisturbed stream
q	kinetic pressure of undisturbed stream
s	semi-span at the trailing edge
v	volume of the wing
x,y,z	Cartesian coordinates with origin at the apex of the wing, x axis measured in the direction of the undisturbed stream; the z axis normal to the chordal plane of the wing
β^2	$= \lvert(M^2 - 1)\rvert$
γ	adiabatic index
ξ	x/c_o, the chordwise station as a fraction of the centre-line chord and measured from the apex
ϕ	velocity potential

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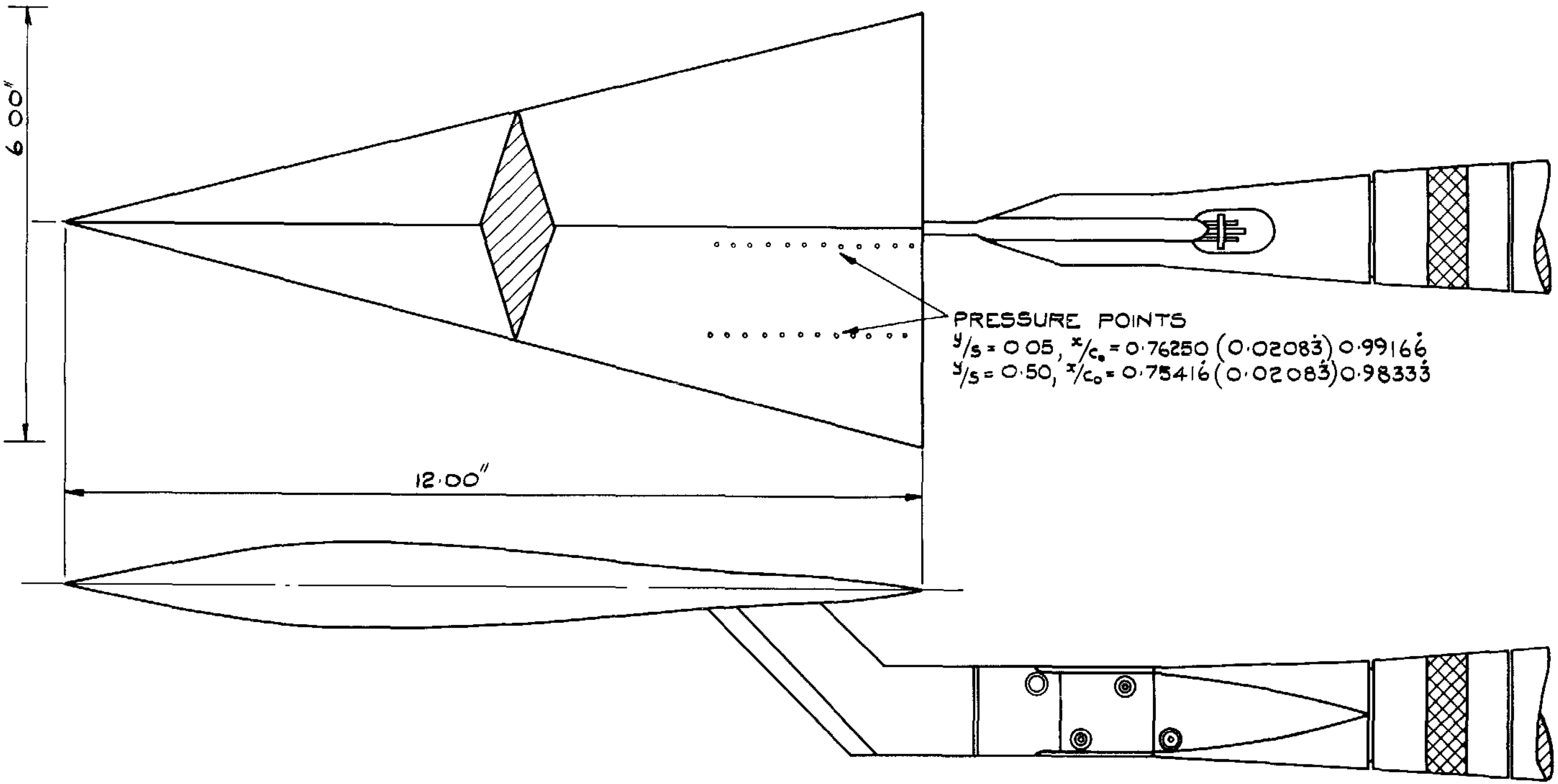

SCALE – $\frac{1}{2}$

FIG. 1 MODEL DETAILS

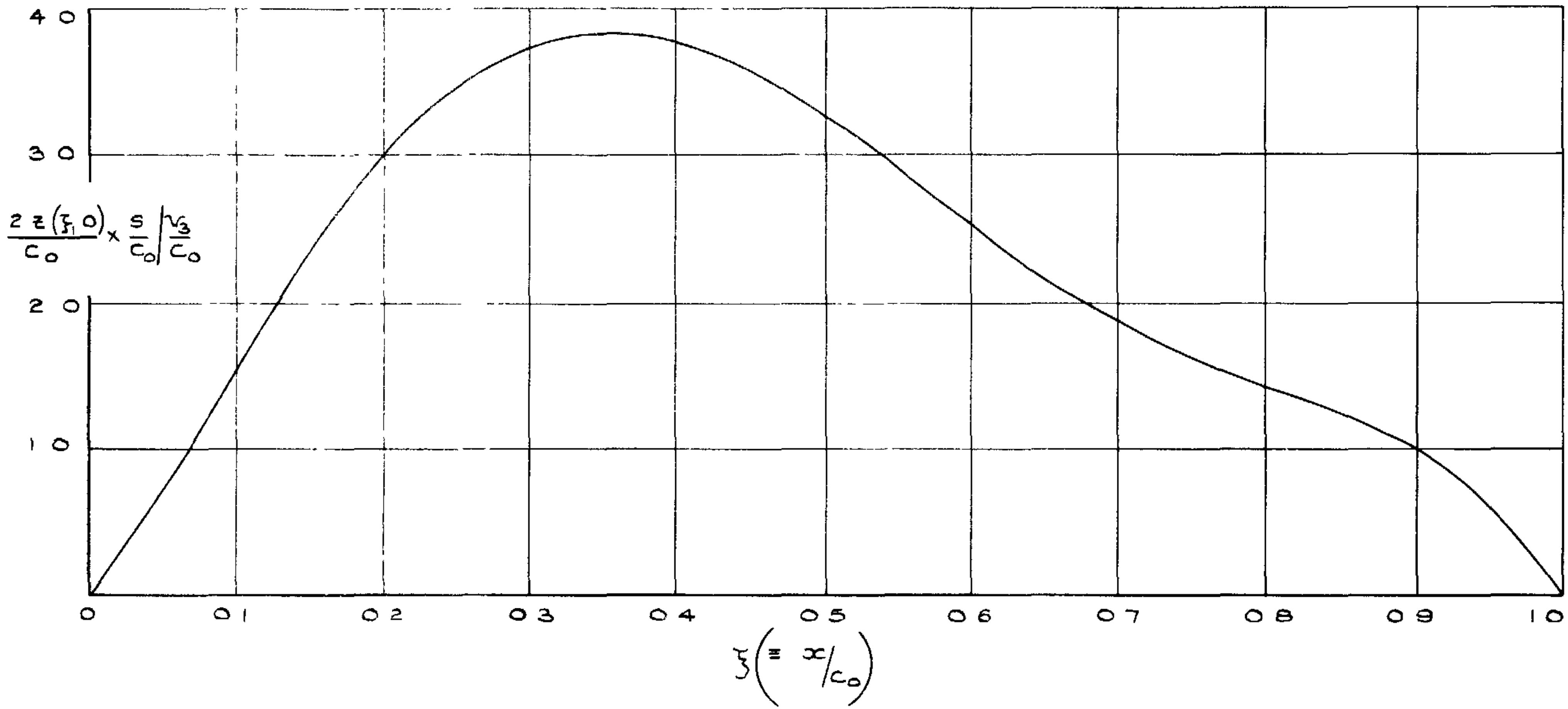

FIG 2 NON-DIMENSIONAL CENTRE LINE THICKNESS DISTRIBUTION

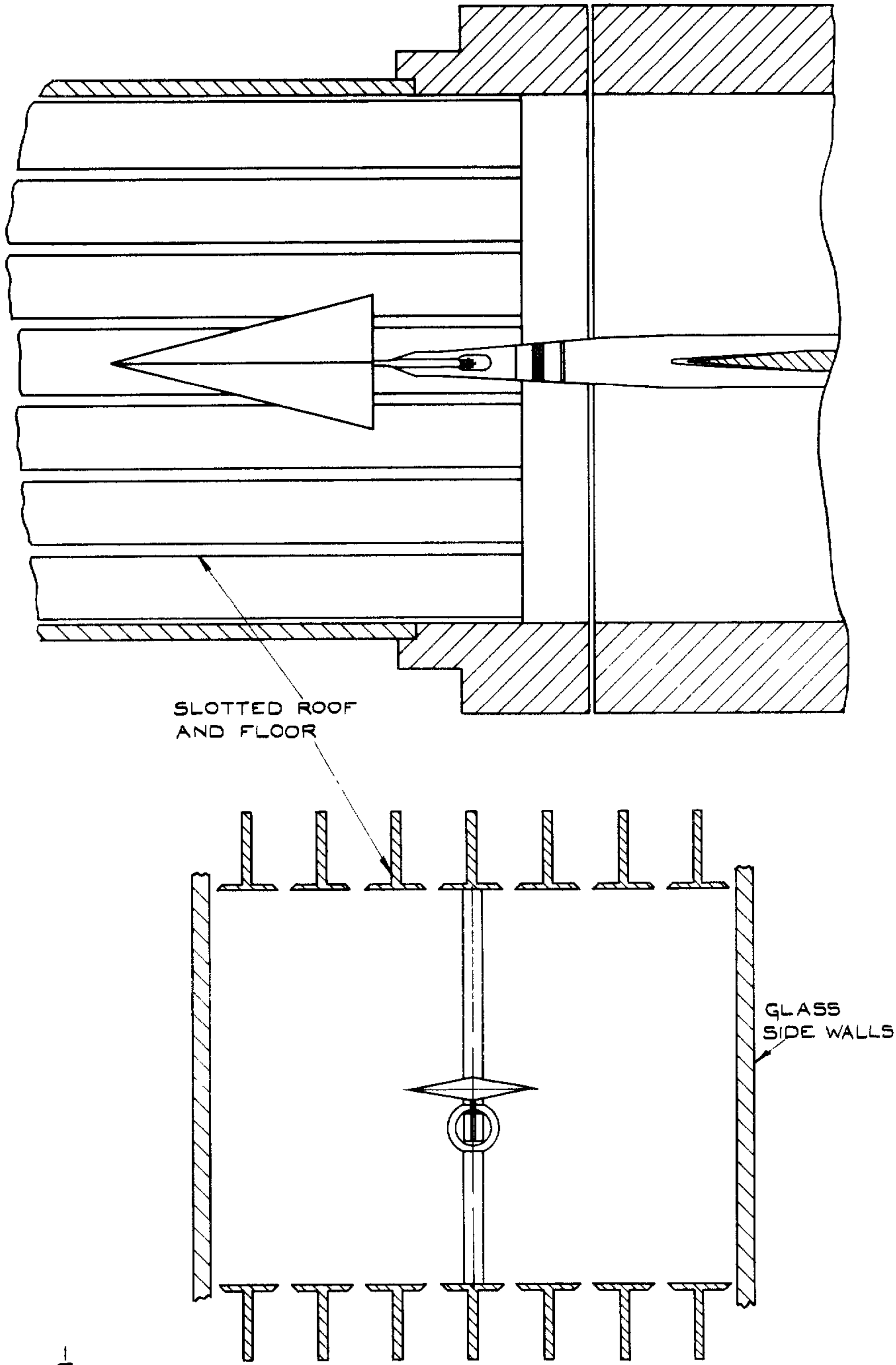

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FIG. 3 DETAILS OF MODEL IN 24" X 18" TUNNEL

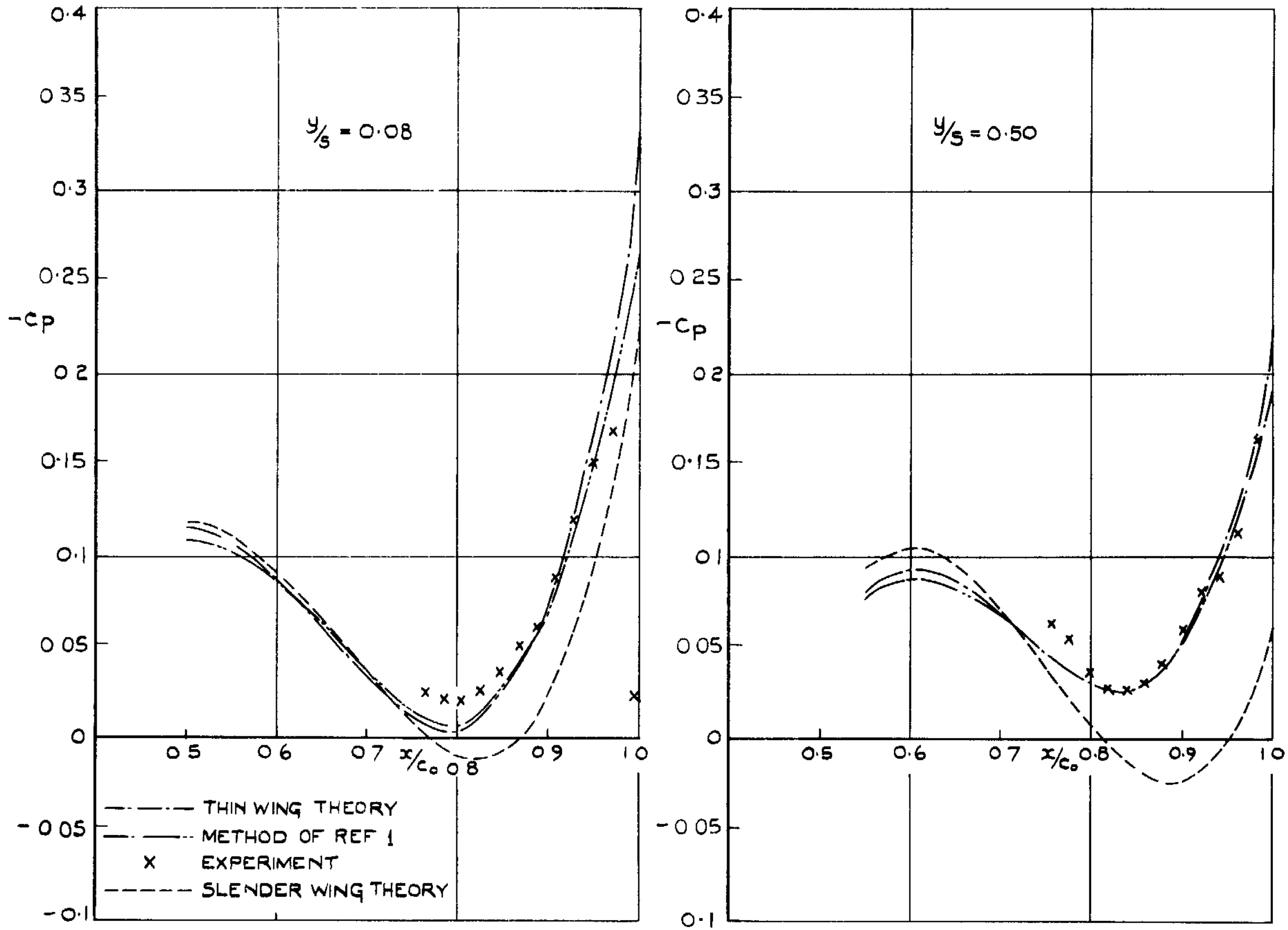

FIG. 4 (a)

FIG. 4 PRESSURE DISTRIBUTION AT $M=1\cdot30$, $C_p^* = 0\cdot392$

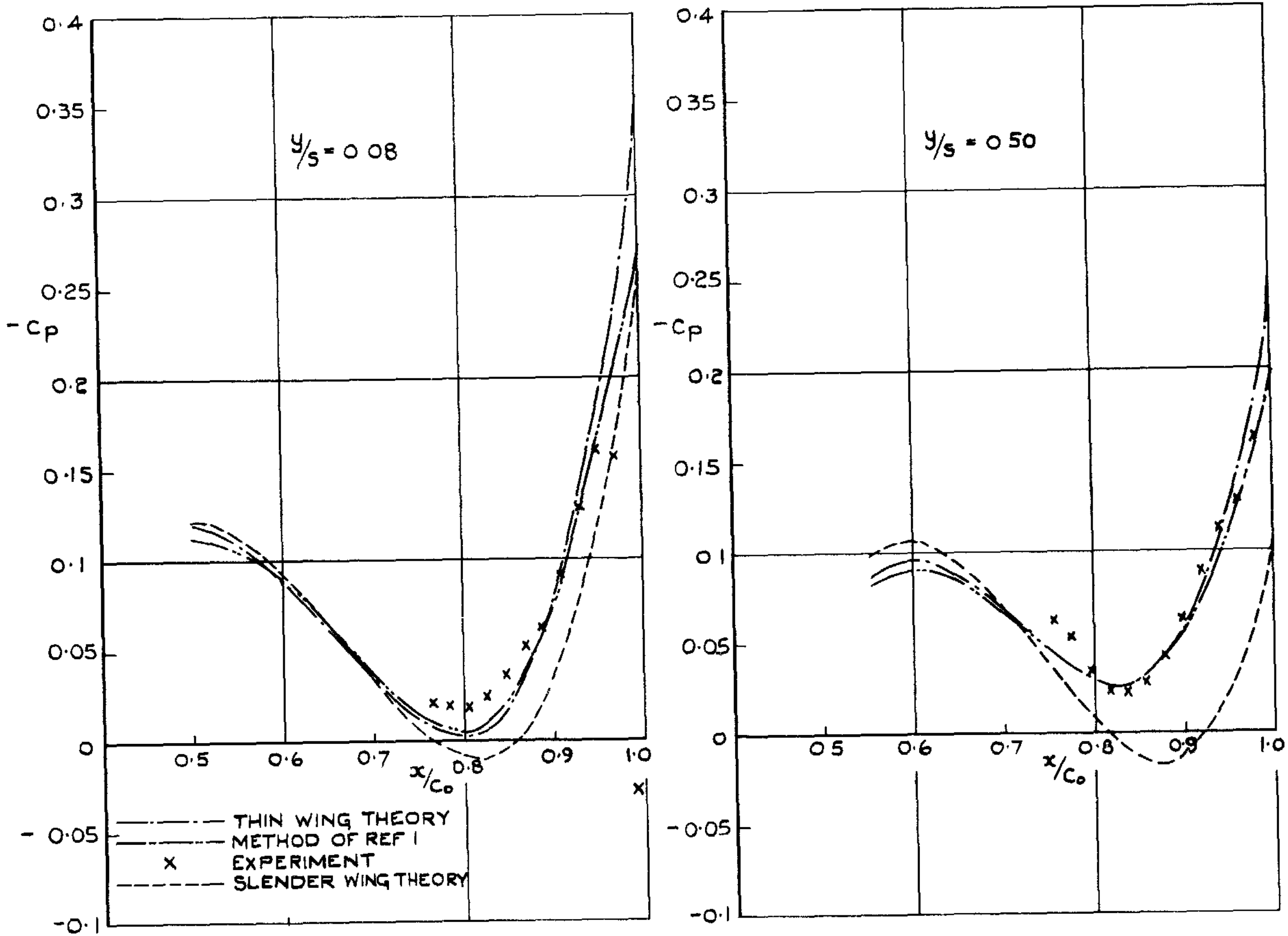

FIG. 4 (b)

FIG. 4 PRESSURE DISTRIBUTION AT $M = 1·25$, $C_p^* = 0·337$

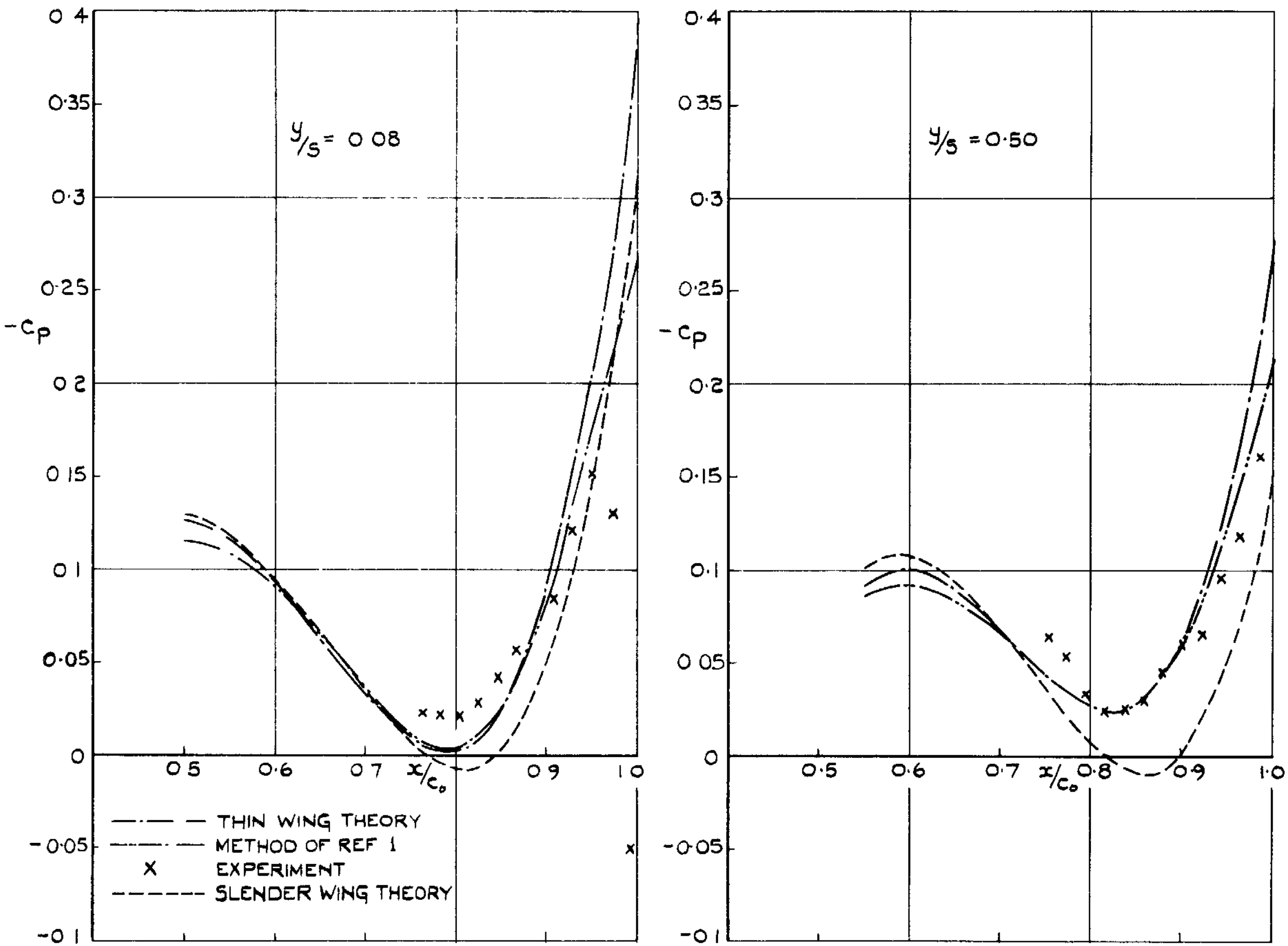

FIG 4 (c)

FIG. 4 PRESSURE DISTRIBUTION AT $M = 1{\cdot}20$, $C_p^* = 0{\cdot}279$

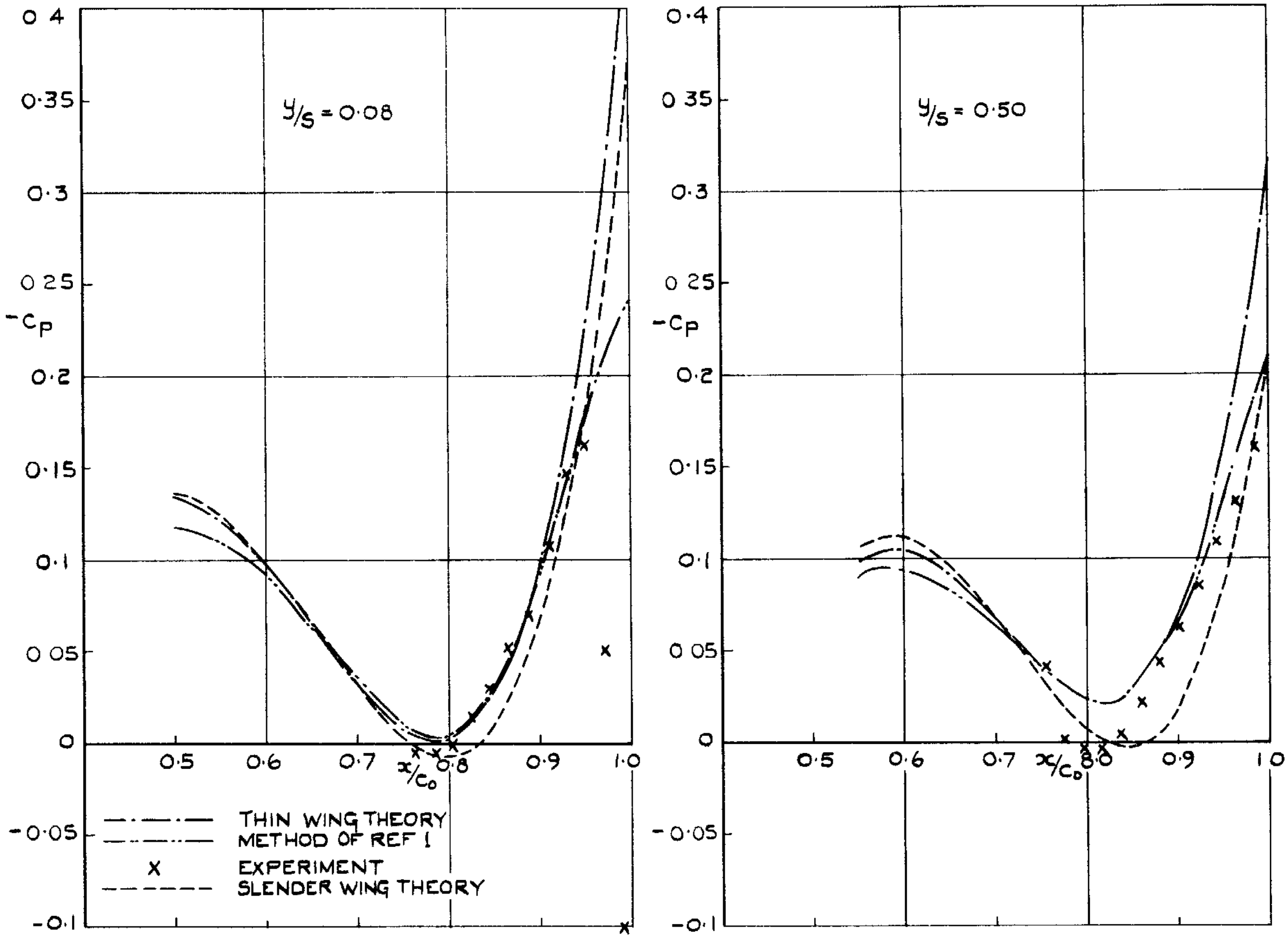

FIG. 4 (d)

FIG 4 PRESSURE DISTRIBUTION AT $M = 1{\cdot}15$, $C_p^* = 0{\cdot}217$

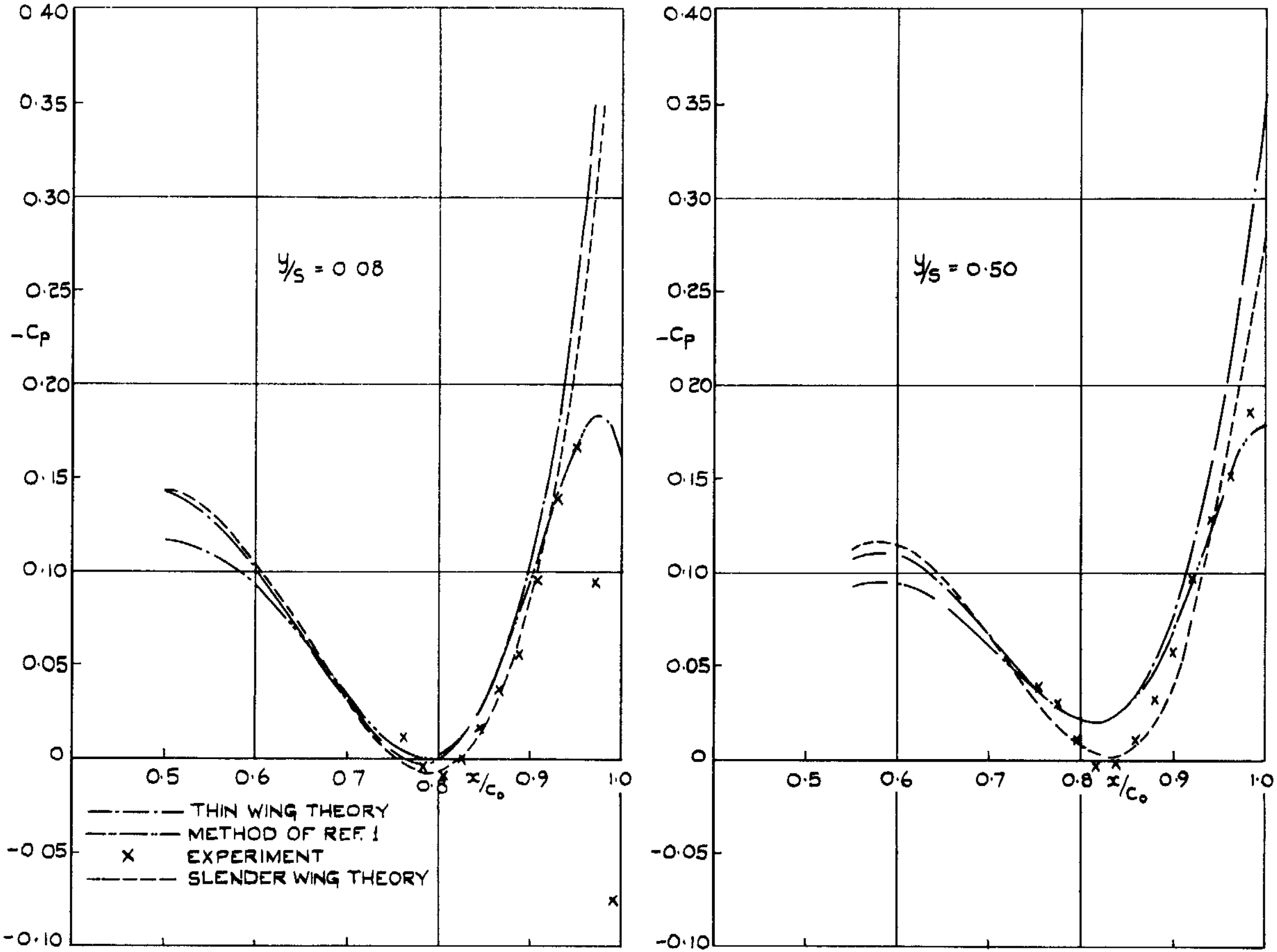

FIG. 4 (e)

FIG. 4 PRESSURE DISTRIBUTION AT $M=1 \cdot 11$, $C_p^* = 0 \cdot 165$

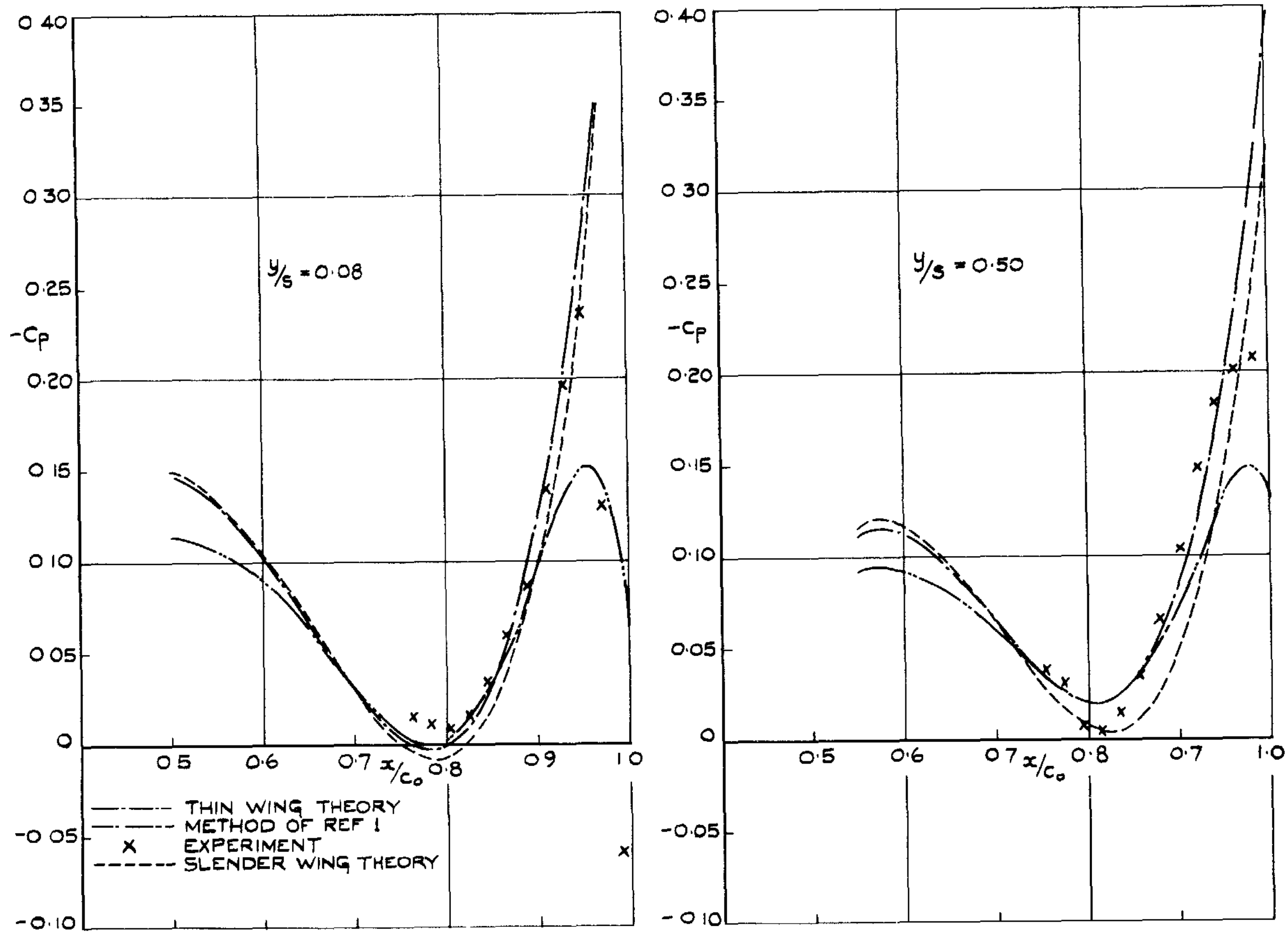

FIG. 4 (f)

FIG. 4 PRESSURE DISTRIBUTION AT $M = 1{\cdot}09$, $C_p^* = 0{\cdot}137$

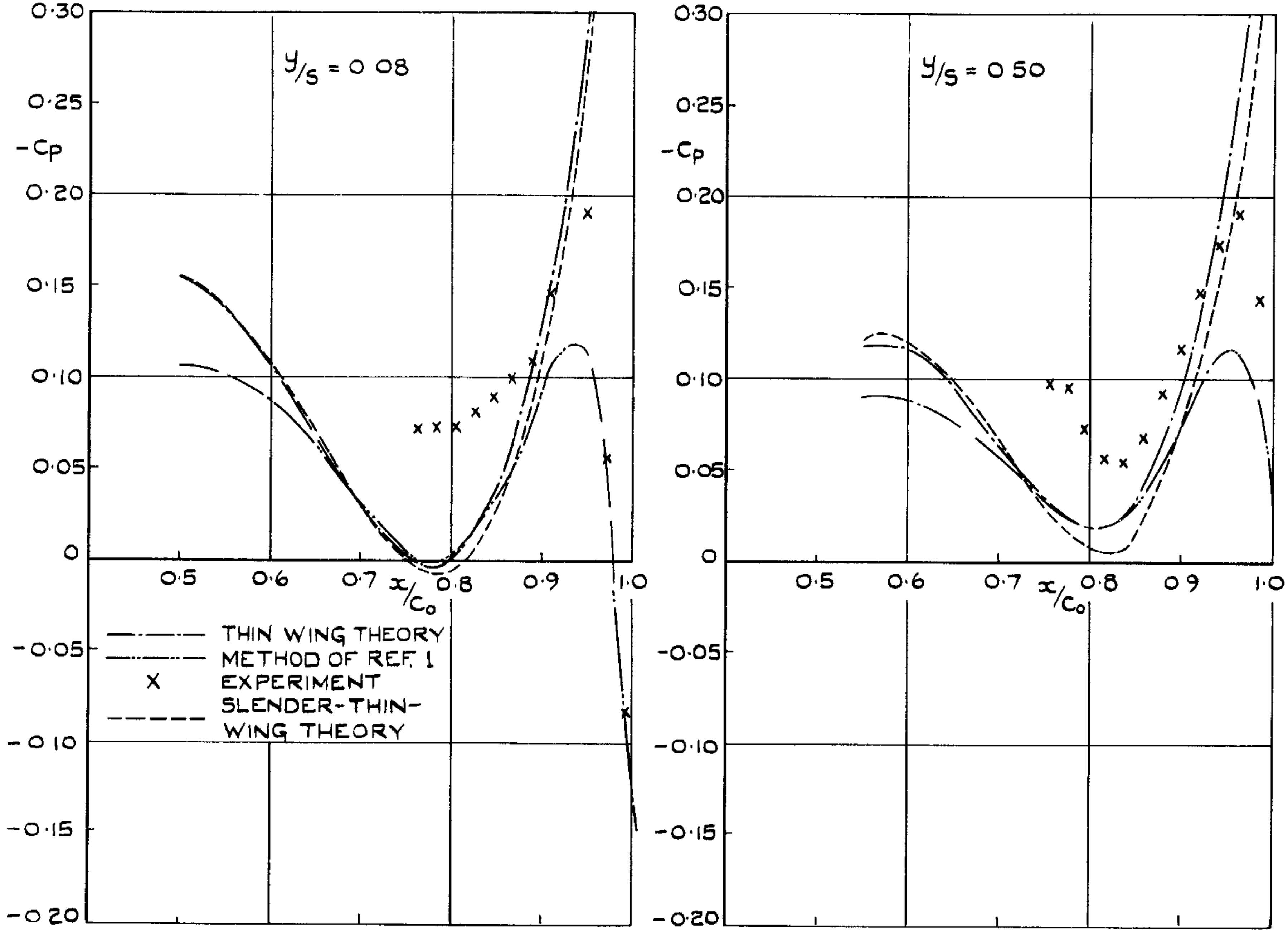

FIG 4 (g)

FIG.4 PRESSURE DISTRIBUTION AT M = 1 07, $C_p^* = 0\ 109$

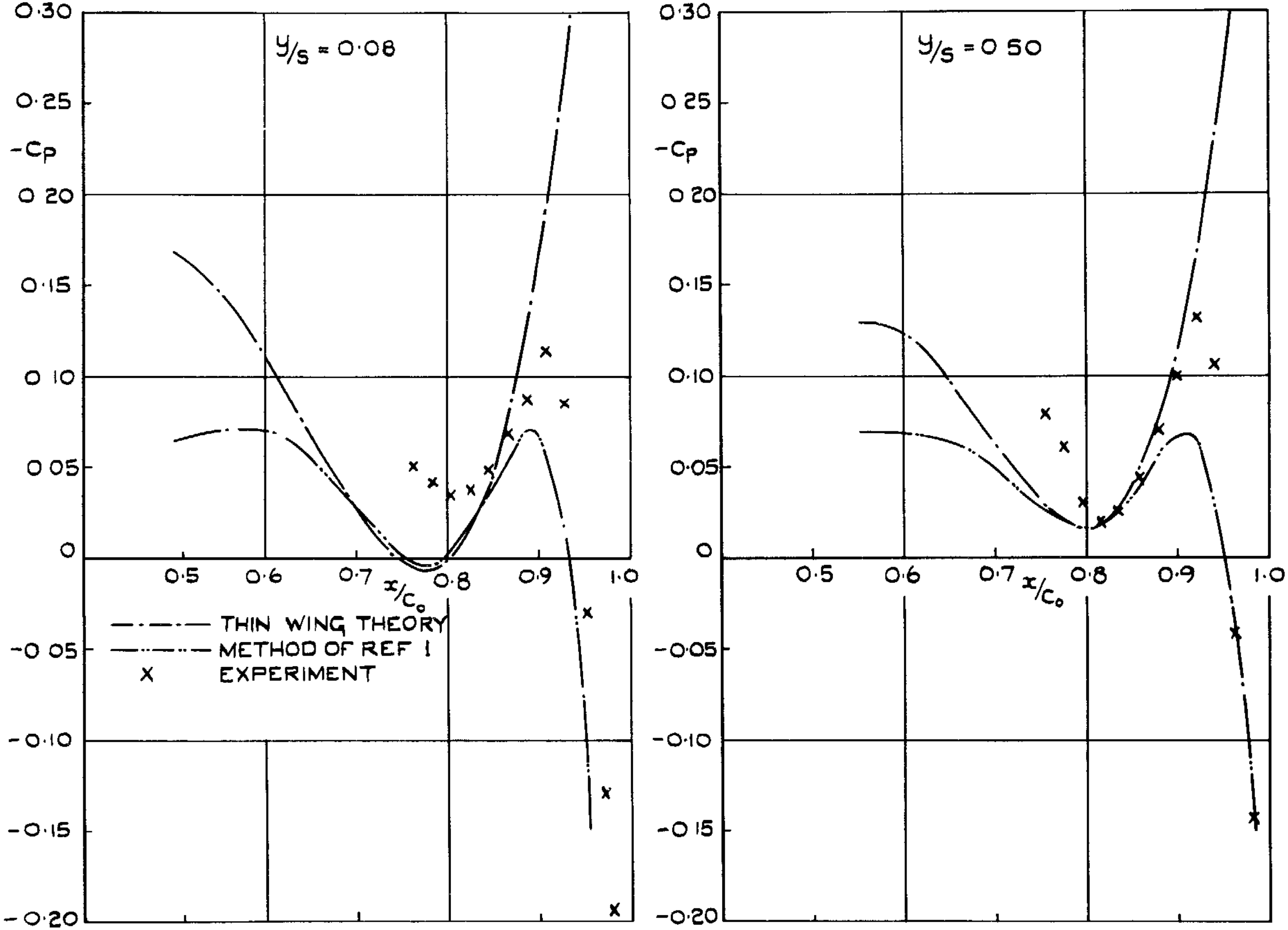

FIG 4 (h)

FIG. 4 PRESSURE DISTRIBUTION AT $M = 1{\cdot}04$, $C_p^* = 0{\cdot}064$

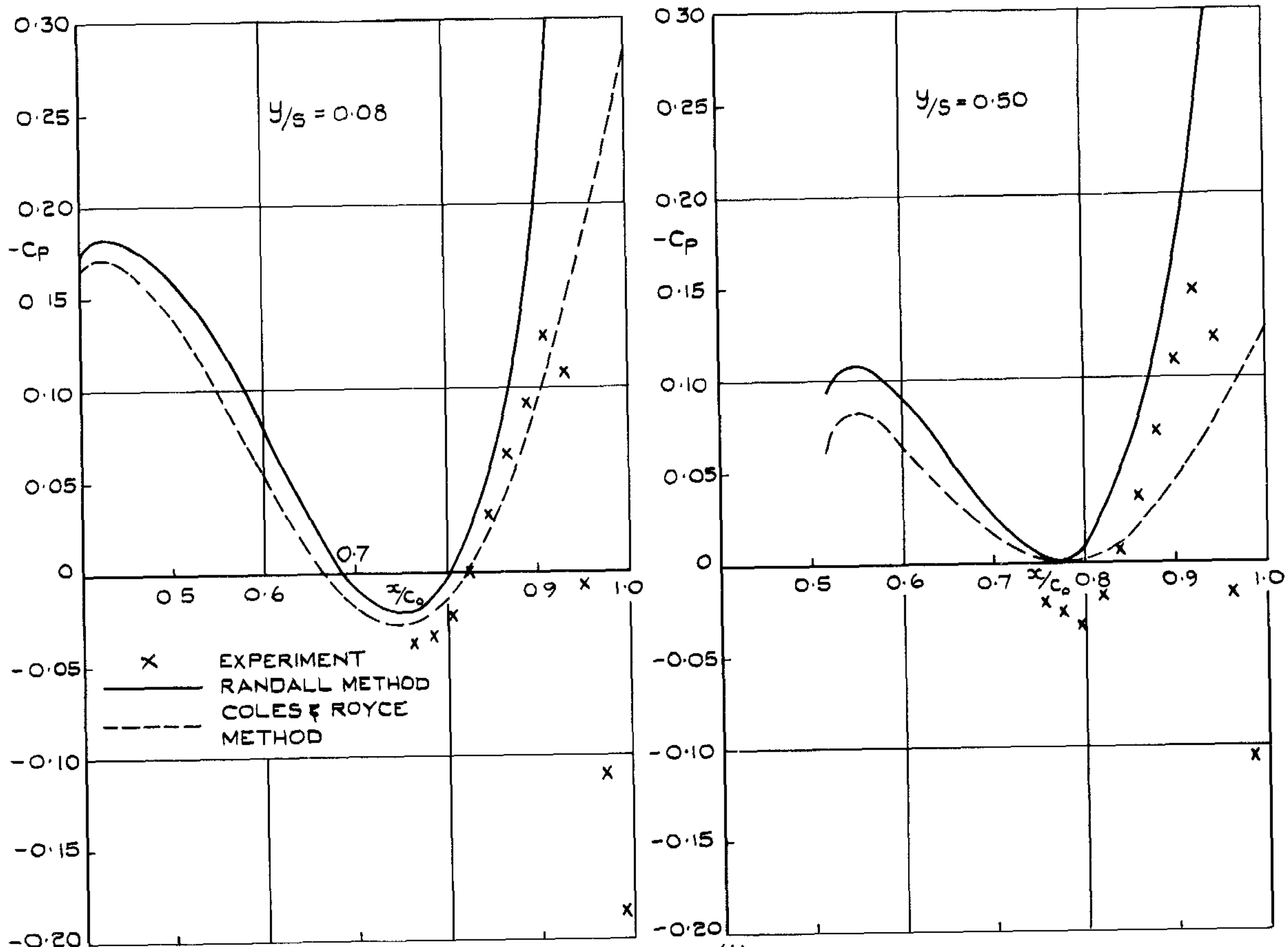

FIG. 4 (i)

FIG 4 PRESSURE DISTRIBUTION AT $M = 1\cdot 00$, $C_p^* = 0$

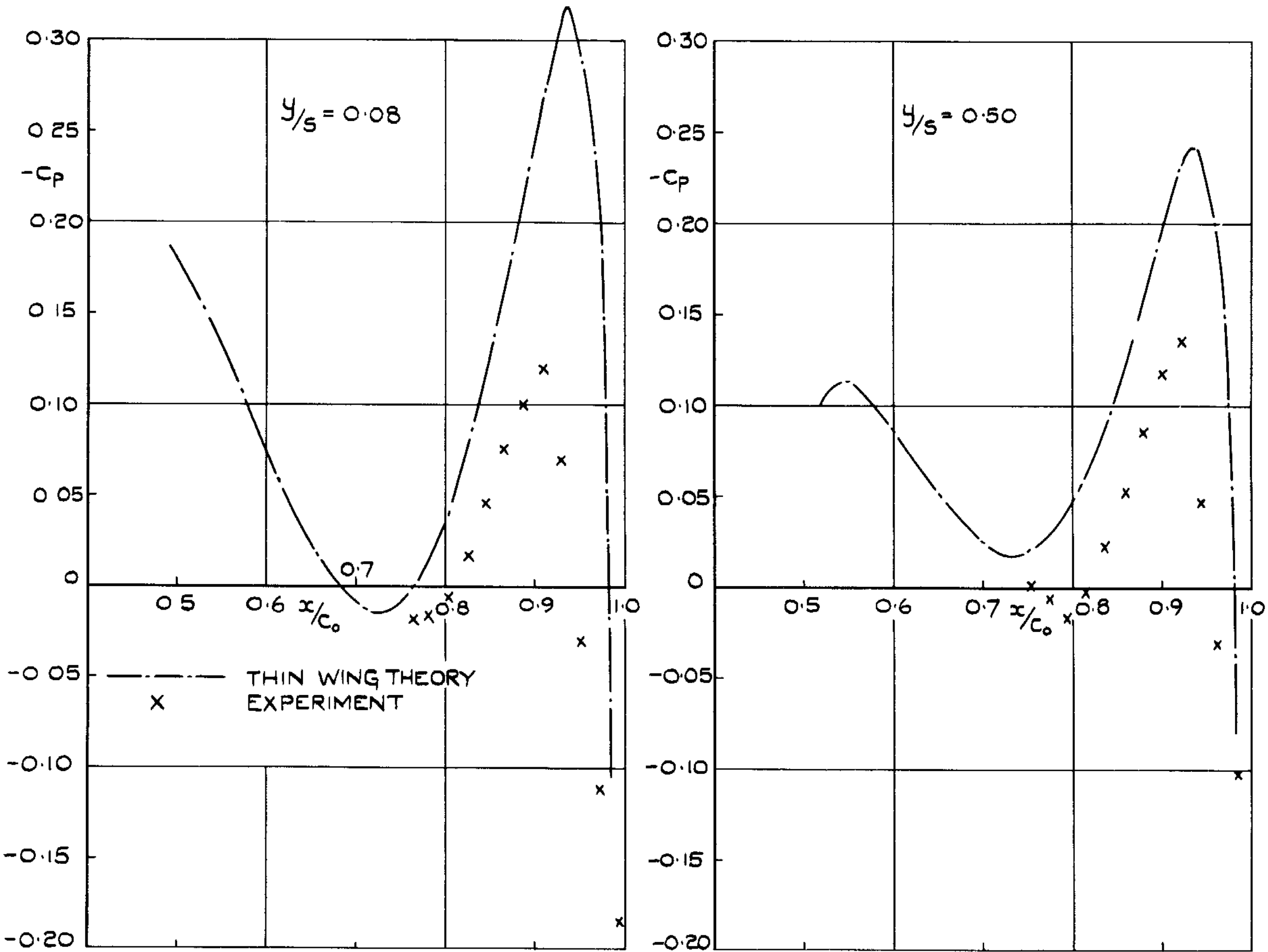

FIG.4 (j)

FIG. 4 PRESSURE DISTRIBUTION AT $M = 0·99$, $C_p^* = -0·017$

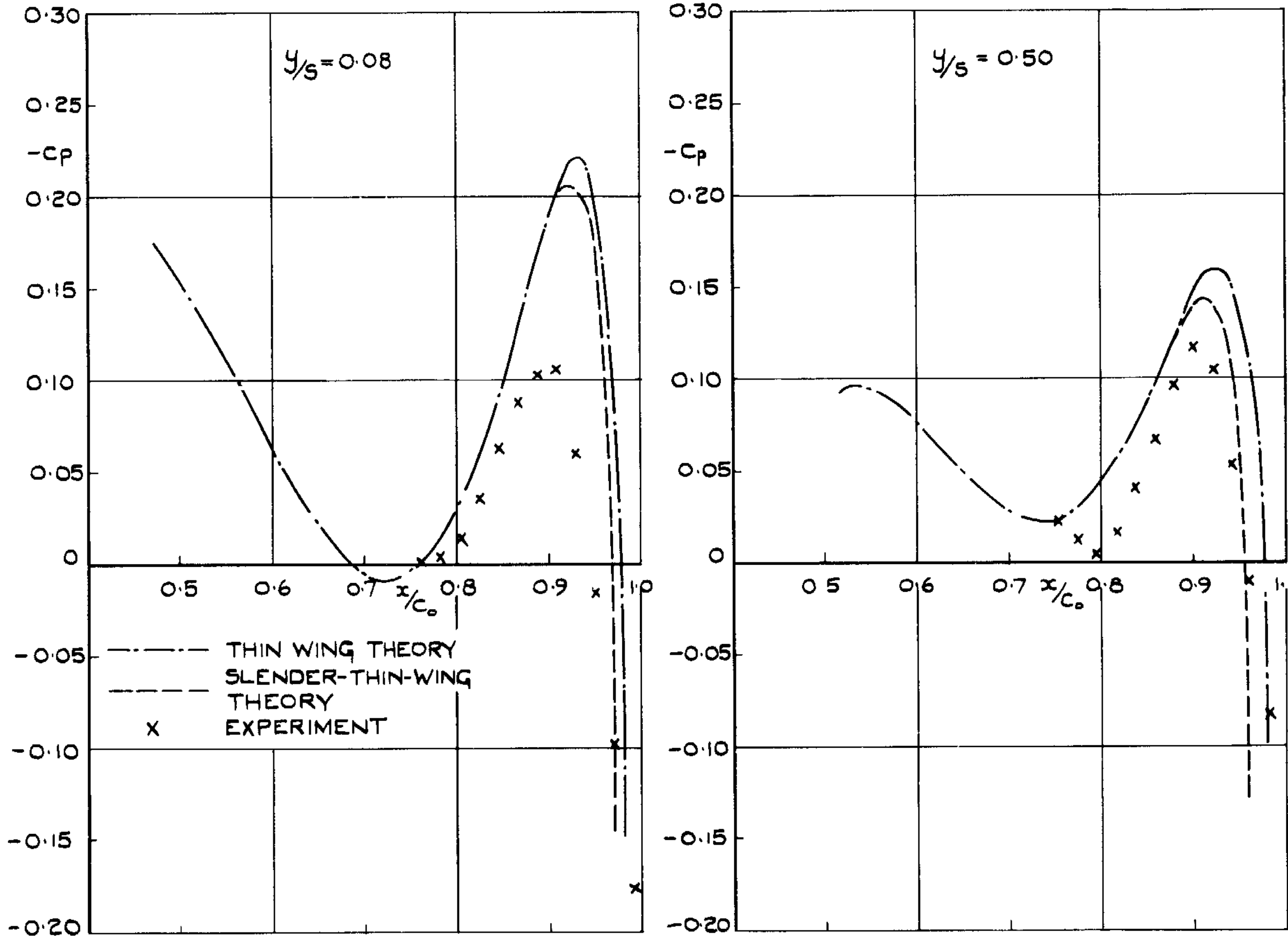

FIG. 4 (k)

FIG. 4 PRESSURE DISTRIBUTION AT $M = 0{\cdot}97$, $C_p^* = -0{\cdot}052$

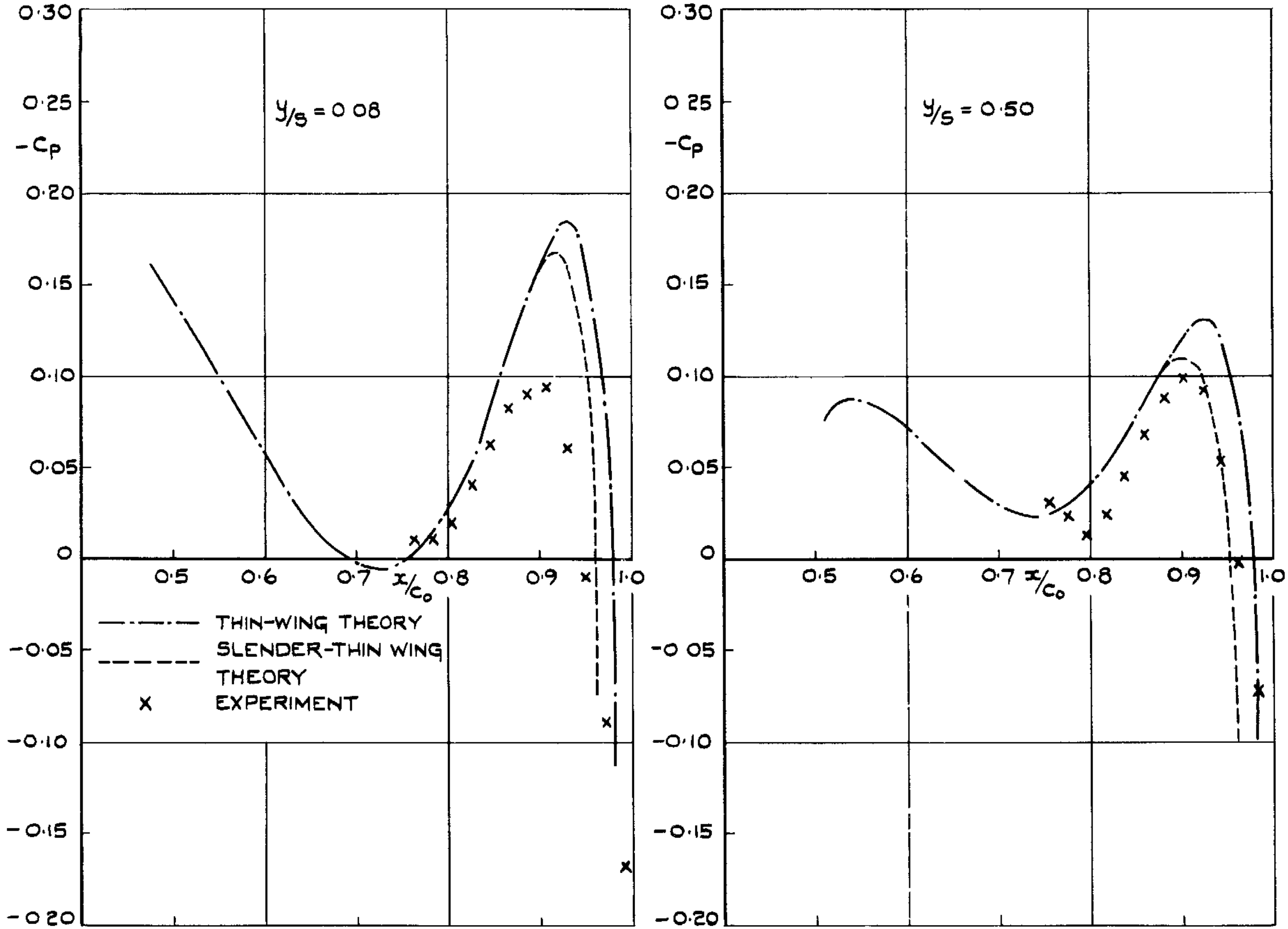

FIG. 4 (ℓ)

FIG. 4 PRESSURE DISTRIBUTION AT $M = 0{\cdot}95$, $C_p^* = -0{\cdot}088$

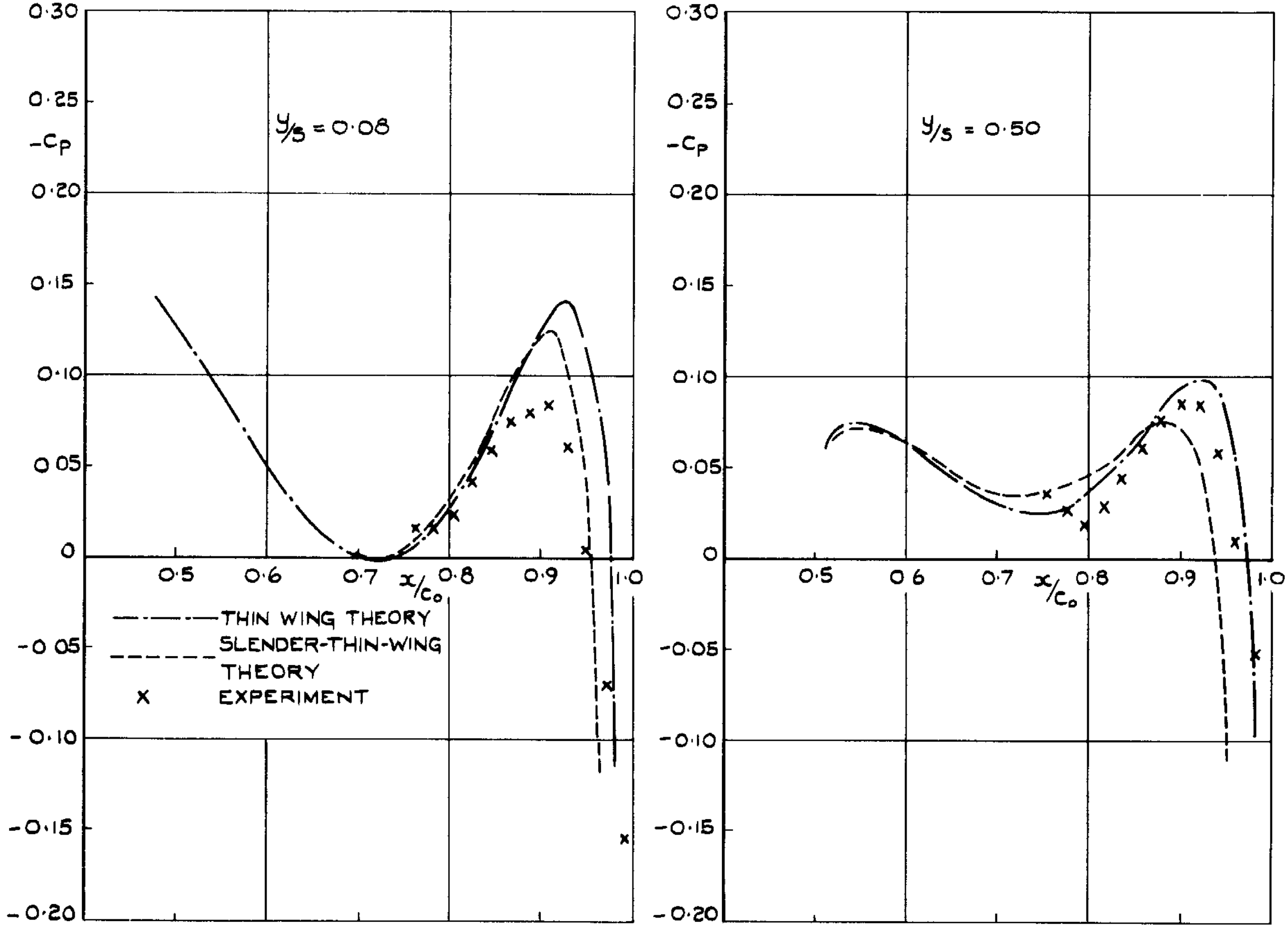

FIG. 4 (m)

FIG. 4 PRESSURE DISTRIBUTION AT M=0·90, $C_p^* = -0·188$

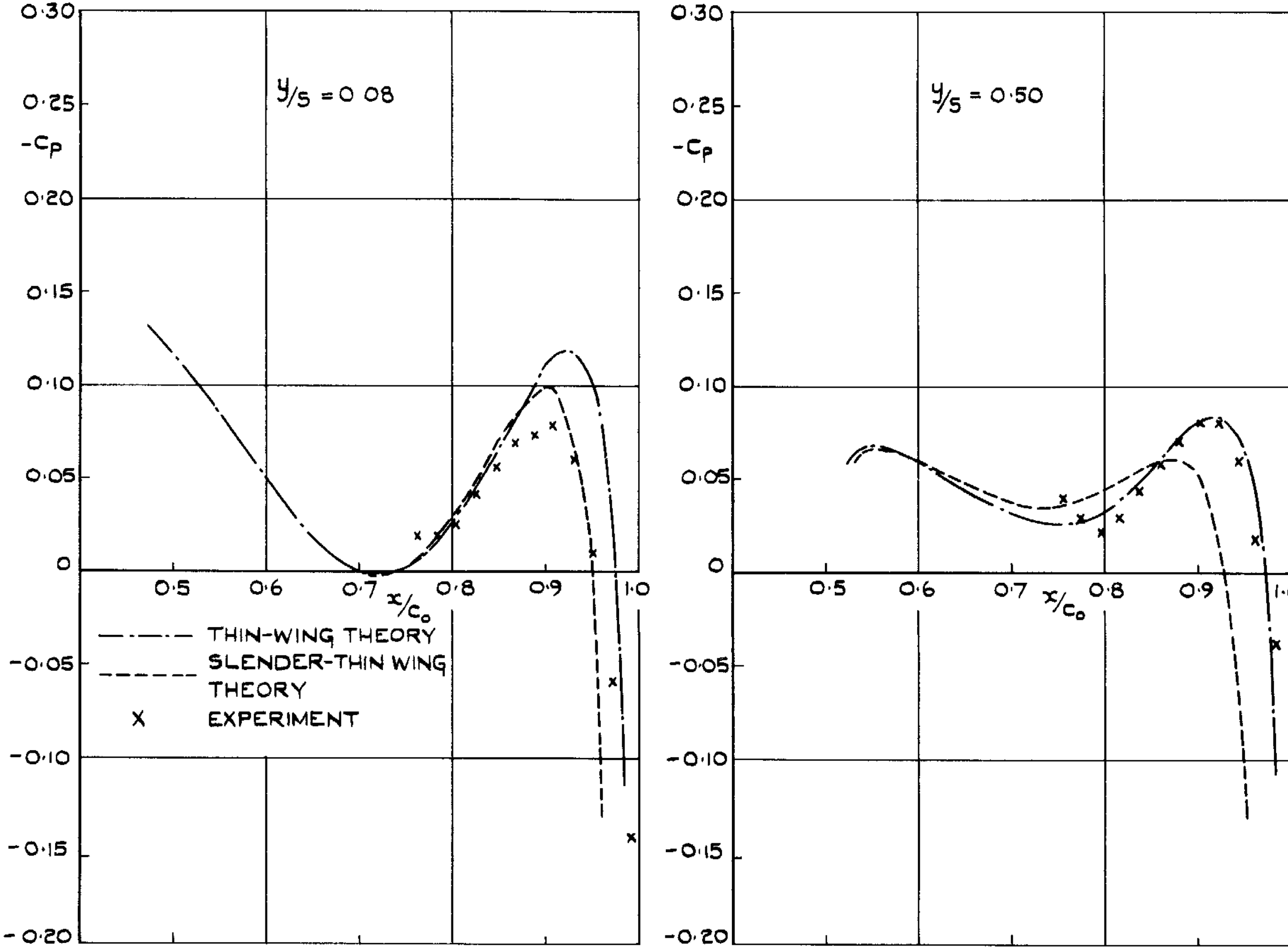

FIG. 4 (n)

FIG. 4 PRESSURE DISTRIBUTION AT M = 0·85, $C_p^* = -0·302$

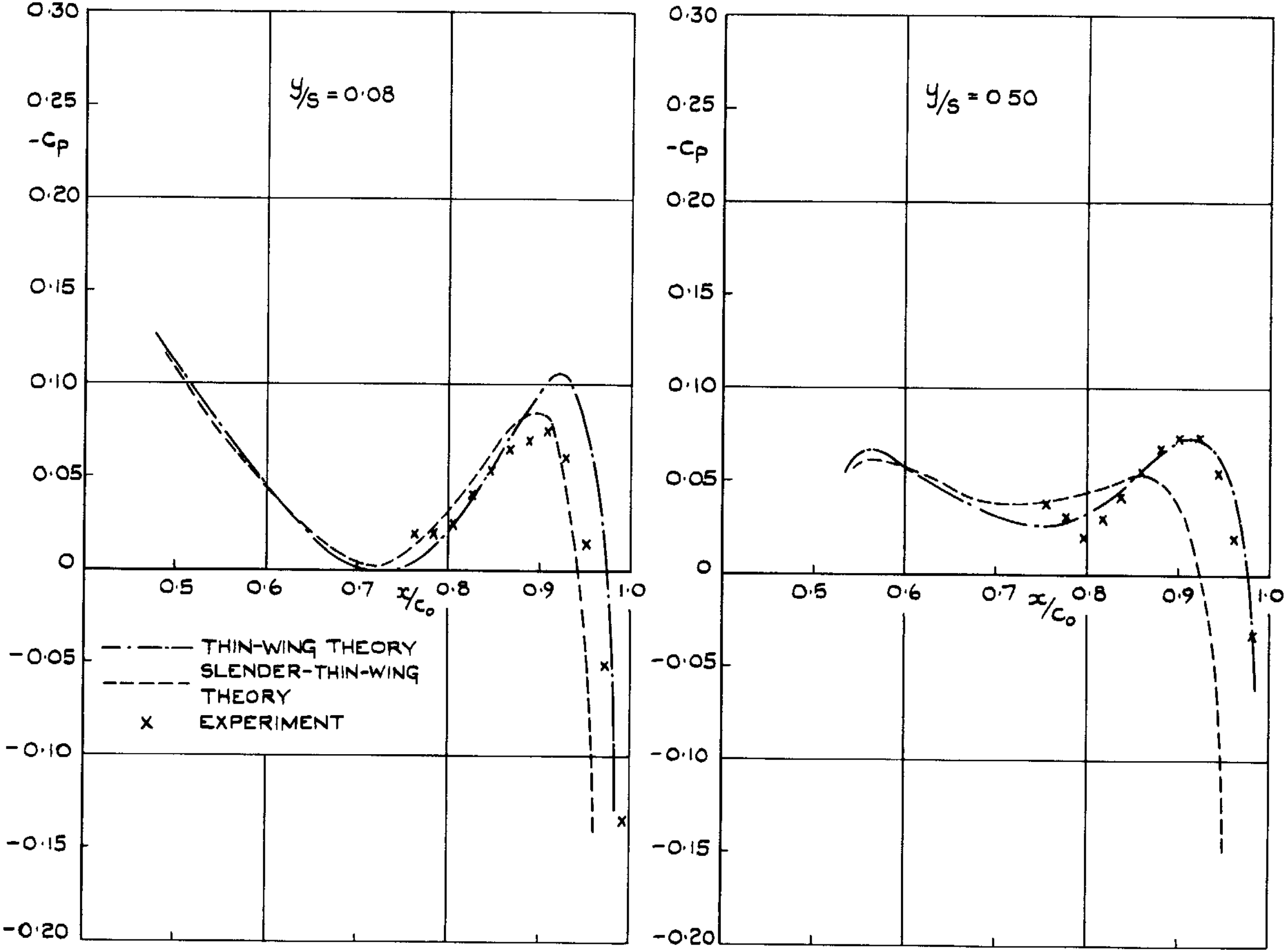

FIG. 4 (a)

FIG. 4 PRESSURE DISTRIBUTION AT $M = 0·80$, $C_p^* = -0·435$

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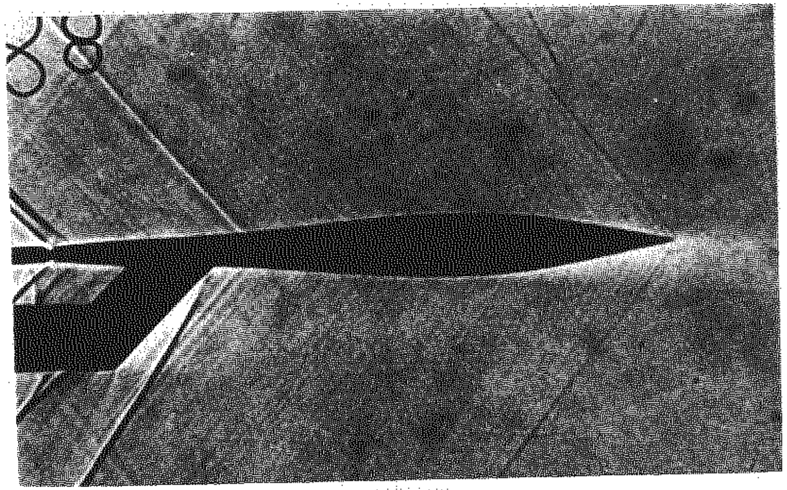
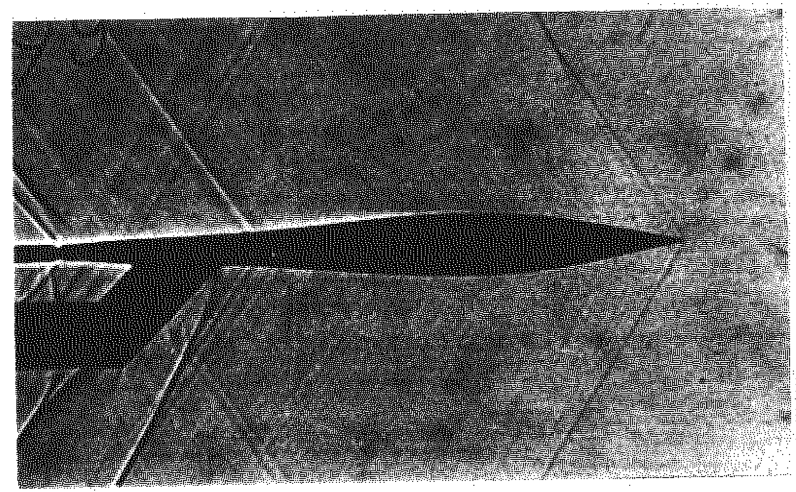
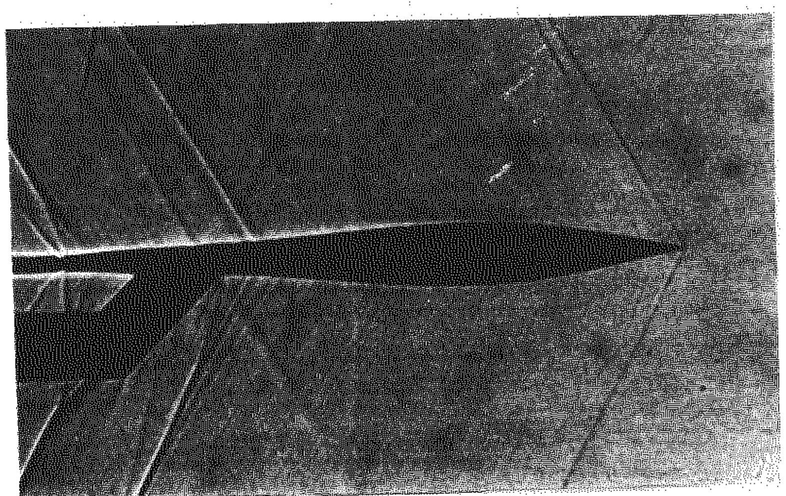


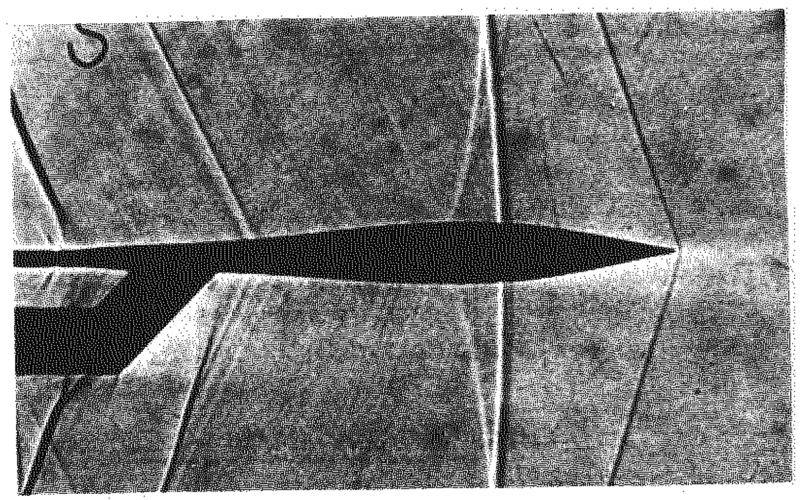
M=1.30



M=1.20

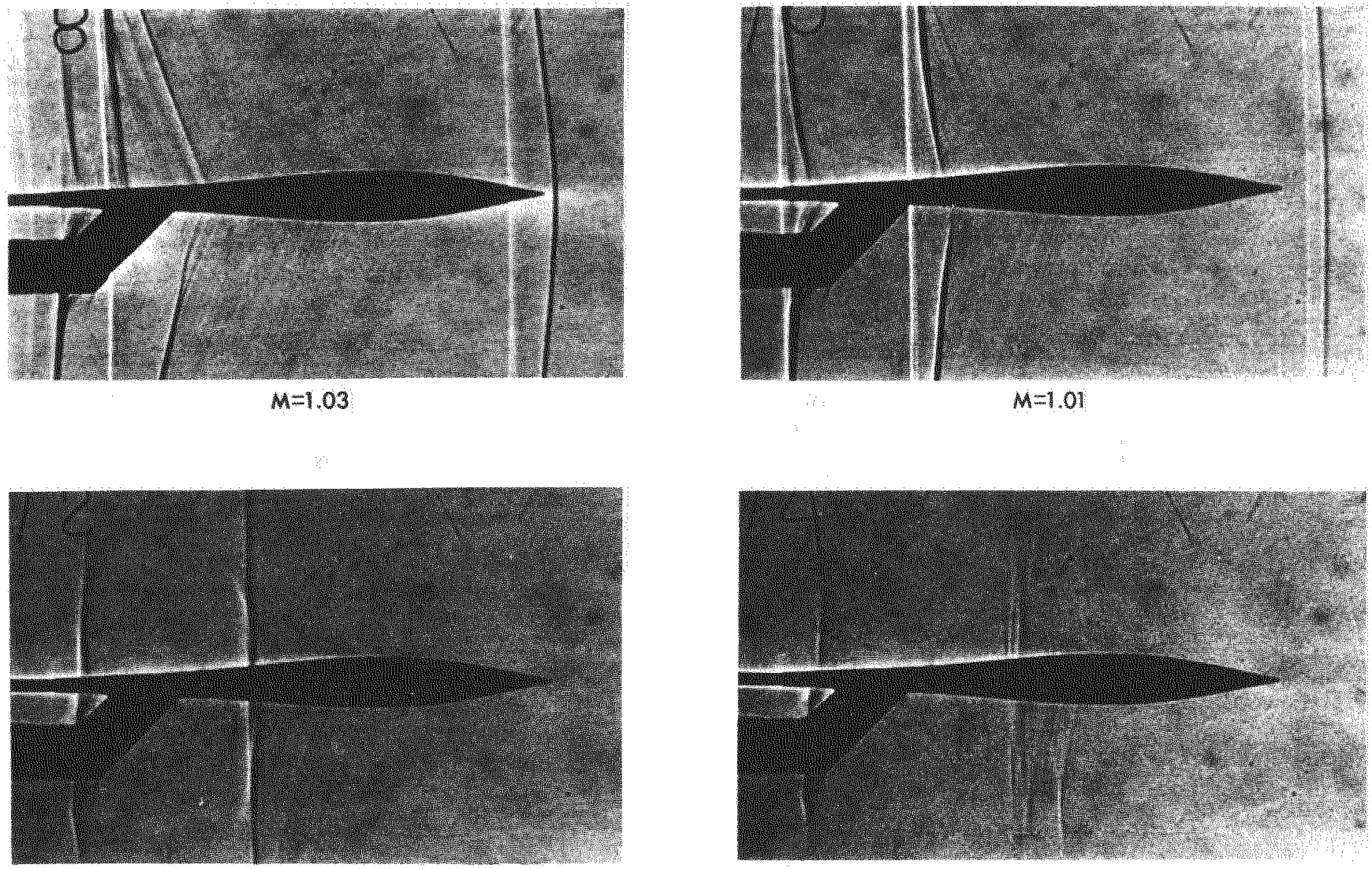


M=1.15



M=1.07

Fig.5 Shadowgraph photographs



M=1.03

M=1.01

M=0.99

M=0.97

Fig.5 Cont'd. Shadowgraph photographs

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