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Structural Aspects of Suction Wings

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Structural Aspects of Suction Wings

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SUMMARY

This report considers the structural design problems arising directly from the use of distributed wing suction. Possible types of construction are discussed and estimates are made of the increase in aircraft all up weight due to these constructions and due to the extra power required for suction.

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1 Introduction

There is reason to believe that low wing drag may be achieved by stabilising the boundary layer by distributed suction. This report is concerned with the structural design problems arising from the use of distributed suction, and with the weight penalties directly incurred.

These structural weight penalties need to be offset against the advantages of reduced drag in the overall study of the economics of wing suction. The weight of the suction machinery would also need to be taken into consideration in such a study. This aspect comes rather outside the scope of the present paper, but since the calculations are simple a brief outline is included for the sake of completeness.

1.1 Standard wing for comparison

In order to obtain quantitative comparison with a wing without suction it will be assumed that the corresponding non-suction wing is 15% of the all up weight and has the following structural breakdown:-

Spars	25% of wing structure weight
Skin	25% of wing structure weight
Stringers	15% of wing structure weight
Ribs	12% of wing structure weight
Remainder	23% of wing structure weight

It will be further assumed that the flexural strength and torsional stiffness of the two wings are the same. Possible effects of changes in flexural stiffness, however, are disregarded in this study, as being normally a secondary factor in design. Torsional strength is likewise disregarded on the grounds that adequate torsional strength is, on modern aircraft, readily obtainable without weight penalties owing to the stringency of the torsional stiffness requirements.

2 General Types of Construction for Suction Wing

There are two distinct types of construction for a suction wing. In one construction (type (a)) the suction is carried out along a number of spanwise strips, so that the porous material covers a small proportion of the wing chord, say 5 - 15%. In the other construction (type (b)) the suction is carried out through 60% or more of the wing chord. In both cases the suction takes place in spanwise compartments; this is simpler structurally and better aerodynamically than an uncompartmented suction system.

Types intermediate between (a) and (b) would present more practical difficulties and have not been considered.

2.1 Structural features of Type (a)

A possible type of construction is shown in Fig. 1. With such a construction the porous material (which may be in convenient lengths) will have a negligible stress in shear because of the sliding fit at its edges and it will have a negligible direct stress when the wing bends if a few of the end fittings of the lengths of porous material allow slight movement.

2.11 Strength and stiffness of Type (a)

The porosint does not contribute to the stiffness of the wing in bending or torsion and the perforated part of the stringer is only partially effective. In bending the rest of the construction will be as effective as the corresponding non-porous construction. In torsion, however, most of the shear will now have to pass through the rivet line (C) connecting the outer skin to the stringer. The rivet line will thus be heavily loaded and, furthermore, the shear strength and stiffness will be reduced unless the thickness of skin and stringer in these regions is increased.

2.12 Weight penalty of Type (a)

An indication of the increase in weight of a porous wing of type (a) over a corresponding non-porous wing can be found from an examination of a particular example. The wing is assumed to have $\lambda\%$ of the top and bottom wing surfaces covered by porous strips. The thickness of the Porosint will be taken to be equal to that of the supporting edges which are 50% thicker than the main skin.

The perforated part of the stringers will be taken to be 70% efficient. As the density of Porosint is about 2.1 times that of dural, the increase in weight, as a fraction of the skin weight is therefore

$$(2.1 \times 1.5 + 0.3 \times 1.5)\lambda\% = 3.6\lambda\%$$

This is practically all due to the Porosint, and therefore the main reductions in weight penalty are to come from a reduction in weight of the porous material itself.

The rib and spar weight are unchanged and so the increase in structure weight expressed as a percentage increase in all up weight will be

$$3.6\lambda \times 0.25 \times 0.15 = 0.135\lambda\% \qquad (1)$$

2.2 Structural Features of Type (b)

Two types of construction, in which most of the wing surface is covered with porous material, are

(i) an inverted stringer-sheet construction with the stringers on the outside of the load-carrying sheet so as to provide support for the porous material stretched over them,

(ii) a sandwich construction, with the outer skin of a porous material (or combination of materials) which will contribute to the stiffness.

2.21 Strength and stiffness of Type (b)

For type (b)(i) the load-carrying skins must be thickened to maintain torsional and flexural stiffness. The requisite increase in skin thickness is considered in Appendix I, where, for example, it is shown that at a section where the mean wing thickness is 20 ins and the stringer depth is 1 in the skin thickness must be increased by 25%.

Types of inverted stringer-sheet construction envisaged for type (b)(i) are as shown in Figs. 3, 4 and 5

Wrinkling of the porous outer skin on the top surface might be prevented by stretching the porous skin while it is being glued to the stringers.

For type (b)(ii), the sandwich construction, the outer porous skin will have to be made up of a perforated load-carrying skin attached to a porous skin. A possible arrangement would be as shown in Fig. 6.

The closeness of the supporting ridges can be adjusted so that the porous outer skin will not tend to buckle under axial strain nor collapse under suction pressure. A diamond pattern for the supporting ridges would be an ideal arrangement in that the porous skin would lie more securely on the correct aerofoil contour.

2.22 Weight penalty of Type (b)

For type (b)(i) with the structure the same as that given in the previous example the weight penalty will be the same as that for type (a) of para. 2.12 (taking $\lambda = 10$) if the porous material has a surface density of $(36 - 25)\% = 11\%$ of the skin.

For type (b)(ii) the increase in all up weight over a corresponding non-suction wing with a similar sandwich construction is due practically entirely to the porous material. A simple expression for this increase can be given in terms of the wing loading.

If w = wing loading in lb/sq ft,
δ = weight of porous skin in lb/sq ft,

$$\text{the percentage increase in all up weight} = \frac{2\delta\lambda}{w}. \qquad (2)$$

Thus if 75% of both top and bottom surfaces will be covered with porous material and $w = 50$ lb/sq ft,

$$\text{the percentage increase in all up weight} = 3\delta. \qquad (3)$$

There are already porous materials with a δ of about 0.4 (see para. 5) which would correspond to an increase in all up weight of 1.2%.

The increase in weight due to the thickness of the air space and porous material reducing the distance between top and bottom sandwiches will be very small. For example, if this thickness is 0.05" and the average distance between top and bottom sandwiches is 20 ins the percentage increase in all up weight is less than 0.1.

As would be expected, type (b)(ii) compares very favourably with the standard stringer-sheet wing, because of reductions in the number of ribs necessary.

3 Suction Duct Sizes

It will be assumed here that suction is to take place over 75% of the wing surfaces and that the suction velocity is constant over the entire porous surface (see Fig. 7).

In order that the pressure drop along the ducts should not be large compared with the drop across the porous skin the air velocity in the ducts must be limited to, say, 100 ft/sec. (This point is considered in greater detail in Appendix III.) In a design for a constant duct velocity of 100 ft/sec the percentage wing section area which is devoted to ducting is Δ_ξ at any section ξ and Δ_ξ is given by

$$\Delta_\xi = \left[\frac{28Av_s(1+k)^2}{K}\right]\left[\frac{(1-\xi)\{1+k-\xi(1-k)\}}{(1+k)\{1-\xi(1-k)\}^2}\right] \qquad (4)$$

The function inside the second square bracket has the value of unity at the root section and has been plotted against ξ for various values of taper ratio in Fig. 10.

To obtain some numerical values, suppose, for example,

$$A = 6$$

$$V = 900 \text{ ft/sec}$$

$$k = \tfrac{1}{2}$$

$$K = 10\%$$

$$\frac{v_s}{V} = 0.0005, \text{ a somewhat pessimistic value.}$$

Substituting in equation (4) gives

$$\Delta_o = 12\%$$

$$\Delta_{\frac{1}{2}} = 10\%.$$

For a 90°-delta wing, for which $k = 0$ and $A = 4$, the corresponding values are

$$\Delta_o = \Delta_\xi = 5\%,$$

and for a 60°-delta wing

$$\Delta_o = \Delta_\xi = 3\%.$$

These examples clearly demonstrate the importance of low aspect ratio and taper ratio k in obtaining low values for Δ_o. The analysis indicates that Δ_ξ will generally be below about 15%, so that the region between the inner skin and the porous skin in types (b)(i) and (b)(ii) will be sufficient for ducting. This also implies that the bulk of the wing section between spars can be used for flexible fuel tanks. In this respect type (b)(ii) offers added advantages because of the greater rib spacing.

If the region between the inner skin and the porous skin is not sufficient for ducting a possible solution, which leaves the region between spars free for tanks, would be to use the D-nose as a "by-pass duct" to serve an outer region of the wing.

If the total suction flow for type (a) is the same as for type (b) it may well be that the ducting available for type (a) will be insufficient.

The effect of duct friction is considered in Appendix III and the possibility of varying porosity in Appendix IV.

4 Suction Power

The suction pump will have to provide a suction pressure sufficient to overcome the pressure drop across the porous skin and an additional suction pressure sufficient to balance the maximum value of the localised lift pressure over the wing surface. Thus, ignoring the pressure drop due to duct friction, we can write

$$p_p = p_s + \alpha w \qquad (5)$$

where α is some constant depending on the aerofoil characteristics and will usually be about 2.

Along ducts where the lift pressure is not at its maximum the suction pump pressure p_p will have to be throttled down, and it might be advantageous to use more than one suction pump to minimise the losses due to throttling.

The total volume of flow/second at the root sections is 1.5 Sv_s and, if ft lb sec units are used throughout, the H.P. required for suction and ejection at the speed of the aircraft is therefore

$$P = \frac{Sv_s}{370} \left(p_p + \tfrac{1}{2}\rho V^2\right). \qquad (6)$$

Thus if p_s = 25 lb/sq ft

S = 1500 sq ft

all up weight = 100,000 lb

α = 2

ρ = 0.0005 (corresponding to a height of about 45,000 ft)

and the other dimensions are as for the first example on page 6,

$$P = 320 + 370$$

$$= 690.$$

Allowing an increase in weight of 3 lb per H.P. developed this represents an increase in all up weight of 2.1%. For the general case the percentage increase in all up weight can be expressed conveniently in the form

$$\left(\frac{v_s/V}{0.0005}\right)\left(\frac{100}{w}\right)\left(\frac{V}{1000}\right)\left\{\frac{(p_s + \alpha v)}{250} + \left(\frac{\rho}{0.0005}\right)\left(\frac{V}{1000}\right)^2\right\} \qquad (7)$$

5 Porous Materials

The porous material available for type (a) is not unsatisfactory but a lower density and the ability to withstand a compressive strain up to about 0.005 would result in structural improvements.

For types (b)(i) and (b)(ii) a porous material which consists of a nickel plated phosphor bronze gauze, and can be rolled to give a good surface might be satisfactory. A nickel plated 120 mesh gauze weighs 0.4 lb/sq ft and is 0.01 in. thick. Tension tests parallel to and at 45° to a mesh line indicate yield strains of the same order as for dural, but this point needs further investigation. It is also possible that suitable heat treatment may result in a higher yield strain. Such a thin material would buckle unless supported at $\frac{1}{2}$ inch pitch; this would be possible for the sandwich type of construction, but for the inverted stringer-sheet type it would mean that the porous skin would have to be either thicker or, for example, supported on a sheet of perforated dural.

Non-metallic porous materials, such as Durestos, have been disregarded on the grounds that they are more liable to become affected by humidity conditions and that they will be difficult to clean.

6 Normal Loading on the Porous Surface

When the suction is operating the normal pressure on the outer skin at any point will be equal to the pressure drop across the porous skin at that point and will act inwards. This loading is of no significance for type (a) because of the relatively high thickness/width ratio of the porous strips, but for type (b) inward-quilting may take place depending on the pressure, support spacing, curvature and stiffness of the porous skin. An approximate analysis to predict the suction pressure at which inward quilting will take place is given in Appendix II. Figs. 11 and 12 have been based on this analysis, in which it is shown that

$$p_{crit} = \left(\frac{Et^3}{Rw^2}\right) f_1\left(\frac{w}{\sqrt{Rt}}\right) \qquad (8)$$

where $f_1\left(\frac{w}{\sqrt{Rt}}\right)$ lies between 1.0 and 3.3, and the maximum allowable unsupported width of porous skin is given by

$$w_{max} = \left(\frac{Et^3}{p_s R}\right)^{\frac{1}{2}} f_2\left(\frac{p_s R^2}{Et^2}\right) \qquad (9)$$

where $f_2\left(\frac{p_s R^2}{Et^2}\right)$ lies between 1.0 and 1.8.

For example, for a structure in which

p_s = 25 lb/sq ft

E = 10^7 lb/sq in

R = 100 ins

t = 0.01 in.

it will be found that

$$w_{max} = 0.76 \text{ in.}$$

and if t = 0.02 in.

$$w_{max} = 2.1 \text{ in.}$$

On the top surface of the wing it is probable that considerations of buckling under direct load will determine w_{max}.

7 Acknowledgement

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8 Conclusions

The structural aspects of suction wings have been considered briefly in this report. Little attention has been paid to actual detail design or to some of the technical difficulties which will arise in manufacture, but it is possible to draw the following general conclusions:-

(i) for values of v_s/V less than about 0.0005 the channels formed by skin and stringers, or between the two skins of a sandwich construction, are sufficient for ducting purposes,

(ii) the power required for suction through the porous skin is appreciably less than the total power required for suction and ejection, and it follows that a high suction pressure, say 25 lb/ft^2 or more, is advisable,

(iii) for a given wing loading, V, v_s and ρ the increase in weight due to the total suction power is a constant fraction of the all up weight. Assuming an extra weight of 3 lb per H.P. this constant fraction is about 2%,

(iv) whether suction is to take place along a number of spanwise lines or over most of the wing surface, the percentage increase in all up weight is unlikely to exceed 2%. For a given wing loading this percentage increase is practically independent of the aircraft size and characteristics,

(v) the most promising type of construction appears to be a sandwich type, such as that shown in Fig. 6,

(vi) the main reductions in weight penalty will come from a reduction in weight of the porous material itself, rather than from design modifications.

List of Symbols

λ = percentage of top and bottom wing surfaces covered by porous material

w = wing loading in lb/sq ft

δ = weight of porous skin in lb/sq ft

v_s = suction velocity through porous skin

V = velocity of aircraft

k = wing taper ratio in plan form

A = aspect ratio

K = percentage thickness/chord ratio

c_o = root chord (kc_o = tip chord)

ξ = distance from root chord ÷ semi-span

Δ_ξ = percentage wing section area to be devoted to ducting at any section ξ

p_s = suction pressure in lb/sq ft

p_p = suction pump pressure in lb/sq ft

α = maximum value of the localised lift pressure ÷ wing loading

P = H.P. required for suction and ejection

S = wing area

ρ = density of air in slugs/cu ft

p_{crit} = suction pressure sufficient to cause buckling of porous skin

E = Young's modulus for the porous skin

t = thickness of porous skin

w = unsupported width of porous skin

R = radius of curvature of porous skin

APPENDIX I

Increase in Skin Thickness to Maintain Torsional and Flexural Stiffness in Inverted Stringer-Sheet Construction

Torsion

If the wing section of the equivalent non-suction wing is represented by a thin-walled doubly-symmetrical cylinder of rectangular section as in Fig. 8 it can be shown that the torsional stiffness is proportional to

$$\frac{a^2\, b^2}{\left(\dfrac{a}{t_2} + \dfrac{b}{t_1}\right)}$$

If, due to the inversion of sheet and stringers, the effective distance between top and bottom sheets becomes $b(1-n)$ the thickness of these sheets must therefore be increased to $t_2(1+m)$ where

$$\frac{a^2\, b^2}{\dfrac{a}{t_2} + \dfrac{b}{t_1}} = \frac{a^2\, b^2\, (1-n)^2}{\dfrac{a}{t_2(1+m)} + \dfrac{b(1-n)}{t_1}}$$

i.e.

$$1 + m = \frac{1}{(1-n)\;(1-n-nr)} \qquad (10)$$

where

$$r = \frac{bt_2}{at_1}$$

For example, suppose $b = 20$ ins

$a = 100$ ins

$t_1 = 2t_2$

so that $r = 0.1$

then if stringer depth $= 1$ in.

$n = 0.1$ and from equation (10) $m = 0.25$.

Flexure

The contribution of the spars to the stiffness and strength will be unaltered if there is no porous material along the line of the spar booms, and the contribution from the stringers will be lowered by a negligible amount due to the thickness of porous material and air space reducing the effective distance between them.

If the skin thickness is increased by an amount determined by equation (10) it can be shown that the flexural stiffness and strength contribution from the skin is increased by a small amount, so that no further thickening is necessary.

APPENDIX II

Critical Suction Pressures Causing Inner-Quilting

The method developed here for estimating p_{crit} is based on the following simplifying assumptions:-

(i) the distance between supports does not change,

(ii) the connection between support and skin is pin-jointed rather than clamped,

(iii) chordwise supports, if any, are so widely spaced as not to influence the deflected shape of the skin,

(iv) the skin curvature is constant for any particular pressure,

(v) instability will take place when either (a) the pressure is sufficient to flatten the skin or (b) the pressure is sufficient to cause a compressive strain (from support to support) equal to that which would cause it to buckle as an Euler strut. The fundamental Euler mode is stable and accordingly the 2nd Euler mode is chosen.

A strain-energy method will be used. For type (a) instability:-

chordwise strain when curvature vanishes $= \dfrac{w^2}{24\ R^2}$

so that the chordwise stress when curvature vanishes $= \dfrac{Ew^2}{24\ R^2}$

and the direct strain-energy stored $= \dfrac{Etw^5}{1152\ R^4} = W_1$, say.

Similarly the bending energy stored $= \dfrac{Ewt^3}{24\ R^2} = W_2$, say.

Work done by pressure $= \dfrac{pw^3}{24\ R}$

$= W_1 + W_2.$

Simplifying and introducing $\beta = Rt/w^2$ gives

$$p_{crit} = E\left(\frac{w}{R}\right)^4\left(\frac{\beta}{48} + \beta^3\right) \qquad (11)$$

For type (b) instability:-

buckling will occur at a chordwise strain of $\dfrac{\pi^2}{3}\left(\dfrac{t}{w}\right)^2 = \varepsilon$, say.

This will occur before type (a) instability if

$$\frac{\pi^2}{3}\left(\frac{t}{w}\right)^2 < \frac{w^2}{24\ R^2}$$

i.e. $$\beta < \frac{1}{2\pi\sqrt{2}} = 0.112.$$

If this is so, it can be shown that the radius of curvature R_ε at the strain ε is related to the original radius of curvature by

$$\frac{1}{R_\varepsilon^{\,2}} = \frac{1}{R^2} - \frac{8\pi^2 t^2}{w^4}.$$

Direct strain-energy stored $$= \frac{\pi^4 E t^5}{18 w^3} = W_1$$

Bending energy stored $$= \frac{E w t^3}{24}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2} - \frac{8\pi^2 t^2}{w^4}}\right]^2 = W_2$$

Work done by pressure $$= \frac{p w^3}{24}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2} - \frac{8\pi^2 t^2}{w^4}}\right]$$

$$= W_1 + W_2.$$

Simplifying and introducing β gives

$$p_{crit} = E\left(\frac{w}{R}\right)^4 \beta^3 \left[\frac{\pi^2}{6} + 1 + \left(\frac{\pi^2}{6} - 1\right)\sqrt{1 - 8\pi^2\beta^2}\right] \qquad (12)$$

Figs. 11 and 12 have been based on equations (11) and (12).

APPENDIX III

Flow Along a Rough Duct with a Porous Wall

The problem considered here is the effect of duct friction on the distribution of pressure and suction velocity along the ducts. The problem is aggravated by the fact that a drop in suction pressure along the duct means that, if the suction pressure at the wing tip is to be maintained, inboard sections will be sucking more than necessary thus increasing the duct velocity and therefore the loss of head along the duct.

Notation (see Fig. 9)

x = distance along duct measured from tip inboard

h_x = height of rectangular sectioned duct

w_x = width of rectangular sectioned duct

= width of porous side of duct

v_s = critical suction velocity

v_x = suction velocity at section x

μ = porosity coefficient

= v_s/p_s

p_s = critical suction pressure

p_x = suction pressure at section x

V_x = duct velocity at section x

ℓ = length of duct, $x = \ell$ at a/c ℄, $x = 0$ at wing tip

$C_{D,x}$ = friction coefficient in duct at section x (see equations (14) and (19))

$\lambda = \mu C_{D,\ell}/h_\ell$

$\phi = \mu C_{D,\ell}\, \ell^3\, v_s/h_\ell^{\,2} = C_{D,\ell}\, \ell^3\, v_s^{\,2}/h_\ell^{\,2}\, p_s$

$\gamma = \dfrac{\alpha W + \frac{1}{2}\rho V^2}{p_s}$ (see para. 4)

As p_s is appreciably greater than variations in $\frac{1}{2}\rho V_x^{\,2}$ there is no need to distinguish between the dynamic and static pressures.

The velocity through the porous surface is determined by

$$v_x = \mu p_x$$

$\therefore$ volume of flow in/unit length $= \mu p_x w_x$

volume of flow in duct $= V_x h_x w_x$

$$\therefore \frac{d}{dx}(V_x h_x w_x) = \mu p_x w_x \tag{13}$$

If we ignore the slight effect of varying Reynolds' number we can write

$$\frac{dp_x}{dx} = C_{D,x} V_x^2, \tag{14}$$

where $C_{D,x}$ is a drag coefficient for the duct.

Eliminating p_x from equations (13) and (14) gives

$$C_{D,x} V_x^2 = \left[\frac{d}{dx}\left(\frac{1}{\mu w_x}\right)\frac{d}{dx}(V_x h_x w_x)\right] \tag{15}$$

Two cases will now be considered; in the first there will be no taper of the duct, in the second a taper ratio k is considered.

Case (i) No taper

The differential equation for V_x reduces to

$$\frac{d^2V_x}{dx^2} = \lambda V_x^2 \tag{16}$$

where $$\lambda = \frac{\mu C_D}{h}$$

Now V_0 is zero and a solution may therefore be sought in the form

$$V_x = \alpha_1 x + \alpha_4 x^4 + \alpha_7 x^7 + \alpha_{10} x^{10} + \ldots..$$

Substituting in equation (16) and equating coefficients of like powers of x gives

$$\alpha_4 = \alpha_1 \frac{(\lambda\alpha_1)}{12}$$

$$\alpha_7 = \alpha_1 \frac{(\lambda\alpha_1)^2}{252}$$

$$\alpha_{10} = \alpha_1 \frac{(\lambda\alpha_1)^3}{6048}$$

Also at $x = 0$

$$\frac{dV_x}{dx} = \frac{v_s}{h} = \alpha_1$$

$$\therefore \quad V_x = \frac{v_s x}{h} \left\{ 1 + \frac{\lambda \alpha_1 x^3}{12} + \frac{(\lambda \alpha_1 x^3)^2}{252} + \frac{(\lambda \alpha_1 x^3)^3}{6048} + \ldots\ldots \right\}$$

and from equation (13)

$$p_x = p_s \left\{ 1 + \frac{\lambda \alpha_1 x^3}{3} + \frac{(\lambda \alpha_1 x^3)^2}{36} + \frac{(\lambda \alpha_1 x^3)^3}{604.8} + \ldots\ldots \right\}$$

If we introduce

$$\phi = \frac{\mu C_D \ell^3 v_s}{h^2} = \frac{C_D \ell^3 v_s^2}{h^2 \rho_s}$$

we can write

$$V_\ell = \frac{v_s \ell}{h} \left\{ 1 + \frac{\phi}{12} + \frac{\phi^2}{252} + \frac{\phi^3}{6048} + \ldots\ldots \right\} \tag{17}$$

and

$$\frac{v_\ell}{v_s} = \frac{p_\ell}{p_s} = \left\{ 1 + \frac{\phi}{3} + \frac{\phi^2}{36} + \frac{\phi^3}{604.8} + \ldots\ldots \right\} \tag{18}$$

The factors inside the braces represent the extra effort to overcome duct friction, and have been plotted in Fig. 13.

A suitable value for $C_{D,x}$ is given by

$$C_{D,x} = \frac{0.03\ \rho\ (\text{duct perimeter})}{R^{\frac{1}{4}}\ (\text{duct area})} \tag{19}$$

where R is Reynolds' number and is approximately given by

$$R = V_x\ (w_x + h_x)/4\ (\text{kinematic viscosity})$$

Assuming a duct speed of about 100 ft/sec and $\rho = 0.0005$ it may be accurate enough to take an average value for R of about 10^4 so that

$$C_{D,x} = \frac{3 \times 10^{-6}\ (w_x + h_x)}{w_x\ h_x} \tag{19a}$$

For example, consider a duct for which

$w = 0.1$ ft

$h = 0.15$ ft

$\ell = 40$ ft

$v_s = 0.5$ ft/sec

$p_s = 25$ lb/sq ft

(If duct friction is ignored this would give rise to a duct velocity of zero at the far end increasing linearly to 133 ft/sec at the root.)

Substituting in equation (19a) gives

$$C_D = 5 \times 10^{-5}$$

whence

$$\phi = 1.4.$$

The factors inside the braces of equations (17) and (18) are therefore 1.12 and 1.52 respectively, and the extra energy required for suction and ejection is therefore $100 \times \left\{ 1.12 \left(1 + \frac{0.52}{1 + \gamma} \right) - 1 \right\}, = 16\%$ if $\gamma = 15$, say. But if w and h are increased by 20% so that $\phi = 1.4 \div 1.2^3 = 0.81$, these factors become 1.07 and 1.30 and the total extra energy required is 9%.

Case (ii) Taper ratio k

The differential equation (15) does not now reduce to a form that can be readily integrated but a method of successive approximation yields the following expressions for the suction pressure and duct velocity at the root chord necessary to ensure that the suction velocity nowhere falls below v_s.

$$\left.\begin{aligned} V_c &= \frac{\ell v_s (1+k)}{2h_c} \left\{ 1 + \phi F_1(k) + 0(\phi^2) \right\} \\ \frac{v_c}{v_s} &= \frac{p_c}{p_s} = \left\{ 1 + \phi F_2(k) + 0(\phi^2) \right\} \end{aligned}\right\} \qquad (20)$$

where

$$\left.\begin{aligned} F_1(k) &= \frac{\frac{1}{2}k^2 + \frac{k^4}{8} - \frac{5}{8} - (\frac{1}{2} + k^2)\log k}{2(1+k)(1-k)^4} \\ F_2(k) &= \frac{k^2 - \frac{1}{4}k^4 - \frac{3}{4} - \log k}{4(1-k)^3} \end{aligned}\right\} \qquad (21)$$

Values of F_1 and F_2 are given below.

k	0.25	0.4	0.6	1.0
F_1	0.236	0.174	0.130	0.083
F_2	0.415	0.370	0.346	0.333

As an example, consider a duct for which

$k = 0.25$

$w_\ell = 1$ in.

$h_\ell = 1\frac{1}{2}$ in.

$\ell = 40$ ft

$v_s = 0.5$ ft/sec

$p_s = 25$ lb/sq ft

so that $V_\ell = 100$ ft/sec if duct friction is ignored.

Substituting in equation (19a) gives

$$C_{D.\ell} = 6 \times 10^{-5}$$

whence $$\phi = 2.46 .$$

The factors inside the braces of equation (20) are therefore $\left\{1 + 2.46 \times 0.236 + ...\right\}$ and $\left\{1 + 2.46 \times 0.415 + ...\right\}$ or, say, 1.7 and 2.2 respectively, and taking $\gamma = 15$ the extra energy required for suction and ejection is about 75%.

If w_ℓ, h_ℓ and ℓ are increased in the same proportion these factors are unaltered.

If w_ℓ and h_ℓ are increased by a factor of 2, ℓ remaining unaltered, it will be found that $\phi = 0.31$ so that the factors inside the braces of equation (20) are now 1.08 and 1.14 respectively, and the extra energy required for suction and ejection is about 9%.

APPENDIX IV

Variable Porosity to Give Correct Suction Flow

It was shown in Appendix III that the pressure drop due to friction along the duct is not negligible and may in some cases necessitate a suction flow near the root that is appreciably higher than the design value v_s. It will be shown here how the suction flow along the whole length of the duct can be kept constant at the value v_s by varying the porosity coefficient μ. For convenience the variation along the duct of v_s/μ_x ($= p_x$) will be considered.

Equation (13) may now be written

$$\frac{d}{dx}(V_x h_x w_x) = w_x v_s \tag{22}$$

and integrating gives

$$V_x = \frac{v_s}{h_x w_x} \int_0^x w_x \, dx \tag{23}$$

Substituting in equation (14) to determine p_x gives

$$p_x = p_o + v_s^2 \int_0^x \frac{C_{D,x}}{h_x^2 w_x^2} \left\{ \int_0^x w_x \, dx \right\}^2 dx \tag{24}$$

where p_o is an arbitrary suction pressure at the tip from which the tip porosity $\mu_o = v_s/p_o$ may be determined.

If the duct has a taper ratio k such that

$$\left.\begin{aligned} h_x &= h_\ell \{k + (1-k)\, x/\ell\} \\ w_x &= w_\ell \{k + (1-k)\, x/\ell\} \end{aligned}\right\} \tag{25}$$

we can write equation (24) in the form

$$p_x = p_o \{1 + \phi F(k,x)\} \tag{26}$$

where
$$\phi = C_{D,\ell}\, \ell^3\, v_s^2 / h_\ell^2\, p_o \tag{27}$$

and
$$F(k,x) = \frac{1}{4(1-k)^3} \left[\log(1+\psi) - \frac{\psi(2+\psi)(2+6\psi+3\psi^2)}{4(1+\psi)^4} \right] \tag{28}$$

where
$$\psi = \frac{(1-k)x}{k\ell}.$$

At the root section $(x = \ell)$ equation (26) reduces to

$$\mu_o/\mu_i = p_i/p_o = 1 + \phi F_2 (k) \quad (29)$$

where F_2 (k) is defined in equation (21).

Values of the function inside the square brackets of equation (28) are given below.

ψ	0	0.5	1.0	1.5	2.0	3.0	4.0	6.0
[ ] of eq. (28)	0	0.051	0.178	0.320	0.458	0.700	0.900	1.216

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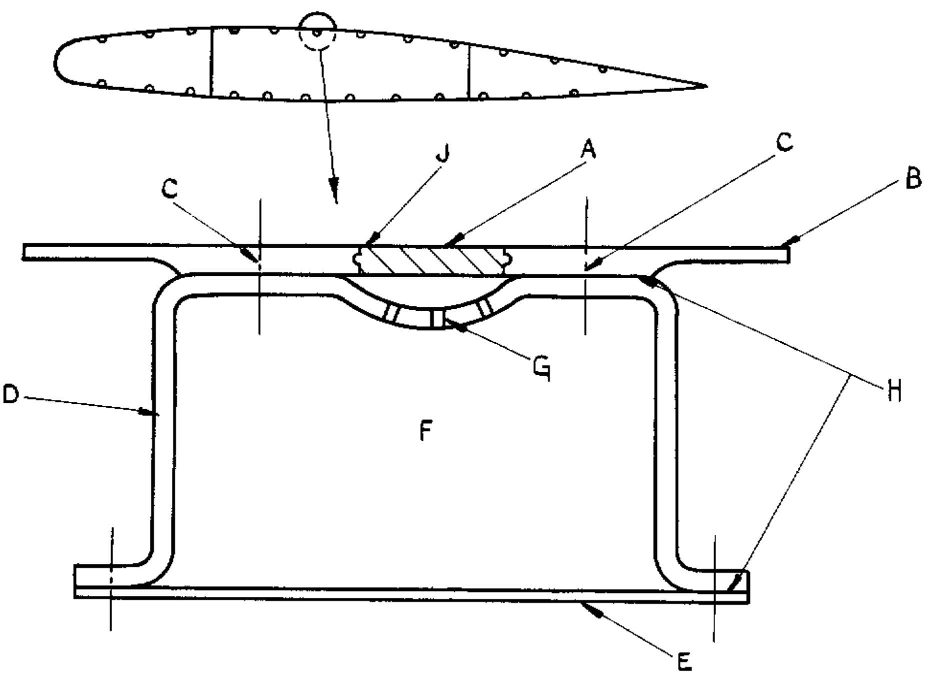

LEGEND

A = POROSINT (A SINTERED BRONZE)
B = OUTER SKIN
C = LINE OF RIVETS
D AND E = BOX STRINGER
F = AIR SUCTION DUCT
G = PERFORATIONS
H = AIR TIGHT JOINTS
J = SLIDING FIT

FIG. 1. CONSTRUCTION FOR SUCTION ALONG SPANWISE STRIPS.

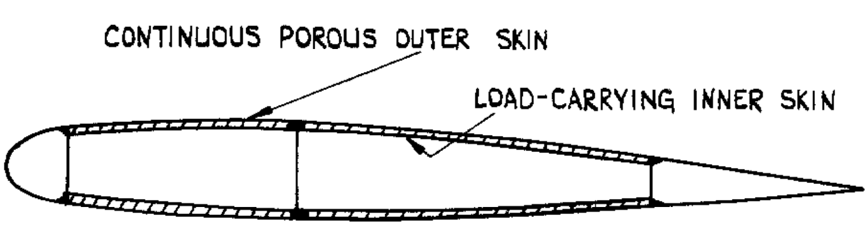

FIG. 2. GENERAL ARRANGEMENT OF WING SECTION IN TYPES (b) (i) AND (b) (ii).

FIG.3,4 & 5.

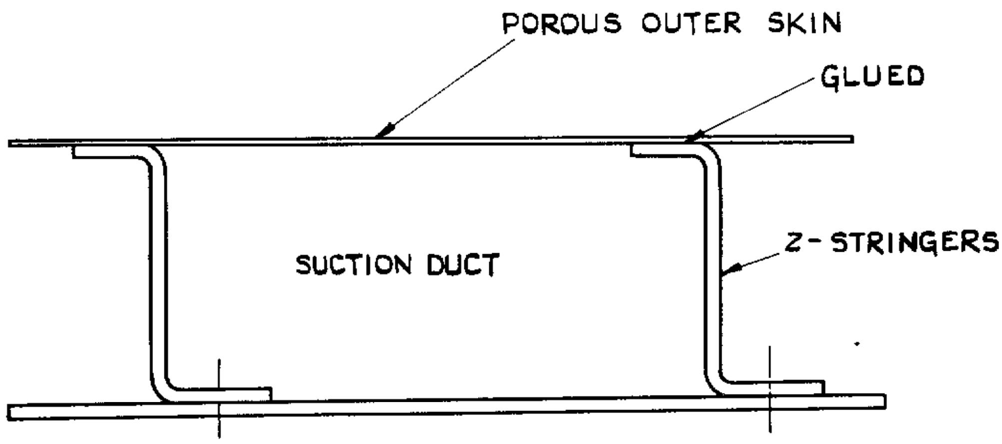

FIG.3. INVERTED STRINGER-SHEET CONSTRUCTION.

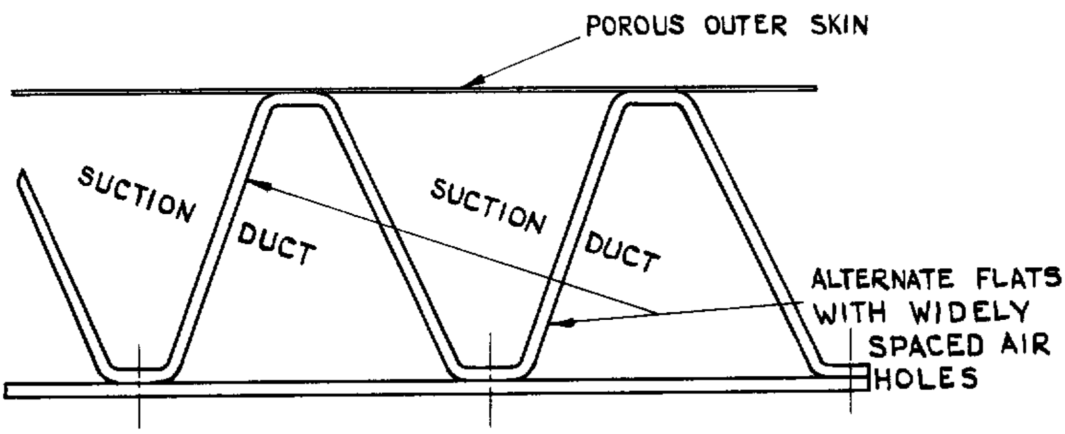

FIG.4. INVERTED CORRUGATION-SHEET CONSTRUCTION.

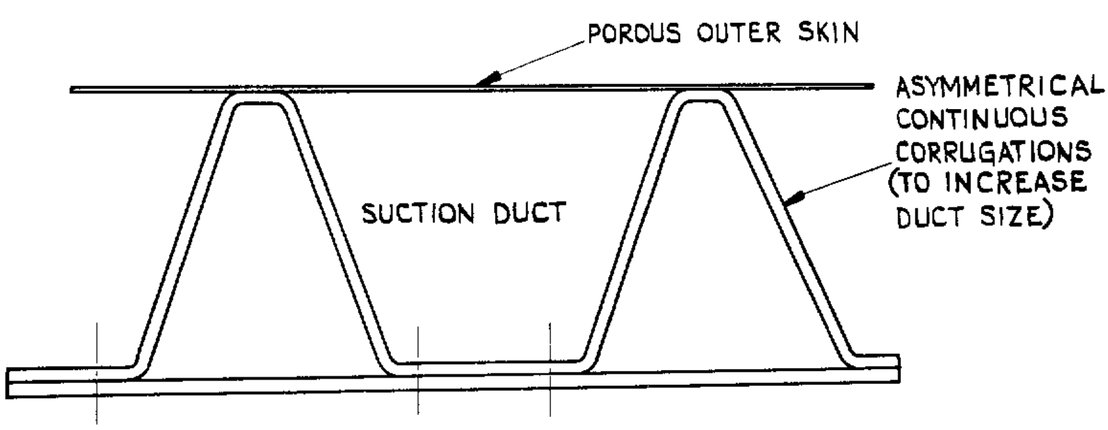

FIG.5. INVERTED ASYMMETRICAL CORRUGATION-SHEET CONSTRUCTION.

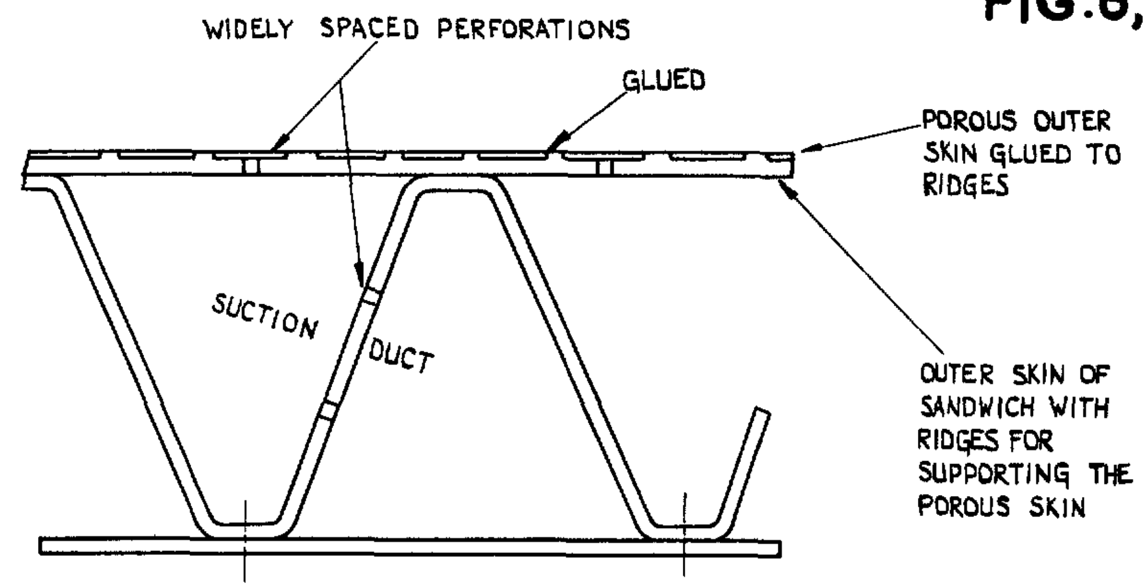

FIG.6. SANDWICH CONSTRUCTION.

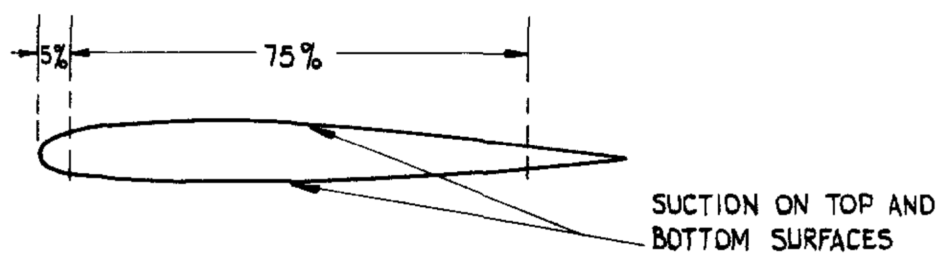

FIG.7. SUCTION ON TOP AND BOTTOM SURFACES

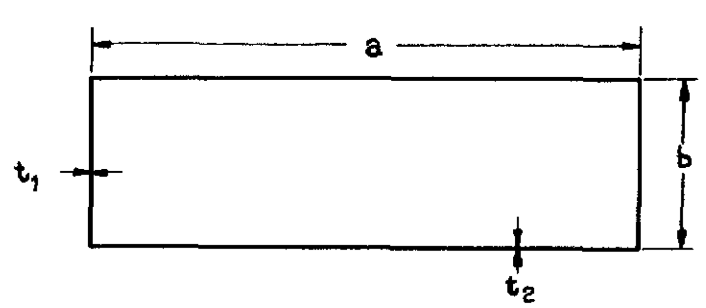

FIG. 8. ILLUSTRATIVE SKETCH.

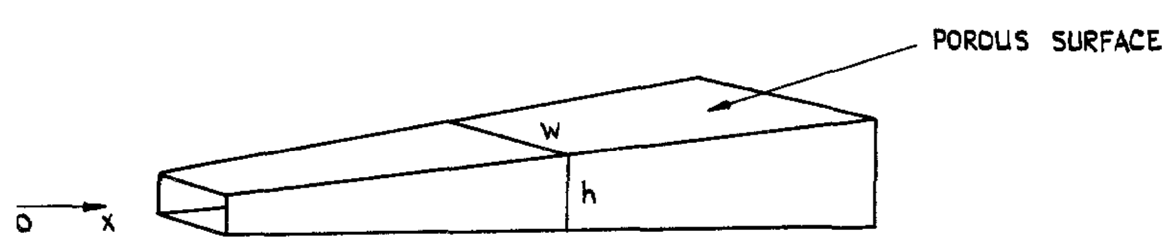

FIG.9. ILLUSTRATIVE SKETCH.

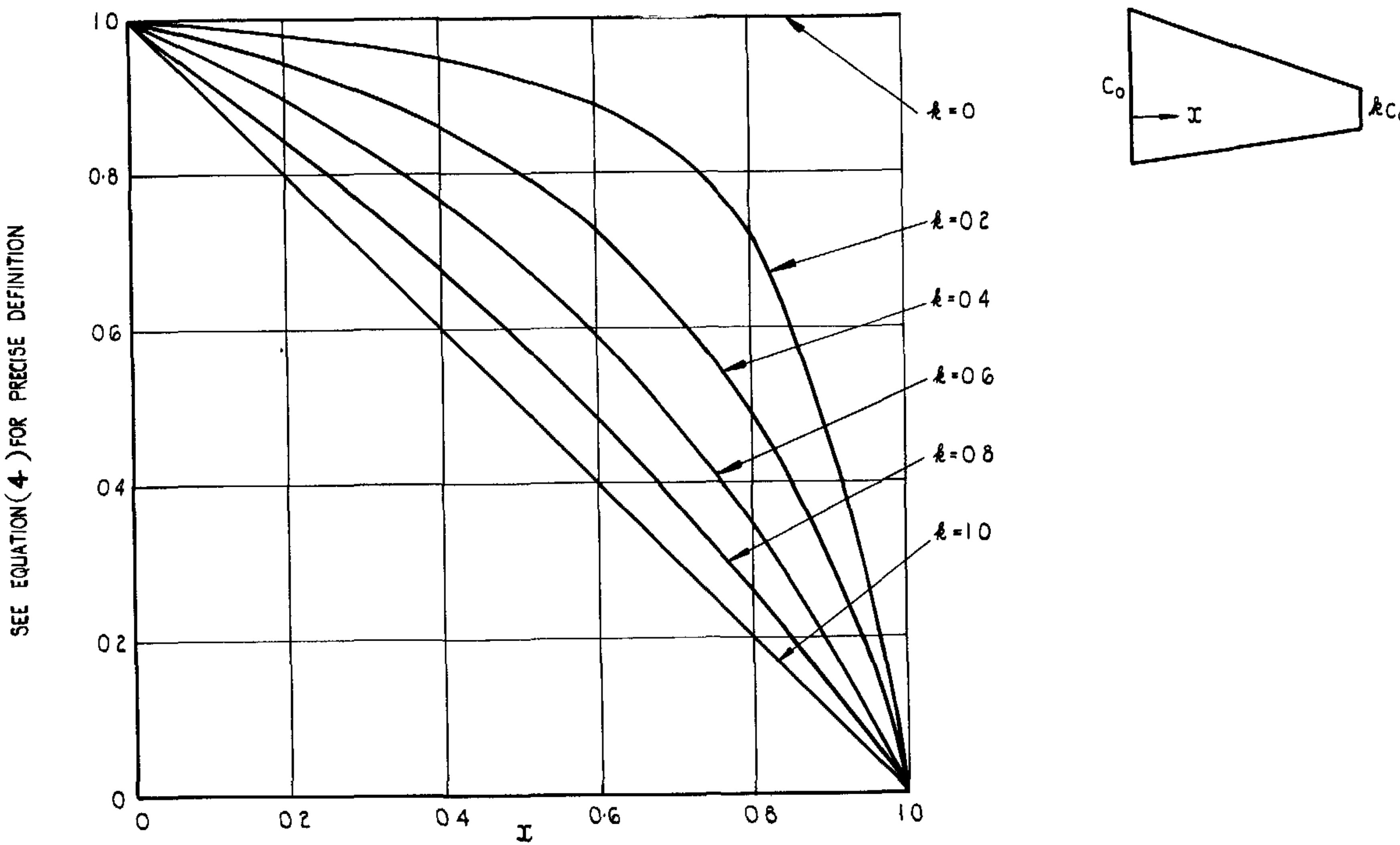

FIG.IO RELATIVE SPANWISE DISTRIBUTION OF $\frac{\text{DUCT AREA}}{\text{SECTION AREA}}$ FOR CONSTANT DUCT VELOCITY.

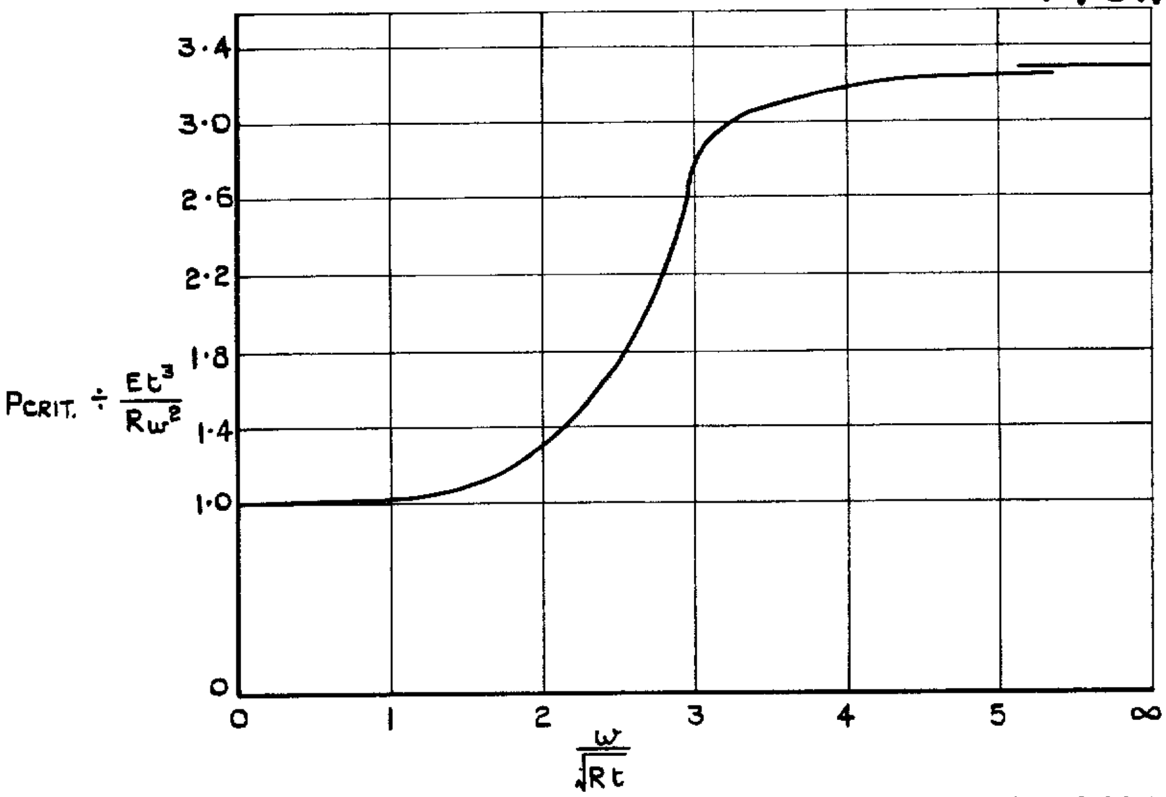

FIG. 11. SUCTION PRESSURES CAUSING INNER-QUILTING.

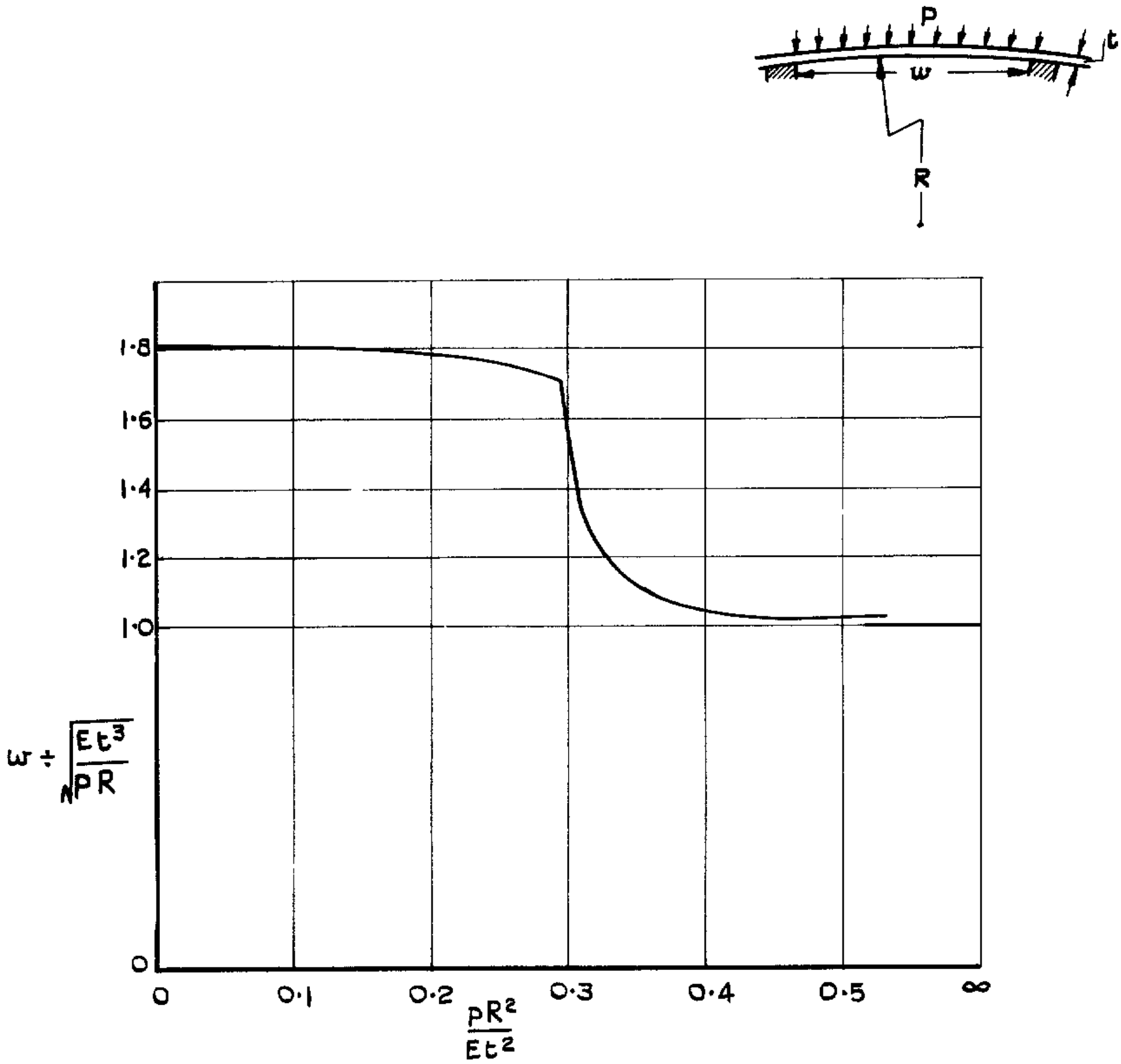

FIG. 12. MAXIMUM ALLOWABLE UNSUPPORTED WIDTH OF POROUS SKIN.

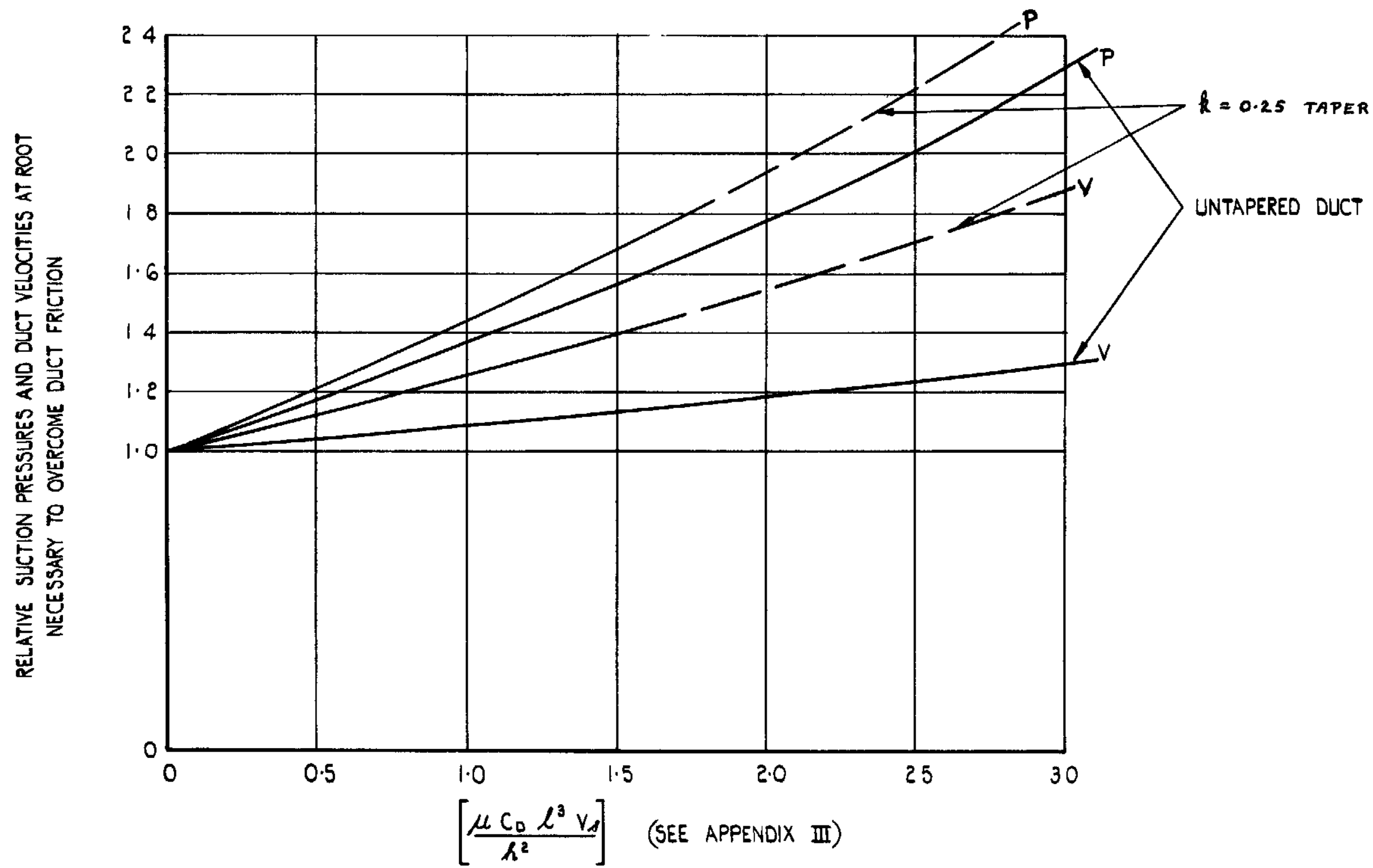

FIG.13 INFLUENCE OF POROSITY AND DUCT FRICTION

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