

C.P. No. 505
(19,502)

A.R.C. Technical Report

ROYAL AIR FORCE ESTABLISHMENT
BEDFORD.

C.P. No. 505
(19,502)

A.R.C. Technical Report



MINISTRY OF AVIATION
AERONAUTICAL RESEARCH COUNCIL
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The Effect of Bend Outlet Conditions on the Pressure Losses in Bent Circular Pipes

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1960

FIVE SHILLINGS NET

June, 1957

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Losses in Bent Circular Pipes

- by -

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Summary

In an experiment made with a three-coil helical pipe a value was found for the bend deflection corresponding to a complete oscillation of the secondary circulation. From this result a bend radius may be determined for a given pipe diameter such that certain secondary flow conditions are attained at the outlet of a bend of given deflection. A 2" pipe bent on a radius to produce maximum secondary circulation at the outlet of a 30 deg bend was used for further experiments. The build up of pressure losses in both the bend itself and in the transition region downstream from the bend was investigated for three values of the bend deflection. Similar calculations were made for flow in a helical pipe at approximately the same Reynolds number. The effect of varying the downstream transition length was also considered.

Introduction

Pressure losses in pipe bends have been measured by a number of investigators and widely varying results have been reported (Gray 1945). This is probably due not merely to their use of differing inlet velocity distributions but also to the conditions at the outlet of the pipe.

In a theoretical discussion of flow in bent pipes Hawthorne (1951) shows that the secondary circulation which appears in bends is of an oscillatory nature with a period proportional to $\sqrt{d/R}$, where d is the diameter of the pipe and R is the radius of the bend. Because of increasing frictional effects in the bend the secondary flow is subject to large damping, and, provided that the bend deflection is large enough, there will be a region of fully developed curved flow after perhaps one or two complete periods. In such a region the velocity profile is similar throughout, but what that profile is like will depend on the geometry of the bend and the deflection at which fully developed curved flow is attained. A theory for laminar flow in the fully developed curved flow region was first presented by Dean (1927), who described the pressure losses in terms of a parameter depending only on the ratio $d/2R$ and the Reynolds number. The relevance of this parameter has been confirmed by the experiments of Adler (1934) and White (1929) and in a recent contribution to the theory made by Barua (1955). It does not however describe the flow when turbulent conditions are obtained, nor has a method yet been suggested for extending the theory to these more practical cases. In a system for which there is no such region of fully developed curved flow the conditions in the downstream transition region will vary with the phase of the secondary flow oscillations attained at the bend outlet; this in turn being determined by the geometry of the bend and the inlet conditions. Furthermore the "stilling length" - i.e., the transition length in which a symmetric velocity is recovered - may well be in excess of sixty diameters, whereas pipe-flow tests are frequently made with comparatively short transition lengths, or even no transition length at all. With such a large range of possible outlet conditions it is clear that any sort of correlation between the results of different experimenters is likely to be very difficult.

The first group of experiments reported here was designed to study the important effect on the build up of pressure losses of the value of the secondary circulation at the bend outlet. A three-coil helical pipe was used first to determine the bend deflection corresponding to a complete oscillation of the secondary circulation. Using this result a ratio of d/R may be calculated to give a maximum value of the secondary circulation at the outlet of a bend of, say 30° . The same value of d/R will produce zero secondary circulation at a deflection of 60° , and a small circulation in the opposite direction at 70° . Three such systems were used to investigate the difference in the build up of pressure losses in both the bend itself and in the downstream transition region. The results are compared with losses calculated for the helical pipe in which (of course) there is a region of fully developed curved flow. All the tests were made at approximately the same Reynolds number with approximately the same inlet velocity profiles, and in each case the downstream transition length was sufficient for a nearly symmetrical velocity distribution to be attained at the pipe outlet.

The second aspect of the problem considered is the effect caused by varying the downstream transition length. Except for one test made on the helical pipe attention here was confined to the 2" pipe with the 70° bend, using six different transition lengths between 62 diameters and zero.

Flow in Bent Circular Pipes

In a given system consisting of a circular pipe of diameter d , bent on a circular arc of radius R , with a length of straight pipe of the same diameter downstream of the bend, three flow regions may be distinguished. Firstly there is the inlet transition region in which the inertia forces are more important than those due to viscosity. The main effect here is the generation of a component of vorticity ξ in the direction of flow. Using an inviscid fluid theory Hawthorne (1955) shows how, when the inlet velocity profile is linear, to a first approximation the effect of the secondary flow is to rotate the entire streamline pattern about the axis of the pipe. Having previously made this assumption Hawthorne (1951) showed that the nature of the secondary flow is oscillatory, the angular displacement α being given approximately by the equation:

$$\frac{d}{R} \frac{d^2 \alpha}{d\phi^2} = \cos \alpha \quad \dots (1)$$

where ϕ is the bend deflection.

Horlock (1955) considered the secondary flow as a perturbation on the main potential flow and, with a linear velocity profile, established the first order result that ξ is uniform over any plane cross-section of the pipe and is a function of ϕ only. The circumferential velocity is then given by:

$$v = -\frac{1}{4} \xi d \quad \dots (2)$$

The analysis may be extended to more general pipe flows where the inlet velocity profile is symmetrical about the pipe axis if it is assumed that the pipe be divided into two halves by the plane of the bend and that in each half ξ is replaced by a mean vorticity ξ_m whose product with the cross-sectional area of the pipe gives the total circulation. Then

$$\frac{\xi_m}{q} = -2 \int_0^\phi \left| \text{grad} \frac{h}{\rho} \right| \cos \alpha \frac{d\phi}{q^2} \quad \dots (3)$$

where h , ρ , and q are the stagnation pressure, density and velocity respectively.

Also/

Also, approximately

$$\frac{v}{q} = \frac{d}{2r_c} \frac{d\alpha}{d\phi} \quad \dots (4)$$

where r_c is the distance from the particle to the centre of the bend.

Initially $\frac{1}{q^2} \left| \text{grad} \frac{h}{\rho} \right| = \frac{1}{q} \left| \text{grad} q \right|$. If U_m is the maximum velocity,

by the mean value theorem there is a point in the cross-section at which $\left| \text{grad} q \right| = U_m/d$. It will be assumed that $\left| \text{grad} q \right|$ across the section may be replaced by this mean value, so that

$$\frac{1}{q^2} \left| \text{grad} \frac{h}{\rho} \right| \approx \frac{U_m}{qd}$$

Following the particle of highest total pressure, where $q = U_m$ and $r_c = R + \frac{1}{2} d \sin \alpha$, equations (2) to (4) reduce as before to:

$$\frac{d}{R} \frac{d^2 \alpha}{d\phi^2} = \cos \alpha$$

provided that R is large compared with d . An exact solution of this equation, with the condition $d\alpha/d\phi = 0$ when $\alpha = \pi/2$, gives the period of oscillation of the fluid in the bend as $1.18 \times 2\pi \sqrt{d/R}$.

The introduction of ξ_m and the other assumptions made above are not regarded as a satisfactory basis for the extension of Hawthorne's (1951) theory. In the absence of an alternative analysis however, they do at least suggest the presence of secondary flow oscillations.

This oscillation has been observed in practice though there is a large frictional damping effect which depends on the Reynolds number and the roughness of the pipe. The period of oscillation may be written $2\pi B \sqrt{d/R}$ where B is a constant. Because of the assumptions made in the analysis it seems likely that B will depend to some extent on the inlet velocity profile.

When the secondary flow oscillations have been completely damped - if the bend deflection is sufficiently large for this to happen - a region is reached in which there is fully developed curved flow. Here there is no further displacement of the Bernoulli surfaces and total pressure profiles are similar. The extent of the region of fully developed curved flow will depend on the bend deflection, the value of d/R , the Reynolds number (since this affects the damping of the secondary flow) and the inlet velocity profile (insofar as this affects the value of B).

Finally, there is the outlet "transition" region; that is the region downstream of the bend outlet. Conditions here can be very varied since in many bends no region of fully developed curved flow is ever attained. In such a case the flow enters the straight part of the pipe with some value of the secondary circulation depending on the geometry of the bend and the inlet conditions: if this is a maximum there will be a tendency for the fluid of high stagnation pressure to be spread over the walls of the pipe: if it is zero a region somewhat similar to a region of fully developed curved flow will exist near the bend outlet.

Unless a symmetrical velocity profile is attained at, or before, the pipe outlet the length of the outlet transition region appears to have an effect not only on the flow in the region itself but also in the bend: moreover the length of pipe required to recover a symmetrical velocity profile depends on the conditions at the bend outlet.

Determination of Pressure Losses

Consider a system in which the bend deflection is ϕ and the downstream transition length is l .

The overall losses in the system may be calculated from the difference between the values of total pressure at bend inlet and pipe outlet. A mass averaged stagnation pressure H may be defined by the integral

$$H = \frac{4}{\pi \bar{q} d^2} \int_0^{d/2} \int_0^{2\pi} hqr \, dr \, d\theta$$

where r and θ are polar co-ordinates in the cross-section of the pipe

- h is the stagnation pressure at the point (r, θ)
- q is the velocity at the point (r, θ)
- \bar{q} is the mean velocity.

At the bend inlet, assuming that the straight entry segment is long enough to ensure fully developed turbulent flow there, the velocity profile is symmetrical about the pipe axis and therefore h is independent of θ .

$$H_I = \frac{8}{\bar{q} d^2} \int_0^{d/2} hqr \, dr$$

At the pipe outlet a surface traverse may be made which will give $h = h(r, \theta)$. A numerical value for H_0 may therefore be determined.

The mean overall loss is then $\frac{H_I - H_0}{\frac{1}{2} \rho \bar{q}^2}$.

With the straight part of the pipe is associated a friction coefficient λ_s given by:

$$\lambda_s = \frac{\Delta p}{\frac{1}{2} \rho \bar{q}^2} \cdot \frac{d}{x}$$

where Δp is the pressure difference measured over a length x . Then, the loss in a length of straight pipe equivalent to the length of the given system = $\lambda_s (\phi R/d + l/d)$.

Two loss coefficients may then be defined:

An excess loss coefficient $\Delta \zeta = \frac{\text{Total pressure loss}}{\text{Mean dynamic head}}$

$\frac{\text{Total pressure loss in equivalent length of straight pipe}}{\text{Mean dynamic head}}$

And a loss ratio coefficient $\zeta_r =$

$\frac{\text{Total pressure loss}}{\text{Total pressure loss in equivalent length of straight pipe}}$

$$\Delta \zeta = \frac{H_I - H_0}{\frac{1}{2} \rho \bar{q}^2} - \frac{\lambda_s}{d} (\phi R + l)$$

$$\zeta_r = \frac{H_I - H_0}{\frac{1}{2} \rho \bar{q}^2} / \frac{\lambda_s}{d} (\phi R + l)$$

Pressure losses may also be estimated over parts of the system separately. In a region of fully developed curved flow the inlet and outlet velocity profiles are similar, hence the total pressure loss is constant across the cross-section of the pipe. If H_{cI} and H_{cO} are the values of H at inlet and outlet to this region respectively and if h_{cI} and h_{cO} are the values of h at the corresponding cross-sections for the same values of r and θ ,

$$\begin{aligned}
 H_{cI} - H_{cO} &= \frac{4}{\pi \bar{q} d^2} \left\{ \int_0^{d/2} \int_0^{2\pi} q h_{cI} r \, dr \, d\theta - \int_0^{2\pi} q h_{cO} r \, dr \, d\theta \right\} \\
 H_{cI} - H_{cO} &= \frac{4(h_{cI} - h_{cO})}{\pi d^2 \bar{q}} \int_0^{d/2} \int_0^{2\pi} q r \, dr \, d\theta \\
 &= h_{cI} - h_{cO}.
 \end{aligned}$$

Downstream of the bend the static pressure becomes constant over a cross-section and the actual total pressure loss here will be equal to the static pressure loss. This value may differ slightly from the mean loss defined above since H depends on the velocity profile and this is changing through the region.

For the bend transition region as a whole a mean loss may be estimated equal to the overall loss minus the losses in the other regions. A more detailed determination of the build up of pressure losses in this region however requires the making of total head traverses in several planes at each cross-section, and this technique must also be applied for information about the losses in the immediate vicinity of the bend outlet. An alternative method when only comparative results are needed is to define at each cross-section a mean static pressure calculated from readings taken at a number of points on the pipe wall.

Let θ be measured from the radius directed towards the bend centre and let $p = p(\theta, z)$ be the static pressure on the wall, where z is the distance in diameters from the bend inlet. Then associated with the cross-section $z = \text{constant}$ a mean pressure may be defined by the equation:-

$$\bar{p}(z) = \frac{1}{\pi d} \int_0^{2\pi} p(z, \theta) \frac{d}{2} \, d\theta. \quad \dots (5)$$

Also useful for comparative purposes, such as are wanted in the second group of experiments described below, are the actual static pressure differences $p(z_1) - p(z_2)$ for constant θ .

Loss in Availability

The availability corresponding to a state of a system for which its energy is E , its volume V and its entropy S is given by

$$(E + p_0 V - T_0 S) - (E_0 + p_0 V_0 - T_0 S_0)$$

where p_0 and T_0 denote respectively the pressure and temperature of the surrounding atmosphere and E_0 , V_0 and S_0 denote respectively the energy, volume and entropy of the system in the dead state: that is the state in which there is no possibility of obtaining work from the system, the surrounding atmosphere, or from interaction between the two. (Keenan (1949)).

Consider/

Consider a steady stream of fluid entering a pipe with velocity U .

The energy e per unit mass is given by: $e = u + \frac{1}{2g} U^2$ where u is the internal energy per unit mass. But $i = u + pV$ where i is the enthalpy.

∴ The availability per unit mass becomes

$$(i - T_0 S + \frac{1}{2g} U^2) - (i_0 - T_0 S_0)$$

If the subscript 1 refers to conditions at entry, and the subscript 2 to those at outlet, and if it is assumed that U is constant down the pipe, the loss of availability per unit mass between entry and outlet is:

$$\beta = (i_1 - i_2) - T_0(S_1 - S_2)$$

For an ideal, polytropic gas $pV = RT$ and $pV^\gamma = A \exp \frac{S-S_0}{c_v}$ where A

is a constant. From these equations it may be shown that

$$i = \frac{\gamma}{\gamma-1} RT$$

and
$$\frac{S_1 - S_2}{C_v} = \log (p_1/p_2) + \gamma \log (V_1/V_2)$$

or
$$S_2 - S_1 = R \log (p_1/p_2) - c_p \log (T_1/T_2)$$

If further we assume that the change from state 1 to state 2 takes place adiabatically, $di = 0$ and there will be no change of temperature along the pipe.

Hence
$$\beta = RT_0 \log (p_1/p_2)$$

Writing
$$p_1 = p_{1e} + p_0$$

$$p_2 = p_{2e} + p_0$$
 where p_{1e} and p_{2e} are the pressures in excess of atmospheric pressure at states 1 and 2 respectively. For values of p_{1e} and p_{2e} small compared with p_0

$$\begin{aligned} \frac{p_1}{p_2} &\simeq \frac{1}{p_0} (p_{1e} + p_0) \left(1 - \frac{p_{2e}}{p_0}\right) \\ &\simeq 1 + \frac{p_{1e} - p_{2e}}{p_0} = 1 + \frac{p_s}{p_0} \end{aligned}$$

where p_s is the static pressure difference between states 1 and 2 and

$$\log \left(\frac{p_1}{p_2} \right) \simeq \frac{p_s}{p_0}$$

i.e.,
$$\beta \simeq RT_0 \left(\frac{p_s}{p_0} \right) \quad \dots (6)$$

Let subscripts b and s refer respectively to a bent pipe and to an equivalent length of straight pipe.

The excess loss of availability coefficient $\Delta\beta$ is defined by

$$\Delta\beta = \frac{\beta_b - \beta_s}{RT_o} = \frac{1}{P_o} \left((P_s)_b - (P_s)_s \right)$$

$$= \frac{\Delta Z}{Cp_o}$$

where

$$Cp_o = \frac{P_o}{\frac{1}{2}\rho U^2}$$

Compressibility effects have been ignored in equation (6) by the neglect of second order terms. This is certainly justifiable in the present tests where $U \approx 100$ ft./sec.

Some Effects of the Secondary Circulation

Experiments with a helical pipe

The pipe used in these tests was made of steel and had an approximately circular cross-section with a nominal diameter of 1.9". It consisted of a three-coil helical section bent on a mean radius of 9.5", to which were flanged pieces of straight pipe made of similar material forming an entry segment 7' 5" long and an outlet length of 9' 3". Holes were drilled in the pipe at a point 6" before the commencement of the bend and then at 45° intervals in the first complete turn of the helix; also at various points near the end of the bend and in the straight section downstream of it. Total head traverses were made across the centre line of the pipe in the plane of the bend only: for these a pitot tube of 0.043" outside diameter was used.

Traverses were made at fifteen points along the pipe and the corresponding total head profiles are shown in Fig.1. The location of the pipe wall is indicated for each traverse since, due to flattening in the bend, the diameter of the pipe varies between 1.90" and 1.66" there.

It will be noticed that the profiles after 360° and 1045° are very nearly similar, from which it may be deduced that fully developed curved flow is attained in the bend at a deflection of about 360°.

The position of the particle of highest total pressure determines to a first approximation the angle of displacement of the fluid in the bend. Fig.2 shows the oscillatory nature of this angle α and gives the value of the bend deflection for a complete oscillation of the secondary circulation as 195°. The oscillation is apparently completely damped after about two periods. Writing the period of oscillation as $2\pi B\sqrt{d/R}$ this gives an experimental value of $B = 1.21$ compared with the theoretical value of 1.18. Comparison may also be made with the experimental results of Squire (1954) who, operating at a Reynolds number of 3×10^5 , obtained a value of $1.16 < B < 1.45$.

Of particular note is the remarkable change of total head profile at 90° deflection: there is an apparent "caving in" on the inner side of the bend which occurs shortly before the direction of the secondary circulation is reversed. Such a clogging effect is observed by Eichenberger (1952) in his experiments with a 8" x 8" curved duct. It occurs again here - though to a very much smaller extent - immediately downstream of the end of the bend. With both points are associated positions of approximately maximum displacement of the particle of highest total pressure.

For the determination of H_o , defined above, a surface traverse was made at the pipe outlet, the result of which is shown in Fig.3.

Static pressures were measured from wall tappings in the plane of the bend on the outer wall of the pipe, and values obtained in the inlet transition region are shown in Fig.4(a). Values in the straight pipe following the bend show an almost exactly linear decrease to atmospheric pressure over the whole length: these are given in Fig.4(b) and are there compared with the fall of pressure in the same straight pipe flanged directly to the straight entry segment and tested under the same inlet conditions. The Reynolds number based on the nominal pipe diameter ($d = 1.9$) and the mean inlet velocity was estimated to be approximately 9.4×10^4 throughout these tests.

The mean losses in the system are presented in the following table and are also shown graphically in Fig.5:

	$\Delta\zeta$	Excess loss per unit length	ζ_r
Bend transition ($0-2\pi$)	0.40	0.0127	1.72
Region of fully developed curved flow ($2\pi-6\pi$)	0.58	0.0092	1.51
Downstream straight (58 diam)	0.01		1.01
Overall	0.99	Overall	1.36

The mean excess loss per unit length for the complete bend = 0.0104 and the mean value for ζ_r for the complete bend = 1.58. Also, the excess loss of availability coefficient $\Delta\beta = 0.0050$. These figures have all been obtained using a friction coefficient $\lambda_s = 0.0180$.

Experiments with 2" pipes bent on a mean radius of 26.4"

From the tests on the helical pipe where $R/d = 5$, an experimental value of B was found equal to 1.21. The secondary circulation in the bend will be a maximum after a quarter period of the oscillation, i.e., at a deflection

$$\phi_m = \frac{\pi}{2} B \sqrt{d/R} = \frac{1.21 \times \pi}{2} \sqrt{d/R}$$

Using this equation it may be shown that, for a 2" pipe bent on a mean radius of 26.4", maximum secondary circulation is theoretically attained in the bend at a deflection of 30° . At 60° the secondary circulation will be zero and at 70° it will have a small value in the opposite direction to that at 30° .

In each of the three tests reported here the length of the pipe from bend inlet to pipe outlet was 78 diameters: the experiments were made at a Reynolds number of approximately 9.0×10^4 and the friction coefficient in the straight part of the pipe was measured equal to 0.0166.

(i) 30° Bend In this test traverses were made in the plane of the bend at six points along the pipe, and the corresponding total head profiles are shown in Fig.6(a). Indication of the existence of strong secondary vorticity - as theoretically predicted - is given by the continued displacement of the particle of highest total pressure downstream of the bend outlet. In Fig.6(b) are profiles of four traverses made in the plane perpendicular to the plane of the bend at points near the bend outlet. They give evidence that there is swirling in the downstream transition region, the tendency being for the fluid of high stagnation pressure to be spread over the walls of the pipe (Hawthorne 1951).

The mean losses in the system were calculated as follows:-

		$\Delta\zeta$	Excess loss per unit length	ζ_r
Bend	(0-30°)	0.04	0.0053	1.36
Downstream (1)	(0-26 diam.)	0.15	0.0042	1.34
"	(2) (26-44 diam.)	0.09		1.30
"	(3) (44-71 diam.)	0.06		1.13
Overall		0.34	Overall	1.26

with an excess of availability coefficient $\Delta\beta = 0.0016$.

Overall losses are calculated from total pressure losses but otherwise the figures above are based on static pressure readings. These were taken at points on the wall in both the plane of the bend and in the plane perpendicular to it. Compared with the change between two cross-sections in the mean total pressure $H(z_1) - H(z_2)$, and mean static pressure $\bar{p}(z_1) - \bar{p}(z_2)$, the local total pressure difference $h(r, z_1) - h(r, z_2)$ (measured in the plane of the bend) gives a very low estimate of the pressure loss in the downstream transition region. This would be expected to occur in a section where there is swirling flow, while high velocity fluid near the walls of the pipe is a source of increased loss, the spread of such fluid to the "inside" wall ($\theta = 0$) will reduce the total pressure drop there. It is remarkable that even with 71 diameters of straight pipe after the bend the total head profile at outlet is not exactly symmetrical.

(ii) 60° Bend Total head profiles for this system are shown in Fig.7. The displacement of the particle of highest total pressure appears to be approximately stationary at the bend outlet, as expected, but the change of profile on the inside of the pipe is most marked a short distance downstream. The outlet profile after a transition region of 64 diameters is seen to be still some way from being symmetrical. Mean losses calculated as before give:

		$\Delta\zeta$	Excess loss per unit length	ζ_r
Bend	(0 - 60°)	0.17	0.0124	1.74
Downstream (1)	(0-19 diam.)	0.10	0.0036	1.30
"	(2) (19-37 diam.)	0.09		1.29
"	(3) (37-64 diam.)	0.04		1.09
Overall		0.40	Overall	1.31

$\Delta\beta = 0.0019$

Since the secondary circulation is theoretically zero at the bend outlet no effort was made to locate swirling in that region.

(iii) 70° Bend (See Fig.8). The direction of the secondary vorticity is now reversed before the bend outlet and an immediate recovery towards a symmetrical profile is apparent in the downstream transition region. Pressure losses here are given by:

$\Delta\zeta/$

	$\Delta\zeta$	Excess loss per unit length	ζ_r
Bend (0°-70°)	0.22	0.0136	1.82
Downstream (1) (0-17 diam.)	0.12	0.0039	1.43
" (2) (17-35 diam.)	0.07		1.23
" (3) (35-62 diam.)	0.05		1.11
Overall	0.46	Overall	1.35

$$\Delta\beta = 0.0021.$$

Values of the rate of change of $\Delta\zeta$ with distance are plotted in Fig.9 against distance from the bend inlet. These are compared there with the corresponding build up of pressure losses in the other tests, including those made with the helical pipe.

Measurements of mean static pressures in the downstream transition length are presented for comparison in Fig.10, together with the corresponding inlet static pressures.

Effect of the Downstream Transition Length

The 2" pipe with the 70° bend described above was also used to study the effect on the pressure losses of a variation in downstream transition length. The original pipe had a transition length of 62 diameters and this was gradually cut back (in 5 stages) until only the bend itself was left. Holes for measuring static pressures were made in the pipe wall at a number of cross-sections in the neighbourhood of the bend outlet and downstream from it: in all eight holes were drilled at each cross-section at 45° intervals around the perimeter. Special care was taken to ensure that inlet conditions were as near as possible identical for each system, the Reynolds number throughout being calculated approximately equal to 8.6×10^4 . Total pressure traverses made in each case at the pipe outlet are shown in Fig.11. From each of these the mean dynamic head was calculated and a corresponding value found for the mean overall loss: this procedure constituted a check on the overall static pressure loss - it being assumed that the static pressure was constant over the inlet cross-section. Figs.12(a)-(d) record the wall static pressures $p(\theta, z, \ell/d)/\frac{1}{2}\rho\bar{q}^2$ for constant θ , and Fig.13 the same results at two different values of z with the pressures plotted radially as functions of θ .

The mean static pressure, defined above by equation (5), was used to determine the mean excess loss coefficient $\Delta\zeta^*$ at any cross-section. Values of this coefficient are given in Fig.14 for different values of z and ℓ/d : in particular the variation of $\Delta\zeta^*$ with ℓ/d at a point 3 diameters downstream from the bend outlet is presented in Fig.15 and compared there with some results obtained with the helical pipe operating at a slightly higher Reynolds number.

Total pressure traverses were made at the bend outlet for the 70° bend and at 3 diameters downstream from the helical pipe. Compared in Fig.16 are profiles in the plane of the bend showing their variation with downstream transition length. It will be observed that changes occur on the inside of the bend, where, as the transition length decreases, the total pressure maximum becomes more pronounced. This effect is not observed, however, at the open end of the pipe when the transition length is zero: here there is no pressure maximum at all, but on the outside there is an increased displacement of the particle of highest total pressure.

The hydrogen sulphide technique may be more useful in the investigation of two-dimensional phenomena for, in a series of tests carried

out by the author (Percival (1958)) in curved ducts, results similar to this one were obtained even in cases where separation was suspected.

A brief visualisation test was carried out on the 70° bend in an attempt to show up any tendency towards separation or reversed flow near the bend outlet. Here the downstream transition length was detached from the bend so that the inside of the pipe, for approximately 2 diameters on either side of the bend outlet, could be coated with a lead carbonate paint: the two pieces of pipe were then fitted smoothly together and held firmly in place by a close-fitting sleeve (of internal diameter $2\frac{3}{8}$ ") screwed on to either side of the join. Hydrogen sulphide was injected into the system both in the main stream and at points on the pipe wall. No sign of reversed flow could be detected from the traces, but the existence of the secondary circulation was apparent and also a slight rotation of the plane of symmetry. This effect may be seen in the outlet total pressure traverses (Fig.11), and in particular for the system with a 30 diameter long transition length where there is sufficient time before the outlet for an appreciable angle of rotation to develop.

Discussion

The above results show that there is indeed a marked difference in the build up of pressure losses according to the conditions at the bend outlet. For the helical pipe, where there is a region of fully developed curved flow, there is little or no excess loss in the downstream transition region; this compares with the observations of Keulegan and Beij (1937). Although the excess loss is so small the stilling length is something more than 58 diameters. When the transition length is considerably less than this the overall excess loss appears to be increased, the change occurring either in the bend itself or within 3 diameters of the bend outlet.

At outlet from the 30° bend the secondary circulation is tending to invert the normal stagnation pressure distribution and losses are expected to remain high. In fact the results indicate that they differ little from those in the bend itself for as much as 44 diameters downstream. By this time frictional damping will have reduced the rotational effects and the excess loss drops accordingly: so too does the rate at which the velocity profile is changing so that even after 70 diameters it is still not entirely symmetrical.

The 60° bend shows an increase in the loss per unit length in the bend. At bend outlet the value drops, appearing to be roughly constant over 37 diameters. Although there is (theoretically) no secondary circulation at the bend outlet the particles there are displaced by a maximum amount and losses must be expected in association with the distorted pressure distribution. The excess loss becomes small after about 40 diameters, but although the loss in the downstream transition length as a whole is less than for the 30° bend the stilling length may well be longer.

A further increase in the excess loss per unit length in the bend is observed for the 70° system. It will be noticed though, that this increase with bend deflection does not continue indefinitely: the mean loss in the bend transition region of the helical pipe is slightly less than for a deflection of 70°. For the 70° bend the rate of loss immediately after the bend appears to be higher than for the other two systems: this is presumably due to the fact that there is some secondary circulation there as well as near maximum distortion of the pressure distribution. The secondary circulation however is now in a direction which tends to help the recovery of a normal distribution and losses consequently drop more quickly than in the other cases. Also the stilling length appears to be slightly decreased.

For the 70° bend it appears that, as with the helical pipe, the calculated pressure losses are least for long transition lengths; furthermore, the effect seems to be felt well upstream of the bend outlet (Fig.13). It is

difficult/

difficult to assess the accuracy of the present results but the inlet static pressures check well with the total pressure differences, and the other readings, besides being reasonably consistent among themselves, are only used for comparative purposes. In the light of this it would certainly seem that the recorded increase is significant.

The changes occurring in these tests with variable transition length are clearly extremely complex. The problem is essentially a three-dimensional one, and the approximate theory outlined above cannot give information on pressure differences when its assumptions imply that these are negligible. Such assumptions are reasonable in the early stages of the bend, but can hardly be expected to apply at the bend outlet.

Some observations can be made from the experimental results, however. Firstly there is the existence of a region immediately upstream of the bend outlet in which there is an adverse pressure gradient. In every system this region extends over at least a quarter of the circumference of the pipe wall but for the shortest transition lengths (zero and 1 diameter) it appears to have spread to more than half. This adverse pressure gradient probably indicates a partial choking of the pipe just as if the flow had been separated from the wall. If choking is also associated with the development of a pressure maximum on the inside of the bend the changes in the total pressure profiles at the bend outlet (Fig.16) also point to a variation (with transition length) in the extent and position of the region of "separation".

It is noticeable that almost immediately after the bend wall static pressures become independent of θ , (though this does not mean that static pressures are necessarily constant over the whole cross-section). For long transition lengths it may be assumed that the exit pressure is uniform and atmospheric; a pressure variation may however exist at the outlet when l is small. Consider the displacement of the particle of highest total pressure at the cross-section $z = l_1/d$ for two systems with transition lengths l_1 and l_2 ($l_1 < l_2$). A greater displacement in the pipe with the shorter transition length suggests there may be a reduction in the value of the axial velocity when outlet conditions are imposed. The variation of transition length must then cause changes in the pressure distribution in the pipe, with even the possibility of altering the value of the "constant" B and with it the period of the secondary oscillation.

Then also there is the question of the rotation of the plane of symmetry. This may perhaps be connected with the consistently low static pressure reading at $\theta = 45^\circ$, 1 diameter downstream from the bend outlet: if a region of "separation" exists, it is plausible to suppose that, due to imperfections in the pipe, the region will favour one side of the plane of symmetry rather than the other - (compare, for example, the separation from the walls of a diffuser).

These phenomena are clearly interconnected and are likely as well to depend on the secondary flow conditions attained at the bend outlet.

Conclusion

In assessing pressure losses in a bent circular pipe both bend outlet conditions and pipe outlet conditions are important as well as the inlet velocity profile. Loss coefficients depend on some parameter which varies with the phase of the secondary circulation at the bend outlet. Also there is a contribution to the loss in the downstream transition region of a given system not merely associated with the value of the secondary circulation at the bend outlet but also the displacement of the fluid particles there. Due in part, perhaps, to changes in displacement but more particularly to choking effects in the pipe an increase in the loss coefficient is observed as the downstream transition length is shortened. For purposes of correlation therefore it would seem to be necessary to have transition lengths of 60 diameters or more.

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Notation

- A Constant in equation of state.
- B Constant associated with period of secondary oscillation.
- d Pipe diameter.
- E Energy.
- e Energy per unit mass.
- H Mean stagnation pressure over cross-section.
- h Stagnation pressure.
- i Enthalpy.
- ℓ Downstream transition length.

p	Static pressure on pipe wall.
$\bar{p}(z)$	Mean static pressure at cross-section $z = \text{constant}$.
Δp	Pressure difference over length x .
q	Magnitude of velocity vector.
\bar{q}	Mean velocity.
R	Bend radius.
r_c	Distance from particle to bend centre.
r	Polar co-ordinate in cross-section.
S	Entropy.
T	Temperature.
U	Axial component of velocity.
U_m	Maximum velocity.
u	Internal energy per unit mass.
V	Volume.
v	Circumferential velocity.
x	Distance along pipe.
z	Distance in diameters from bend inlet.
α	Angular displacement.
β	Loss of availability per unit mass.
$\Delta\beta$	Loss of availability coefficient.
$\Delta\zeta$	Excess loss coefficient.
$\Delta\zeta^*$	Excess loss coefficient based on mean static pressures.
ζ_r	Loss ratio coefficient.
θ	Polar co-ordinate in cross-section.
λ_s	Friction coefficient.
ϕ	Bend deflection.
ρ	Density.
ξ	Secondary vorticity.
ξ_m	Mean secondary vorticity.

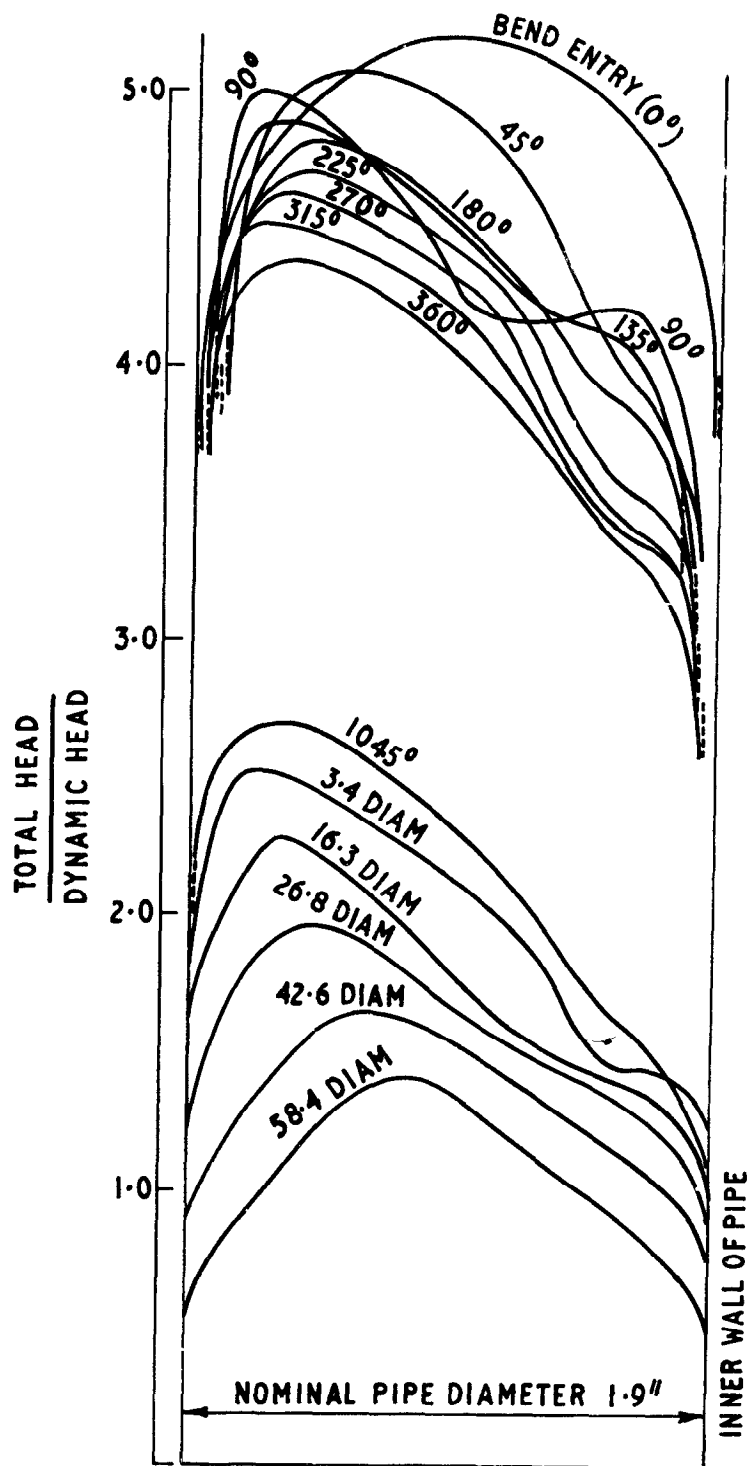


FIG. 1. TOTAL HEAD PROFILES ACROSS ϵ OF PIPE IN PLANE OF BEND HELICAL PIPE

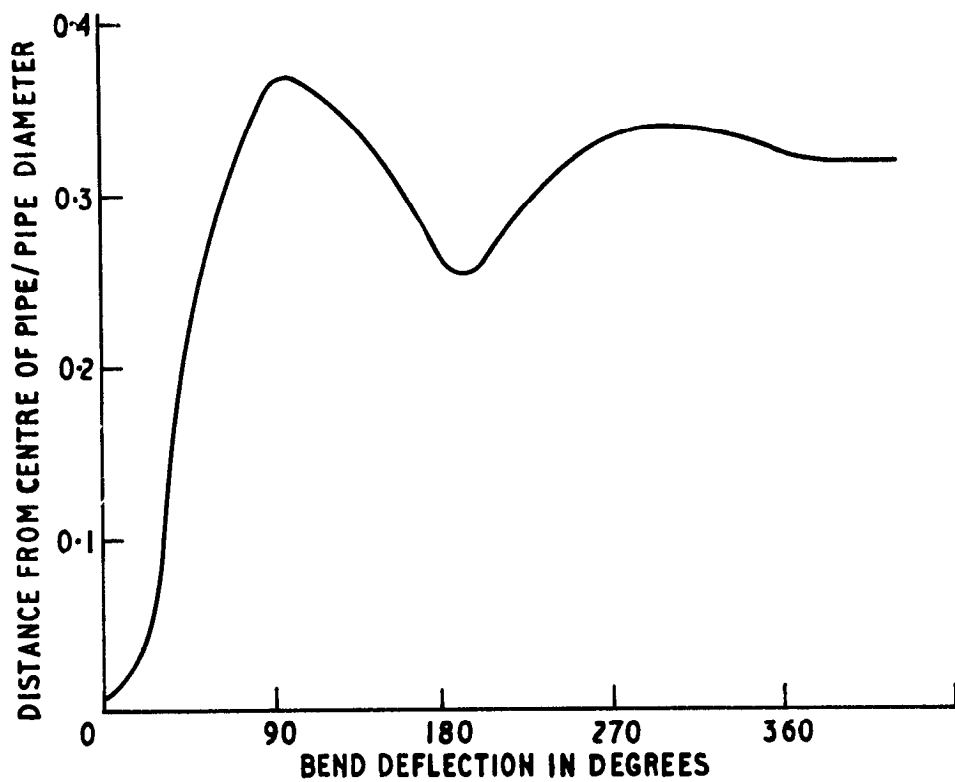


FIG. 2. POSITION OF PARTICLES WITH HIGHEST TOTAL PRESSURE

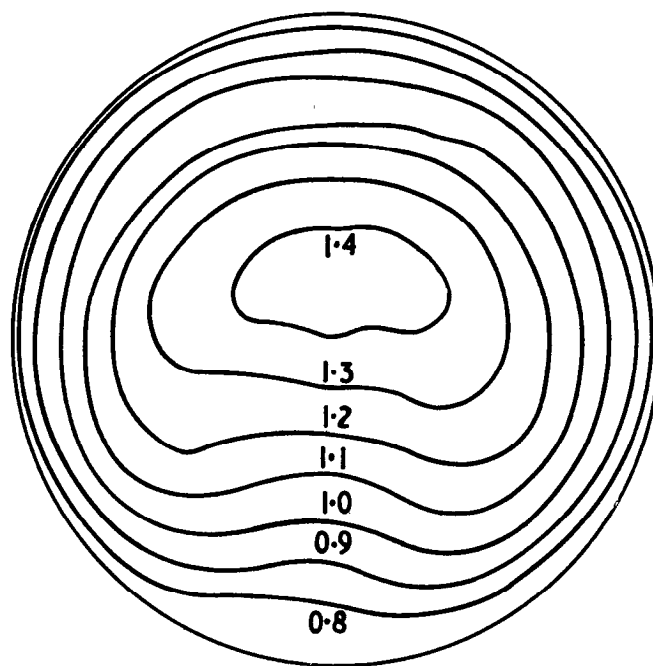


FIG. 3. LINES OF CONSTANT TOTAL PRESSURE AT PIPE OUTLET

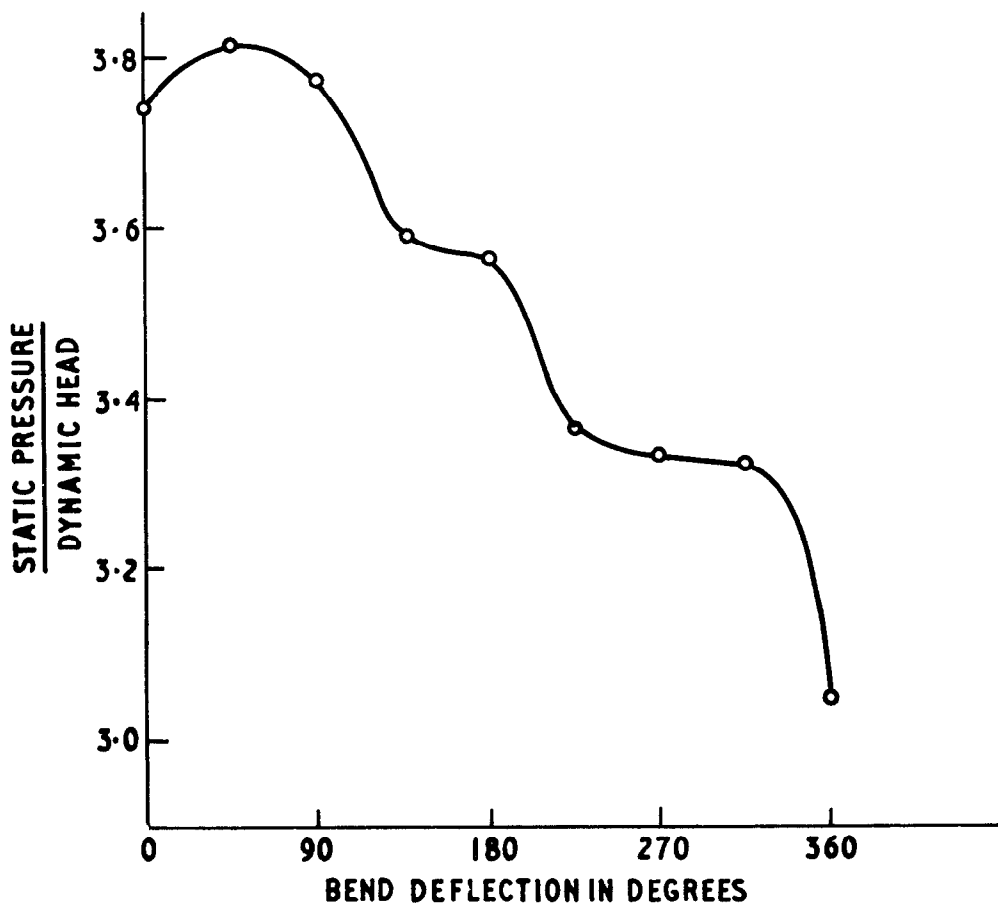


FIG. 4 (a) STATIC PRESSURES ON OUTER WALL OF HELICAL PIPE

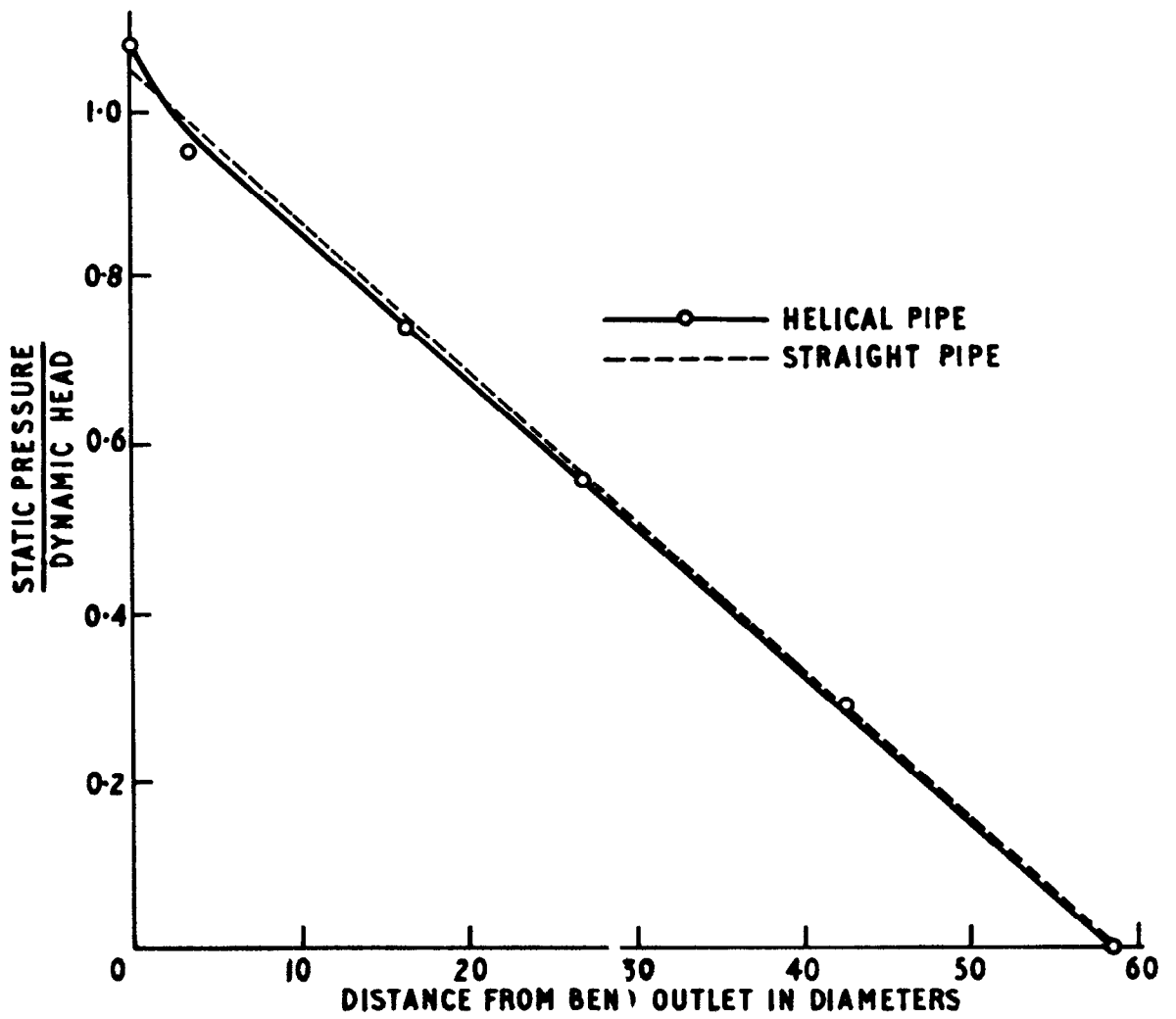


FIG. 4 (b) STATIC PRESSURES IN DOWNSTREAM TRANSITION REGION

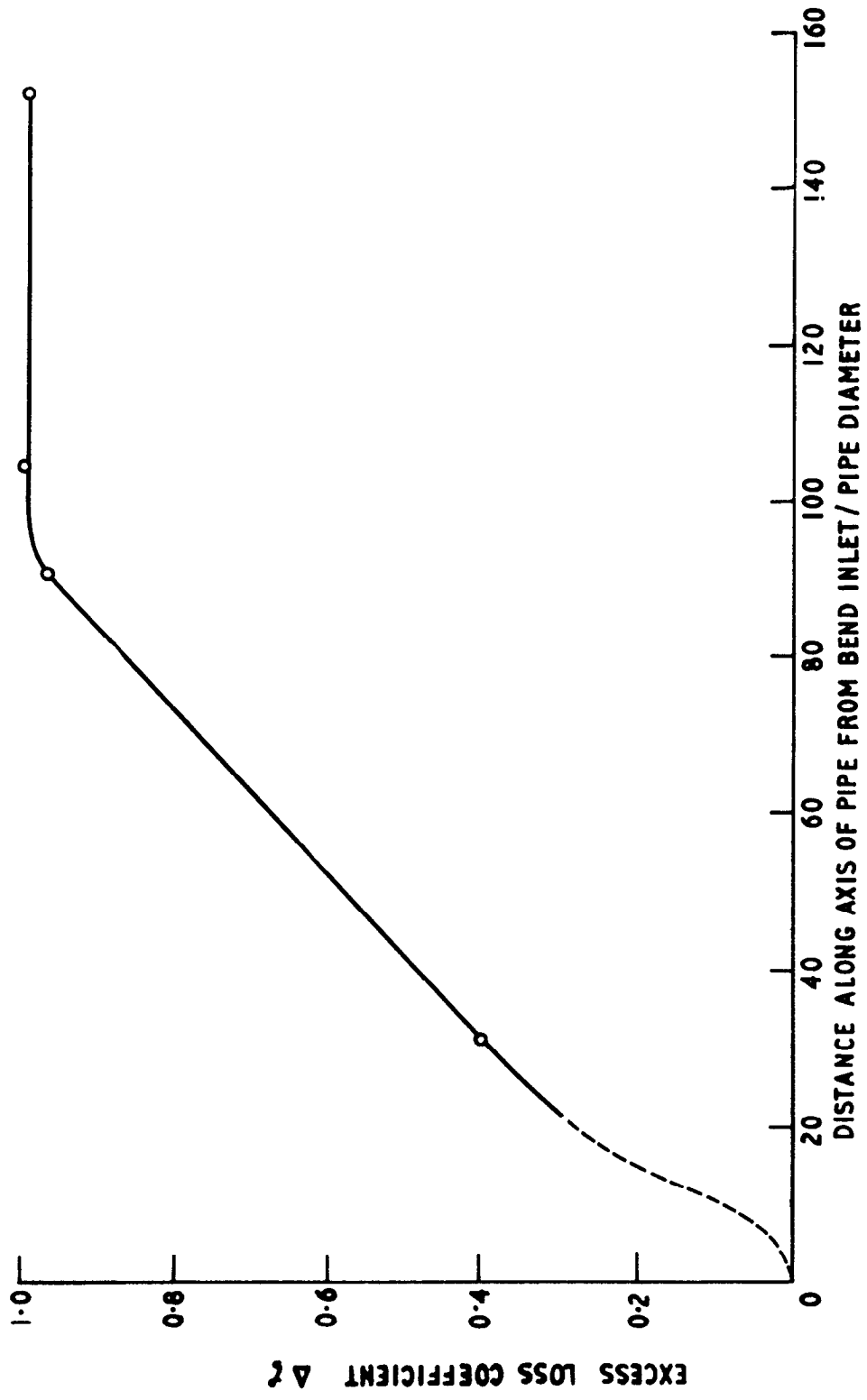


FIG. 5 PRESSURE LOSS IN HELICAL PIPE WITH 9' -3" STRAIGHT PIPE DOWNSTREAM OF BEND

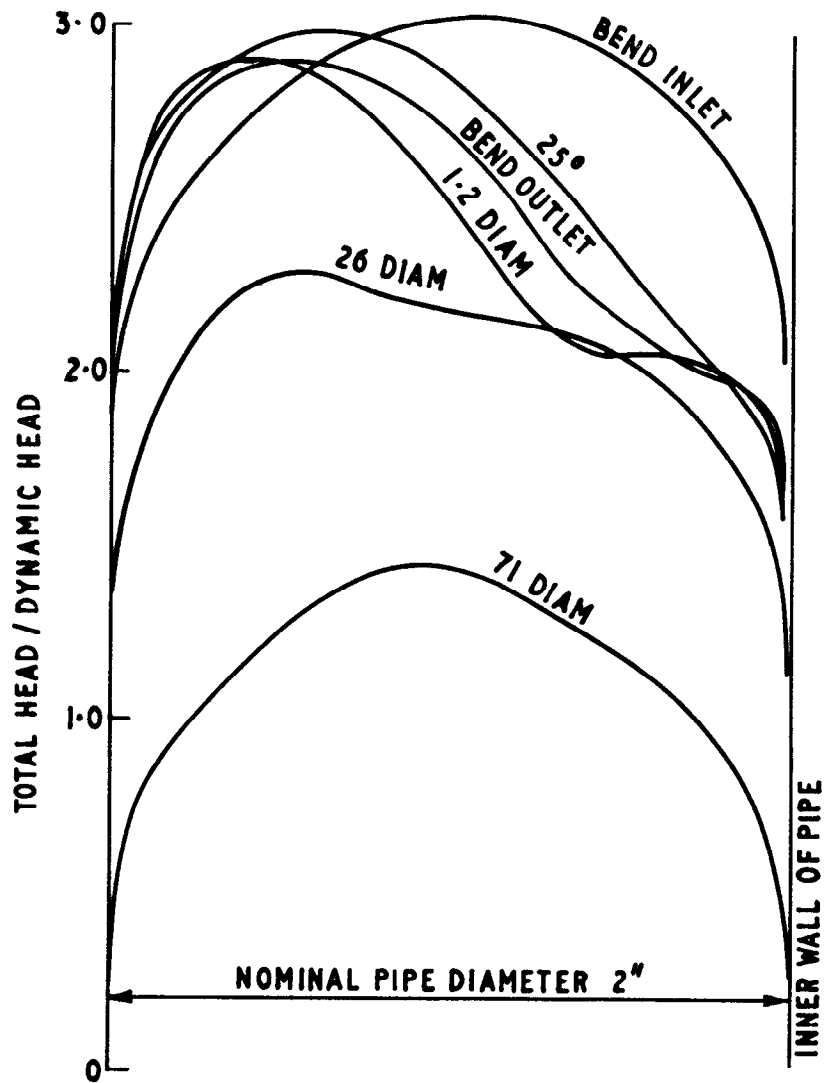


FIG. 6 (a) TOTAL HEAD PROFILES ACROSS $\frac{1}{2}$ OF PIPE IN PLANE OF BEND. 30° BEND

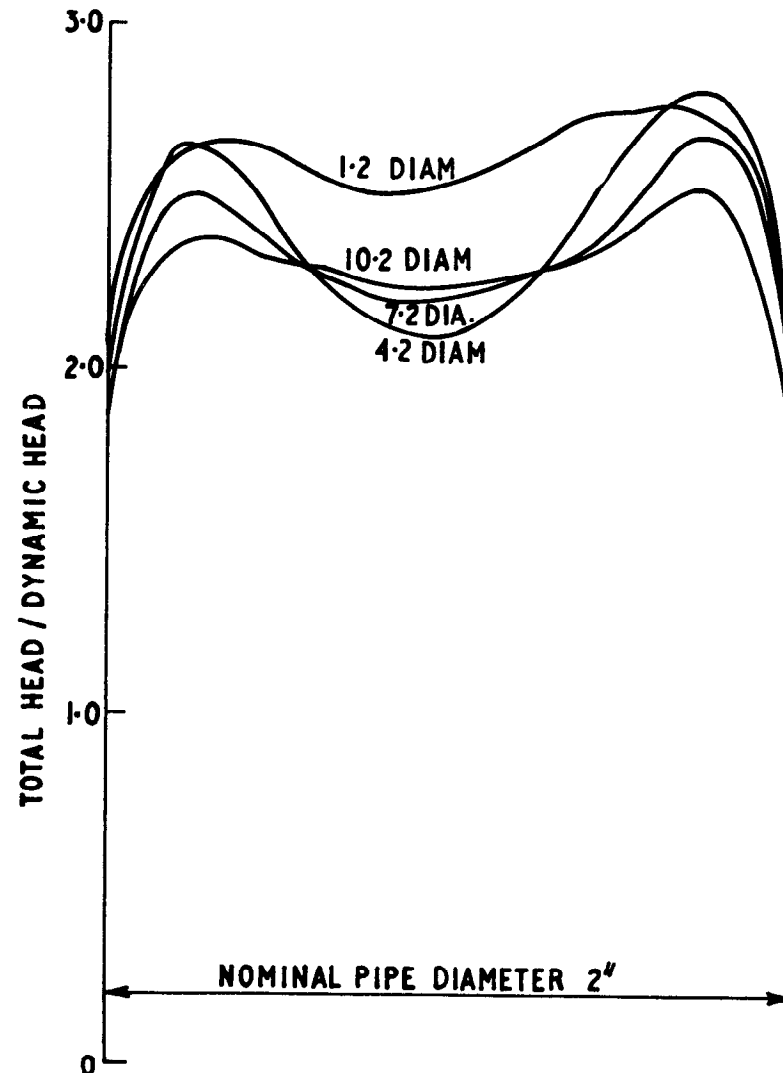


FIG. 6 (b) TOTAL HEAD PROFILES IN PLANE \perp TO PLANE OF BEND. 30° BEND

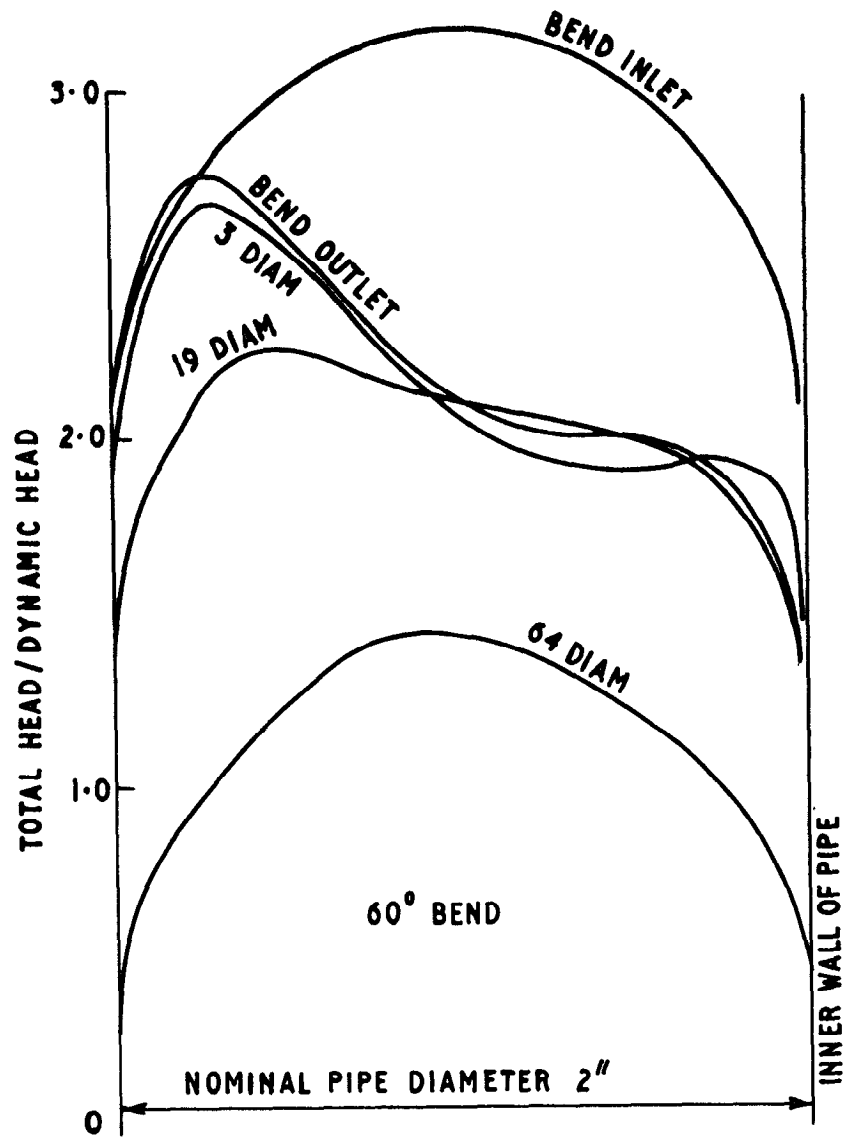


FIG. 7. TOTAL HEAD PROFILES ACROSS ϵ OF PIPE IN PLANE OF BEND

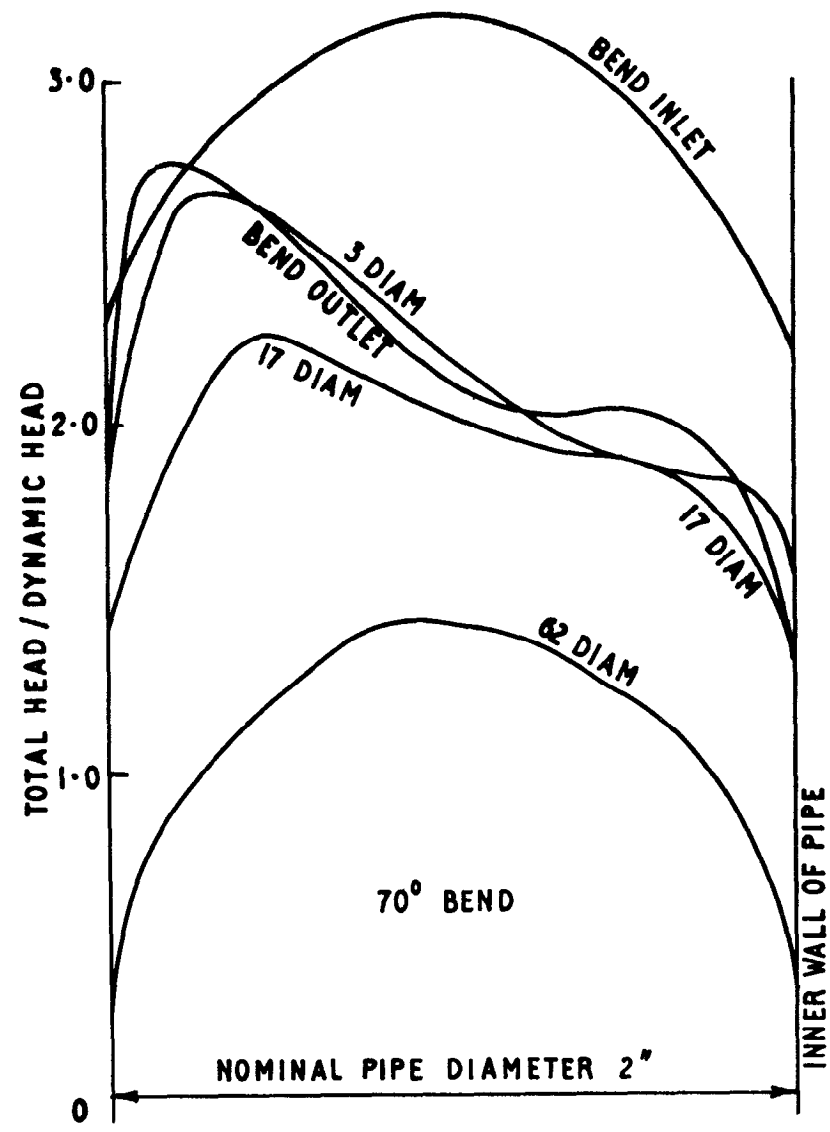


FIG. 8. TOTAL HEAD PROFILES ACROSS ϵ OF PIPE IN PLANE OF BEND

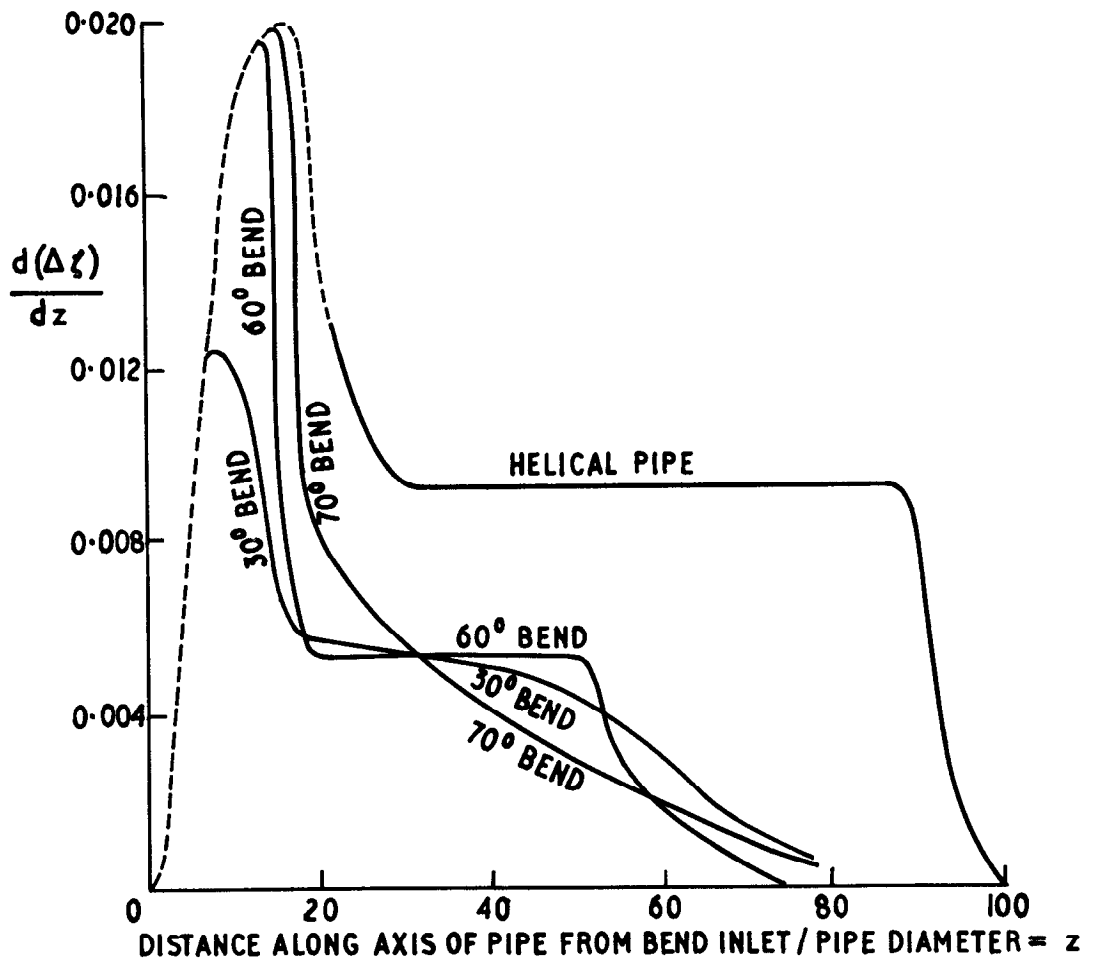


FIG. 9. PRESSURE LOSSES IN DIFFERENT SYSTEMS

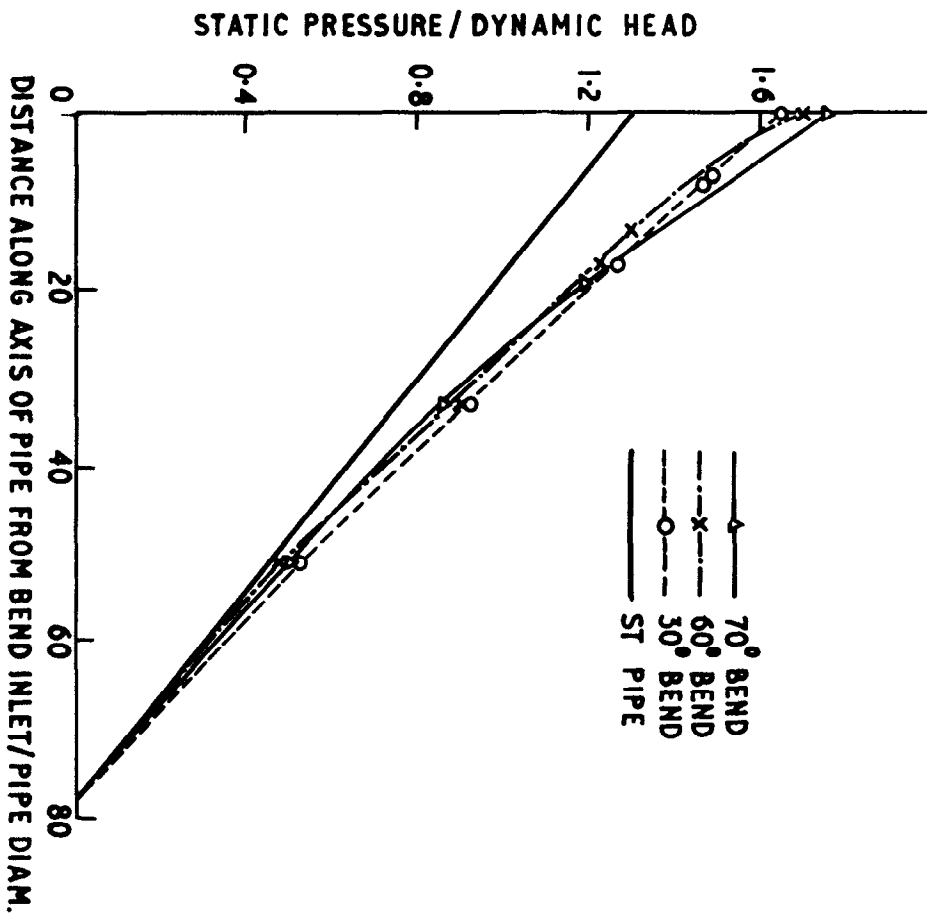
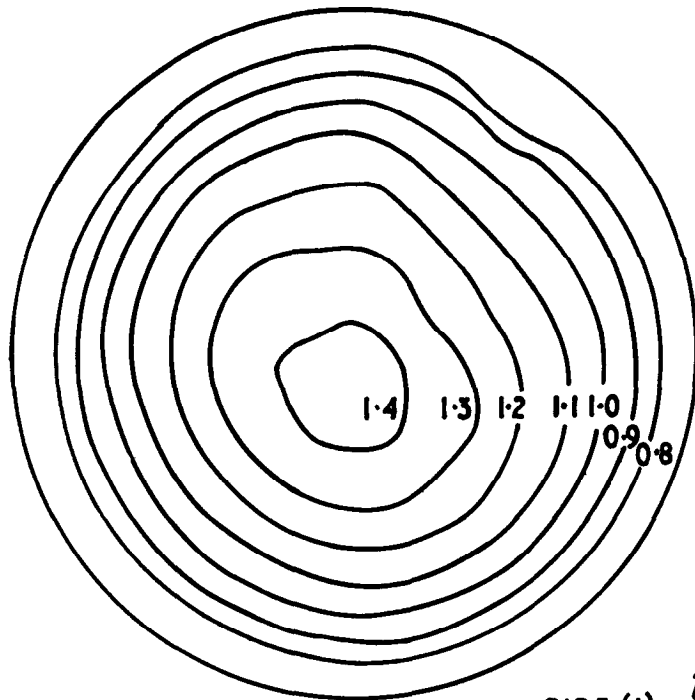
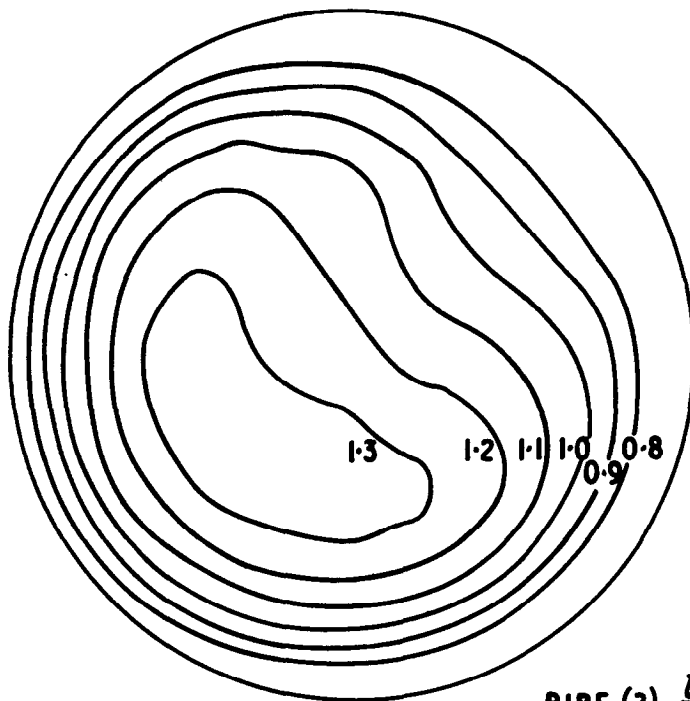


FIG.10. STATIC PRESSURES IN DIFFERENT SYSTEMS

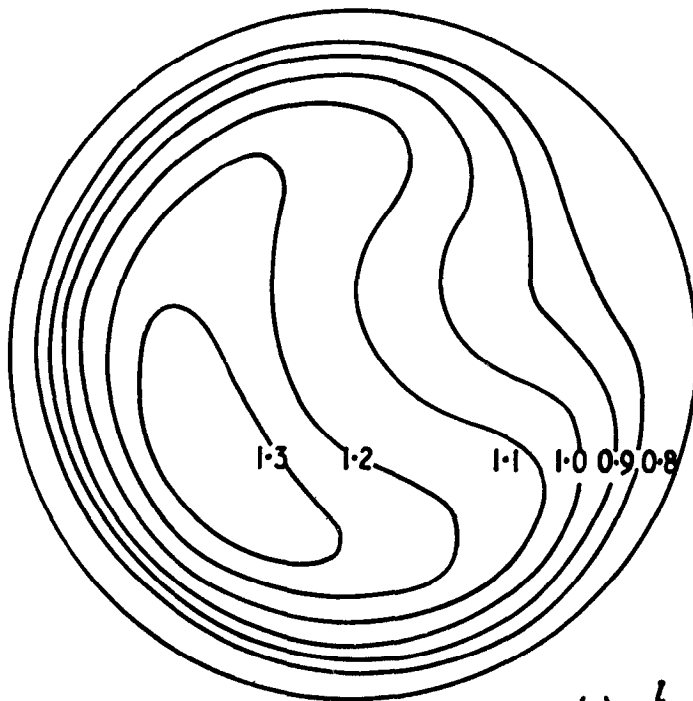


PIPE (1), $\frac{L}{D} = 62$

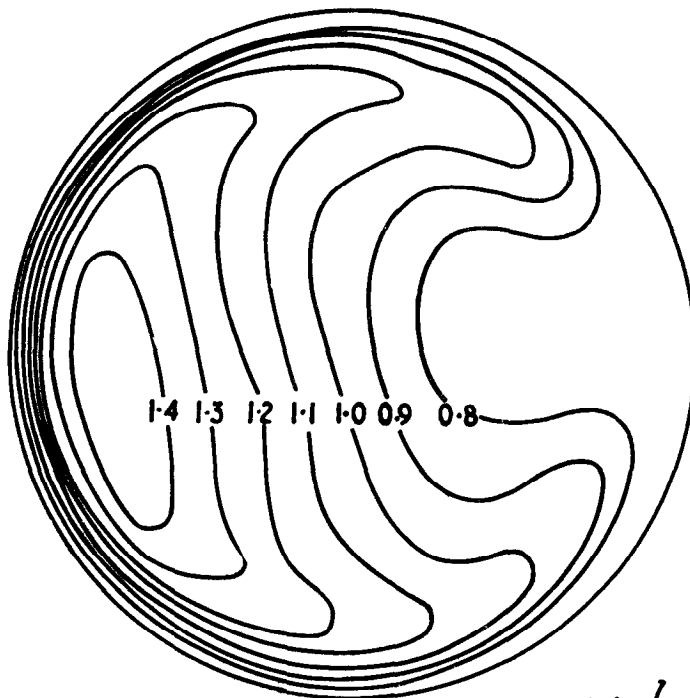


PIPE (2), $\frac{L}{D} = 30$

FIG. 11 (a). LINES OF CONSTANT TOTAL PRESSURE AT PIPE OUTLET

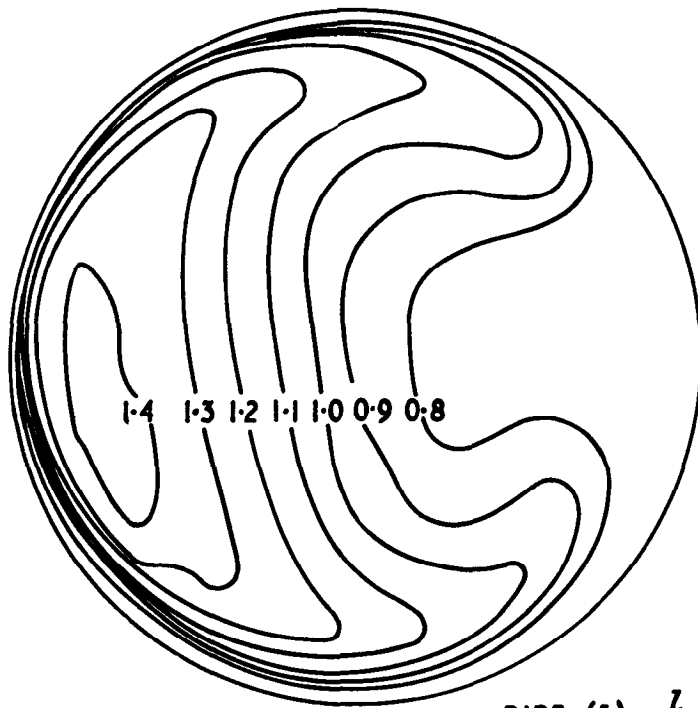


PIPE (3), $\frac{l}{d} = 17$

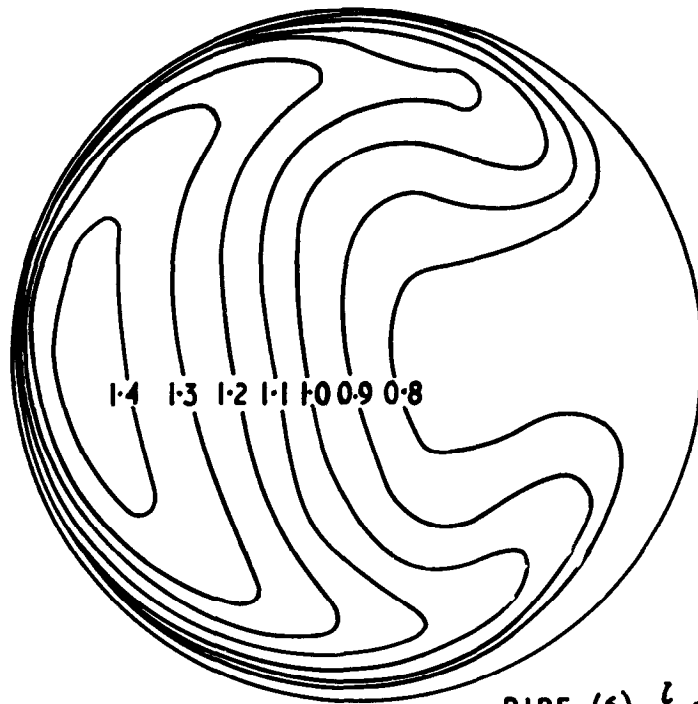


PIPE (4) $\frac{l}{d} = 3$

FIG. 11(b) LINES OF CONSTANT TOTAL PRESSURE AT PIPE OUTLET



PIPE (5), $\frac{z}{d} = 1$



PIPE (6) $\frac{z}{d} = 0$

FIG. 11(c) LINES OF CONSTANT TOTAL PRESSURE AT PIPE OUTLET

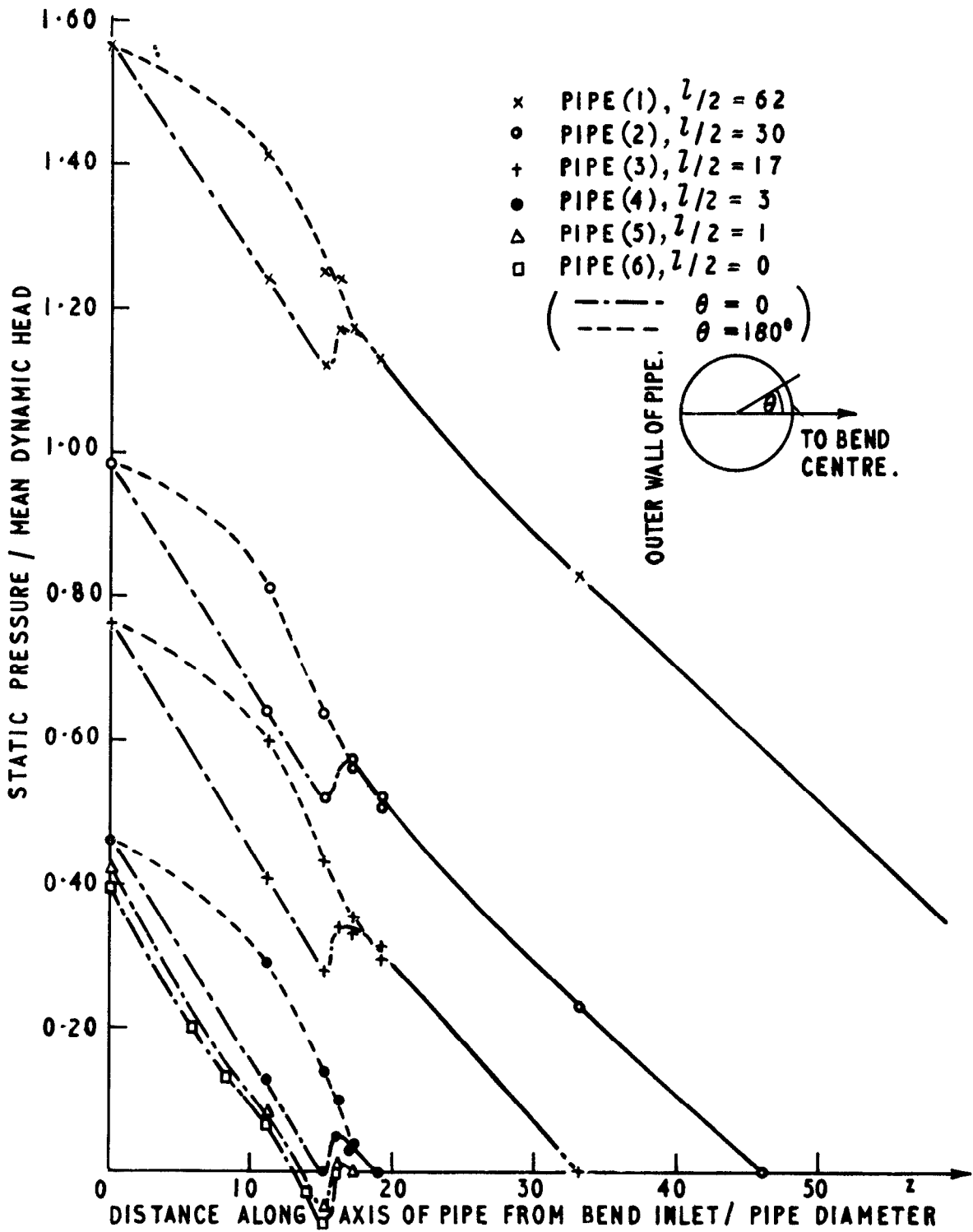


FIG.12 (a) STATIC PRESSURES FOR CONSTANT θ , ($\theta=0$ AND 180°)-
 70° BEND.

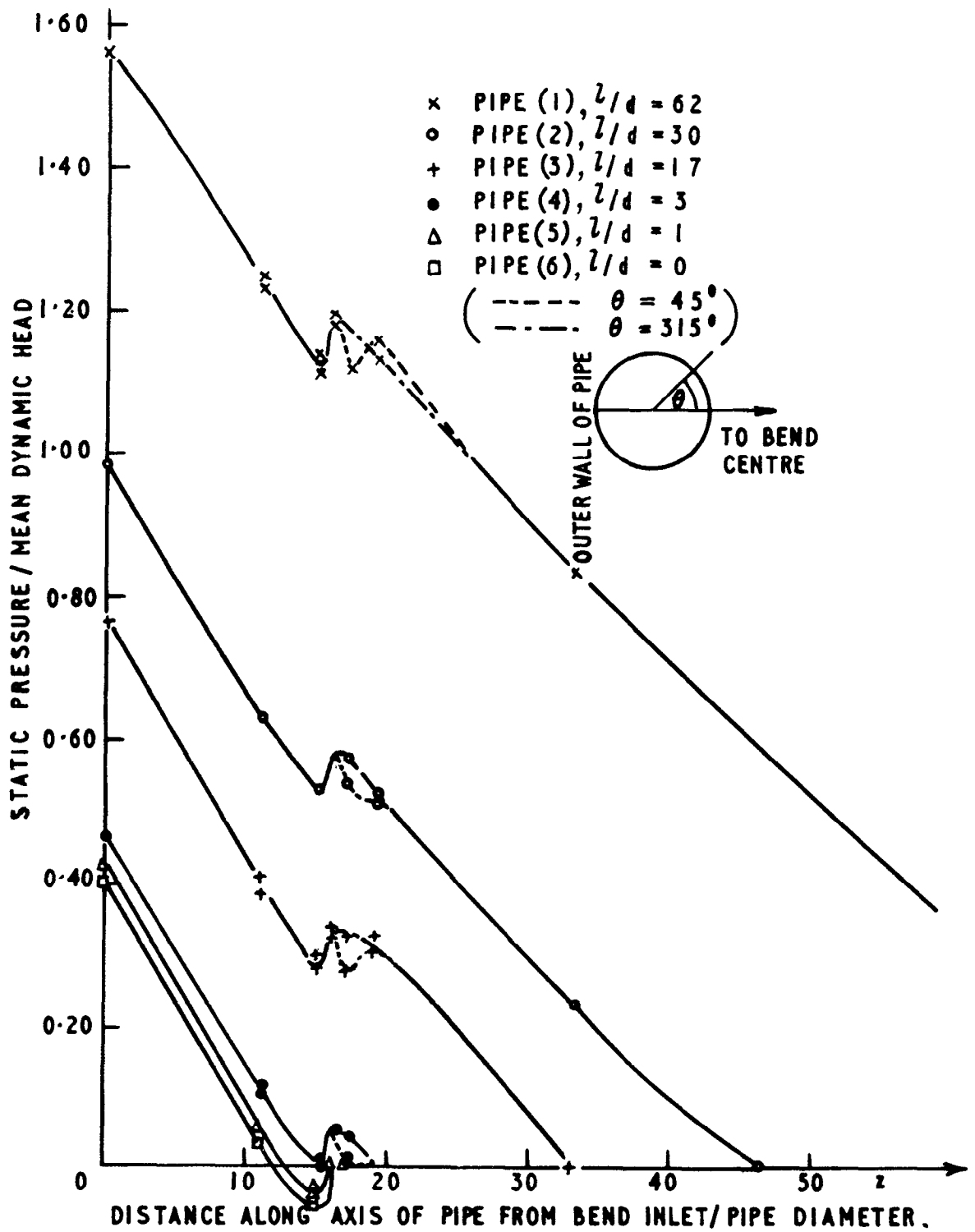


FIG.12(b) STATIC PRESSURES FOR CONSTANT θ ($\theta = 45^\circ$ AND 315°) - 70° BEND.

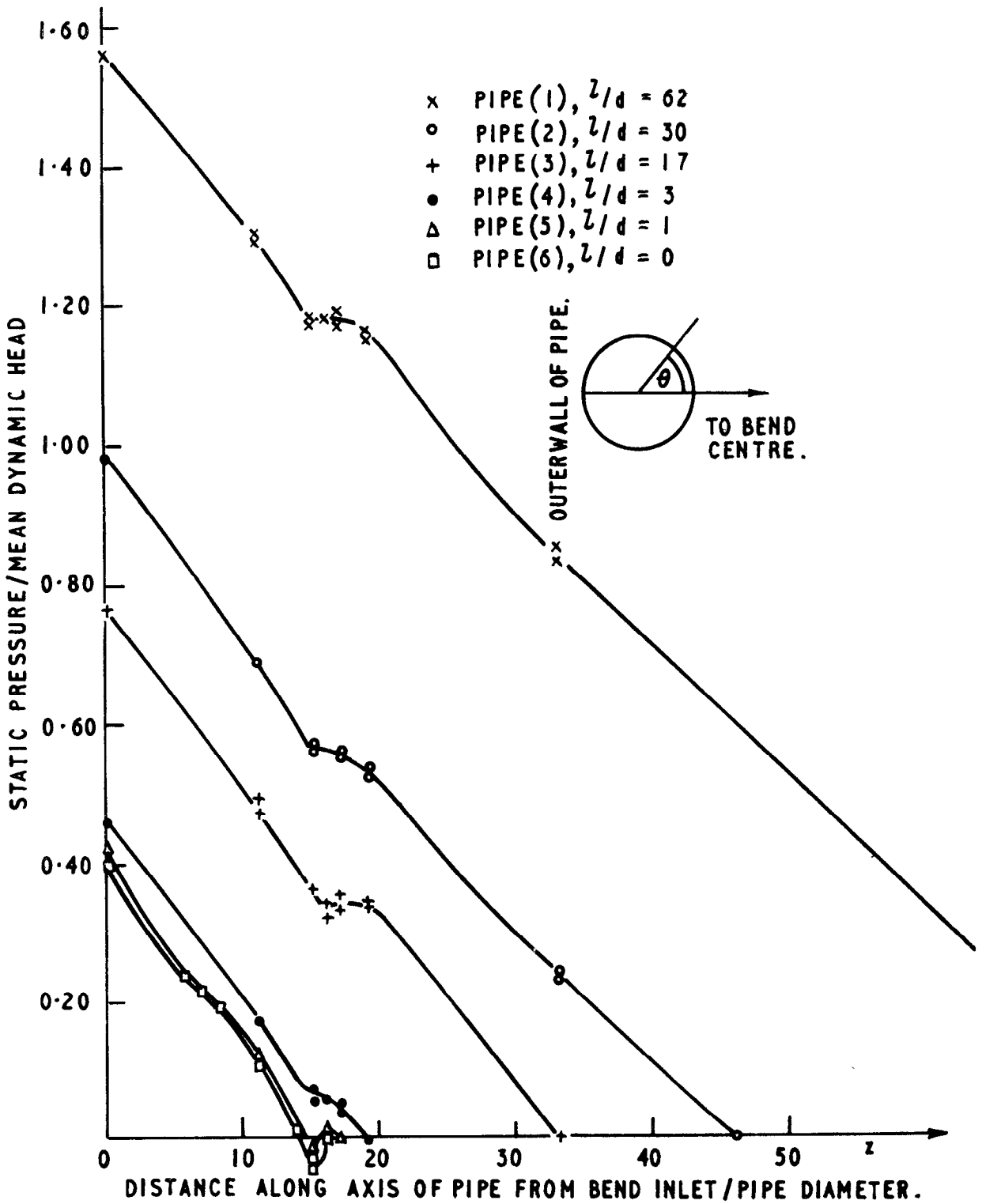


FIG. 12(c) STATIC PRESSURES FOR CONSTANT θ , ($\theta=90^\circ$ AND 270°)—
 70° BEND.

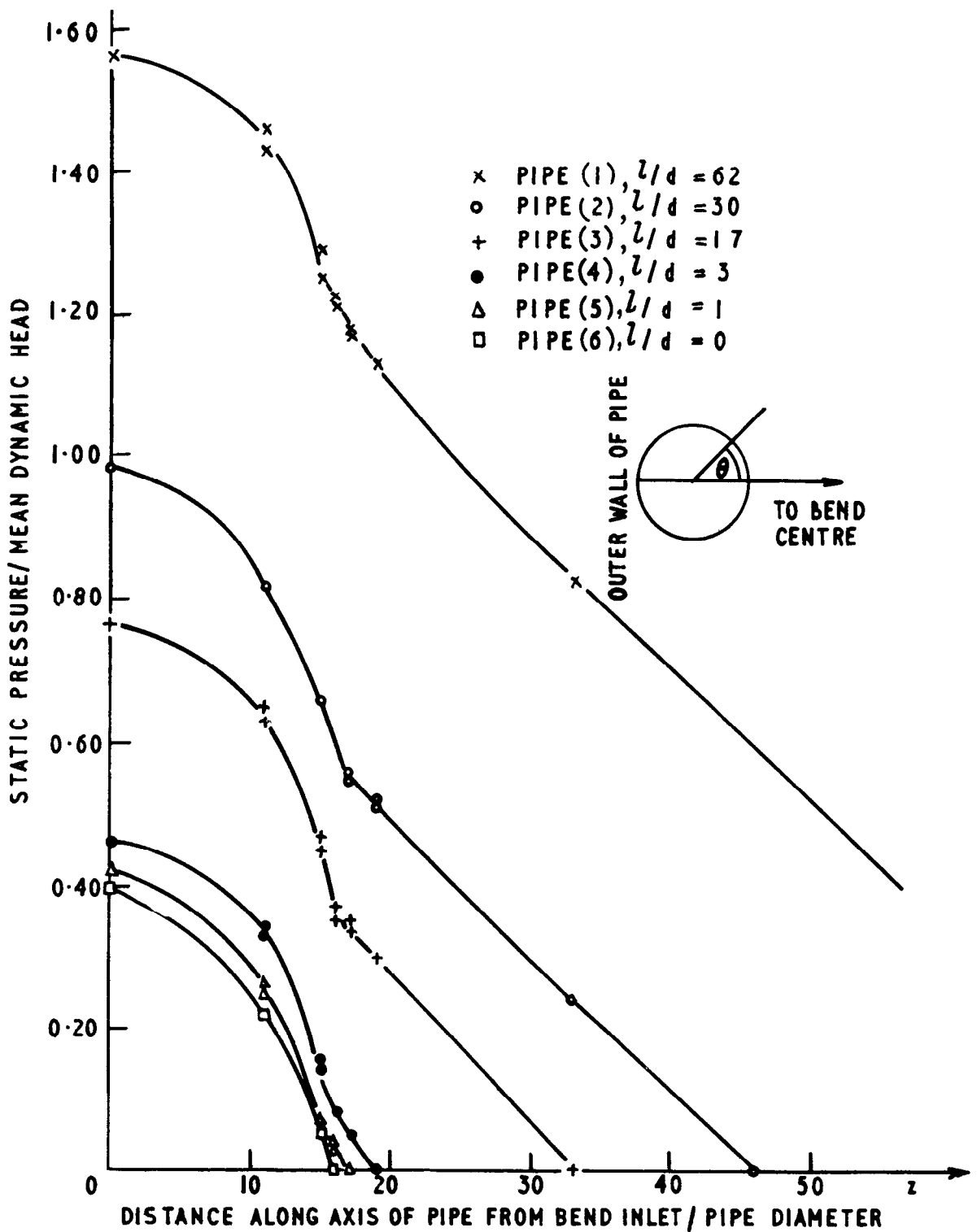


FIG.12(d) STATIC PRESSURES FOR CONSTANT θ ($\theta = 135^\circ$ AND 225°) —
70° BEND

- x PIPE (1), $L/d = 62$
- o PIPE (2), $L/d = 30$
- + PIPE (3), $L/d = 17$
- Δ PIPE (5), $L/d = 1$

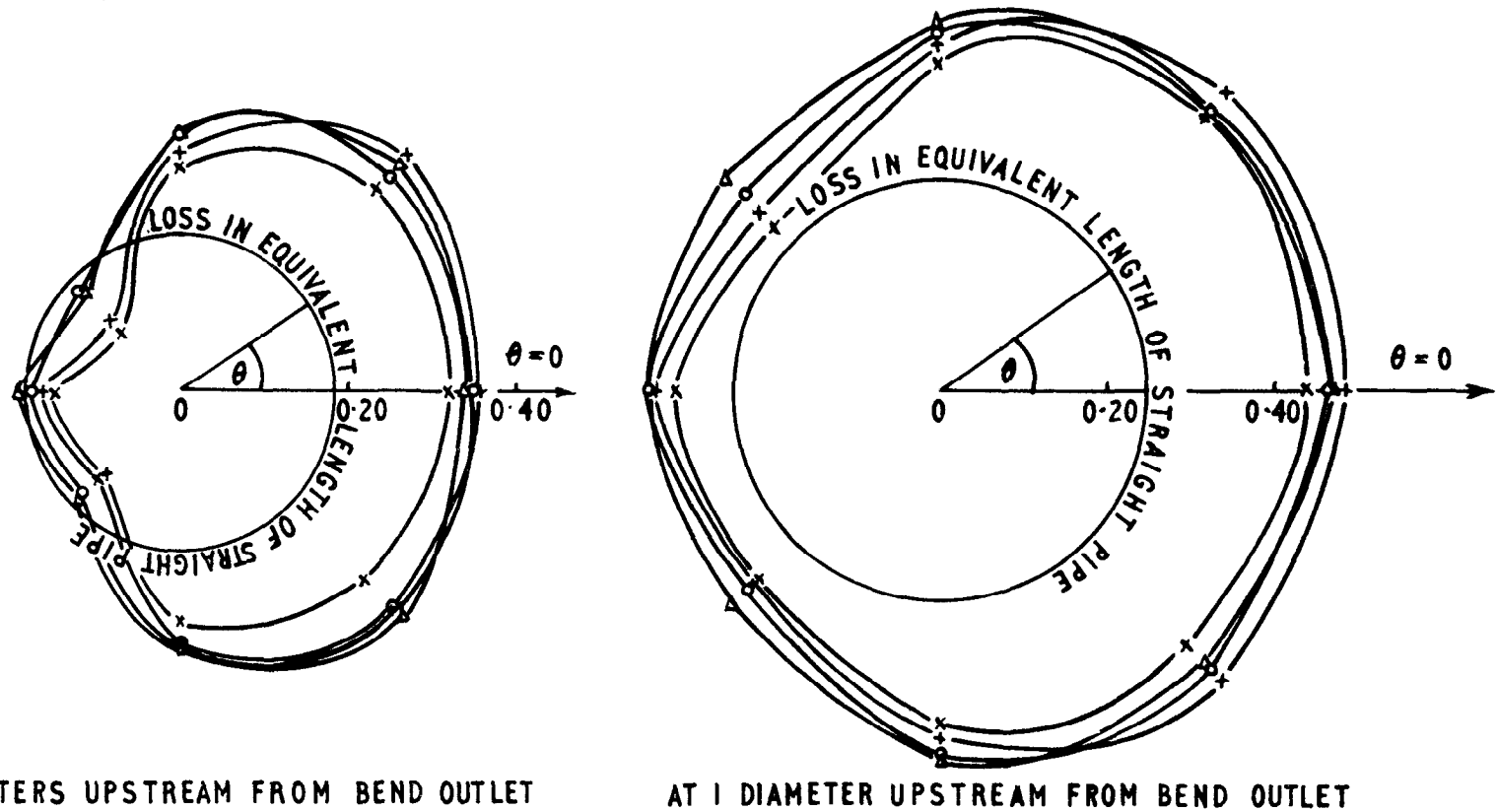
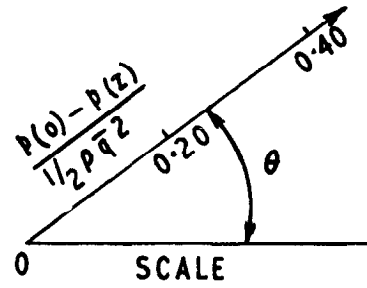
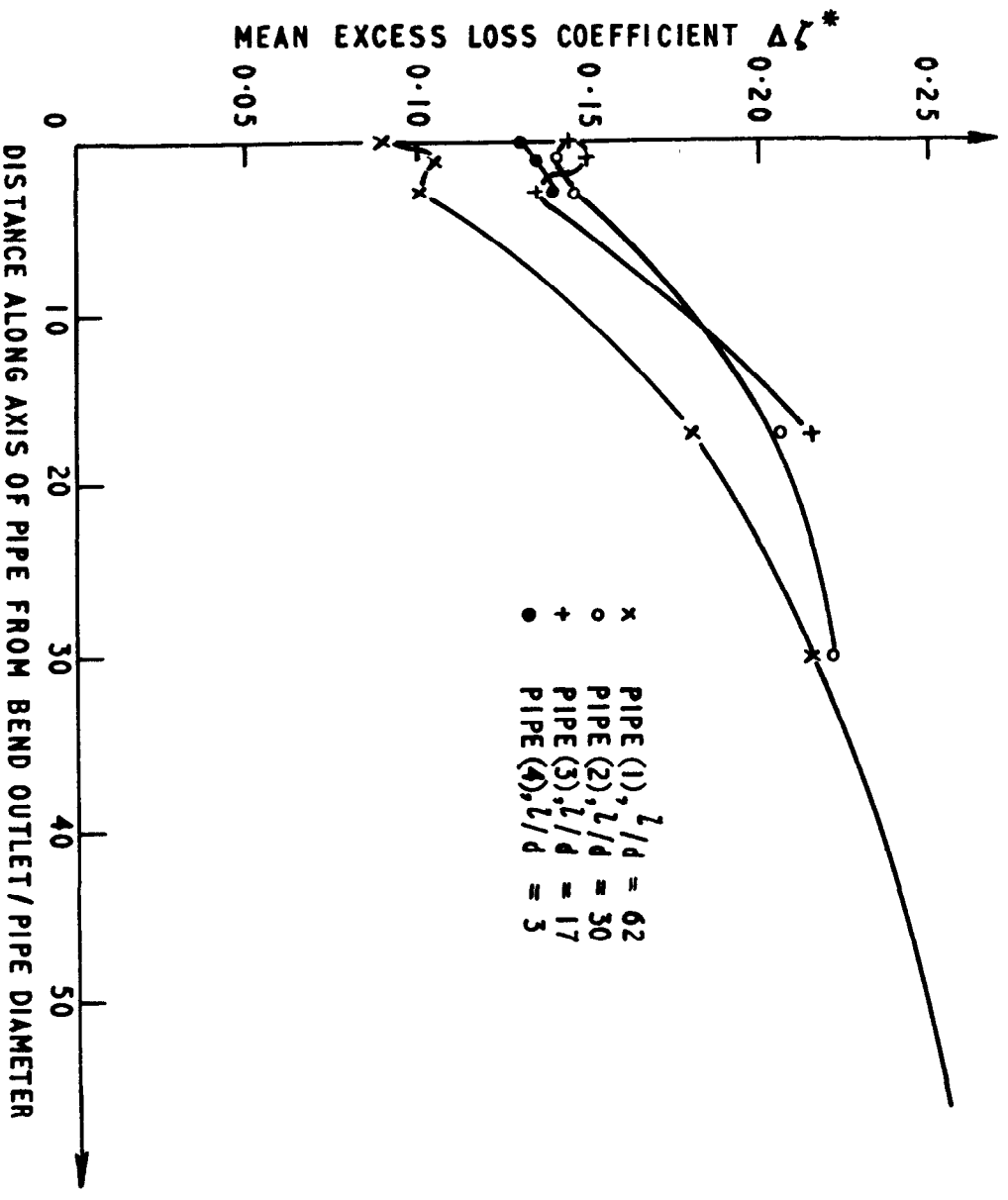


FIG.13 COMPARISON OF WALL STATIC PRESSURE LOSSES PLOTTED AS A FUNCTION OF θ .



**FIG.14 VARIATION OF PRESSURE LOSS WITH POSITION
DOWNSTREAM OF BEND OUTLET.**

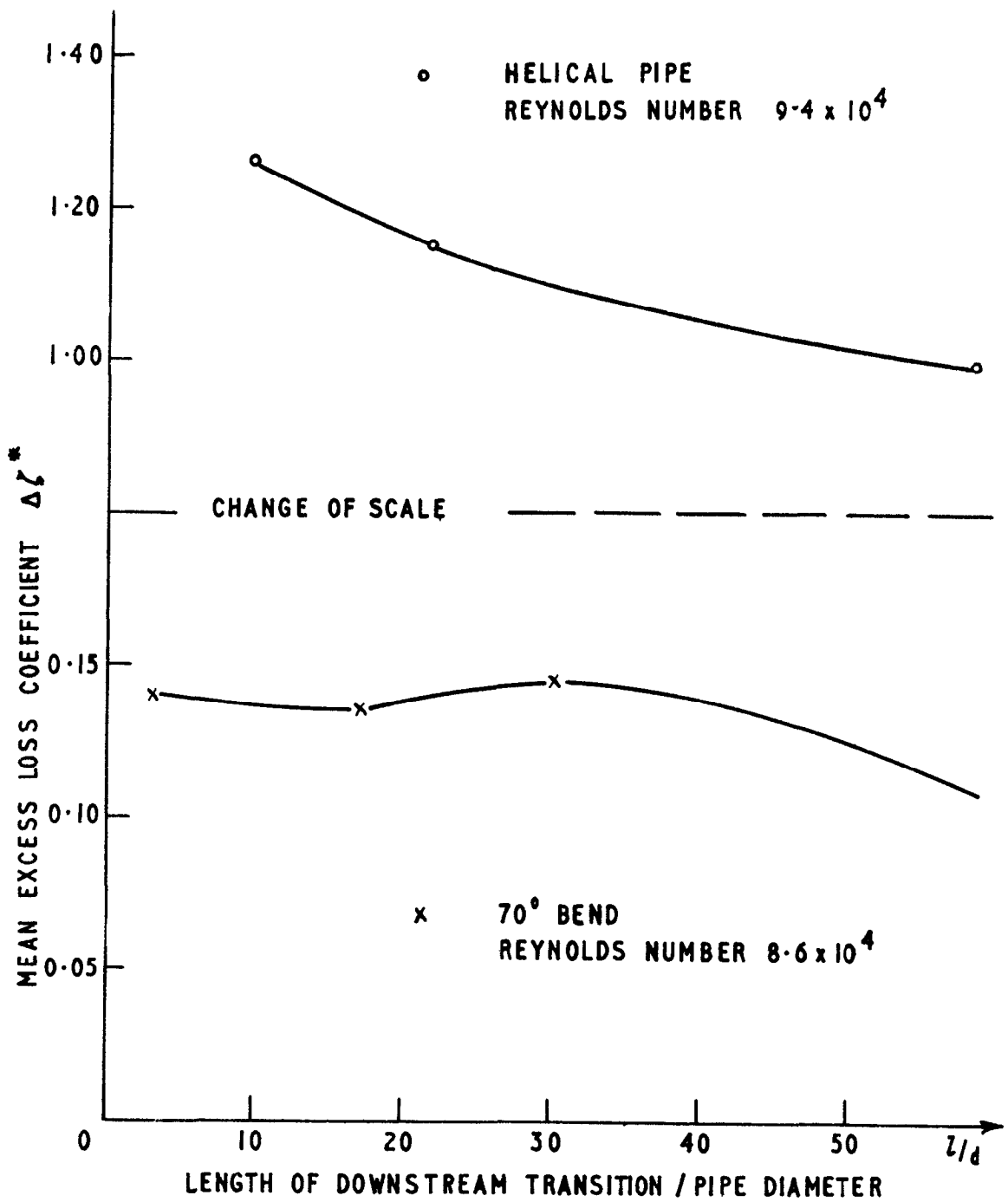


FIG. 15 VARIATION OF PRESSURE LOSSES WITH TRANSITION LENGTH (EVALUATED 3 DIAMETERS DOWNSTREAM OF BEND OUTLET.)

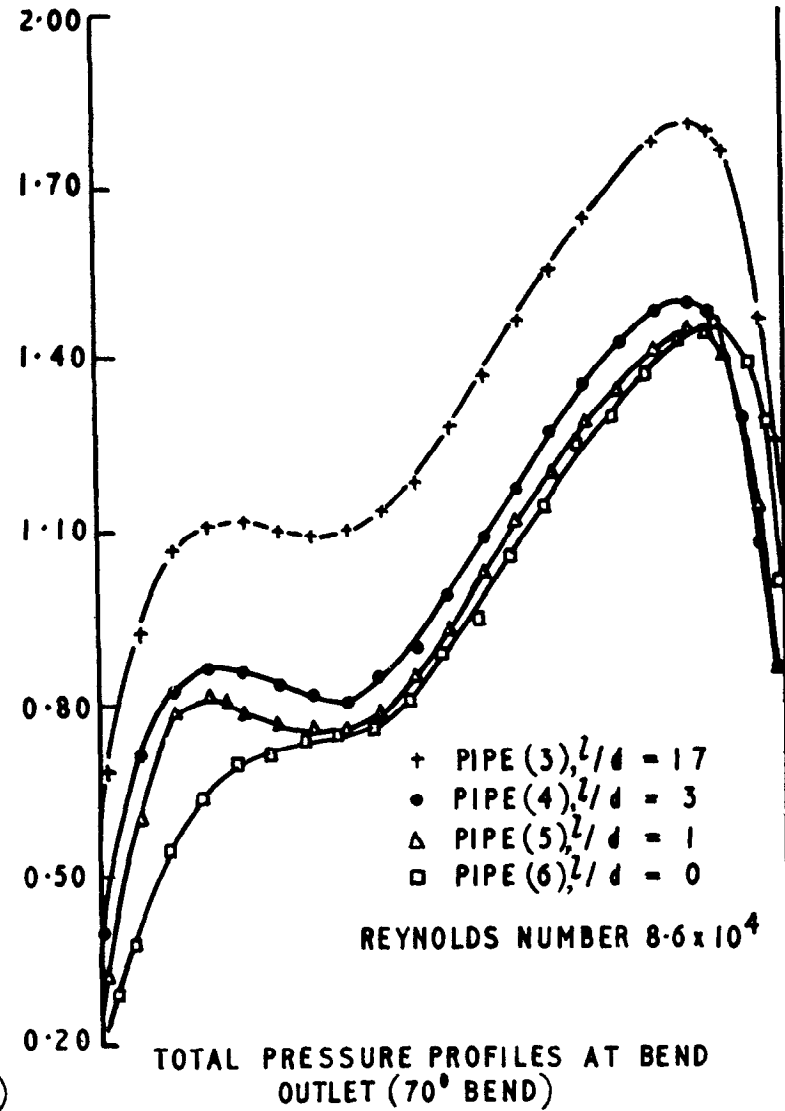
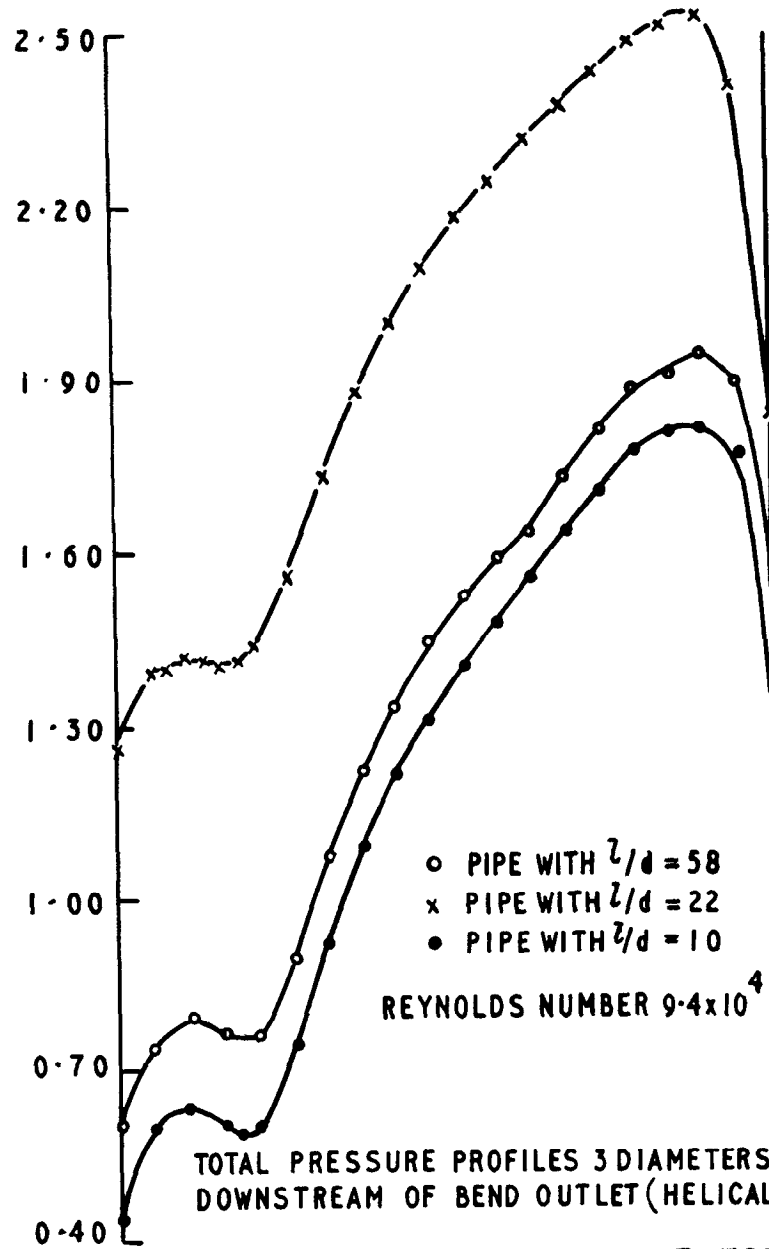


FIG. 16 VARIATION OF TOTAL PRESSURE PROFILES WITH TRANSITION LENGTH.

C.P. No. 505
(19,502)

A.R.C. Technical Report

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