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Interference on Characteristics of Aerofoil in Wind Tunnel of Rectangular Section

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AERODYNAMIC SYMBOLS.

1. GENERAL

- m mass
- t time
- V resultant linear velocity
- Ω resultant angular velocity
- ρ density, σ relative density
- ν kinematic coefficient of viscosity
- R Reynolds number, $R = lV/\nu$ (where l is a suitable linear dimension), to be expressed as a numerical coefficient x 10^6

Normal temperature and pressure for aeronautical work are 15° C. and 760 mm.

For air under these conditions $\begin{cases} \rho = 0{\cdot}002378 \text{ slug/cu. ft.} \\ \nu = 1{\cdot}59 \times 10^{-4} \text{ sq. ft./sec.} \end{cases}$

The slug is taken to be 32·2 lb.-mass.

- α angle of incidence
- ϵ angle of downwash
- S area
- c chord
- s semi-span
- A aspect ratio. $A = 4s^2/S$
- L lift, with coefficient $k_L = L/S\rho V^2$
- D drag, with coefficient $k_D = D/S\rho V^2$
- γ gliding angle, $\tan \gamma = D/L$
- L rolling moment, with coefficient $k_l = L/sS\rho V^2$
- M pitching moment, with coefficient $k_m = M/cS\rho V^2$
- N yawing moment, with coefficient $k_n = N/sS\rho V^2$

2. AIRSCREWS

- n revolutions per second
- D diameter
- J V/nD
- P power
- T thrust, with coeff
- Q torque, with coeff
- η efficiency, $\eta = T$

THE INTERFERENCE ON THE CHARACTERISTICS OF AN AEROFOIL IN A WIND TUNNEL OF RECTANGULAR SECTION.

By H. GLAUERT, F.R.S.

Communicated by the Director of Scientific Research, Air Ministry.

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20*th February*, 1932*.

Summary.—Approximate formulae for the interference on an aerofoil in a rectangular wind tunnel have been known and used for several years. More accurate formulae have been developed by Terazawa and Rosenhead, but their results are given in very complicated forms which are unsuitable for numerical computation. In this paper Rosenhead's formulae are reduced to a more convenient form and numerical results are derived for square and Duplex wind tunnels. The correction to the approximate formula is comparatively unimportant for a square tunnel, but important for a Duplex tunnel.

1. *Introduction.*—Formulae for the interference on the characteristics of an aerofoil in a wind tunnel of rectangular section, based on the author's approximate theory (Ref. 1), have been known and used for several years. More recently attempts have been made to increase the accuracy of these formulae by a closer analysis of the problem. Terazawa (Ref. 2) has developed the analysis rigorously for an aerofoil with constant circulation across its span and has determined the mean value of the induced velocity experienced by the aerofoil. Rosenhead (Ref. 3) has repeated Terazawa's analysis for uniform loading, obtaining the same result but in a very different mathematical form, and he has also developed the corresponding analysis for an aerofoil with elliptic distribution of lift across the span. These authors have not deduced general numerical values from their formulae, and indeed Rosenhead's formulae are not suitable for numerical computation unless the span of the aerofoil is only a small fraction of the width of the tunnel. In this paper the formulae given by Terazawa and Rosenhead are examined and recast into a form suitable for direct numerical computation, and the numerical results are derived for the two shapes of practical interest, the square and the rectangle whose width is double its height.

* R.A.E. Report December, 1931.

2. *Uniform loading.*—Let v be the upward normal induced velocity, due to the constraint of the tunnel walls, at any point of the aerofoil, and let—

$$\frac{v}{V} = \eta \frac{L}{C \varrho V^2} = \eta \frac{S}{C} k_L \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (1)$$

where C is the area of the tunnel section, S the area of the aerofoil, and k_L its lift coefficient.

The formula of the approximate theory, as used at present, represents solely the value of v at the centre of the tunnel and is derived from the assumption of an aerofoil of very small span. This formula may be expressed as—

$$\eta_0 = \lambda \left\{ \frac{\pi}{12} + 2\pi \sum_{p=1}^{\infty} \frac{p\, q^p}{1 + q^p} \right\} \quad \cdots \quad \cdots \quad \cdots \quad (2)$$

where—

$$\left. \begin{aligned} \lambda &= h/b \\ q &= e^{-2\pi\lambda} \end{aligned} \right\} \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (3)$$

Thus λ is the ratio of height to breadth of the tunnel, and the two important practical values are :—

$$\left. \begin{aligned} &\text{Square } \lambda = 1 \qquad q = 0{\cdot}00187 \\ &\text{Duplex } \lambda = \tfrac{1}{2} \qquad q = 0{\cdot}04321 \end{aligned} \right\} \quad \cdots \quad \cdots \quad \cdots \quad (4)$$

It is then found that η_0 has the same value $0{\cdot}274$ for both tunnels.

Terazawa calculated the value of v at any point of the aerofoil, on the assumption of uniform loading, and deduced the mean value across the span. After a slight algebraic adjustment* his result may be expressed in the form—

$$\bar{\eta} = \frac{\lambda}{2\pi\sigma^2} \left\{ \log \frac{\pi\sigma}{\sin \pi\sigma} + \sum_{n=1}^{\infty} (-)^{n+1} \log \frac{1 - 2q^n \cos 2\pi\sigma + q^{2n}}{(1 - q^n)^2} \right\} \quad (5)$$

where σ is the ratio of the span of the aerofoil to the width of the tunnel, or—

$$\sigma = \frac{2\,s}{b} \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (6)$$

Rosenhead, attacking the same problem on rather different lines, obtained a result which is indeed identical with Terazawa's but is expressed in a very different form as follows :—

$$\bar{\eta} = \frac{\lambda}{2\sigma} \sum_{n=0}^{\infty} \frac{f_{2n}}{(2n + 1)!} \left(\frac{\sigma}{2} \right)^{2n} \quad \cdots \quad \cdots \quad \cdots \quad (7)$$

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* Terazawa's q is the square root of q as used by Rosenhead and throughout this paper.

where—

$$f_{2n} = \sum_{r=0}^{\infty} \frac{Q_{n+r}}{(2r+1)!}\left(\frac{\sigma}{2}\right)^{2r+1} \quad \cdots \quad \cdots \quad \cdots \qquad (8)$$

and*—

$$\left.\begin{aligned} Q_m &= \frac{2}{\pi}(2m+1)!\,P_m + (-)^m (2\pi)^{2m+1} R_m \\ P_m &= \sum_{p=1}^{\infty} p^{-2(m+1)} \\ R_m &= \sum_{p=1}^{\infty} \frac{4p^{2m+1} q^p}{1+q^p} \end{aligned}\right\} \qquad (9)$$

Both expressions (5) and (7) for the value of $\bar{\eta}$ are highly complex and inconvenient for numerical computation, but Rosenhead's formulae can be reduced without difficulty to a more convenient form. After substituting the expression for f_{2n} and collecting the coefficient of σ^{2n}, we obtain—

$$= \frac{\lambda}{4} \sum_{m=0}^{\infty} \left(\frac{\sigma}{2}\right)^{2m} \sum_{n=0}^{m} \frac{Q_m}{(2n+1)!\,(2m-2n+1)!}$$

$$= \frac{\lambda}{2} \sum_{m=0}^{\infty} \frac{Q_m\,\sigma^{2m}}{(2m+2)!}$$

since—

$$\sum_{n=0}^{m} \frac{(2m+2)!}{(2n+1)!\,(2m-2n+1)!} = 2^{2m+1}$$

Now—

$$\sum_{m=0}^{\infty} \frac{P_m\,\sigma^{2m+2}}{m+1} = \log \frac{\pi\,\sigma}{\sin \pi\,\sigma}$$

and—

$$\begin{aligned} \sum_{m=0}^{\infty} (-)^m \frac{R_m\,(2\pi\,\sigma)^{2m+2}}{(2m+2)!} &= \sum_{p=1}^{\infty} \frac{4\,q^p}{p\,(1+q^p)} \sum_{m=0}^{\infty} (-)^m \frac{(2\pi\,p\,\sigma)^{2m+2}}{(2m+2)!} \\ &= \sum_{p=1}^{\infty} \frac{4\,q^p}{p\,(1+q^p)}\,(1 - \cos 2\pi\,p\,\sigma) \\ &= \sum_{p=1}^{\infty} \frac{8pq^p}{1+q^p}\left(\frac{\sin \pi\,p\sigma}{p}\right)^2 \end{aligned}$$

Hence finally—

$$\bar{\eta} = \frac{\lambda}{2\pi\,\sigma^2} \log \frac{\pi\,\sigma}{\sin \pi\,\sigma} + 2\pi\,\lambda \sum_{p=1}^{\infty} \frac{pq^p}{1+q^p}\left(\frac{\sin \pi\,p\sigma}{\pi\,p\sigma}\right)^2 \qquad (10)$$

and this form is suitable for direct numerical calculation with any value of σ owing to the rapid convergence of the terms of the series. When σ tends to zero, this formula gives the value η_0 of equation (2) as derived from the approximate analysis.

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* Correcting an obvious misprint in the index of $2\,\pi$ in equations (3) and (5) on page 143 of Rosenhead's paper, which gives $(2\,r+1)$ instead of $(2\,n+2\,r+1)$.

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3. *Elliptic loading.*—Rosenhead has also developed the analysis on the assumption of elliptic distribution of lift across the span of the aerofoil, and his results may be summarised as follows. At any point y of the span—

$$\eta = \frac{\lambda}{2\sigma} \sum_{n=0}^{\infty} \frac{l_{2n}}{(2n!)} \left(\frac{y}{b}\right)^{2n} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

where—

$$l_{2n} = \frac{4}{\pi} \int_0^{\pi/2} \sum_{r=0}^{\infty} \frac{Q_{n+r}}{(2r+1)!} \left(\frac{\sigma}{2}\right)^{2r+1} (\sin\theta)^{2r+2}\, d\theta$$

$$= \sum_{r=0}^{\infty} \frac{2\, Q_{n+r}}{r!\,(r+1)!} \left(\frac{\sigma}{4}\right)^{2r+1} \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

Writing—

$$y = s \sin\phi\,,$$

the mean value of η, weighted according to the lift distribution across the span, is

$$\bar{\eta} = \frac{4}{\pi} \int_0^{\pi/2} \eta \cos^2\phi\, d\phi$$

$$= \frac{2\lambda}{\pi\sigma} \int_0^{\pi/2} \sum_{n=0}^{\infty} \frac{l_{2n}}{(2n)!} \left(\frac{\sigma}{2}\right)^{2n} (\sin\phi)^{2n} (\cos\phi)^2\, d\phi$$

$$= \frac{2\lambda}{\pi\sigma} \int_0^{\pi/2} \sum_{n=0}^{\infty} \frac{l_{2n}}{(2n+1)!} \left(\frac{\sigma}{2}\right)^{2n} (\sin\phi)^{2n+2}\, d\phi$$

$$= \frac{\lambda}{2\sigma} \sum_{n=0}^{\infty} \frac{l_{2n}}{n!\,(n+1)!} \left(\frac{\sigma}{4}\right)^{2n} \ldots \quad \ldots \quad \ldots \quad (13)$$

In order to reduce this result to a more convenient form it is convenient to consider separately the parts of $\bar{\eta}$ due to the coefficients P_m and R_m. Substituting for l_{2n} and collecting the coefficient of σ^{2m}, the first part of $\bar{\eta}$ becomes—

$$\bar{\eta}_1 = \frac{\lambda}{2\pi} \sum_{m=0}^{\infty} P_m \left(\frac{\sigma}{4}\right)^{2m} \sum_{n=0}^{m} \frac{(2m+1)!}{n!\,(n+1)!\,(m-n)!\,(m-n+1)!}$$

and after summation with respect to n we obtain—

$$\bar{\eta}_1 = \frac{\lambda}{2\pi} \sum_{m=0}^{\infty} \frac{(2m+1)!\,(2m+2)!}{m!\,(m+1)!\,(m+1)!\,(m+2)!} P_m \left(\frac{\sigma}{4}\right)^{2m} \ldots \quad (14)$$

The series in this expression is a function of σ only and converges with reasonable rapidity. The result may be expressed as—

$$\bar{\eta}_1 = \lambda\, F(\sigma) \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

and numerical values of $F(\sigma)$ are given in the accompanying table.

TABLE 1.

Values of $F(\sigma)$.

σ	$F(\sigma)$	σ	$F(\sigma)$
0	0·2618	0·5	0·280
0·1	0·2624	0·6	0·290
0·2	0·2645	0·7	0·304
0·3	0·2679	0·8	0·325
0·4	0·2730	0·9	0·358

Turning next to the second part of $\overline{\eta}$, involving the coefficients R_m, it is best to start with the expressions for $\overline{\eta}$ and l_{2n} as integrals. Collecting the coefficient of σ^{2m}, we obtain—

$$\overline{\eta}_2 = \frac{8\lambda}{\pi}\int_0^{\pi/2}\int_0^{\pi/2}\sum_{m=0}^{\infty}(-)^m R_m(\pi\sigma)^{2m} \times \sum_{n=0}^{m}\frac{(\sin\theta)^{2m-2n+2}(\sin\phi)^{2n+2}}{(2n+1)!\,(2m-2n+1)!}\,d\theta\,d\phi$$

$$= \frac{4\lambda}{\pi}\int_0^{\pi/2}\int_0^{\pi/2}\sum_{m=0}^{\infty}(-)^m\frac{R_m(\pi\sigma)^{2m}}{(2m+2)!} \times\{(\sin\theta+\sin\phi)^{2m+2}-(\sin\theta-\sin\phi)^{2m+2}\}\sin\theta\sin\phi\,d\theta\,d\phi$$

But from the analysis of the previous section—

$$\sum_{m=0}^{\infty}(-)^m\frac{R_m x^{2m+2}}{(2m+2)!} = \sum_{p=1}^{\infty}\frac{8p\,q^p}{1+q^p}\left(\frac{\sin\frac{1}{2}p\,x}{p}\right)^2$$

and hence—

$$\overline{\eta} = \frac{32\,\lambda}{\pi^3\sigma^2}\int_0^{\pi/2}\int_0^{\pi/2}\sum_{p=1}^{\infty}\frac{q^p}{1+q^p}\,\frac{W}{p}\sin\theta\sin\phi\,d\theta\,d\phi$$

where—

$$W = \sin^2\{\tfrac{1}{2}\pi p\sigma(\sin\theta+\sin\phi)\} - \sin^2\{\tfrac{1}{2}\pi p\sigma(\sin\theta-\sin\phi)\}$$
$$= \sin(\pi p\sigma\sin\theta)\sin(\pi p\sigma\sin\phi)$$

Also—

$$\int_0^{\pi/2}\sin(\pi p\sigma\sin\theta)\sin\theta\,d\theta = \frac{\pi}{2}J_1(\pi p\sigma)$$

where J_1 is the Bessel function of the first order, and thus the final expression for $\overline{\eta}_2$ becomes—

$$\overline{\eta}_2 = 8\pi\lambda\sum_{p=1}^{\infty}\frac{p\,q^p}{1+q^p}\left\{\frac{J_1(\pi p\sigma)}{\pi p\sigma}\right\}^2 \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

On comparing this result with the second part of formula (10), which gives the value of $\bar{\eta}$ for uniform loading, it will be noticed that the only change is to replace $\sin \pi p \sigma$ by $2 J_1 (\pi p \sigma)$. The series of $\bar{\eta}_2$ converges rapidly and can be easily evaluated. Numerical values of the required Bessel function are given in the accompanying table.

TABLE 2.

Values of $\{ J_1 (\pi \sigma)/\pi \sigma \}^2$

σ	$\{J_1(\pi\sigma)/\pi\sigma\}^2$	σ	$\{J_1(\pi\sigma)/\pi\sigma\}^2$
0	0·250	0·6	0·095
0·1	0·244	0·7	0·064
0·2	0·227	0·8	0·038
0·3	0·200	0·9	0·020
0·4	0·167	1·0	0·008
0·5	0·130	1·2	0·000

The final value of $\bar{\eta}$, due to elliptic loading, is—

$$\bar{\eta} = \lambda \, F(\sigma) + 2\pi \lambda \sum_{p=1}^{\infty} \frac{p \, q^p}{1 + q^p} \left\{ \frac{J_1 (\pi p \sigma)}{\pi p \sigma} \right\}^2 \quad \ldots \quad (17)$$

and numerical values can be derived by assistance of the subsidiary tables 1 and 2.

4. *Discussion of results.*—The formulae (10) and (17) can be used to derive values of $\bar{\eta}$ for any rectangular wind tunnel, but the only two shapes of practical interest at present are the square ($\lambda = 1$) and the Duplex ($\lambda = \frac{1}{2}$) tunnels. Numerical values for these two tunnels are given in Table 3 and are shown graphically in the figure. The difference between the values deduced from the assumptions of uniform and elliptic distributions of lift is not great, but there is an important difference between the two shapes of tunnel. The value of $\bar{\eta}$ increases with the span of the aerofoil in a square tunnel, the increase being 12 per cent. of the initial value for a span of $0{\cdot}7\,b$ and elliptic loading, whilst in a Duplex tunnel the value of $\bar{\eta}$ decreases and reaches a minimum value, 33 per cent. below the initial value, for a span slightly less than $0{\cdot}8\,b$.

The corrections to the aerodynamic characteristics of an aerofoil tested in a wind tunnel, to deduce those which would occur in free air, are expressed conveniently as—

$$\left.\begin{aligned} \Delta \, \alpha &= \delta \frac{S}{C} k_L \\ \Delta \, k_D &= \delta \frac{S}{C} k_L^2 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

and it has been shown in another paper (Ref. 4) that the best value to assign to the coefficient δ is the weighted mean value of η calculated on the assumption of elliptic distribution of lift across the span of the aerofoil. These corrections are small in all practical applications and it usually suffices to know them with an accuracy of $\pm$ 20 per cent. On this basis the approximate value of δ (0·274) would suffice in a square tunnel, since the span of the aerofoil will not exceed 80 per cent. of the width of the tunnel, but in a Duplex tunnel it is necessary to take account of the decrease of the value of δ revealed by the more detailed analysis, and in all cases the best course is to take for δ the value of $\bar{\eta}$, at the appropriate value of $2\,s/b$, on the basis of elliptic distribution of lift.

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3. L. Rosenhead.—The effect of wind tunnel interference on the characteristics of an aerofoil. Roy. Soc. Proc. (A) 129, p. 135 (1930).

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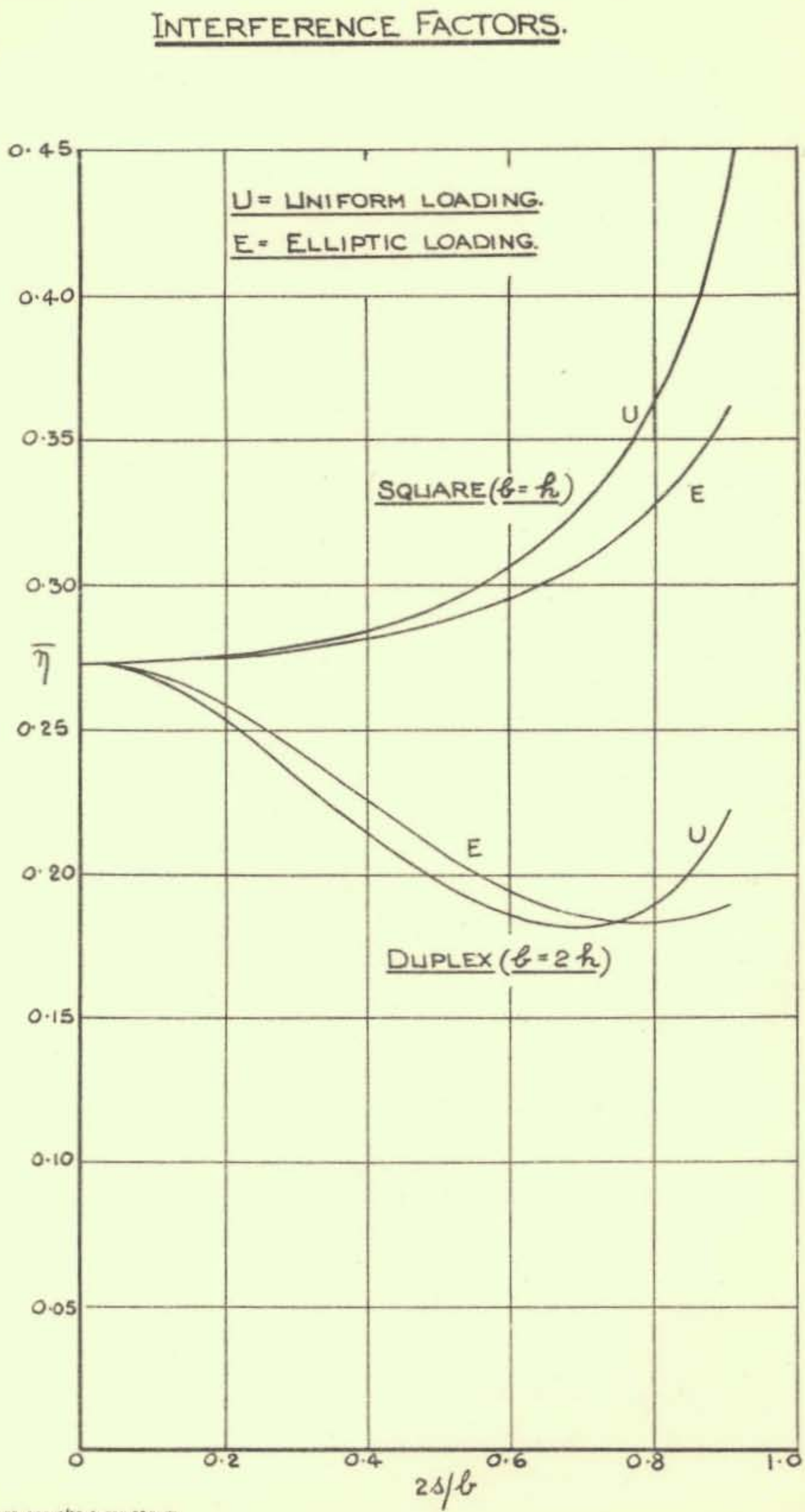
TABLE 3.

Values of $\bar{\eta}$.

$2\,s/b$.	$b = h$.		$b = 2\,h$.	
	Uniform.	Elliptic.	Uniform.	Elliptic.
0	0·274	0·274	0·274	0·274
0·2	0·276	0·275	0·254	0·258
0·4	0·284	0·281	0·214	0·225
0·5	0·292	0·286	0·197	0·208
0·6	0·305	0·295	0·185	0·194
0·7	0·326	0·307	0·181	0·185
0·8	0·362	0·327	0·188	0·183
0·9	0·435	0·359	0·219	0·189



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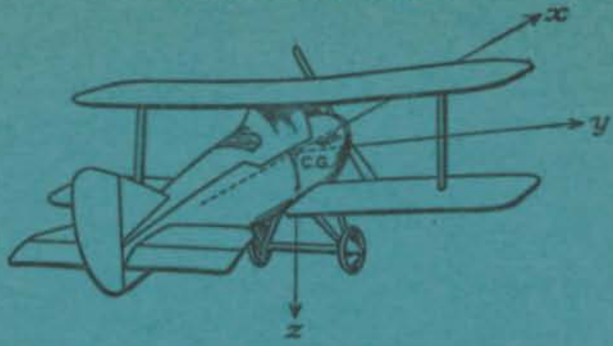
SYSTEM OF AXES.



Axes	Symbol Designation. Positive direction	x longitudinal forward	y lateral starboard	z normal downward
Force	Symbol	X	Y	Z
Moment	Symbol Designation	L rolling	M pitching	N yawing
Angle of Rotation	Symbol	ϕ	θ	ψ
Velocity	Linear Angular	u p	v q	w r
Moment of Inertia		A	B	C

Components of linear velocity and force are positive in the positive direction of the corresponding axis. Components of angular velocity and moment are positive in the cyclic order y to z about the axis of x, z to x about the axis of y, and x to y about the axis of z.

The angular movement of a control surface (elevator or rudder) is governed by the same convention, the elevator angle being positive downwards and the rudder angle positive to port. The aileron angle is positive when the starboard aileron is down and the port aileron is up. A positive control angle normally gives rise to a negative moment about the corresponding axis. The symbols for the control angles are :—

- ξ aileron angle
- η elevator angle
- η_T tail setting angle
- ζ rudder angle