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Upwash Interference in a Rectangular Wind Tunnel with Closed Side Walls and Porous Slotted Floor and Roof

By D. R. HOLDER, B.Sc.

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Summary.

Solutions have been obtained for the upwash interference on two-dimensional aerofoils, and small-span wings, in a rectangular wind tunnel with viscous flow through the slotted floor and roof.

A discussion is included on the application of results to tests on wings of moderate span.

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1. *Introduction.*

The problem of determining the flow interference due to the partially open walls of a wind tunnel has received a great deal of attention. Davis and Moore[1] dealt with many slotted-wall configurations, representing the flow at the walls by the homogeneous boundary condition $\phi + K(\partial\phi/\partial n) = 0$, where K is a function of slot geometry. In order to take viscous effects in the slots into account, Baldwin and Turner[2] introduced a further term, and derived the boundary condition

$$\frac{\partial \phi}{\partial x} + K \frac{\partial^2 \phi}{\partial x \, \partial n} + \frac{1}{R} \frac{\partial \phi}{\partial n} = 0 .$$

The parameter R is defined by the equation $\Delta p = (\rho U/R)(\partial\phi/\partial n)$ and is a measure of the porosity of the screens which are placed in the slots in many wind tunnels. The quantity $\rho U/R$ can be determined experimentally by measuring the mass flow and pressure drop through a sample of the screen under conditions corresponding to zero stream velocity.

Baldwin and Turner derived the blockage and upwash interference on a small-span wing in a circular wind tunnel, together with the blockage effect on a two-dimensional aerofoil in a rectangular wind tunnel, with slotted floor and roof. The present author has extended the analysis to deal with the upwash interference on a two-dimensional aerofoil, and on a small-span wing in the latter type of tunnel.

The application of results to evaluating corrections for wings with moderate span is discussed in Section 4.

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* Replaces the de Havilland Aircraft Co. Ltd., Wind Tunnel Report No. 91—A.R.C. 23,784.

2. *Upwash Interference on a Two-dimensional Aerofoil.*

The upwash interference on a two-dimensional aerofoil in a rectangular tunnel with solid side walls and slotted floor and roof is derived as follows:

The velocity potential due to a two-dimensional aerofoil in free air is

$$\phi_1 = \frac{\Gamma}{2\pi} \tan^{-1} \frac{z}{x} \tag{1}$$

which transformed into a Fourier integral gives

$$\phi_1 = -\frac{\Gamma}{4} - \frac{\Gamma}{2\pi} \int_0^\infty \frac{e^{-qz}}{q} \sin qx \, dq \,, \quad z > 0 \,. \tag{2}$$

The interference velocity potential may be expressed in the form

$$\phi_2 = \frac{\Gamma}{2\pi} \int_0^\infty [\{A(q) \sin qx + B(q) \cos qx\} \sinh qz] \, dq \,. \tag{3}$$

The boundary condition which must be satisfied is

$$\left(\frac{\partial}{\partial x} + K \frac{\partial^2}{\partial x \, \partial z} + \frac{1}{R} \frac{\partial}{\partial z} \right) (\phi_1 + \phi_2) = 0 \,, \quad z = h \,. \tag{4}$$

On differentiating (2) and (3) and substituting in (4) the terms in $\sin qx$ and $\cos qx$ must vanish independently for all values of q. Hence

$$\left\{ (1 - Kq) e^{-qh} - q A(q) (\sinh qh + Kq \cosh qh) - \frac{1}{R} q B(q) \cosh qh \right\} \cos qx = 0$$

and

$$\left\{ \frac{1}{R} e^{-qh} - q B(q) (\sinh qh + Kq \cosh qh) + \frac{1}{R} q A(q) \cosh qh \right\} \sin qx = 0 \,.$$

Substituting q for qh and c for K/h and solving for $A(q)$ and $B(q)$ we have

$$A(q) = \frac{e^{-q} \left\{ (\sinh q + cq \cosh q)(1 - cq) - \left(\frac{1}{R^2} \cosh q \right) \right\}}{\left\{ \frac{q}{h} (\sinh q + cq \cosh q)^2 + \left(\frac{1}{R} \cosh q \right)^2 \right\}}$$

and

$$B(q) = \frac{1/R}{\frac{q}{h} \left\{ (\sinh q + cq \cosh q)^2 + \left(\frac{1}{R} \cosh q \right)^2 \right\}} \,.$$

On substituting for $A(q)$ and $B(q)$ in equation (3) the interference at any point can be obtained. In particular at the origin $x = z = 0$,

$$\frac{\partial \phi_2}{\partial z} = \frac{\Gamma}{2\pi} \int_0^\infty \frac{q}{h} B(q) \frac{dq}{h} = \frac{\Gamma}{2\pi} \int_0^\infty \frac{(1/R) dq}{h \left\{ (\sinh q + cq \cosh q)^2 + \left(\frac{1}{R} \cosh q \right)^2 \right\}} \,. \tag{5}$$

As $1/R \to 0$ in equation (5) a limiting process yields

$$\frac{\partial \phi_2}{\partial z} = \frac{+\Gamma}{4h(1+c)}. \tag{6}$$

Equation (6) is in agreement with the results of Ref. 1.

3. *Upwash Interference on a Small-Span Wing.*

The upwash interference on a three-dimensional model can be divided into two separate parts: (*a*) the effect of the solid side walls, which may be evaluated by means of a simple image system; (*b*) the effect of the slotted floor and roof.

For a small-span wing in a tunnel of width unity and height λ the latter effect is determined as follows:

The velocity potential due to a small-span wing in free air is

$$\phi_1 = -\frac{\Gamma s}{2\pi}\left\{1 + \frac{x}{(x^2+y^2+z^2)^{1/2}}\right\}\frac{z}{(y^2+z^2)} \tag{7}$$

which together with the images in the solid side walls gives

$$\begin{aligned}\phi_R &= -\frac{\Gamma s}{2\pi}\sum_{k=-\infty}^{+\infty}\left\{1 + \frac{x}{[x^2+(y+2k)^2+z^2]^{1/2}}\right\}\frac{z}{[(y+2k)^2+z^2]} \\ &= f_R(y, z) + g_R(x, y, z).\end{aligned} \tag{8}$$

We may express g_R in the form

$$g_R = -\frac{\Gamma s}{2\pi}\sum_{m=0}^{\infty}\int_0^{\infty} C_m(q)\sin qx \cos m\pi y \, dq \tag{9}$$

where

$$\alpha^2 = q^2 + m^2\pi^2.$$

Now

$$\frac{\partial g_R}{\partial x} = -\frac{\Gamma s}{2\pi}\sum_{k=-\infty}^{+\infty}\frac{z}{[x^2+(y+2k)^2+z^2]^{3/2}}.$$

Hence

$$\left.\begin{aligned}C_m(q) &= \frac{4p}{\pi}\int_0^1\int_0^{\infty}\sum_{k=-\infty}^{+\infty}\frac{z\cos qx\cos m\pi y}{q[x^2+(y+2k)^2+z^2]^{3/2}}\,dx\,dy \\ &= \frac{4p}{\pi}\int_0^1\sum_{k=-\infty}^{+\infty}\frac{zK_1[q\{(y+2k)^2+z^2\}^{1/2}]}{[(y+2k)^2+z^2]^{1/2}}\cos m\pi y\,dy\end{aligned}\right\}\begin{aligned}p &= \tfrac{1}{2}(m=0) \\ &= 1(m\neq 0)\end{aligned}$$

where K_1 is the modified Bessel function of the second kind order one.

The integral has been evaluated in Ref. 3 and gives

$$C_m(q) = \frac{2pe^{-\alpha z}}{q}.$$

In attempting to express f_R in the form

$$f_R = \frac{\Gamma s}{2\pi}\sum_{m=0}^{\infty}\int_0^{\infty} D_m(q)\cos qx\cos m\pi y\,dq$$

it is found that $D_m(q) = 0$, $q \neq 0$, and that the interference velocity potential must be of the form

$$\phi_s = f_s(y, z) + g_s(x, y, z)$$

where

$$g_s = \frac{\Gamma s}{2\pi} \sum_{m=0}^{\infty} \int_0^{\infty} 2p\{A_m(q)\sin qx + B_m(q)\cos qx\} \sinh \alpha z \cos m\pi y \, dq. \tag{10}$$

The boundary conditions which must now be satisfied are

$$\frac{\partial}{\partial z}(f_R + f_S) = 0\,, \quad z = h\,, \quad \frac{1}{R} \neq 0 \tag{11}$$

and

$$\left(\frac{\partial}{\partial x} + k\frac{\partial^2}{\partial x\,\partial z} + \frac{1}{R}\frac{\partial}{\partial z}\right)(g_R + g_S) = 0\,. \tag{12}$$

From equations (11) and (12) it appears that f_s represents the interference which would arise at the origin from a closed floor and roof, and g_s gives the difference from the interference in a completely closed wind tunnel.

On substituting for $C_m(q)$ and differentiating (9) and (10), substituting in (12), solving for $B_m(q)$ and putting $q = q\lambda$, $\alpha = \alpha\lambda$ and $c = k/\lambda$ we obtain

$$\begin{aligned} B_m(q) &= \frac{\alpha/qR}{\dfrac{q}{\pi}\left\{(\sinh\alpha + c\alpha\cosh\alpha)^2 + \left(\dfrac{\alpha}{qR}\cosh\alpha\right)^2\right\}} \\ \frac{\partial g_s}{\partial z} &= \frac{\Gamma s}{2\pi}\sum_{m=0}^{\infty}\int_0^{\infty}\frac{2p\alpha}{\lambda q}\,\frac{(\alpha/qR)dq}{\left\{(\sinh\alpha + c\alpha\cosh\alpha)^2 + \left(\dfrac{\alpha}{qR}\cosh\alpha\right)^2\right\}} \end{aligned} \tag{13}$$

at the origin $x = y = z = 0$.

The summation is rapidly convergent, and, putting $h = \lambda$ in equation (5), and remembering that s vanishes on computing δ from equation (13), it appears that the leading term in the summation produces a correction factor equal to that for tests on a two-dimensional aerofoil.

4. *Application of Results to Tests on Wings of Moderate Span.*

Upwash correction factors at the origin, have been computed numerically for two-dimensional aerofoils and for small-span wings ($\lambda = \frac{1}{2}$ and $\lambda = 1$), for various porosity and slot geometry parameters. Curves are presented in Figs. 1 and 2.

A method for determining the upwash correction factor for a wing in a rectangular wind tunnel, from the two-dimensional result, is described by Katzoff and Barger[4]. It appears from Section 3 of this report that, apart from the influence of the side walls, the correction factor for a small-span wing does, in fact, tend to the two-dimensional value, with increasing height/width ratio. However, this method is inadequate for small values of λ.

Now, the correction factor (δ_1) for a wing which spans the tunnel is obtained by adding $\lambda/2\pi$, i.e., the effect of the side walls, to the appropriate two-dimensional correction. An approximate correction factor for a wing of moderate span may be derived by linear interpolation between δ_1, and the small-span correction δ_0, i.e.,

$$\delta = \delta_0\left(1 - \frac{s}{b}\right) + \delta_1\frac{s}{b}. \tag{14}$$

5. *Concluding Remarks.*

The curves in Figs. 1 and 2 and the interpolation described in Section 4 of this report, yield the upwash correction factor at the origin for a wing in a rectangular wind tunnel with viscous flow through a slotted floor and roof.

The analysis of Sections 2 and 3 may also be used to derive camber corrections, and to determine the spanwise variation in upwash.

The solution for a porous wind tunnel is obtained by putting $c = 0$.

NOTATION

U	Freestream velocity
Γ	Vortex strength $= \dfrac{USC_L}{4s}$
C_L =	Lift/$\frac{1}{2}\rho U^2 S$
s	Wing semi-span
S	Wing area
b	Tunnel semi-width
h	Tunnel semi-height
C	Tunnel cross-sectional area $= 4bh$
λ	Tunnel height/width ratio $= h/b$
K	Slot geometry parameter $= \dfrac{d}{\pi}\log_e \operatorname{cosec}\dfrac{\pi r}{2}$ for a rectangular tunnel
δ	Upwash correction factor $= \Delta\alpha\,\dfrac{Ubh}{\Gamma s}$ for a wing
	$= \Delta\alpha\,\dfrac{Uh}{\Gamma}$ for a two-dimensional aerofoil
x, y, z	Cartesian co-ordinates ($x+ve$ downstream and $z+ve$ upwards)
d	Distance between slots
r	Open-area ratio of the tunnel floor and roof

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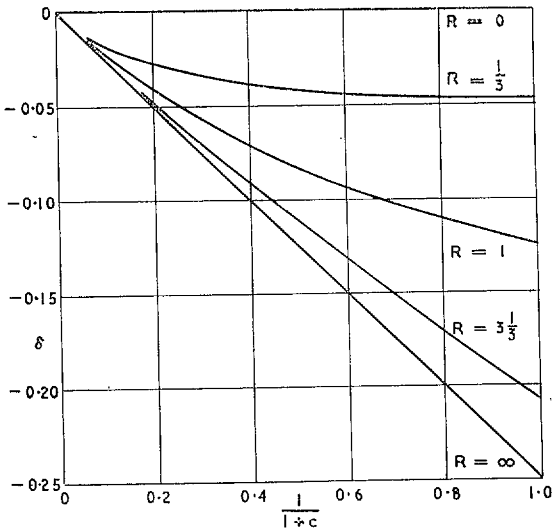

FIG. 1. The upwash correction factor for a two-dimensional aerofoil for various porosity and slot geometry parameters.

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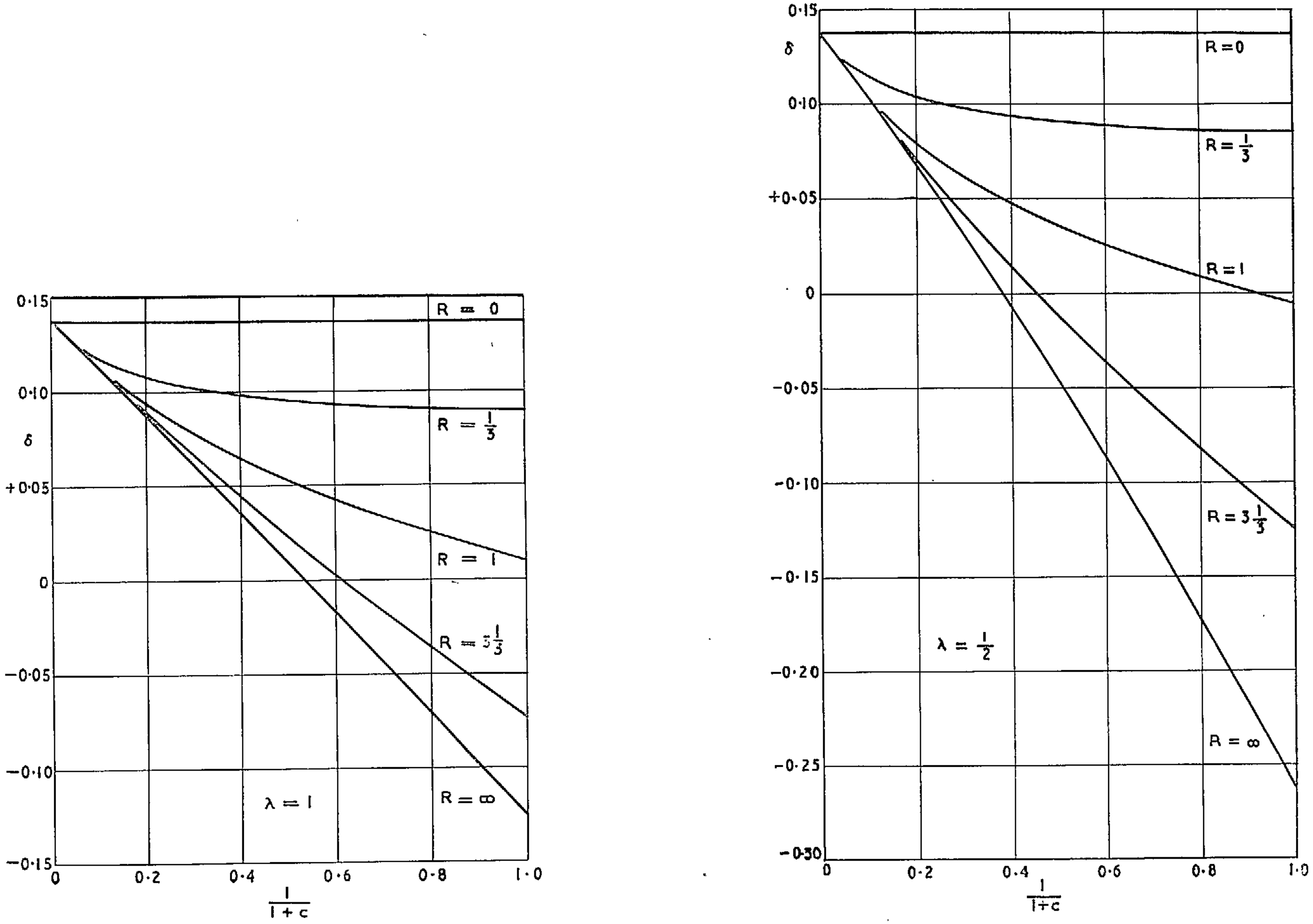

FIG. 2. The upwash correction factor for a small-span wing for various porosity and slot geometry parameters.

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