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Summary.—This report describes the design and construction of an electronic simulator for flutter investigations in any number of degrees of freedom not exceeding six. The principles of the simulation are discussed, and the actual circuits of the machine are described in some detail for those wishing to build similar machines.

The advantages and limitations of this method of solving flutter equations are considered, and other developments and applications are discussed.

1. *Introduction.*—Aircraft flutter is an important problem, and considerable effort is devoted to the theoretical prediction of flutter in order to avoid its occurrence in flight. Up to the present time the solution of the flutter equations has been by mathematical methods using desk calculating machines, but the process is tedious and slow and can involve many weeks of work to obtain a single solution. However, the development of high-speed digital computers promises a considerable reduction in the labour of solution by mathematical methods.

There is another approach to the solution of the flutter equations, that of analogy. There exists a direct analogy between the characteristics of structural components and electrical components. For example, stiffness of a structure may be compared with the impedance of a capacitor, inertia may be compared with the impedance of an inductor, and resistance to structural movement to the damping effect of an electrical resistor. In this manner a structure can be represented by an electrical circuit containing capacitor, inductor and resistor. The electrical oscillations of such a circuit correspond exactly to those of the structure.

Unfortunately, flutter does not involve only one mode of oscillation of the aircraft structure. If this were so the solution of flutter problems would be easy. Flutter, in fact, involves in principle all the possible modes of oscillation of the structure, and it is the aerodynamic coupling between these modes which leads to instability. However, the number of modes of oscillations of the structure, normally referred to as degrees of freedom, can by careful selection be reduced to a small number, usually three or four, without appreciably affecting the accuracy of the solution.

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R.A.E. Internal Memo. Structures 335, received 4th February, 1957.

If the direct analogy method is applied to the complete structural problem in, say, four degrees of freedom, difficulties arise. The coupling capacitors and inductors linking the separate degrees of freedom cannot be inserted without affecting the main circuits, and furthermore the inherent resistance in both capacitor and inductor leads to difficulty. Adjustment of the electrical components to represent the problem is easy in the case of resistors and capacitors, but not in the case of inductors.

Recent developments in electronic amplifiers have led to another approach to the flutter problem, that of indirect analogy. It is possible, by use of electronic amplifiers coupled with electric networks, to produce circuits that are analogues of the flutter equations. The method is similar to that employed in the mechanical differential analyser. Although the whole circuit is analogous to the whole equation of motion in, say, one degree of freedom, there is no direct analogy between any electrical component and any structural characteristic. This leads to some minor limitations of the method, but the advantages considerably outweigh the limitations.

A machine based on the indirect analogy method was proposed by Smith¹ in 1949 for the solution of flutter problems in six degrees of freedom. Since this approach to the flutter problem was novel, and involved many practical difficulties, it was decided to build first a simpler machine to solve problems in two degrees of freedom. This machine was built² and proved to be of considerable use in demonstrating the accuracy to be expected from the larger machine and in indicating the best methods of operation of such a flutter simulator.

Following the experience gained in this prototype machine the large simulator originally proposed¹ has been constructed and has been working satisfactorily since 1953. Considerable interest has been shown in it, both by the research worker in the flutter field and by the aircraft industry. The present report describes the construction and operation of the simulator and, to assist aircraft firms who may be considering the building of a simulator, the circuits are described in some detail.

2. General Principles of the Simulation.—2.1. The Flutter Problem to Be Represented.—The mechanical oscillation of an aircraft structure in a particular mode or degree of freedom may be represented by the differential equation:

$$A\ddot{x} + Ex = 0, \quad \dots \dots \dots (1)$$

where x is a typical displacement in the degree of freedom, A is the equivalent inertia and E the stiffness in the mode. In practice, damping will be present in the structure and if we can assume that the damping is proportional to the velocity of the motion then the complete equation, in still air, is

$$A\ddot{x} + D\dot{x} + Ex = 0, \quad \dots \dots \dots (2)$$

where D represents the proportionality of damping to velocity.

If the structure is subject to aerodynamic loads then there will be additional terms proportional to v^2x , $v\dot{x}$, \ddot{x} , etc., where v is proportional to the air speed. Making the assumption that aerodynamic terms of higher order than two may be neglected, we can write

$$A\ddot{x} + D\dot{x} + Ex + A'\ddot{x} + B\dot{x}v + Cxv^2 = 0, \quad \dots \dots \dots (3)$$

where A' , B , C are aerodynamic coefficients. A' is usually small compared with A and is by definition in flutter work invariable with frequency and air speed so that for simplicity it may be included in the mechanical inertia coefficient. Thus equation (3) becomes

$$A\ddot{x} + (Bv + D)\dot{x} + (Cv^2 + E)x = 0. \quad \dots \dots \dots (4)$$

This is the full equation in one degree of freedom.

If we consider now two separate degrees of freedom of the structure we have

$$A_{11}\ddot{x}_1 + (B_{11}v + D_{11})\dot{x}_1 + (C_{11}v^2 + E_{11})x_1 = 0, \quad \dots \dots \dots (5)$$

$$A_{22}\ddot{x}_2 + (B_{22}v + D_{22})\dot{x}_2 + (C_{22}v^2 + E_{22})x_2 = 0, \quad \dots \dots \dots (6)$$

as the two independent equations representing two independent degrees of freedom x_1 and x_2 . However, in practice, the two modes will not be independent and there will be both mechanical and aerodynamical couplings. We may therefore write the full equations in two degrees of freedom as

$$A_{11}\ddot{x}_1 + (B_{11}v + D_{11})\dot{x}_1 + (C_{11}v^2 + E_{11})x_1 + A_{12}\ddot{x}_2 + (B_{12}v + D_{12})\dot{x}_2 + (C_{12}v^2 + E_{12})x_2 = 0 \quad \dots \quad (7)$$

$$A_{21}\ddot{x}_1 + (B_{21}v + D_{21})\dot{x}_1 + (C_{21}v^2 + E_{21})x_1 + A_{22}\ddot{x}_2 + (B_{22}v + D_{22})\dot{x}_2 + (C_{22}v^2 + E_{22})x_2 = 0, \quad \dots \quad (8)$$

where, for example, $A_{12}\ddot{x}_2$ represents the inertia coupling in degree of freedom 1 due to the motion in degree of freedom 2. If equations (7) and (8) are represented as

$$f_{11}(x_1) + f_{12}(x_2) = 0, \quad \dots \quad (9)$$

$$f_{21}(x_1) + f_{22}(x_2) = 0, \quad \dots \quad (10)$$

then the complete set of six equations in six degrees of freedom are

$$\sum_{j=1}^{j=6} f_{ij}(x_j) = 0 \quad \dots \quad (11)$$

$i = 1 \text{ to } 6.$

These equations are more complete than is usually necessary, but to aid the interchangeability of electronic units in the simulator the complete equations are represented. Thus a total of 180 coefficients are contained in the six-degree-of-freedom flutter problem.

In practice v , which is proportional to air speed, may be expressed as the ratio of air speed V to a reference air speed V_0 , arbitrarily chosen so that v does not normally exceed unity.

On the simulator an extra term, F_i , is added on the right-hand side of each of the equations (11). This is equivalent to adding an additional force to the set of forces arising from and producing the motion in each degree of freedom, and this extra force is used to excite the motion simulated on the machine.

In practice F_i is one of two forms, either a fixed value applied instantaneously or removed instantaneously at the will of the operator (step function), or a sinusoidally varying value whose frequency can be chosen by the operator.

The first type of excitation is normally used to initiate free oscillation, and the second is used to investigate the response of the aircraft structure represented by the equations (11) to external excitation of a sinusoidal character (forced vibration on flight resonance test). Thus the complete equations represented on the simulator are

$$\sum_{j=1}^{j=6} f_{ij}(x_j) = F_i \quad \dots \quad (12)$$

$i = 1 \text{ to } 6.$

2.2. Limitations of the Simulator Representation.—Both the form and the order in which the equations are written are important. If we consider the equations

$$A\ddot{x} + Ex = 0, \quad \dots \quad (13)$$

$$-A\ddot{x} - Ex = 0, \quad \dots \quad (14)$$

then the mathematical solution is the same; they both represent a sinusoidal oscillation of frequency $(E/A)^{1/2}$. Physically, however, they are quite different. The first represents real inertia and stiffness, whereas physically the second equation represents an unreal problem, and since the simulator is an analogue machine it will fail to give a solution to the second equation.

In a similar manner the order in which the equations are written is important. If equation (9) represents the forces producing motion in degree of freedom 1, then $f_{11}(x_1)$ is the principal term. It cannot be regarded as an equation in degree of freedom 2 with $f_{12}(x_2)$ as the principal term; so that, if the equations (9) and (10) are reversed

$$\overline{f_{21}(x_1)} + f_{22}(x_2) = 0, \quad \dots \dots \dots (10)$$

$$f_{11}(x_1) + \overline{f_{12}(x_2)} = 0, \quad \dots \dots \dots (9)$$

with the principal terms marked by a bar, the problem is no longer real and the simulator will fail to give an answer.

These limitations, which do not exist with a high-speed computing machine, are not without their merits. They do indicate that the problem is real or unreal, and since in six degrees of freedom there are many possible ways in which the problem may inadvertently be made unreal during an exploratory variation of coefficients, the limitations can become an asset. If the Lagrangian approach to the problem is taken and Routh's stability criteria satisfied, the problem will be real and the simulator will give the correct answer.

2.3. Accuracy of the Representation.—The accuracy to which a machine such as the flutter simulator is built is a matter of considerable importance. It was decided to aim at an overall accuracy of 1 per cent on flutter speed for the well-conditioned problem, that is, the problem in which there is a marked change from stability to instability for a small change in air speed at the critical flutter speed. Electronic components can be selected to a considerable accuracy, but the stability of the components available for this work is, in general, no better than 0.1 per cent. The main difficulty is thus to maintain an overall accuracy with components only ten times more accurate.

This difficulty is increased by the wide range of numerical values of coefficients to be considered, usually of the order of 1 to 10^6 . The problem can, however, be greatly eased by adjustment of the frequency, amplitude, and overall numerical scales of the problem. Let us first consider the frequency of a sinusoidal oscillation. The equations

$$\frac{\ddot{x}}{100} + x = 0, \quad \dots \dots \dots (15)$$

$$\ddot{x} + x = 0, \quad \dots \dots \dots (16)$$

represent two sinusoidal motions similar in every respect except frequency; but, whereas the coefficients in the second equation are equal, those in the first are in the ratio of 1 : 100. If, therefore, both the numerical values in equation (15) are to be represented by similar units in an electric network the accuracy of representation will suffer. If, however, we slow down the harmonic motion represented by equation (15) in the ratio 10 : 1 by increasing the inertia term 100 : 1 the problem is changed only in that events now happen ten times more slowly (this can be an advantage, incidentally, in demonstrating the flutter phenomenon on the simulator).

In a similar way the equations

$$0.9x_1 + 0.08x_2 = 0.7, \quad \dots \dots \dots (17)$$

$$0.75x_1 + 0.05x_2 = 0.6, \quad \dots \dots \dots (18)$$

are not so accurately represented in a network as

$$0.9x_1 + 0.8x_2 = 0.7, \quad \dots \dots \dots (19)$$

$$0.75x_1 + 0.5x_2 = 0.6, \quad \dots \dots \dots (20)$$

where the definition of the amplitude x_2 has been changed in the ratio 10 : 1. Equally equation (20) is preferable to

$$0.075x_1 + 0.05x_2 = 0.06. \quad \dots \dots \dots (21)$$

In this manner, by scaling the frequency range of the problem to centre around a circular frequency of unity, by choosing suitable definitions of the displacements in each degree of freedom, and by numerical adjustment of the equations as a whole, the range of numerical values of coefficients normally present in practical flutter problems can be considerably reduced, thereby improving overall accuracy. The scale factors used for frequency and amplitude must, of course, be reinserted when applying information obtained from the flutter simulator to the practical problem.

Scaling of the flutter equations in this way has the advantage that an apparently wide range of aircraft problems are brought within a narrow numerical range, so that a large superficial difference between the flutter coefficients of, say, a bomber and a fighter, disappears and the real physical difference becomes more apparent. Thus to the simulator all problems appear to be the same problem with comparatively minor alterations, and hence any numerical errors are more obvious and more easily traced. This characteristic has been experienced on the simulator many times, and with a skilled operator it can save time on an incorrectly calculated problem.

A detailed description of the scaling process as it is applied to an actual set of flutter equations is given in Appendix A.

The accuracy to be obtained with ill-conditioned equations is not so great as 1 per cent on flutter speed, but since the behaviour of the simulator can be examined at speeds other than that of flutter it is readily established that the problem is ill-conditioned. The stability of the motion near the critical flutter condition is not usually obtained from the computer approach to the problem. The stability of the motion can, of course, be of considerable interest to the designer, since he wishes to know not only the flutter speed but also the degree of damping of the structural oscillations excited by gusts or manoeuvres at speeds less than the flutter speed, and how a small change in the aircraft structure can affect the liability to flutter. These are features for which the simulator approach to the problem is well suited.

3. *Electric Representation of the Problem.*—3.1. *Use of High-Gain D.C. Amplifiers.*—The electronic flutter simulator depends mainly on the use of high-gain direct-coupled electronic amplifiers. Direct coupling ensures that the amplifiers will respond to sinusoidal inputs at frequencies down to zero.

Consideration will first be given to the use of a high-gain d.c. amplifier as a summing amplifier (Fig. 1). Here the input is supplied from a series of voltages V_2, V_3, \dots through resistors R_2, R_3, \dots and also from its own output V_1 through a resistor R_1 (in the negative feedback or stable sense). With an input impedance of r and equating the total current at point x (Fig. 1) to zero,

$$\frac{V_1 - V_0}{R_1} + \frac{V_2 - V_0}{R_2} + \frac{V_3 - V_0}{R_3} \dots = \frac{V_0}{r}, \quad \dots \quad (22)$$

or

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \dots = V_0 \left(\frac{1}{r} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \dots \right). \quad \dots \quad (23)$$

But the input voltage, V_0 , is small since

$$V_0 = -\frac{V_1}{G}, \quad \dots \quad (24)$$

where G is the gain of the amplifier, so that, to a first approximation

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \dots = 0, \quad \dots \quad (25)$$

providing the input impedance, r , is large.

Thus it is possible to add together several currents and equate the total to zero, simulating the equation

$$Px_1 + Qx_2 \dots = 0, \dots \dots \dots (26)$$

where the variables x_1, x_2, \dots are represented by V_1, V_2, \dots and the coefficients P, Q, \dots by $1/R_1, 1/R_2, \dots$

Since, in general, the flutter problem requires that the coefficients P, Q , etc., can be positive or negative, it is necessary to be able to reverse the sign of the amplifier output. This can be done either by inserting an extra sign reversing stage in the amplifier or by supplying 'push-pull' outputs from the amplifier. The latter method was chosen since in the simulator as a whole this led to the most economical lay-out.

To prevent trouble with pick-up on the input leads to the amplifier it followed that extension of the push-pull output principle to the input terminals was desirable. Further, the stability of a d.c. amplifier can be much improved by the internal use of push-pull circuits, as this eliminates the effects of fluctuation in the power supplies.

For these reasons, push-pull d.c. amplifiers seemed preferable and a design of amplifier to these requirements was suggested by the Mathematical Services Department at the Royal Aircraft Establishment.

For simplicity the amplifier is shown in Figs. 1 to 9 as single-sided. In practice all input and output leads are duplicated.

Returning to equation (23), if the gain of the amplifier were sufficiently high the terms on the right-hand side of the equation could be neglected within the accuracy to which it was proposed to represent the flutter problem, although it must be noted that each additional input introduces an additional error term. In practice, the gain of the amplifiers used on the present simulator was not sufficiently high to justify the neglect of these error terms but a method of correction has been adopted, using the push-pull output of the amplifier.

If, in addition to the normal feed-back across the amplifier through resistor R_1 , we also insert a feed to point x through resistor R_x from the other output of the amplifier, $-V_1$ (not shown in Fig. 1), then the equation (23) becomes

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \dots + \frac{(-V_1) - V_0}{R_x} = V_0 \left(\frac{1}{r} + \frac{1}{R_1} + \frac{1}{R_2} \dots \right) \dots \dots (27)$$

or

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \dots = V_0 \left(\frac{1}{r} + \frac{1}{R_x} + \frac{1}{R_1} + \frac{1}{R_2} \dots \right) + \frac{V_1}{R_x} \dots \dots (28)$$

But

$$V_0 = -\frac{V_1}{G}; \dots \dots \dots (24)$$

therefore if R_x is adjusted so that

$$R_x = \frac{(G - 1)}{\left(\frac{1}{r} + \frac{1}{R_1} + \frac{1}{R_2} \dots \right)}, \dots \dots \dots (29)$$

then the terms on the right-hand side of equation (28) disappear and equation (26) has been represented without error.

It might be thought that, because of this method of correction, amplifiers of quite low gain could be used with the consequent reduction in cost.

Unfortunately, fluctuations in amplifier gain mean that the terms on the right-hand side of equation (28) do not exactly cancel each other since

$$V_0 = \frac{-V_1}{G \pm \Delta G} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (30)$$

An accepted value for the fluctuation in gain of such an amplifier is ± 10 per cent of the standard gain. Thus the residue on the right-hand side of equation (28) is ± 10 per cent of the original error. This method of correction can be used to improve the accuracy of representation by the order of 10 to 1 and in fact this is adequate for the present simulator. With a total of thirty terms to be represented in equation (26) the total number of inputs to the summing amplifier is equivalent to 38 terms allowing for the potentiometer representation to be described later and the additional inputs for excitation of the circuits. The basic error term is thus $(38/G)V_1$. With the correcting feed-back resistor the error term becomes $(3.8/G)V_1$. If the error is not to exceed 0.1 per cent of V_1 , then G must be greater than 3800. Actually a gain of 5000 was used for the amplifiers in this simulator.

These high-gain amplifiers can also be used as integrators. If the resistor feed-back of Fig. 1 is replaced by a capacitor and only one external input from a voltage V_2 is present (Fig. 2) then we have

$$\frac{V_2 - V_0}{R} + \bar{c}(\dot{V}_1 - \dot{V}_0) = \frac{V_0}{r} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (31)$$

or

$$V_2 + R\bar{c}\dot{V}_1 = V_0 + R\bar{c}\dot{V}_0 + \frac{V_0 R}{r} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (32)$$

With a gain of 5000 the error terms on the right-hand side of the equation can be neglected. Thus

$$V_2 + R\bar{c}\dot{V}_1 = 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (33)$$

or

$$V_1 = - \int \frac{V_2 dt}{R\bar{c}} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (34)$$

Thus V_1 is the integral of the input voltage V_2 , the factor $R\bar{c}$ being known as the time constant of the circuit. Since the amplifier output is push-pull the negative sign can be omitted.

Three values of time constant are available in the simulator, namely, 1, 0.1 and 0.01. The basic value is 1.0 which gives a time scale on the simulator equal to the true time scale of the problem after the scaling process described in Section 2.3 and in Appendix A.

3.2. Representation of the Flutter Equations.—Let us now consider the circuit shown in Fig. 3 where one summing amplifier and two integrating amplifiers ($R\bar{c} = 1$) have been coupled together. Denoting the amplifier outputs by V_1 , V_2 and V_3 respectively we have, neglecting errors,

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = 0, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (35)$$

$$V_2 = \int V_1 dt, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (36)$$

$$V_3 = \int V_2 dt, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (37)$$

$$S_1V_1 + S_2V_2 + S_3V_3 + S_4V_4 + S_5V_5 = 0, \quad \dots \quad (46)$$

$$V_2 = \int V_1 dt, \quad \dots \quad (47)$$

$$V_4 = vV_2, \quad \dots \quad (48)$$

$$V_3 = \int V_2 dt, \quad \dots \quad (49)$$

$$V_5 = v^2V_3, \quad \dots \quad (50)$$

or eliminating all voltages other than V_3 ,

$$S_1\ddot{V}_3 + S_2\dot{V}_3 + S_3V_3 + S_4v\dot{V}_3 + S_5v^2V_3 = 0, \quad \dots \quad (51)$$

corresponding to the complete equation for one degree of freedom

$$A\ddot{x} + (Bv + D)\dot{x} + (Cv^2 + E)x = 0. \quad \dots \quad (4)$$

The circuit of Fig. 7 thus represents a single-degree-of-freedom equation.

Two circuits such as that of Fig. 7 will present two uncoupled degrees of freedom. If in addition to the supplies to amplifier 1 in either circuit from the amplifiers in its own circuit we also add proportions of the outputs of the five amplifiers in the other circuit, then we have represented the cross-coupling terms (Fig. 8), and have simulated the flutter equations in two degrees of freedom.

Further extension of the principle that each summing amplifier shall have an input from every amplifier in the system leads us to the circuit shown in Fig. 9 for six degrees of freedom. There are 30 amplifiers in the circuit (6×5), each amplifier having six potential divider outputs in parallel, one for each summing amplifier. Each summing amplifier has 30 inputs (6×5) and there are a total of 180 potential divider units ($6 \times 6 \times 5$) representing the 180 coupling terms in the equations (11).

The terms F_i in equations (12) representing external excitation are obtained in two ways. Firstly, the step function is represented by adding an extra input into the summing amplifier giving a proportional current from a fixed voltage supply. Secondly, a proportional input into the summing amplifier is supplied from a sinusoidally varying voltage. The latter is obtained from two integrating amplifiers so arranged that their outputs oscillate sinusoidally, the frequency depending on the value of the coupling potentiometers (see Section 5.10).

4. *General Design of the Simulator.*—Before considering the detail design of the simulator it will be of some assistance to discuss the general arrangement of the machine, since this is important when describing the interconnections between amplifiers and circuits.

4.1. *General Description.*—The machine consists of a central control desk flanked on either side by three standard racks raised on a wooden base. A general view of the front and rear of the flutter simulator with access panels removed is shown in Fig. 10. The coefficient settings and amplifiers appropriate to each of the equations (11) are housed in the separate racks, numbered 1 to 3 on the left-hand side of the control desk and 4 to 6 on the right-hand side. Intercoupling between the racks on either side of the control desk is made through a panel in the centre of the control desk (rear view Fig. 10 and Fig. 19). Variations required in each rack, e.g., air-speed variations, are made from the control desk by remote control, with suitable precautions against a possible error. The control desk also contains indicating meters, phase and frequency measuring equipment, and units capable of giving either a step function or sinusoidal excitation to the problem. We will now consider the parts of the simulator in more detail.

4.2. *Control Desk.*—The central control desk is mainly of wooden construction having a central table backed by a sloping indicator panel (Figs. 11 and 12). On either side of the table are two control and indicator panels at 45-deg angle to the centre panel, giving easy access to the operator. The racks on either side of the desk follow this 45-deg line, so that the rows of racks are at right angles to each other (Fig. 10).

The centre indicator panel (Fig. 12) has six indicating meters, one for each of the degrees of freedom, six balancing controls for each of the summing amplifiers, a clock, and a visual indication of air speed in the centre. Under the edge of the table (lower part of Fig. 12) are five press buttons. The three centre buttons control the setting of air speed in three decade settings. The left-hand button is used to inject a pre-selected step function into the problem, and the right-hand button is a master locking control which alternately locks or frees the circuits representing the problem. On each side of the press buttons is a drawer.

The left-hand indicator and control panel (Fig. 13) contains two meters. That on the right can be switched into any of the meter circuits on the centre panel by the knob immediately below the meter. In this way a direct comparison of the amplitudes of x_1, x_2, x_3, x_4, x_5 and x_6 in each degree of freedom can be made in turn on the same meter, thus avoiding calibration errors. The left-hand meter can be switched on to the output of any amplifier in the simulator by means of the two selector knobs immediately below the meter, one for the degree of freedom and one for the amplifier in the degree of freedom.

Immediately below these last two controls there is a knob to select either a step-function disturbance or a sinusoidal excitation. An indicator light on either side of the control gives its position at a glance. The four controls to the right of this selector knob are used to adjust the frequency of the resonator unit which supplies the sinusoidal excitation. When the selector knob is in the disturbance position the resonator unit is locked. The first of the four frequency control knobs gives ranges of circular frequency of values 1, 2, 5, 10 and 20. The three further controls are decade settings giving frequency settings to 0.1 per cent of the range chosen. With a general simulator time constant of 1/100 of a second the frequency ranges are limited to 1, 2 and 5 but each is multiplied by 100 so that the ranges are in fact 100, 200 and 500. The operation of the 100 factor is indicated by a light immediately to the left of the range control. The amplitude of the resonator unit is controlled by a special limiter circuit, the value being adjusted by the control on the extreme left of the panel, to the left of the disturbance-resonator control. The output of the resonator is shown on the left-hand meter when the resonator unit is in use.

Below the resonator controls are six pairs of controls, one pair for each degree of freedom. These are two-decade potentiometers which can be selected to vary the proportion of either the step function or resonator output (F_i in equations (12)) to be injected into each equation. The accuracy of setting is 1 per cent of the maximum value.

The right-hand control and indicator panel (Fig. 14) also has two meters. The meter on the left is used to indicate the frequency of oscillation of any degree of freedom (when the oscillation is sinusoidal), and the meter on the right the phase angle between the oscillations in any two degrees of freedom. The control immediately below the frequency meter selects the degree of freedom for frequency measurement. The control below the phase meter selects a second degree of freedom whose phase is required relative to the first. The phase and frequency unit is operated by the control press button between the meters. Since the method is based on the time of one cycle of oscillation, the meters do not indicate immediately. However, when the timing process is complete the readings of frequency and phase come up instantaneously and remain as steady readings. After a short time interval for observation (about one minute) the meter readings return to zero and the unit is ready for another operation. The completion of this process is indicated by a light above the press button. The readings can be cleared in a shorter period, if required, by moving either of the two selector controls.

Below the phase and frequency unit on the left-hand side are the individual locking controls for each degree of freedom. The number of degrees of freedom to be used can be selected here and, if required, individual degrees of freedom can be removed during a problem. Thus a four-degree-of-freedom problem can, for example, be reduced to a three-degree-of-freedom problem and the effect on flutter due to removing a degree of freedom noted.

To the right of these locking controls is the master time-constant control. The press button gives, in sequence, time constants of one, one-tenth and one-hundredth of a second. The position of the control is indicated by the three indicator lights above the press button. These are marked in frequency range, namely, 1, 10 and 100 corresponding to the time-constant values of 1, 1/10th and 1/100th. The lowest frequency setting gives oscillations of approximately one radian per second (if suitable scaling has been applied to the problem). The next setting gives oscillations exactly (to 0.1 per cent accuracy) ten times faster. The solution is unchanged in every respect except frequency. The third setting is again ten times faster, that is, exactly 100 times the original frequency. Since at this setting of frequency control the oscillation would be too rapid to follow on the meters, those of the centre panel are removed from circuit by a relay whilst the two meters on the left-hand control panel are connected to the appropriate amplifiers through a rectifier. Thus they give a steady indication of the oscillation. A growing reading corresponds to a growing oscillation and a decaying reading to a decaying oscillation.

Change of time constant also controls two other circuits. Firstly, as mentioned before, the resonator unit frequency is increased 100 times on the '100 setting' of the time constant, but is unaffected on the '10 setting'. Secondly the phase-frequency unit only operates on the '1 setting'.

Continuing the description of the controls on the right-hand panel of the control desk, we now come to the velocity range control to the right of the time-constant control. In Section 2 the choice of a reference speed V_0 was mentioned. The air speed is represented on the simulator by the quantity v , which is equal to airspeed divided by V_0 . To avoid the necessity to rescale if v should have to be taken beyond unity, provision has been made for the range of v to be doubled, though with a slight loss of accuracy. For this purpose a change-over switch is provided on the right-hand panel with two indicator lights. Normal scale is designated (1) and double scale (2). This switch changes circuits in each rack by remote control.

Also on this panel are three master switches and indicator meters controlling 230 volts A.C., 300 volts D.C. and 48 volts D.C. The control is by remote relays.

4.3. Degree-of-Freedom Racks.—The coefficient settings and amplifiers associated with each degree-of-freedom equation are in racks (Figs. 15, 16), one for each equation. In each rack there are three units: two identical coefficient units and one air-speed and control unit. Two coefficient units are used for convenience of handling and they are located at the top of the rack. Each coefficient unit consists of fifteen decade units, the two units giving the total of thirty inputs to the summing amplifier in the air-speed unit. Each decade unit consists of four switches: one sign change switch and three switches giving the three-decade setting of the coefficient. These two units are supplied at the rear with the push-pull output of every one of the thirty operating amplifiers. Fig. 16 shows how the units are coupled to the other racks and the control desk by a cable and socket method. The interior of a coefficient unit is shown in Fig. 17. All units are interchangeable. The output of each unit is a pair of push-pull wires to the input of the summing amplifier, the top unit of the two being linked to the lower and this in turn coupled to the air speed and control unit.

The air speed and control unit contains three uniselectors giving the remote three-decade setting of the air speed. Also in this chassis are the five amplifiers, removable from the front of the unit (Fig. 15); and all the remote relay controls. The interior of this unit can be seen in Fig. 18. The controls are supplied from the rear through cables (Fig. 16). There are two classes

of cable; one connecting each rack to each other rack, and the other connecting each rack to the control desk only. The latter class is for individual control of each unit, whilst the former is for controls common to all racks.

Power for the amplifiers is supplied from the wooden base on which the racks stand. The 300-volt H.T. comes from a common supply. 6.3-volt A.C. for the valve heaters is supplied by individual transformers for each rack built into the wooden base. The transformers are fed with 230-volt A.C. The 300-volt and 230-volt supplies to each rack are individually fused on the main connector panel (Fig. 19).

4.4. *Method of Operation.*—The operation of the simulator starts with the appropriate scaling of the coefficients in the equations as described in Appendix A. The coefficients are then set up on the degree-of-freedom racks; if less than six degrees of freedom are involved, those racks used must be noted and the corresponding racks unlocked by the controls on the right-hand panel of the control desk. The machine is then ready for operation.

For the determination of a critical flutter speed the air-speed control should be adjusted to give a low value of air speed setting, a suitable step-function injection to each degree of freedom selected, and a range of frequency chosen. The circuits are then unlocked by the master locking control and a step function injected by pressing the control button on the desk. The circuits should then oscillate in a manner corresponding to the oscillations of the aircraft structure represented. The motion, as indicated by the panel meters, will probably be damped, and if the operation is repeated at a higher air-speed setting the damping may increase. However, with a normal problem properly scaled the damping will decrease above a certain air-speed setting and will continue to decrease as the air speed is further increased, until at a critical setting of the air-speed control the motion becomes undamped. This is the flutter speed for the problem.

The frequency of oscillation and phase relationship between the motions indicated on the meters may be obtained by use of the equipment on the right-hand panel of the desk.

A typical result obtained in this manner is shown in Fig. 53, where critical flutter speed is plotted against a structural parameter for a four-degree-of-freedom problem. This curve was obtained from a systematic series of simulator tests, matching assumed frequency parameters (in the calculation of the aerodynamic coefficients) with derived frequency parameters. The comparison between calculated values and simulator values is shown and the agreement is generally within 1 per cent. Between the parts of the curve A to B and C to D the method used was that described above; namely, to increase air speed for a fixed value of the structural parameter. For that part of the curve lying between B and C (Fig. 53) this approach failed. Here, to obtain a satisfactory curve, the air speed was held fixed and the structural parameter varied until an undamped motion occurred. The simulator did not give the small tongue (at an air speed value of 0.75) shown by calculation. However, the frequency parameter was changing rapidly around this part of the curve and the difficulty of matching assumed to derived frequency parameter probably accounts for this small discrepancy.

For simulation of flight resonance tests the technique is different. To demonstrate the method of operation a solution is given for a four-degree-of-freedom problem (Fig. 54). Here, response amplitude is shown as a function of excitation frequency at three different air-speed settings. The critical flutter speed was first found for this aircraft problem in the manner described above. It corresponded to 660 kt, full scale. Several values of air speed below the flutter speed were chosen for investigation, including zero air speed. A small amount of structural damping was included in the equations so that a finite response could be obtained at zero air speed, corresponding to the ground resonance case. Since in this particular problem the degrees of freedom were the aircraft normal modes, the response at zero air speed showed only in one mode at a time, that is, the one nearest in natural frequency to the excitation frequency. The total output of all four degrees of freedom was therefore summed in a special unit (*see* Section 5.11) and it is this totalled result that is shown in Fig. 54.

To start the investigation the excitation equipment was selected on the left-hand panel of the desk, a steady oscillation initiated on the resonance unit, and the proportion to be injected into each mode chosen. A value for air-speed coefficient was set up and a frequency scale of 'times 100' selected. On unlocking the circuits the meters on the left-hand panel showed excitation amplitude and response amplitude, and as the frequency of excitation was varied there was a corresponding variation in response amplitude for a given excitation amplitude. Near resonance the excitation was reduced to limit the response amplitude, but the results shown in Fig. 54 are scaled to correspond to constant excitation amplitude.

Considering Fig. 54 in more detail, it will be seen that at zero air speed there are four well defined peaks corresponding to the natural frequencies of the four modes. At 500 kt the first, third and fourth peaks are much reduced whilst the second is still considerable but has shifted to a higher frequency. At 610 kt, only 50 kt below the flutter speed, the second peak has grown to the value observed in the ground resonance tests, but with a further increase in frequency. Use of this technique on the simulator can form a very desirable prelude to flight flutter testing on full-scale aircraft.

5. *Detail Design of the Simulator.*—5.1. *D.C. Amplifier.*—The requirements for the amplifier were that it should be directly coupled with push-pull input and output points all at the same voltage level, it should be free from drift, and it should have a gain of at least 5000. The amplifier circuit is given in Fig. 25.

It is a three-stage amplifier with an input stage using a double triode (CV 1988) followed by a high-gain pentode stage (two CV 1091). The third stage uses beam tetrodes (two CV 511) to carry the load of the decade switch units. The stage gains are approximately 12, 100 and 4 respectively.

The 300-volt H.T. supply is arranged to be at ± 150 volts relative to the input and output leads. All cathodes are at roughly -50 volts and all anodes at zero potential relative to the input and output leads. Each stage can be balanced separately by potentiometers on the anode supply, and in addition there is a potentiometer on the supply to the input resistor chain enabling the amplifier to be balanced under two input conditions (see Section 6.2).

Some difficulty was experienced with a high-frequency parasitic oscillation (about 12 kc/s) on the prototype amplifier, although some care had been taken to avoid this possibility by the use of suitably arranged capacitors. The oscillation was suppressed by rearrangement of these capacitors with some loss of gain over the higher frequency band. The frequency range used on the simulator is not affected.

Considerable care was taken to match all components including valves in order to obtain satisfactory balance of each stage.

The low voltage difference across the valves (50 volts anode to cathode) gives linear response for the voltage swing used in the simulator (20 volts push-pull) but leads to non-linear response outside this range. Thus the simulator can be allowed to overload without excessive voltages being applied to the indicating meters.

5.2. *Decade Switch Unit.*—In Section 3 the use of proportional division of all voltages added together in the summing amplifier is described. By this means a direct representation of the coefficients A , B , C , D and E can be obtained. The application is shown diagrammatically in Fig. 4. The proportion S of a voltage V can be obtained in two ways either by a precise mechanical adjustment of a single resistor, or by use of simple decade switches, the setting of each switch representing a digit in the coefficient. The latter method was chosen since it enabled the coefficients to be set by an unskilled operator.

In the decade system the coefficient can be set to three places of decimals by three switches. For example, the coefficient 0.864 can be represented by three decade settings of 8, 6 and 4. This is shown in Fig. 27. If a 1-megohm resistor is tapped into 0.8V, a 10-megohm resistor into 0.6V and a 100-megohm resistor into 0.4V, and the resulting currents added together, then the total current is $(0.8 + 0.06 + 0.004)V$ microamperes or 0.864V microamperes.

In practice a 100-M Ω resistor is difficult to obtain, so that an equivalent network is used (Fig. 27) with a 1-M Ω , 8.18M Ω and a 10-M Ω resistor. A push-pull circuit is used so that all circuits are duplicated. In addition a changeover switch is incorporated so that positive and negative coefficients can be represented.

The intervals of resistance in the potentiometer must be adjusted to obtain equal voltage steps in the decade system. If the total potentiometer resistance is r and the load resistance R and the fractional setting is σ , then :

$$\frac{V - SV}{(1 - \sigma)r} = \frac{SV}{\sigma r} + \frac{SV}{R}, \quad \dots \dots \dots (52)$$

or

$$S = \frac{\sigma}{1 + \sigma(1 - \sigma)\frac{r}{R}} \dots \dots \dots (53)$$

Approximately

$$\sigma = S + S^2(1 - S)\frac{r}{R} \dots \dots \dots (54)$$

From this equation the resistance intervals can be calculated.

High stability resistors are used (0.1 per cent stability) with resistances ranging from 995 to 1005 ohms. The resistances were adjusted so that with the 1-M Ω resistor in position the correct setting was obtained. This means that with the 10-M Ω resistor in the same position instead of the 1-M Ω resistor an error appears. The error, however, lies well within the 0.1 per cent accuracy limit.

The tapping resistors were 1 M $\Omega \pm 0.1$ per cent tolerance and stability and 10 M $\Omega \pm 0.5$ per cent tolerance and stability. The remainder of the tapping resistors were selected to 1 per cent.

5.3. *Air-Speed Variation.*—All coefficients B (see Section 2.1) are associated with air speed and all coefficients C with (air speed)². Section 3.2 describes two special applications of the summing amplifier (equations 43 and 45), to give factors of v and v^2 , where v is proportional to air speed but does not normally exceed unity. The circuits are shown diagrammatically in Figs. 5 and 6. In both circuits a decade switch unit is used and in the latter circuit there is also a variable feed-back to the amplifier, shown in detail in Fig. 29. Equal steps of 100 kilohms ± 0.1 per cent, 10 kilohms ± 0.5 per cent and 1 kilohm ± 0.1 per cent high-stability resistors are used. Thus the decade settings of 0.8, 0.6 and 0.4 respectively give a total resistance of 0.864 M Ω .

The three variations, two decade units and one variable resistance, must be coupled together and operated remotely. Uniselectors are used for this purpose, three to each degree-of-freedom rack to represent the three decade settings. The operation is described in Section 5.7.

In addition, the air-speed range can be doubled if v inadvertently exceeds unity (see Section 4.2). The circuits are shown in Fig. 28. Considering amplifier 4, we have a potential divider across the output giving half the amplifier output voltage. This divider is used for the feed-back. Thus, referring to Fig. 5,

$$vV_2 + \frac{1}{2}V_1 = 0, \quad \dots \dots \dots (55)$$

or

$$V_1 = -2vV_2 \dots \dots \dots (56)$$

In this simple manner the output of amplifier 4 has been doubled. Similarly the output of amplifier 5 is quadrupled, both circuits being altered by a common relay in the air-speed unit (Fig. 37). All relays in the six air-speed units are operated simultaneously by a switch on the right-hand panel of the control desk (Section 4.1, Fig. 14). The values of the potential divider resistors have been chosen to give the correct representation at $v = \frac{1}{2}$ on the 'times 2' scale. At other speed settings there is a slight error.

5.4. *Frequency Ranges.*—The range of frequency of aircraft flutter is roughly from 5 c.p.s. to 100 c.p.s. After the problem has been suitably scaled (Section 2.3), the frequency of all problems is approximately $1/2\pi$ c.p.s., or a circular frequency of unity. If a time constant of one second is chosen in the integrator circuits, the flutter frequency on the simulator is of the order of one radn/sec. Generally speaking, this low frequency is to be welcomed since the characteristics of the flutter oscillation can be examined in detail. For certain investigations, *e.g.*, flight resonance tests, where response to external excitation is required, higher frequencies are, however, desirable.

For this reason three frequency ranges were chosen for the simulator centred around 1 radn/sec, 10 radn/sec and 100 radn/sec and these ranges are produced by simultaneous change of the time constant ($R\bar{c}$ of Fig. 2) of all integrators. The control is on the right-hand panel of the control desk (*see* Section 4.2). The circuit details are shown in Figs. 30 and 31. It will be seen that the 3-position time-constant switch controls two relay circuits, one for 'times 10' frequency range and one for 'times 100' frequency range; 'times 1' corresponds to the normal rest position of both these relays.

The time-constant circuits are given in Fig. 31. A 1-M Ω resistor is used throughout and the time-constant change is obtained by selection of three capacitors, 0.1 + 0.9, 0.1 and 0.01. Adjustment of the time constant to 0.1 per cent was made for the first two conditions by adjustment of a padding resistor and in the third condition by accurate selection of the capacitor. The 0.1 and 0.9 capacitors were specially manufactured with Polystyrene dielectric. The 0.01 capacitor is of silver-mica construction. Both these dielectrics have low dielectric loss factors corresponding to errors of 0.03 per cent in the flutter problem; this is well within the 0.1 per cent accuracy required. Paper dielectric was found to correspond to a 0.7 per cent error at one radn/sec.

5.5. *Correction Circuits.*—5.5.1. *Summing Amplifier.*—In Section 3 it is shown that in each summing amplifier there is an error term, but if an additional feed-back (positive feed-back) R_x were included, this error can be reduced to one-tenth of the original value.

In fact

$$R_x = \frac{G - 1}{38}, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (57)$$

or approximately

$$R_x = \frac{G}{38} \simeq 130 \text{ megohms.} \quad \dots \quad \dots \quad \dots \quad (58)$$

Since a resistor of 130 M Ω is inconvenient, a 10-M Ω resistor was used but tapped into a potentiometer on the amplifier output giving one-thirteenth of the output voltage. The circuit is shown in Fig. 33.

5.5.2. *Correcting Feed-back for Amplifier 5.*—When the simulator was first assembled it was found that an oscillation occurred on amplifier 5, the frequency and amplitude varying with the setting of the air-speed uniselectors (the frequency was of the order of 10 to 20 kc/sec). The cause was traced to interwiring capacity on the variable feed-back, the three-stage resistor-capacitor network forming a very effective phase-shift network. The oscillation was readily suppressed by the addition of a 200 pico-farad capacitor in parallel with each feed-back resistor network (Fig. 34). The error was within 0.1 per cent on both the frequency scales 'times 1' and 'times 10'. The error is outside this limit on the 'times 100' range, but since this range was not intended to be as accurate as the other ranges, a more elegant method of suppression was not sought. The source of the trouble may be removed when an alternative method of air-speed control is adopted (Section 7).

5.5.3. *Balancing Circuit for Summing Amplifiers.*—D.C. amplifiers were chosen so that problems other than flutter could be considered. For example divergence, a problem closely allied to flutter, concerns a non-oscillatory motion, for which response to steady deflections is essential.

However, D.C. amplifiers are well known to be liable to drift and this drift is a major source of trouble in analogue computer work. Fortunately, with the flutter problem, the circuits are to a large extent self-compensating in that either the output of one amplifier corrects for the error of the next in circuit (amplifiers 1, 2 and 3) or the amplifiers are themselves self-compensating (amplifiers 4, 5). If the amplifiers require small input currents, ε_1 , ε_2 , ε_3 , ε_4 and ε_5 respectively, to give zero output then we can rewrite equations (46) to (50) (Section 3) as

$$S_1V_1 + S_2V_2 + S_3V_3 + S_4V_4 + S_5V_5 = \varepsilon_1, \quad \dots \quad (59)$$

$$V_1 + \dot{V}_2 = \varepsilon_2, \quad \dots \quad (60)$$

$$V_2 + \dot{V}_3 = \varepsilon_3, \quad \dots \quad (61)$$

$$vV_2 + V_4 = \varepsilon_4, \quad \dots \quad (62)$$

$$vV_3 + \frac{V_5}{v} = \varepsilon_5, \quad \dots \quad (63)$$

Considering only drift conditions, we can omit derivatives with respect to time. Then

$$V_1 = \varepsilon_2, \quad \dots \quad (64)$$

$$V_2 = \varepsilon_3, \quad \dots \quad (65)$$

$$V_4 = \varepsilon_4 - v\varepsilon_3, \quad \dots \quad (66)$$

$$V_5 = v\varepsilon_5 - v^2V_3, \quad \dots \quad (67)$$

and V_3 is given by equation (59).

This is of course an over-simplified statement of the drift problem, and with all terms in six degrees of freedom represented, the problem is obviously complicated. But the general result is indicated by these equations, namely that the circuits are self-compensating and drift errors appear as small errors in the amplifier outputs.

The one troublesome case is that of the errors in the summing amplifiers, for two reasons. Firstly, the magnitude of the error here is magnified by the large number of input terms; and secondly, since the compensation comes mainly from amplifier 3 through the terms S_3V_3 and $S_5v^2V_3$, the voltage output V_3 of the amplifier must be large if the coupling terms S_3 , S_5 are small.

This difficulty is one encountered frequently since in some types of problems (notably those involving aircraft body freedoms), the coefficients S_3 or S_5 can be zero.

To solve this problem in some measure a special balancing circuit was devised for amplifier 1 (Fig. 32). It is a manually adjusted control of a steady-error current into this amplifier, the object being to compensate for the error term in amplifier 1. The output of amplifier 3 must of course be observed when adjustment is made. In practice, once adjusted, the balance will remain within reasonable limits for periods up to an hour and has been acceptable to staff operating the simulator.

Suggested improvements are discussed in Section 7.

5.6. Locking Circuits.—There are two main requirements for locking the simulator circuit: (a) to limit the number of degrees of freedom in use if the problem being considered has less than six degrees of freedom, and (b) so that the oscillatory motion can be stopped or started at will.

The locking control is by the circuit shown in Fig. 35. Each individual degree of freedom can be chosen separately and those chosen can be locked or unlocked simultaneously by a master control. These controls are on the control desk and operation in the racks is by relay. The

circuit operated in each rack by the relay is given in Fig. 36. When the degree of freedom is locked (rest position of the relay) the inputs and outputs of amplifiers 1, 2 and 3 are shorted individually (in the negative feed-back sense). Thus the outputs of these three amplifiers are all zero, and consequently those of the two dependent amplifiers 4 and 5 are also zero.

The advantage of this system of removing one or more degrees of freedom is that, whilst the appropriate amplifiers are quiescent, the circuits dependent upon them are unchanged and the input impedances of the summing amplifiers in the operative degrees of freedom are unaffected.

5.7. Air-speed Control.—The manner in which air-speed variation is represented is described in Section 5.3. We now consider the circuits by which the eighteen uniselectors (3 in each degree of freedom) are controlled. Operation is by three control buttons under the edge of the table in the control desk (Fig. 12). Each button controls one uniselector in each degree of freedom by the circuit shown in Fig. 38. The relays are denoted by A, B, C, etc., and the circuits they operate by A_1, A_2, B_1, B_2 , etc. The three control buttons are numbered 1, 2 and 3.

With A de-energised, operation of button 1 energises relay B, making contacts B_1 and B_2 . B_1 completes the supply to relays E, F, G, which now operate continuously and consecutively since each controls the next in sequence. The object of this form of control is to make the operation independent of the length of pulse from relay B. The operation of relays E, F, G is slowed down by the shorting of coils in each relay giving a 'slugged' effect.

Each time relay G operates, G_2 makes the circuits of relays H, J, K. However, since button 1 was operated, only H will be energised. If all three buttons are operated simultaneously all three relays will operate. H, J and K have heavy-duty contacts which complete the circuits of the uniselectors, six to each of the relays.

If any uniselector has reached beyond the tenth contact (or, in the case of the coarse-step uniselectors, the eleventh) line W is energised, relay A operates, the press buttons become inoperative through A_1, A_2 and A_3 and the 'slug' coils on relays E, F, G, open-circuited. The pulsing effect is thereby doubled in frequency until the uniselectors have cleared the unwanted contacts 11 (or 12) to 25. Lines X, Y or Z will also be appropriately energised by contacts on the uniselectors and B_2, C_2 or D_2 shorted. If X is the energised line then C_2 and D_2 will be open-circuited and the appropriate uniselectors will remain at rest. On reaching the first contact line W and X, Y or Z are de-energised and the pulsing frequency reverts to normal.

Thus, on operating press button 1 and maintaining pressure, the appropriate uniselectors operate at the rate of roughly 3 contacts per second. On removing the finger from the control the uniselectors cease operating, unless they are now in the automatic clearing position contacts 11 to 25 (or 12 to 25 for button 1). If the button is pressed continuously the uniselectors will cease to operate on their first contact, since A is energised through A_1, A_2 or A_3 . The press button must first be released and then operated again to continue the uniselector movement. This part of the circuit was included to prevent continuous operation of the uniselectors without action on the part of the operator.

We now come to the corrector circuit shown in Fig. 39. If one of the six uniselectors working from the contacts on relay H gets out of step, let us say that it lags by one position, then this error will be shown on the air-speed indicator (Section 5.8). The error must be removed before continuing operations, otherwise a serious error may be introduced into the problem.

When the uniselectors are returned to zero, since line W is energised as long as any of the six relays is on contacts 12 to 25, they will come to rest with one on the first contact (zero setting) and the other five on the second contact (unit setting). If at this stage press button 4 is operated, relay R is energised and contacts R_1, R_2 and R_3 are opened in each degree-of-freedom rack. If, however, any uniselector is in the 'zero' position, the appropriate 'R' contact is shorted, and on operation of the press button 1, that uniselector motor circuit is completed and it will operate once, thus breaking its own circuit. The other uniselectors will not move, and the lagging uniselector is thus moved up correctly.

Any lag error can be cured one step at a time by repeated operation of this process. Since button 4 is rarely used, it has been combined with the master locking button (Fig. 12) on the right of the air-speed control buttons.

5.8. *Air-speed Indicator.*—The indication of air-speed setting is on the centre of the centre panel on the control desk. It operates on the well known principle of outlining numbers by a few light bulbs. To represent the numbers zero to nine, a block of bulbs 5 wide and 7 high are used (Fig. 40) and the individual numbers indicated by use of appropriate bulbs in the pattern (*e.g.*, representation of 'six' is shown in Fig. 40). The three decade settings of air speed are given on three such blocks of lights and in addition, since the first setting can, in fact, record up to 'ten' as a maximum, an additional row of lights giving the extra digit has been added to the left of the three blocks.

This system was chosen for two reasons: firstly, it gives a ready indication of air-speed setting from a distance, and secondly, it can show a fault in air-speed setting by giving a superimposed double number.

Each numeral is indicated by one or two relays. These relays complete the circuits of the appropriate bulbs; details of the circuit are given in Fig. 40. The relays can be operated by any of the six uniselectors in each of the racks. Thus, if one uniselector is out of step, five will make one numeral and the odd uniselector another numeral. The corrector circuit of Section 5.7 can then be used to bring the odd uniselector into step.

5.9. *Disturbance of the Problem.*—In order to excite the oscillations corresponding to flutter, it is necessary to have the equivalent of a gust in flight on the aircraft. This excitation is produced by injecting a step function in the form of a current impulse into one or any of the degrees of freedom. The circuit is shown in Fig. 41. A supply of a few volts is obtained from a potentiometer across the 300 volt H.T. supply. This voltage is applied to six potentiometers in parallel, each being part of a two-decade unit supplying current to each degree of freedom. The voltage supply is operated by a push button on the control desk and the proportion of current injection into each degree of freedom is controlled by the six two-decade switches on the left-hand panel of the control desk.

5.10. *Resonance Unit.*—An added feature of the simulator is the ability to resonate the problem over a range of frequencies. The method is equivalent at zero air speed to ground resonance testing of airframes, and at speeds below the flutter speed to flight resonance testing.

A sinusoidal oscillation is produced by two integrating amplifiers (Fig. 43). The principle is the same as that of the three-amplifier circuit of Fig. 3 but with the summing amplifier omitted. If we denote the amplifier outputs as V_1 and V_2 , the decade switch settings as S , and the time constant as τ , then

$$\tau \dot{V}_2 + SV_1 = 0, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (68)$$

and

$$-\tau \dot{V}_1 + SV_2 = 0, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (69)$$

or

$$\tau^2 \ddot{V}_1 + S^2 V_1 = 0, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (70)$$

where error terms have been omitted and one pair of input wires have been reversed. Equation (68) represents a sinusoidal oscillation of frequency S/τ . S is a three-decade switch unit giving frequency settings to 0.1 per cent of the maximum and settings of time constant of 1, 1/2, 1/5, 1/10, 1/20 are used to give general frequency ranges of 1, 2, 5, 10 and 20. The time constant is

varied by adjustment of capacitance only. Operation of the 'times 100' time-constant setting on the main simulator controls energises a relay giving time constants of 0.01, 0.005 and 0.002, instead of 1, 0.5 and 0.2 respectively. The frequency ranges 10 and 20 are not altered in this manner.

The output of the resonance unit is used to replace the fixed-voltage injection of the circuit in Fig. 41, whilst the excitation to start the oscillation in the resonance unit comes from this fixed voltage. Thus a proportion of the sinusoidal voltage can be injected at will into any degree of freedom to a 1 per cent accuracy, representing to this accuracy the position of a flight resonator on the equivalent aircraft structure.

The amplitude of the resonant frequency is maintained by the control shown in Fig. 43. Amplifier 2 is given a weak resistance feed-back in the negative damping sense (positive feed-back). Amplifier 1 has a similar positive damping feed-back (negative feed-back), but with a non-linear rectifier in the circuit. The feed-back on amplifier 2 causes the oscillation to grow until the other feed-back becomes powerful enough to oppose it. The final amplitude reached as a steady value can be altered by adjustment of the 100-k Ω potentiometer. The error due to non-linear action is less than 0.1 per cent on frequency.

5.11. *Output Addition Unit.*—This unit is particularly useful when using the resonance unit on the 'times 100' frequency scale, when only one meter (the right meter on the left-hand indicator panel), is operative. When seeking peak response of the flutter problem to resonance excitation, it is desirable to be able to observe all degree-of-freedom outputs, since in some cases (notably a problem employing normal modes as the degrees of freedom), only one or two of the degrees of freedom will respond. To overcome this difficulty a single amplifier is used as a summing amplifier adding the outputs and presenting the total on the one operative meter. The circuit is simple (Fig. 42) but useful.

5.12. *Phase and Frequency Unit.*—As far as the authors are aware, methods of measuring phase and frequency had not been developed for the exceptionally low frequencies used in this simulator, and a special unit was therefore designed for this purpose. The basic principle is one of measurement of time interval.

The sinusoidal outputs of amplifier 2 from all degrees of freedom are first taken to the selector switches on the right-hand panel of the control desk, where the output of any degree of freedom can be selected. The sinusoidal inputs are then converted into sharp-edged pulses occurring once per cycle as the signal changes sign from negative to positive relative to the mean level.

Two successive pulses of one signal, A, are used firstly to start and secondly to stop uniselectors synchronised with mains frequency. The uniselectors are arranged to increase a resistance from zero at the first pulse to its final steady value R_1 (Fig. 44a) on the second pulse. R_1 controls the current in a meter supplied from a fixed voltage V_1 . Thus the steady reading of the ammeter is inversely proportional to R_1 and hence it is inversely proportional to the time of one cycle of the oscillation, t_1 . Therefore the meter reading is directly proportional to the oscillation frequency.

The first pulse of signal A also starts a second set of uniselectors which are stopped by the first subsequent pulse of signal B from another degree of freedom. These uniselectors adjust a voltage V_2 , so that V_2 is proportional to the time interval between the two signals A and B. The first set of uniselectors has a duplicate circuit giving a second resistance R_2 proportional to t_1 . A second ammeter is supplied from V_2 through R_2 . Its current is thus equal to V_2/R_2 or t_2/t_1 which equals the phase lag divided by 360 deg, so that the second ammeter gives phase angle directly.

Each of the two sets of uniselectors consists of one fast-running uniselector synchronised to the mains frequency and one slow-moving uniselector which moves one step on completion of 25 steps on the fast uniselector. Each slow uniselector can move a total of 50 contacts. The total time available is thus $(25 \times 50)/50$ at 50 c.p.s. mains frequency, or 25 seconds. If the period of the oscillation is longer than this (four times slower than the basic frequency of the simulator), no reading of current will be available since there is a safety device which returns the unit to its

starting condition. Further, any unintentional adjustment of the panel controls during the operation of the uniselectors will return the unit to its starting condition, thus avoiding possible misreadings.

A block diagram is given in Fig 45 and details are shown in Figs. 46 to 52. The overall accuracy is within ± 1 per cent over the whole range of operation and down to 1 volt sinusoidal signal input. The circuits will now be described in some detail.

5.12.1. *Pulse unit (Fig. 48).*—The outputs of the degrees of freedom of the simulator are push-pull with a maximum of ± 10 volts. There is a standing D.C. level of 150 volts relative to H.T. negative.

In the pulse unit a double-triode valve CV.1988 is arranged in a push-pull network to operate with approximately 150 volts D.C. on the grids. A sensitive polarised relay, connected between the anode load resistances, is adjusted to be on the threshold of operation when the grids are at equal potentials of 150 volts. The adjustment is by means of a potentiometer between the anode resistances and the high-voltage supply.

The relay will operate with a push-pull change of voltage of 0.04 volts on the grids. The unit is thus accurate over a large difference in amplitude between signals A and B.

A capacitor is charged up to 80 volts during the negative half-cycle and discharged when the relay R.L.1/1 operates, giving a pulse output which is fed into the master control circuit.

Shunt rectifiers are connected across the relay to prevent damage through overload.

Resistances are fitted to limit the grid current. The line input from the panel control is terminated with a 50-k Ω resistor to reduce pick-up.

5.12.2. *Master control unit (Fig. 49).*—The circuit consists basically of three thyratrons and four relays which operate uniselector and pre-set circuits. The cathodes of the thyratrons (CV.1128), are connected to a potentiometer network across the high-voltage supply. Relays R.L.3 and R.L.6 are type K3000, R.L.4 and R.L.5 are high-speed relays and each have two separate equal windings.

The operation of the circuit is as follows. A contact on R.L.2. in the panel control unit supplies high voltage to thyatron T1. The first pulse from unit A, after operation of R.L.2, fires T1 operating relays R.L.3, R.L.4 and R.L.5. R.L.3 connects the high voltage supply to T2 and T3, and adjusts the cathode voltage of the thyratrons to suit the new conditions. R.L.4 and R.L.5 operate the uniselector circuits.

The first pulse from unit B, after R.L.3 operates, fires T2. Current flows through the second winding of R.L.4. equal and opposite to that flowing through the first winding. R.L.4. is thus restored to the non-operate position and stops the associated uniselector. In a similar manner, the next pulse from A fires T3, cancels R.L.5 and operates R.L.6. R.L.6. connects the phase and frequency meters to their circuits and sets the time-relay unit into operation.

5.12.3. *Controlled uniselectors (Fig. 50a).*—Two thyratrons T4 and T5 (CV.1128) are used as grid controlled rectifiers.

In the non-operate position, contacts of R.L.2, R.L.4 and R.L.5 are open and the grids of the thyratrons are biased negatively from a rectified A.C. supply, through contact 1 of bank 4 of uniselectors US.A and US.B. The anodes of the thyratrons are connected through the uniselector operating coil, a resistor and a thermal delay to the secondary winding of a transformer.

When R.L.2 is energised, banks 4 of US.A and US.B are shorted out. Contacts of R.L.4 and R.L.5 short circuit the bias of the thyratrons and the uniselectors operate at 50 steps/sec from the mains supply. When R.L.2 drops out the uniselectors will automatically return to zero since the travellers of US.A and US.B will continue to run until a negative bias is applied to the grids of the thyratrons through contact 1 of the bank 4.

5.12.4. *Follower uniselectors (Fig. 50b).*—Contact 1 of US.C bank 4 and US.D bank 2 are used with a lamp circuit to indicate that the uniselectors are set to the non-operate position. The 48-volt D.C. supply is connected through R.L.1/2.

When R.L.1 is operated contacts R.L.1/2 and R.L.1/3 connect the 48-volt supply to contact 25 of bank 3, uniselectors US.A and US.B. Uniselectors US.C and US.D are operated every 25th step of US.A and US.B respectively up to a maximum of 50.

When R.L.1 is released, US.C and US.D automatically return to zero through their own selector contacts and travellers of bank 4. The resistance-capacity combinations across the contacts act as spark suppressors.

5.12.5. *Frequency-indicating circuit (Fig. 52).*—A stabilised high-voltage supply is connected through contact R.L.6/3 and an ammeter (500 microamps full-scale deflection) to the variable-resistance chain of banks 1 US.A and US.C.

At every 25th step of the controlled uniselector US.A the follower uniselector US.C is operated, from the 48-volt D.C. supply, up to a maximum of 50 steps. The 50th step is used to operate an overriding device which restores all circuits to the normal position.

The maximum number of effective steps is 1249. At 50 steps/sec, time intervals up to 25 sec can be measured, 25 sec being equivalent to a circular frequency of 0.25. The lower time intervals are electrically restricted, with a 150-k Ω resistor to a minimum measurement of 2.5 sec, *i.e.*, 125 contacts equivalent to a circular frequency of 2.5.

The simulator has been designed to operate at a circular frequency of approximately unity and at this frequency the accuracy of measurement is 1 step in 314 (approximately 0.33 per cent).

5.12.6. *Phase-indicating circuit (Fig. 51).*—The operation is similar to that of the frequency measuring circuit.

A stabilised voltage is applied across a resistance chain connected to the contacts of banks 1 US.B and US.D. The voltage tapped off is connected through contact R.L.6/1 and an ammeter (50 microamps full-scale deflection) to the variable resistance chain of banks 2 US.A and US.C.

5.12.7. *Time-delay circuit (Fig. 47).*—The valve CV.1091 acts as a half-wave rectifier to charge the electrolytic capacitor C_1 . In the initial position the current through the valve is reduced almost to cut-off by the large value of cathode bias resistance.

When R.L.6/1 is open capacitor C_2 starts charging through R_3 and R_4 . The voltage drop along R_3 and R_4 keeps the grid negative, with respect to the cathode, and retards the rate of charge of C_1 . When the voltage across C_1 has risen to about 100 volts the relay R.L.7 operates and breaks the relay circuit in the panel control section.

With values shown the time delay can be adjusted between 8 and 100 sec.

The anode resistor R_6 is used to limit the voltage that can be applied between cathode and heater.

5.13. *Power Supplies.*—The power supplies fall into three categories: high-tension D.C., low-tension D.C. and low-tension A.C.

The high-tension D.C. supply is nominally 300 volts and is obtained from a motor-generator operating from 3 phase 440-volt A.C. supply. The voltage output is dependent on mains frequency and it is therefore not liable to excessive fluctuations. The output is through a commutator and there is a commutator ripple of several volts at 1200 c.p.s. No attempt is made either to stabilise the output voltage or to remove the commutator ripple, and the freedom from trouble in the D.C. amplifiers due to these causes can be attributed to the push-pull design. The simulator requires 2 Amp at 300 volt.

The low-tension D.C. supply is 48 volts supplied from 230-volt A.C. through a transformer and rectifier. A 48-volt D.C. battery is connected across this supply to reduce the ripple. This supply is solely for the remote-operation circuits and the peak load is 10 Amp.

The third supply, 6.3-volt A.C., is required for the valve heaters and is supplied individually to each rack from transformers operating from 230-volt A.C. The current taken by each rack is 10 Amp.

Other miscellaneous power requirements are for the resonance unit and the phase and frequency unit.

The power supplies are controlled remotely by relay from the right-hand panel of the control desk and the 300-volt H.T. supply has also a thermal delay to prevent voltage overload in the amplifiers whilst the valve heaters are warming up.

6. *Test Procedure.*—It is apparent that, with an electronic machine of such complexity and with an accuracy that represents the present limit of stability of some of the components, considerable care has had to be taken to ensure adequate testing of the simulator. For convenience we shall consider the testing in two parts, namely, tests during construction and tests after completion.

6.1. *Tests during Construction.*—All components for the amplifiers were measured and matched and this procedure was extended to include the valves. The special high-stability resistors used in the machine were measured and selected at the manufacturing stage after special ageing treatment. On arrival at the R.A.E. a sample (about 10 per cent) was checked using equipment in the Radio Department.

The Polystyrene capacitors were made in four parts, three 0.3 μ F and one 0.1 μ F. The 0.1- μ F capacitors were selected and the 0.3- μ F capacitors matched in threes to give 0.9 μ F during manufacture. Values slightly under the nominal value were required to allow for padding the high-stability resistor used in conjunction with each capacitor. The three 0.3 μ F and the 0.1 μ F capacitors were then assembled in sealed cans with glass insulated terminals to prevent leakage.

On arrival at the R.A.E. the capacitors were checked in the Radio Department for value. Leakage was checked by measuring the rate of discharge on open circuit after charging.

For assembly, small rapid soldering irons were used, with copper clips placed on the ends of resistors during soldering to reduce heating of the carbon track. All components were checked where possible after soldering. A special machine was used for soldering of the Mk. IV plugs, and sockets used through the simulator and the cables were checked for circuit and leaks after completion. All insulating material was checked for leakage and some, in fact, was rejected as being too poor for the machine. Every attempt was made to reduce leakage by careful arrangement of the components and by keeping high-impedance cables short at the expense of the low-impedance cables.

The method of testing amplifiers will be described in Section 6.2 but some mention must be made of a unit to test three amplifiers in a group. One amplifier acted as a summing amplifier, the other two as integrators. The time constant was unity and feed-back resistors were arranged to represent

$$\ddot{x} + x = 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (71)$$

After initial disturbance, the frequency and damping of the unit could be checked over a long period of time. By this means, overall amplifier performance and capacitor leakage effects were checked. In fact, in some cases it was only by use of this unit that satisfactory tests of some components could be made. Since completion of the simulator an air-speed and control unit can be used for this purpose.

6.2. *Tests after Completion.*—Since completion there have been several major checks of component values at fairly infrequent intervals. For this purpose and to cover the possibility of failure, spares of the air-speed unit and coefficient units are available. To avoid putting the simulator out of action, it is intended to replace each unit one by one with the spare unit after it has been completely checked.

To a large extent, however, the machine is checked at frequent intervals during oscillation. By setting up equations of the form

$$\ddot{x} + x = 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (71)$$

in each degree of freedom, the overall accuracy of components in the air-speed unit can be checked by observing the rate of damping. Frequency can be checked by comparing one degree of freedom with any other, the same problem being set up on both. The resonance unit can also be brought into this procedure by setting up the correct (theoretical) frequency. Since it is unlikely that all seven systems will develop a fault simultaneously, this is thought to be a very useful check. The resonance unit can be used to check the frequency unit, and the phase meter can be tested by measuring the phase difference between a signal and itself (0 or 360 deg) and by measuring the phase difference between two signals at known phase angle. This latter test can be made by arranging that one degree of freedom is coupled to another primary degree of freedom (in oscillation) in a simple way, giving, for example, 90, 180, or 270 deg.

The main check, made every week, is by use of a special test problem, designed to check as many items as possible in the simulator. Referring to equations (11),

$$\sum_{j=1}^{j=6} f_{ij}(x_j) = 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (11)$$

$$i = 1 \text{ to } 6,$$

each expression $f_{ij}(x_j)$ is given by

$$f_{ij}(x_j) \equiv A_{ij}\ddot{x}_j + (B_{ij}v + D_{ij})\dot{x}_j + (C_{ij}v^2 + E_{ij})x_j \quad \dots \quad \dots \quad (72)$$

Now in the test problem we make

$$\frac{1}{2}A_{ij} = -B_{ij} = C_{ij} = D_{ij} = E_{ij} = 0.250 \quad \dots \quad \dots \quad \dots \quad (73)$$

when $i \neq j$ and

$$\frac{1}{2}A_{ij} = -B_{ij} = C_i = D_{ij} = E_{ij} = 0.500 \quad \dots \quad \dots \quad \dots \quad (74)$$

when $i = j$.

This problem satisfies the rules discussed in Section 2 and it will be seen that when $v = 1$, each degree of freedom will oscillate at a circular frequency of unity. At speed settings of slightly less than 1.000 the oscillations will damp, whilst at speeds slightly higher the oscillations grow. Thus the test problem behaves exactly as a real flutter problem but has the special merit that all coefficients are represented and are large. Any error in the coefficients is indicated by a form of beating between the oscillations in the six degrees of freedom. Any substantial lack of gain or fault in the amplifiers is also apparent, and with experience it is a relatively simple task to track down a fault.

At regular intervals the performance and gain of each amplifier is checked by a test unit. A twelve-pin miniature Jones plug is used to couple the amplifier to the air-speed unit. For operation in the simulator eight wires are used: two heater wires, two H.T. wires, two input and two output wires.

In addition, the push-pull outputs of each of the first two stages of the amplifier are wired to the Jones plug so that under test each stage as well as the overall performance can be examined.

Three checks are made for each stage: (a) balance on open circuit, that is, with no connection to the stage input, (b) balance with short-circuited inputs, (c) stage gain using a low-impedance source.

The unit for carrying out these tests is designed for quick operation so that, if an amplifier performance is in doubt, it can be checked in one or two minutes. It might be argued that the test conditions are unrepresentative of operating conditions, and during the development stage of the simulator other methods, involving negative feed-back, were tried. However, this test unit together with the simulator itself has proved satisfactory in practice.

7. *Discussion.*—The flutter simulator has now been in operation since January 1952. During the first year of use about 150 problems were solved in addition to a considerable amount of demonstration to representatives of industry. Any one of the larger problems might have taken some two or three months to solve by a desk calculating machine. The problems have ranged from those in two degrees of freedom to those in six. In the past very few problems have been undertaken in five or six degrees of freedom because such problems would have required an excessive amount of time for solution. Now that the simulator is available aircraft firms are extending their calculations to six degrees of freedom, with a consequent improvement in the reliability of the solution. Six degrees of freedom have so far appeared to be adequate for most problems.

Users of the machine have commented favourably on its ability to investigate the effects of variation of parameters and the effect of removing one or more degrees of freedom from the problem. Generally speaking the accuracy has been of the order of 1 per cent where calculated values of flutter speed have been available; and on frequency between 1 and 2 per cent.

Criticisms have included the limitations due to drift, leading to difficulties with certain problems. Further, if there is a divergence speed below the flutter speed the machine cannot investigate beyond the lower critical speed. In some cases of flutter two modes may be undamped or poorly damped and this leads to considerable beating between the degrees of freedom.

Both of the latter cases, however, represent real problems on aircraft and the simulator cannot fairly be criticised for giving a representative answer. The simulator drift and the effect of divergence could be removed by the use of A.C. amplifiers, with considerable care being taken over the low-frequency cut-off value. After some consideration of the problem the authors feel that so far as the present machine is concerned the use of A.C. amplifiers might lead to misrepresentation of the problem. They do not feel that their reservations should necessarily prevent the use of A.C. amplifiers in any other machine built on the lines of the R.A.E. machine.

It is felt that the main criticism of the machine could be met by the use of amplifiers with specially reduced drift. A solution³ involving the use of an A.C. amplifier coupled to the input of the D.C. amplifier has been developed in the United States. By this means the gain of the D.C. amplifier can be increased by a factor of 2000 at zero frequency, whilst the gain at normal operating frequency is unchanged. Unfortunately, in the case of the present machine, all amplifiers would need to be modified to this dual purpose amplifier and it is not thought to be necessary to go to this extreme. As stated earlier, the machine is to a large extent self-compensating as far as drift is concerned. Thus an improvement in drift qualities of the existing amplifier together with this special feature should lead to a satisfactory solution.

It is proposed to improve the drift qualities of the existing amplifiers in two ways: firstly, to remove the input resistance network to the grids of the first stage, and secondly, to increase the amplitude of the oscillation in the flutter condition, that is, to improve the ratio of A.C. output to D.C. drift. Both these proposals will lead to some changes in amplifier design and methods of testing the amplifiers. Opportunity will also be taken to improve the lay-out of the amplifier components.

A further improvement to be made in the near future is to replace the uniselectors controlling the air-speed change. It was originally intended to use a type of remote-operated wafer switch. However, this switch was not available in production and the uniselectors were used as a temporary substitute. The main criticism of uniselectors for this machine is the poor quality of the insulation (it is no doubt quite adequate for the purpose for which they were designed). It is proposed to replace the uniselectors at an early date by an improved method of control.

A question commonly asked by representatives of industry concerns the choice of a circular frequency of unity. This low frequency was chosen to enable the flutter oscillation to be studied in detail and to enable the principles of the machine to be more easily grasped by non-electronic users. Broadly speaking, there is no limit to the choice of frequency range, but the obviously desirable alternative to the low value chosen for the present machine is one in which several cycles of the oscillation can be displayed on a cathode-ray tube, the whole group of cycles being repeated at a frequency high enough to avoid flicker. These higher frequencies would, however, introduce other problems such as the effect of stray capacity upon the flutter problem.

A simulator such as the one described here can be used to solve problems other than flutter. The problem of divergence is already represented on the machine, although inconvenient for our purpose. Sharp-edged pulses are used to excite the flutter condition and it can be seen that with a slight adaptation response to gusts can be investigated. If necessary, the simulator could be extended to represent the build-up of aerodynamic forces consequent upon a gust rather than to use the present 'constant' derivatives. Aerodynamic stability equations are similar to the flutter equations and could also be solved.

A further application of this machine is the inclusion in the problem of real components. For example the response of a power-operated control unit may be required in the solution of a flutter problem and it might be desirable to include the actual unit in the problem. This case could also be covered by a simulator similar to the existing one, although it must be noted that the machine would need to operate at the correct aircraft frequencies.

Thus the principles of electronic simulation can be used to solve a wide range of aircraft problems but it should be borne in mind that the wider the range of problems covered by one machine, the less is the efficiency of the machine on any one problem. If there is sufficient work of a particular type, a simulator should preferably be designed to solve that type of problem only.

Considerable interest has been shown by aircraft firms in the R.A.E. simulator and it appears likely that similar machines may be built elsewhere. In view of this possibility the authors feel that they should give a warning. A machine of this type should not be attempted unless adequate facilities for accurate measurement and high-quality electronic work are available and unless a determined attempt is made to maintain a high standard of accuracy throughout the machine.

8. Further Developments.—The solution of flutter equations represents only a part, although an important part, of the flutter problem as a whole. In general, the solution by desk calculating machine takes roughly half the total time of solution; the rest of the time is required for obtaining the equations. Thus, although the time saved by the flutter simulator is considerable, the total time to obtain a solution is only halved. Attention is now being paid to the preparation of the flutter equations by electronic methods.

The major part of the additional work can be split into two parts: the calculation of the aircraft normal modes and the integration, spanwise and chordwise, required to obtain coefficients for both the normal mode and flutter equations.

The vibration problem and the flutter problem are similar and in some respects the former is simpler since aerodynamic effects are absent. But the wide range in the values of the coefficients in vibration problems, even after suitable scaling of the form described in Section 2 has been made, and the large number of terms in the equations, lead to a greater stress on accuracy. To cover the range of problem now envisaged, it is thought that problems in 24 degrees of freedom will need to be solved. Allowing for direct-damping terms a total of 1,176 coefficients needs to

be represented. A machine on the lines of the present flutter simulator has been proposed for the solution of vibration problems in 24 degrees of freedom. The circuits would be fundamentally the same as for the flutter simulator but greater emphasis would need to be placed on the resonance testing procedure.

For both the vibration and the flutter problem it is necessary to integrate, spanwise and chordwise, variables such as mass and stiffness distribution, taking into account the assumed or known mode of deflection. This work is possibly the most tedious although the easiest part of the flutter problem. The task is fundamentally to sum functions which are themselves products of physical variables. The solution can either be by digital or analogue methods. Whichever method is used there is a full-time need for such a machine and it would seem desirable to build a special machine for this one problem.

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APPENDIX A

The Setting-up of the Flutter Equations

by

E. G. Broadbent

1. *Introduction.*—The principle of the R.A.E. flutter simulator, and the form of the equations which it is intended to solve are described in Section 2. The coefficients as calculated in standard flutter work will usually need some modification before being ready for the simulator. Experience has shown that widely differing procedures are followed in setting up the flutter equations, and this Appendix has therefore been written in order to suggest a standard procedure. Use of a standard non-dimensional form of the equations facilitates comparative assessment of different problems and helps in the accumulation of experience that may indicate the existence and source of any errors in the calculation of the coefficients.

The importance of this check, which can only be achieved by standardisation, is greatly increased by use of the flutter simulator. This is because the equations can now be solved so rapidly that important decisions may have been taken and acted upon before the mistake is discovered, whereas when the solution took several weeks (or months) there was a good chance that the mistake would be discovered before the work was complete. It should only be on rare occasions that this last minute check does reveal a mistake, as all the coefficients should have been carefully checked beforehand. Nevertheless, experience shows that even when this has been done, mistakes occur too often for the possibility to be neglected.

2. *The Lagrangian Equations before Modification.*—The standard form of the equations used at the R.A.E. is a non-dimensional one. The co-ordinates are defined as follows :

- (1) For an aircraft normal mode (including roll or vertical translation)

$$q = z_{it}/c_r,$$

where q is the generalised co-ordinate

z_{it} is the normal displacement of the wing tip at the leading edge

c_r is the reference chord.

- (2) For aircraft pitch, control-surface rotation or tab rotation,

$$q = \alpha, \beta, \text{ or } \gamma \text{ whichever is appropriate,}$$

where α is the angle of pitch

β is the control-surface angle

γ is the tab angle.

This system would be disregarded in any case where there was a special reason for choosing some other system, but it is suitable for most problems.

The matrix of structural inertia coefficients is denoted by $[\hat{a}]$, where \hat{a}_{rs} is the inertia coefficient of q_s in the r th equation. Similarly the matrix of aerodynamic inertia coefficients (if they are required) is denoted by $[\bar{a}]$, and the totals by $[a]$, i.e.,

$$a = \hat{a} + \bar{a}.$$

The aerodynamic damping coefficients are denoted by $[b]$ and the aerodynamic stiffnesses by $[c]$. The form of the elastic coefficients varies; for normal modes they are most suitably quoted in terms of the natural frequency, thus

$$e_{rr} = \frac{\omega_r^2 c_r^2 a_{rr}}{V_0^2}$$

where e is the elastic coefficient

ω_r is the natural frequency of the r th mode

V_0 is a reference speed.

The equations have all been divided through by the factor

$$\rho V^2 s c_r^2,$$

where ρ is the air density

s is the wing semi-span,

so that the Lagrangian equations may be written

$$\left[a\lambda^2 + b\lambda + c + e \left(\frac{V_0}{V} \right)^2 \right] [q] = 0, \quad \dots \dots \dots \quad (\text{A.1})$$

where $\lambda = i\nu$

ν is the frequency parameter ($\omega c_r/V$) based on the chord c_r ,

V is the critical flutter speed.

In this form the coefficients a , b , c and e are all non-dimensional. It may be remarked here that current practice is to use small letters for non-dimensional coefficients and capitals for dimensional coefficients. The elastic coefficients are quoted in the form $e(V_0/V)^2$, since this is convenient for modification for the simulator; it is apparent that they can easily be converted to this form from any other when V_0 is chosen.

3. *Modification for the Flutter Simulator.*—The modification consists of three steps:

- (i) Writing the equations in the form for which the simulator was designed
- (ii) Adjusting the frequency scale of the problem
- (iii) Adjusting the numerical size of the coefficients.

Steps (i) and (ii) are carried out together for convenience, and are now discussed.

3.1. *Bringing the Equations to the Required Form.*—The required form is given in equation (4) (see Section 2.1) and may be expressed in the non-dimensional form

$$[a' \ddot{q} + vb' \dot{q} + (c'v^2 + e')q] = 0, \quad \dots \dots \dots \quad (\text{A.2})$$

where $v = V/V_0$, and V_0 is chosen such that most of the solutions for v will lie in the range from 0 to 1; the maximum value of v attainable is 2×1.099 , but the factor of 2 here is intended as an emergency facility. The frequency of the solutions to equation (A.2) should be about 1 radn/sec and should certainly lie in the range 0.5 to 2 radn/sec. It is also undesirable that any of the natural frequencies of the equations should exceed 2 radn/sec as otherwise the simulator will tend to overload in the affected degree(s) of freedom.

The relation between equations (A.1) and (A.2) can be seen by writing equation (A.2) in terms of its frequency, $\bar{\nu}$ say, and dividing through by v^2

$$\left[-\frac{a'\bar{\nu}^2}{v^2} + i\frac{b'\bar{\nu}}{v} + c' + \frac{e'}{v^2} \right] [q] = 0. \quad \dots \dots \dots \quad (\text{A.3})$$

TABLE 2

Freedom	e_{rr}	a_{rr}	$\sqrt{(e/a)}$
1	4.21	107	0.1984
2	520	2040	0.5049
3	38.2	108	0.5947
4	169×10^{-6}	0.354	0.02185
5	150×10^{-6}	0.00109	0.3710
6	10×10^{-6}	0.000390	0.1601

Clearly these frequencies need to be increased, and it is the first 3 freedoms (the normal modes) which are important. In this case freedoms 4, 5 and 6 had large aerodynamic stiffness so that their frequencies increased rapidly with speed. To raise the frequency of freedoms 1 to 3 a value of k of 0.3 would be reasonable; this gives frequencies of 0.661, 1.68 and 1.98 respectively. In practice, however, it was found that the flutter involved mode 3 primarily, so that with $k = 0.3$ the frequencies were for the most part inconveniently high, and as a large programme of work was involved the problem was rescaled to $k = 0.5$.

3.2. *Adjustment of the Numerical Size of the Coefficients.*—It is now necessary to scale the coefficients so as to make maximum use of the 3-digit accuracy of the machine. The process is best illustrated by a numerical example. The coefficients (in this case those used in the problem of Table 2) are arrayed in matrix form after being scaled for frequency (*see* Table 3*). The coefficients consist of inertia terms, aerodynamic damping and stiffness terms, and elastic stiffness terms. All structural dampings are omitted, for although it is often advisable to include structural damping in the problem for solution, this can simply be done from the scaled coefficients (*see* Section 4, Appendix A).

The matrix of coefficients (*e.g.*, Table 3) occurs in the Lagrangian equations as

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} q_1 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = 0$$

where α_{11} is $-a_{11}\bar{v}^2 + ib_{11}\bar{v}v + c_{11}v^2 + e_{11}$, etc. and q are the generalised co-ordinates.

Each row of the matrix, therefore, represents the coefficients of a single linear equation, *i.e.*, the coefficients to be set in one of the racks of the simulator.

The scaling is first done by rows. The largest number which the machine can accommodate is 1.110 in increments of 0.001, but it is customary to ignore the decimal point and quote coefficients up to 1,110 in unit increments. In Table 3 the coefficients are quoted in terms of a variable mass-balance weight m^\dagger , for which the range was 0 to 60. In the first row the factor 39.974 is obtained from the term $a_{11} = 24.572 + 0.05327m$, with $m = 60$, as on multiplication by the factor 39.974 this gives 1,110. Each row is dealt with in the same manner, leading to Table 4, where each row contains one coefficient of 1,110 (possibly for the maximum value of m). The process is now repeated for such columns as do not contain terms of magnitude 1,110 (in this case columns 4, 5 and 6). Table 5 represents a matrix fully scaled for the simulator, but some refinements may still be desirable. In Table 5, for instance, the coefficients in the last 3 columns to the right of the principal diagonal are much smaller than the corresponding terms in the last 3 rows to the left. This has been roughly balanced out by the additional factors on rows and columns given in Table 5, to give (on rounding off the numbers) Table 6 ready for the

* Tables 3 to 7 are given at the end of this Appendix A.

† In practice tables of the inertia matrix would be prepared beforehand for the required values of m .

simulator. In many cases a complete calculation cannot be planned beforehand, and certain variants of the calculation may be decided on after the first results from the simulator. A matrix of scaling factors is then useful for converting coefficients from the form of Table 3 to that of Table 6 or *vice versa*; such a matrix is given in Table 7. Also in Table 7 are the factors used for each row and column. This distinction is important as the former are merely constant multipliers throughout the different equations, but the latter represent changes in the co-ordinates. Thus, if the amplitude ratios on the simulator have been measured and found to be $\alpha_1, \alpha_2 \dots \alpha_6$, then the amplitude ratios in terms of the original co-ordinates are $\gamma_1\alpha_1, \gamma_2\alpha_2 \dots \gamma_6\alpha_6$ where $\gamma_1 \dots \gamma_6$ are the column factors (in this case 1, 1, 1, 7.8325, 15.985, 36.7598).

4. *Some Additional Points.*—4.1. *Co-ordinates with Little or No Direct Stiffness.*—If the direct coefficients of a particular co-ordinate involve little or no elastic stiffness, then that co-ordinate will prove difficult to trim on the simulator at low speeds since the simulator will give both divergence and flutter solutions; e.g., co-ordinate 6 of Table 6 which has only 3 per cent elastic stiffness. In this example no difficulty occurred in practice because the aerodynamic stiffness is so large that at all practical speeds the co-ordinate could be kept trimmed without trouble. But for some freedoms, usually pitch, vertical translation, or roll, this is not so and it may be impossible to trim the simulator at any speed. In such circumstances some artificial stiffness may be added and balanced by a suitable increase in inertia. In any case where this may need to be done, the inertia coefficient should be kept well below 1,110 to allow for the increase, e.g., if required extra stiffness = δe , and expected simulator flutter frequency = \bar{v} radn/sec, then required addition to inertia is $\delta a = \delta e/\bar{v}^2$. This has a direct parallel with the use of springs in wind-tunnel tests.

4.2. *Transformations in Cases where both Elastic and aerodynamic Stiffnesses are Zero.*—If the aerodynamic stiffness coefficients of a particular co-ordinate are zero as well as the elastic stiffness coefficients, then a transformation can be used to avoid the difficulties of trimming discussed in Section 4.1. Consider the body freedom roll. If the angle of roll is denoted by q_1 and the aerodynamic derivatives in use are such that the z derivatives, e.g., l_z , are zero, then all the stiffness coefficients of q_1 in the flutter equations will be zero. The co-ordinate q_1 , in fact, will not appear explicitly in the equations at all; only the first and second derivatives will do so. q_1 , therefore, can be replaced by the co-ordinate $\bar{q}_1 = \dot{q}_1/v$ (this equation refers to the co-ordinates as scaled for frequency, i.e., $\dot{q}_1 = i\bar{v}q_1$) and the matrices appropriate to the new system of co-ordinates become†

$$a^* = \begin{bmatrix} 0 & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & a_{22} & & \dots & a_{2n} \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ 0 & a_{n2} & & \dots & a_{nn} \end{bmatrix}$$

$$b^* = \begin{bmatrix} a_{11} & b_{12} & b_{13} & \dots & b_{1n} \\ a_{21} & b_{22} & & \dots & b_{2n} \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ a_{n1} & b_{n2} & & \dots & b_{nn} \end{bmatrix}$$

† The coefficients which follow as a_{rs} , etc., correspond to the coefficients denoted by a_{rs}' , etc., in the remainder of this Appendix; the dashes are omitted here for reasons of clarity.

$$c^* = \begin{bmatrix} b_{11} & c_{12} & c_{13} & \dots & c_{1n} \\ b_{21} & c_{22} & & \dots & c_{2n} \\ \vdots & \vdots & & & \vdots \\ \vdots & \vdots & & & \vdots \\ b_{n1} & c_{n2} & & \dots & c_{nn} \end{bmatrix}$$

and $[d]$ and $[e]$ are unchanged (it is, of course, essential that the first column of d and e shall be null). There is no reason why the zero coefficient a_{11}^* should lead to trouble as long as b_{11}^* is positive, as it must be. If trimming difficulties still occur they can be overcome by introducing e_{11} and $a_{11}^* = e_{11}/\bar{v}^2$, the full range of these two coefficients is now available without impairing the accuracy of the representation of any other coefficients through having to scale down a row or column.

If the symmetric body-freedoms are involved instead of roll, and if the z derivatives are all zero and the \dot{z} derivatives equal to the α derivatives, e.g., $l_z = l_\alpha$, then a similar transformation is possible. If q_1 represents vertical translation and q_2 represents pitch, then the aerodynamic stiffness coefficients will be of the form

$$\begin{aligned} c_{11} &= \int l_z \\ c_{12} &= \int \left\{ \phi l_z + \left(\frac{c}{c_r} \right) l_\alpha \right\} \\ c_{13} &= \int \left\{ f l_z + \left(\frac{c}{c_r} \right) F l_\alpha \right\} \\ c_{21} &= \int \left\{ \phi l_z + \left(\frac{c}{c_r} \right) (-m_z) \right\} \\ c_{22} &= \int \left\{ \phi^2 l_z + \frac{c}{c_r} \phi (l_\alpha - m_z) + \left(\frac{c}{c_r} \right)^2 (-m_\alpha) \right\} \\ c_{23} &= \int \left\{ \phi f l_z + \frac{c}{c_r} (\phi F l_\alpha - f m_z) + \left(\frac{c}{c_r} \right)^2 F (-m_\alpha) \right\} \\ c_{31} &= \int \left\{ f l_z + \left(\frac{c}{c_r} \right) F (-m_z) \right\} \\ c_{32} &= \int \left\{ \phi f l_z + \frac{c}{c_r} (f l_\alpha - \phi F m_z) + \left(\frac{c}{c_r} \right)^2 F (-m_\alpha) \right\}, \text{ etc.} \end{aligned}$$

Here the co-ordinates are defined as $q_1 = z/c_r$ and q_2 is the angle of pitch, and the integrals taken over the span.

- c_r is the reference chord
- ϕ is a function defining the leading-edge displacement in the mode of pitch
- f defines the leading-edge displacement in the remaining modes
- F defines the incidence in the modes.

When the substitution $\bar{q}_1 = \dot{q}_1/v$ is made, the first and second columns become

$$k \int l_z \left(\frac{c}{c_r} \right) \quad : \quad \int \left(\frac{c}{c_r} \right) l_\alpha$$

$$k \int \left\{ \phi l_z + \frac{c}{c_r} (-m_z) \right\} \left(\frac{c}{c_r} \right) : \int \left\{ \left(\frac{c}{c_r} \right) \phi l_\alpha + \left(\frac{c}{c_r} \right)^2 (-m_\alpha) \right\}$$

$$k \int \left\{ f l_z + \left(\frac{c}{c_r} \right) F(-m_z) \right\} \left(\frac{c}{c_r} \right) : \int \left\{ \left(\frac{c}{c_r} \right) f l_\alpha + \left(\frac{c}{c_r} \right)^2 F(-m_\alpha) \right\}, \text{ etc.,}$$

where k is as defined by equation (A.4).

It is evident that these two columns are now directly proportional, since the α derivatives equal the corresponding z derivatives. The first and second co-ordinates are now transformed again:

First,
$$\bar{q}_1 = \bar{q}_1 + q_2/k,$$

(so that c_{r2} becomes zero),

secondly,
$$\bar{q}_2 = \dot{q}_2/v.$$

With the new co-ordinates, the matrices become

$$a^* = \begin{bmatrix} 0 & 0 & a_{13} & \dots & a_{1n} \\ 0 & 0 & a_{23} & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ 0 & 0 & a_{n3} & \dots & a_{nn} \end{bmatrix}$$

$$b^* = \begin{bmatrix} a_{11}, & a_{12}, & b_{13} & \dots & b_{1n} \\ a_{21}, & a_{22}, & b_{23} & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ a_{n1}, & a_{n2}, & b_{n3} & \dots & b_{nn} \end{bmatrix}$$

$$c^* = \begin{bmatrix} b_{11}, & b_{12} - a_{11}/k, & \dots & c_{1n} \\ b_{21}, & b_{22} - a_{21}/k, & \dots & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ b_{n1}, & b_{n2} - a_{n1}/k, & \dots & c_{nn} \end{bmatrix}$$

and $[d]$ and $[e]$ are unchanged, provided the first two columns are null.

Again, if necessary, terms e_{11} and e_{22} may be introduced and balanced by $a_{11}^* = e_{11}/\bar{v}^2$ and $a_{22}^* = e_{22}/\bar{v}^2$ respectively.

It may also be possible to apply this treatment in the more general case when $l_z \neq 0$ and $l_z \neq l_\alpha$. For wings of finite aspect ratio it will usually be acceptable to approximate to l_z and l_z by the relations

$$l_z \simeq \kappa_1 v^2$$

$$l_z \simeq l_\alpha + \kappa_2 v^2.$$

If this is so, the quantities l_z and $l_z - l_\alpha$ can be reduced to zero by adding the terms in κ to the aerodynamic inertias. Similar considerations apply to the other z and z derivatives; this will make the new aerodynamic inertia matrix unsymmetrical, but this effect is unimportant.

4.3. *Structural Damping*.—In practice all structural modes contain some inherent structural damping, and it is wise always to include some in the coefficients for the simulator. This may help to avoid bad beating (with the attendant difficulty of reading the flutter speed accurately), and will also help to avoid unreal cases of zero flutter speeds.

Critical damping in a single degree of freedom at zero air speed is given by

$$d_c = 2\sqrt{ae}$$

Practical values of structural damping for resonance modes of an aircraft are unlikely to fall below

$$d = 0.005\sqrt{ae}.$$

If no measurement of damping exists, it is still preferable, in the author's opinion, to use an arbitrary low value (such as the 0.5 per cent quoted) than to use zero. Care should, of course, be taken that too large a value is not assumed.

4.4. *Required Properties of the Final Coefficients*.—The final coefficients must observe certain properties (which they automatically will if the Lagrangian equations have been used correctly, and the signs of the coefficients left unchanged during scaling) for the simulator to work satisfactorily.

(1) The inertia matrix must be positive definite†. This cannot normally be seen at a glance, but at least all the direct inertia terms should be positive.

(2) The elastic matrix must be positive definite. This should be checked in cases where the determinant of two degrees of freedom is mathematically zero, *e.g.*, an aileron and servo-tab may have an elastic matrix

$$\begin{bmatrix} N^2e & -Ne \\ -Ne & e \end{bmatrix}$$

where N is the follow-up ratio. This may result, after scaling, in the coefficients

$$\begin{bmatrix} 8.4 & -15.9 \\ -586.4 & 1110 \end{bmatrix}$$

which after rounding off would give a determinant

$$\begin{vmatrix} 8 & -16 \\ -586 & 1110 \end{vmatrix} = -496$$

In practice it would prove necessary to take the first direct elastic coefficient as 9 rather than 8.

(3) The aerodynamic damping matrix need not be positive definite, but usually will be in practice.

(4) The aerodynamic stiffness matrix will not, in general, be positive definite. It should be remembered that the simulator will not work at speeds near to or beyond the divergence speed given by the coefficients put into the machine (not necessarily corresponding to a physical divergence speed).

(5) As regards structural damping, only the direct coefficients will normally be used.

† 'Positive definite' is used in the sense that the determinants of all the principal minors of the matrix must be positive or zero. The scaling process will in general destroy the symmetry of the inertia (and elastic) matrix, but the rule quoted should still apply.

TABLE 3

		1	2	3	4	5	6	Row factor to obtain Table 4
1	<i>a</i>	24.572	— 9.2630	5.9469	0.19500	0.002068	0.6416×10^{-3}	39.97378
		+ 0.05327 <i>m</i>	— 0.11168 <i>m</i>	— 0.07557 <i>m</i>	— 0.005988 <i>m</i>			
	<i>b</i>	1.9542	— 2.9025	— 0.24524	0.26604	— 0.04580	— 0.029565	
	<i>c</i>	0.5630	— 2.1809	— 0.04932	1.3018	0.34327	0.19788	
	<i>e</i>	4.2100						
2	<i>a</i>	— 9.2630	515.83	— 13.992	— 0.29645	— 0.00430	— 0.001273	2.0948
		— 0.11168 <i>m</i>	+ 0.23409 <i>m</i>	+ 0.1584 <i>m</i>	+ 0.01255 <i>m</i>			
	<i>b</i>	— 1.1075	14.913	2.7115	— 0.43092	0.032897	0.027200	
	<i>c</i>	0.39289	— 14.726	0.73890	— 1.7071	— 0.51266	— 0.32171	
	<i>e</i>		520.45					
3	<i>a</i>	5.9469	— 13.992	22.771	— 0.17579	— 0.002953	— 0.8051 $\times 10^{-3}$	29.026
		— 0.07557 <i>m</i>	+ 0.1584 <i>m</i>	+ 0.1072 <i>m</i>	+ 0.008494 <i>m</i>			
	<i>b</i>	— 0.27568	2.9755	4.0035	— 0.28195	0.043689	0.034578	
	<i>c</i>	— 0.092969	1.1180	1.2029	— 1.3309	— 0.35997	— 0.24849	
	<i>e</i>			38.242				
4	<i>a</i>	0.19500	— 0.29645	— 0.17579	0.032173	0.61435×10^{-3}	0.1725×10^{-3}	2,430.96
		— 0.005988 <i>m</i>	+ 0.01255 <i>m</i>	+ 0.008494 <i>m</i>	+ 0.001407 <i>m</i>			
	<i>b</i>	0.0028079	0.0038025	— 0.0019615	0.010808	0.0010614	0.2824×10^{-3}	
	<i>c</i>	— 0.017937	0.084222	0.025714	0.030767	0.024620	0.01150	
	<i>e</i>			0.16905×10^{-3}	$— 0.1595 \times 10^{-3}$	$— 0.00954 \times 10^{-3}$		
5	<i>a</i>	0.002068	— 0.00430	— 0.0029533	0.61435×10^{-3}	0.2713×10^{-3}	0	258,140.0
	<i>b</i>	0.2138×10^{-3}	$— 0.3796 \times 10^{-3}$	$— 0.2315 \times 10^{-3}$	0.30511×10^{-3}	0.14424×10^{-3}	0	
	<i>c</i>	$— 0.2428 \times 10^{-3}$	0.001139	0.3492×10^{-3}	0.6158×10^{-3}	1.3450×10^{-3}	0	
	<i>e</i>				$— 0.1595 \times 10^{-3}$	0.1595×10^{-3}	0	
6	<i>a</i>	0.64158×10^{-3}	— 0.001273	$— 0.8051 \times 10^{-3}$	0.17246×10^{-3}	0	0.09749×10^{-3}	871,956.0
	<i>b</i>	$— 0.04164 \times 10^{-3}$	0.0002294	0.07205×10^{-3}	0.07460×10^{-3}	0	0.027715×10^{-3}	
	<i>c</i>	$— 0.09351 \times 10^{-3}$	0.0004580	0.1548×10^{-3}	0.1283×10^{-3}	0	0.3463×10^{-3}	
	<i>e</i>				$— 0.009541 \times 10^{-3}$	0	0.009541×10^{-3}	

TABLE 4

		1	2	3	4	5	6
1	<i>a</i>	982.236 + 2.129 <i>m</i>	- 370.277 - 4.464 <i>m</i>	237.72 - 3.0208 <i>m</i>	7.7949 - 0.2394 <i>m</i>	0.08267	0.02565
	<i>b</i>	78.117	- 116.02	- 9.804	10.635	- 1.8308	- 1.1818
	<i>c</i>	22.505	- 87.179	- 1.9707	52.038	13.7218	7.9100
	<i>e</i>	168.290					
2	<i>a</i>	- 19.404 - 0.2340 <i>m</i>	1080.56 + 0.4904 <i>m</i>	- 29.310 + 0.3318 <i>m</i>	- 0.62100 + 0.02629 <i>m</i>	- 0.009008	- 0.002667
	<i>b</i>	- 2.3200	31.240	5.6801	- 0.9027	0.068913	0.05698
	<i>c</i>	0.82303	- 30.848	1.5478	- 3.5760	- 1.0739	- 0.67392
	<i>e</i>		1090.24				
3	<i>a</i>	172.615 - 2.1936 <i>m</i>	- 406.132 + 4.5977 <i>m</i>	660.957 + 3.1116 <i>m</i>	- 5.1025 + 0.2465 <i>m</i>	- 0.085714	- 0.023369
	<i>b</i>	- 8.0019	86.367	116.206	- 8.1839	1.26812	1.00366
	<i>c</i>	- 2.6985	32.451	34.915	- 38.631	- 10.4485	- 7.2127
	<i>e</i>			1110			
4	<i>a</i>	474.04 - 14.557 <i>m</i>	- 720.66 + 30.509 <i>m</i>	- 427.34 + 20.649 <i>m</i>	78.2113 + 3.420 <i>m</i>	1.4935	0.41934
	<i>b</i>	6.8259	9.2437	- 4.7683	26.2738	2.5802	0.68650
	<i>c</i>	- 43.604	204.740	62.510	74.793	59.8502	27.9560
	<i>e</i>				0.41095	- 0.38774	- 0.0232
5	<i>a</i>	533.834	- 1110	- 762.36	158.59	70.033	0
	<i>b</i>	55.190	- 97.990	- 59.759	78.761	37.234	0
	<i>c</i>	- 62.676	294.02	90.142	158.963	347.201	0
	<i>e</i>				- 41.173	41.173	0
6	<i>a</i>	559.43	- 1110	- 702.01	150.38	0	85.01
	<i>b</i>	- 36.308	200.03	62.82	65.05	0	24.17
	<i>c</i>	- 81.537	399.36	134.98	111.87	0	301.96
	<i>e</i>				- 8.319	0	8.319
Column factor to obtain Table 5		1	1	1	3.91624	3.1970	3.67598

TABLE 5

Row and Column Factors shown are those to obtain Table 6.

		1	2	3	4	5	6	Row Factor
1	<i>a</i>	982.2 + 2.13 <i>m</i>	- 370.3 - 4.46 <i>m</i>	237.7 - 3.02 <i>m</i>	30.53 - 0.938 <i>m</i>	0.2643	0.09429	1
	<i>b</i>	78	- 116	- 10	41.65	- 5.8531	- 4.3443	
	<i>c</i>	23	- 87	- 2	203.8	43.8686	29.0770	
	<i>e</i>	168						
2	<i>a</i>	- 19.4 - 0.234 <i>m</i>	1080.6 + 0.490 <i>m</i>	- 29.3 + 0.332 <i>m</i>	- 2.43 + 0.103 <i>m</i>	- 0.028799	- 0.009804	1
	<i>b</i>	- 2	31	6	- 3.5352	0.22031	0.20946	
	<i>c</i>	1	- 31	2	- 14.004	- 3.4333	- 2.4773	
	<i>e</i>		1090					
3	<i>a</i>	172.6 - 2.19 <i>m</i>	- 406.1 + 4.60 <i>m</i>	660.96 + 3.11 <i>m</i>	- 19.983 + 0.965 <i>m</i>	- 0.27403	- 0.08590	1
	<i>b</i>	- 8	86	116	- 32.050	4.0542	3.68943	
	<i>c</i>	- 3	32	35	- 151.29	- 33.404	- 26.5137	
	<i>e</i>			1110				
4	<i>a</i>	474.0 - 14.56 <i>m</i>	- 720.7 + 30.51 <i>m</i>	- 427.34 + 20.65 <i>m</i>	306.29 + 13.39 <i>m</i>	4.7747	1.54149	0.5
	<i>b</i>	6.83	9.24	- 4.77	102.89	8.2489	2.5236	
	<i>c</i>	- 43.61	204.74	62.5	292.91	191.341	102.766	
	<i>e</i>				1.6094	- 1.2396	- 0.0853	
5	<i>a</i>	533.83	- 1110	- 762.4	621.08	223.896	0	0.2
	<i>b</i>	55.19	- 97.99	- 59.76	308.45	119.037	0	
	<i>c</i>	- 62.68	294.02	90.14	622.53	1110	0	
	<i>e</i>				- 161.24	131.63	0	
6	<i>a</i>	559.43	- 1110	- 702.01	588.92	0	312.50	0.1
	<i>b</i>	- 36.31	200	62.82	254.75	0	88.848	
	<i>c</i>	- 81.54	399.4	134.98	438.11	0	1110	
	<i>e</i>				- 32.579	0	30.580	
Column factor		1	1	1	2	5	10	

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TABLE 6

		1	2	3	4	5	6
1	<i>a</i>	982.2 + 2.13 <i>m</i>	- 370.3 - 4.46 <i>m</i>	237.7 - 3.02 <i>m</i>	61.06 - 1.876 <i>m</i>	1 - 29 219	1 - 43 291
	<i>b</i>	78	- 116	- 10	83		
	<i>c</i>	23	- 87	- 2	408		
	<i>e</i>	168					
2	<i>a</i>	- 19.4 - 0.234 <i>m</i>	1080.6 + 0.490 <i>m</i>	- 29.3 + 0.33 <i>m</i>	- 4.86 + 0.206 <i>m</i>	0 1 - 17	0 2 - 25
	<i>b</i>	- 2	31	6	- 7		
	<i>c</i>	1	- 31	2	- 28		
	<i>e</i>		1090				
3	<i>a</i>	172.6 - 2.19 <i>m</i>	- 406.1 + 4.60 <i>m</i>	660.96 + 3.11 <i>m</i>	- 39.97 + 1.93 <i>m</i>	- 1 20 - 167	- 1 37 - 265
	<i>b</i>	- 8	86	116	- 64		
	<i>c</i>	- 3	32	35	- 302		
	<i>e</i>			1110			
4	<i>a</i>	237.0 - 7.28 <i>m</i>	- 360.35 + 15.26 <i>m</i>	- 213.67 + 10.33 <i>m</i>	306.29 + 13.40 <i>m</i>	12 20 478 - 3	8 13 514 0
	<i>b</i>	3	5	- 2	103		
	<i>c</i>	- 22	102	31	293		
	<i>e</i>				2		
5	<i>a</i>	107	- 222	- 152	248	224 119 1110 132	0 0 0 0
	<i>b</i>	11	- 20	- 12	123		
	<i>c</i>	- 12	59	18	249		
	<i>e</i>				- 64		
6	<i>a</i>	56	- 111	- 70	118	0 0 0 0	312 89 1110 31
	<i>b</i>	- 4	20	6	51		
	<i>c</i>	- 8	40	13	88		
	<i>e</i>				- 7		

TABLE 7

39.974 2.0948 29.026 1,215.48 51,627.9 87,195.6	39.974 2.0948 29.026 1,215.48 51,627.9 87,195.6	39.974 2.0948 29.026 1,215.48 51,627.9 87,195.6	313.09 16.4075 227.34 9,520.22 404,374 682,958	638.98 33.485 463.98 19,492.5 825,272 1,393,822	1,469.43 77.004 1,066.99 44,680.8 1,897,831 3,205,293
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68

Row	Factor	Column	Factor
1	39.974	1	1
2	2.0948	2	1
3	29.0261	3	1
4	1,215.48	4	7.8325
5	51,627.9	5	15.9850
6	87,195.6	6	36.7598

APPENDIX B

Frequency and Phase measurement Equipment

by

W. D. Hicks

1. *Introduction.*—The R.A.E. six-degree-of-freedom flutter simulator is designed so that stable flutter oscillations are represented by sinusoidal voltages at a frequency that lies within the range from $\omega = 1$ radn/sec to $\omega = 100$ radn/sec. The flutter solution requires an accurate measurement of this frequency and also of the phase difference between any two of the degrees of freedom in the problem. For efficient operation these measurements should be obtained quickly and easily. An instrument that satisfies these requirements and which is now installed in the R.A.E. simulator is described below.

2. *Principle of Operation.*—At very low frequencies a more accurate, and rapid, result is obtained by measuring the time taken by one complete cycle of the oscillation, rather than by counting the number of complete cycles in a pre-determined time. This equipment, therefore, times the interval between similar points on the periodic wave form and obtains phase differences by timing the interval between similar points on the two wave forms examined. The unit is based on the dekatron counter described in Ref. 4 with certain modifications and additional circuits.

2.1. *Frequency Measurement.*—A block diagram of the component lay-out is shown in Fig. 55. In this diagram, switches S.1 and S.2 are used to make connections to the oscillating voltage output of any one of the thirty amplifiers in the flutter simulator. Thus an oscillating voltage is passed through switch S.1 to the Pulse Unit. This unit produces positive pulses of one tenth of a millisecond duration and at the same frequency as the incoming sine wave. One of these pulses closes the Electronic Switch Unit and an immediately following pulse opens it. The electronic switch has thus been closed for one complete cycle of the incoming signal, permitting the passage to the Register Unit of a controlled number of complete cycles from the 1,000-c.p.s. generator. The Register Unit provides a visual count of the 1,000-c.p.s. pulses and thus indicates the time taken for one complete cycle of the incoming low-frequency signal, measured to within one millisecond.

The Automatic Reset shown in Fig. 55 returns the Register Unit to zero and resets the Electronic Switch. This is done after a pre-set delay period and the instrument will then repeat its action throughout.

2.2. *Phase Measurement.*—Continuing to refer to Fig 55, the switch S.2 can be utilised to bring another oscillating voltage signal, of different phase to that passing through S.1, into the Input Change-over Unit. This signal has no effect on the time counting circuit, however, unless switch S.3 is closed.

With switch S.3 closed, the first positive pulse from the Pulse Unit is made to operate both the Electronic Switch and the Input Change-over Units. The input from switch S.1 is then replaced by the signal from switch S.2. As a result of this change-over, the next positive pulse applied to the electronic Switch arises from the action of the signal from S.2 on the Pulse Unit. The Electronic Switch then closes and the Register Unit presents the time interval, in milliseconds, between the two positive pulses. If T_1 is the measured time interval for a complete cycle and T_2 the interval between the two positive pulses from different sources, then the phase difference is $(T_2/T_1)2\pi$ radians, referred to the signal from S.1. T_1 and T_2 are not measured at the same instant of time, so that the reliability of the measurement will depend on the stability of the input oscillations. The unit is unlikely to be reliable if 'beating' takes place.

3. *Components.*—3.1. *The Pulse Unit.*—This consists of a double triode valve arranged in a circuit as shown in Fig. 56. The output of a chosen amplifier in the simulator is applied between the two valve grids. A sensitive, centre-stable, relay is connected as shown between the two anodes. The signal from the flutter simulator is amplified and applied across the relay, closing the relay contacts at the signal frequency. While the contacts are open the $0.1\text{-}\mu\text{F}$ capacitor is charged through a 750-kilohm resistor from the valve supply voltage. When the contacts close this capacitor is discharged through a resistor, resulting in a positive voltage pulse. This pulse is the output of the unit and is used to trigger the Electronic Switch. Initial balance of the relay tongue position is obtained by adjustment of the potentiometer between the anode circuits. Normally, no further adjustment is needed.

3.2. *Electronic Switch.*—This circuit is shown at Fig. 57 and includes the change-over unit needed for phase measurement. The two valves shown are screen-grid thyratrons. Initially T.1 is off and T.2 is passing an anode current which holds the contacts of relay R.1 open. The first pulse to arrive switches T.1 on and, in doing so, switches T.2 off through the capacitor coupling the two cathodes. This closes the relay contact on R.1. The second positive pulse switches T.2 on but T.1 remains unaffected. Hence the relay contacts will open and the unit remains inoperative until the action of the Reset Unit at some pre-determined delay time returns both valves T.1 and T.2 to their initial conditions by opening the cathode circuit of T.1.

Phase measurement is effected by closing the manually operated switch S.3. The additional relay R.2 is then operated by a pair of contacts of relay R.1 (Fig. 57). Relay R.2 completes a change over from S.1 to S.2 when the first operating pulse arrives. Hence the second operating pulse is derived from the output of a different amplifier through switch S.2 in the flutter simulator.

4. *Concluding Remarks.*—This system has been installed in the R.A.E. six-degree-of-freedom flutter simulator and works satisfactorily. With the addition of the subsidiary units described herein the instrument described in Ref. 4 could be adapted to measure phase in oscillating structures at typical frequencies. It should be noted that the accuracy falls off as the frequency measured increases. This could be ameliorated by an increase in the time-base frequency. With the existing instrument the top limit is approximately 2,000 c.p.s. for the time base.

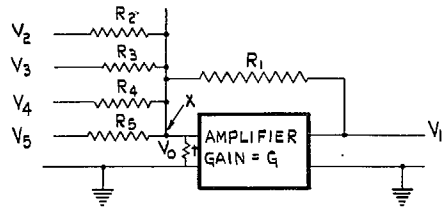


FIG. 1. Summing unit.

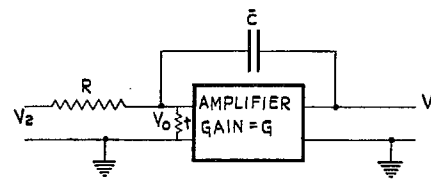


FIG. 2. Integrating unit.

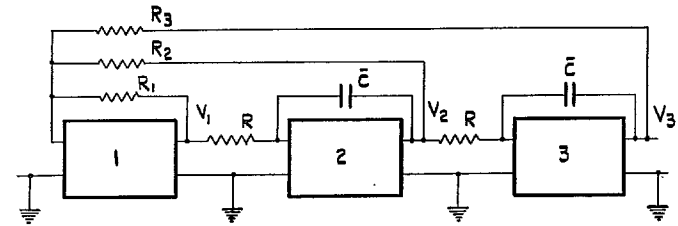


FIG. 3. Circuit representing $A\ddot{x} + D\dot{x} + Ex = 0$.

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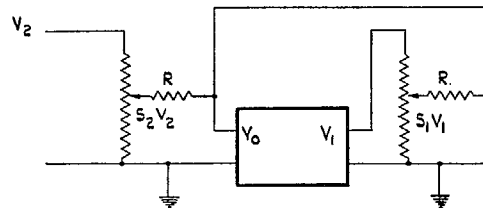


FIG. 4. The use of potentiometers to represent aircraft coefficients.

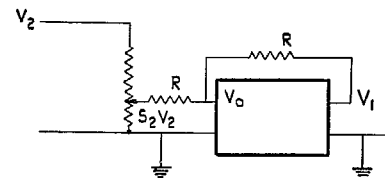


FIG. 5. Variation of a coefficient with air speed.

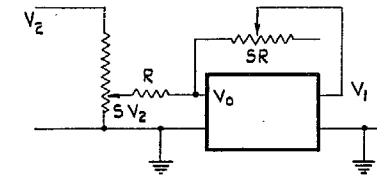


FIG. 6. Variation of a coefficient with $(\text{air speed})^2$.

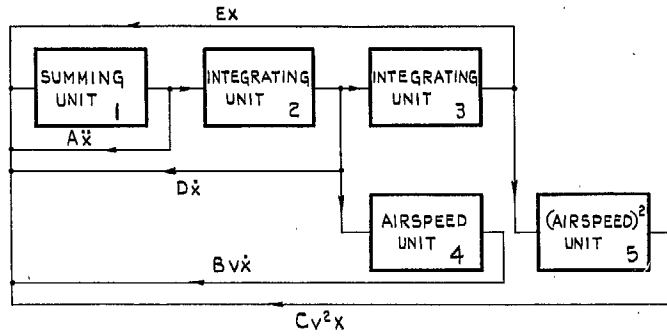


FIG. 7. Circuit simulating equation in one degree of freedom: $A\ddot{x} + (Bv + D)\dot{x} + (Cv^2 + E)x = 0$.

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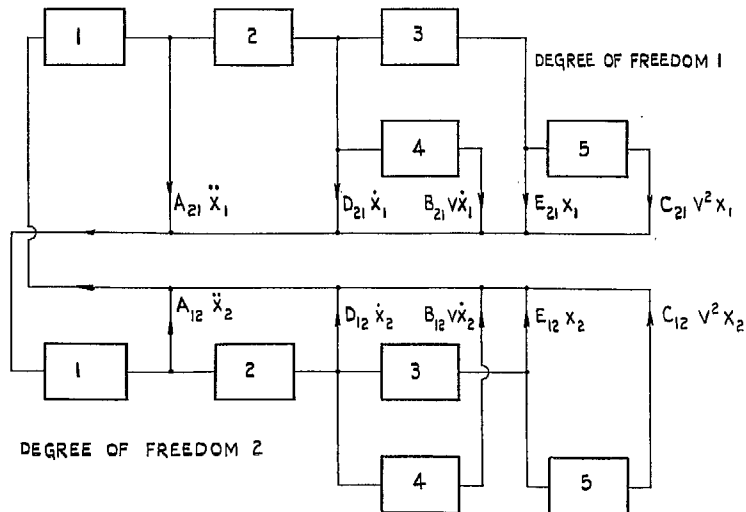
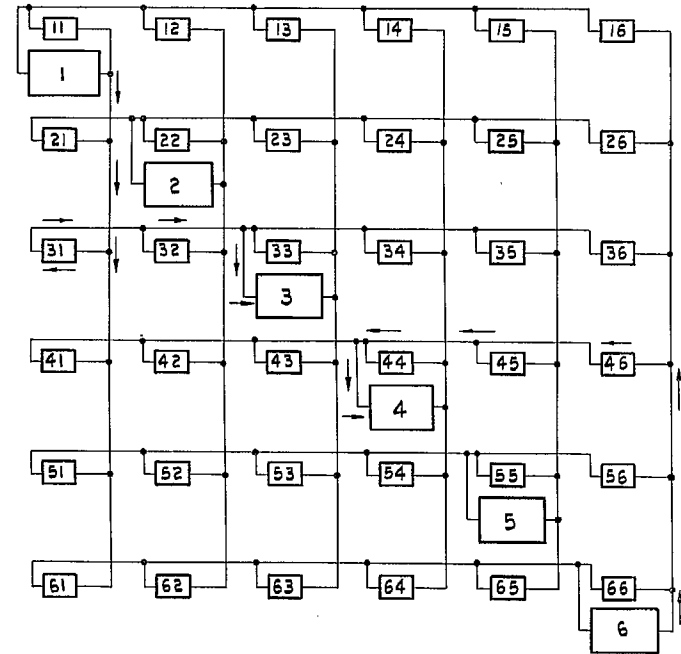




FIG. 8. Coupling between two circuits representing the coupling between two degrees of freedom.



 = ONE DEGREE OF FREEDOM UNIT AS SHOWN IN FIG. 7.

 = POTENTIOMETER REVERSING UNIT CONNECTING OUTPUT OF UNIT q TO INPUT OF UNIT p.

ARROWS INDICATE REPRESENTATIVE FLOW OF CURRENT FROM OUTPUT OF UNIT 1 TO INPUT OF UNIT 3 AND FROM OUTPUT OF UNIT 6 TO INPUT OF UNIT 4. ONLY ONE OUTPUT FROM EACH UNIT SHOWN FOR SIMPLICITY. THERE ARE ACTUALLY FIVE PUSH-PULL OUTPUTS FROM EACH UNIT.

FIG. 9. Basic circuit for six degrees of freedom.

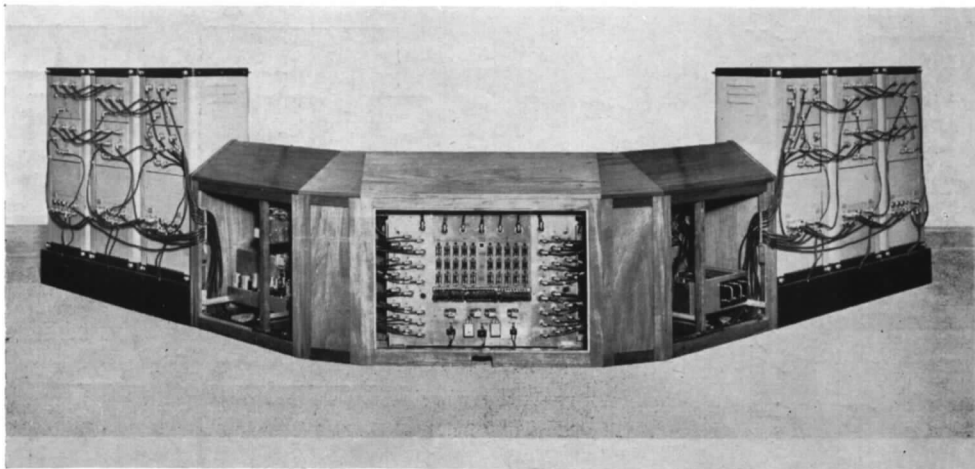
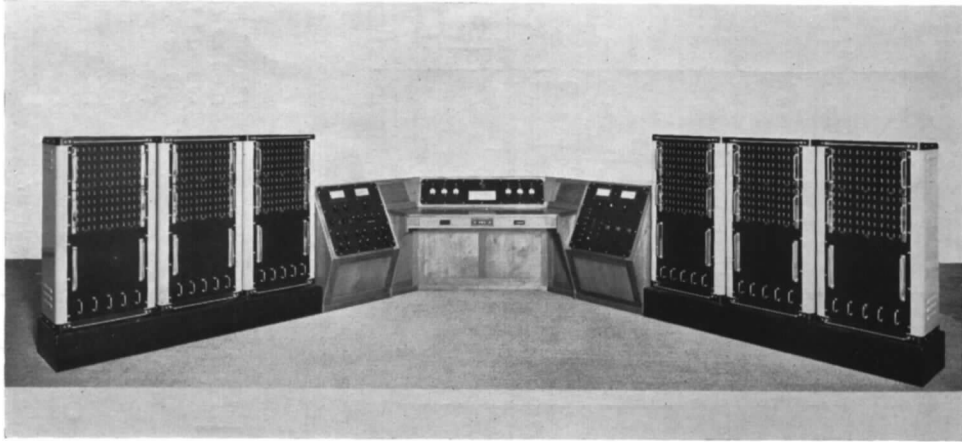


FIG. 10. Front and rear views of complete simulator.

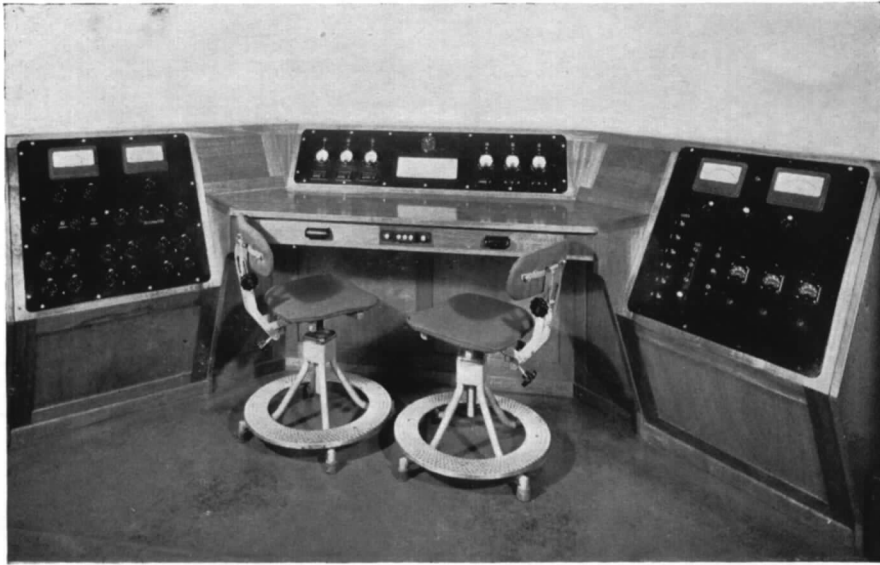


FIG. 11. Control desk.

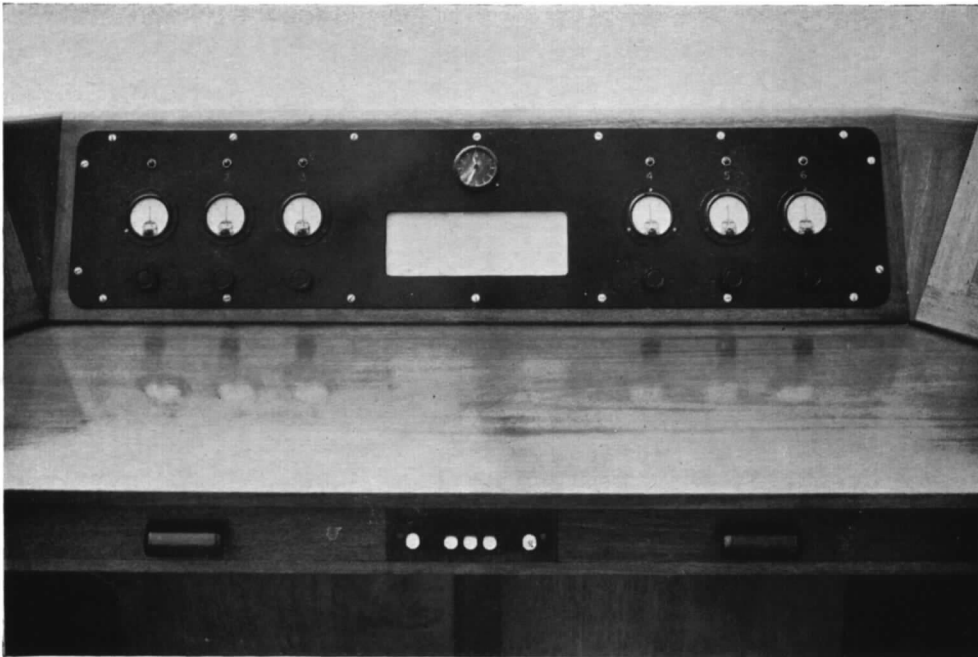


FIG. 12. Centre indicator panel.



FIG. 13. Left-hand indicator panel.

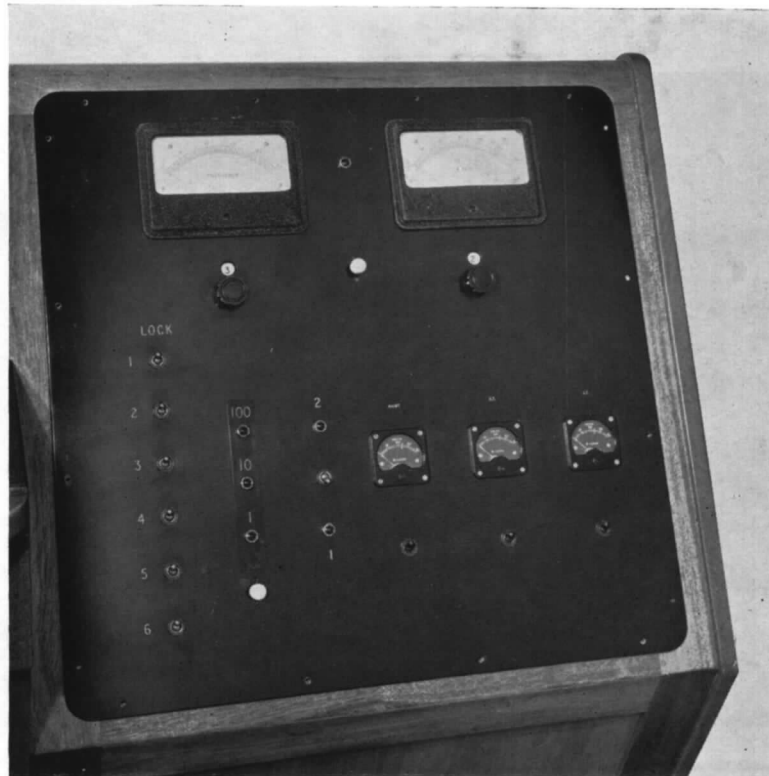


FIG. 14. Right-hand indicator panel.

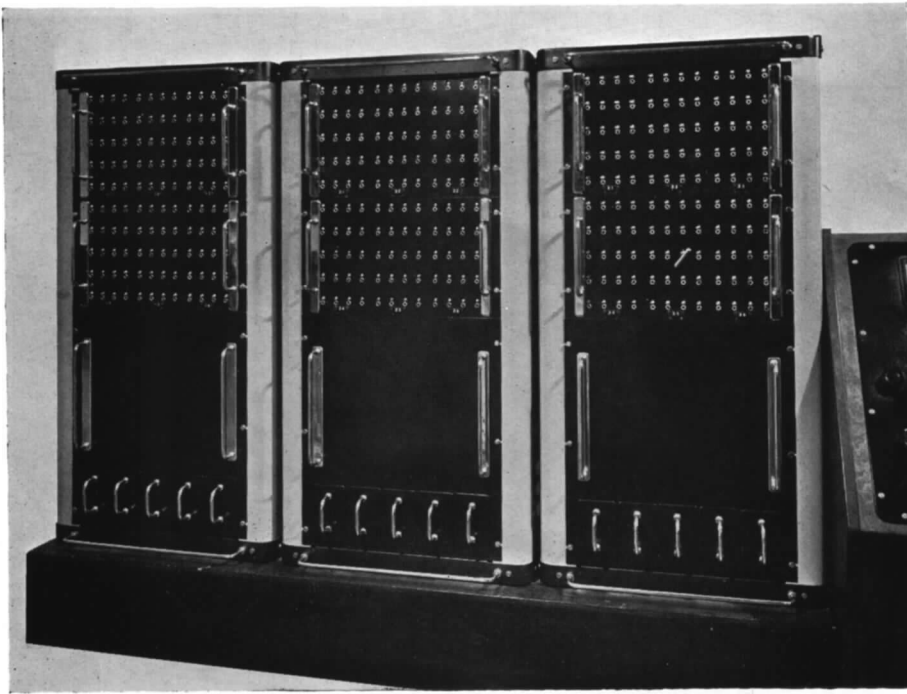


FIG. 15. Front view of three racks.

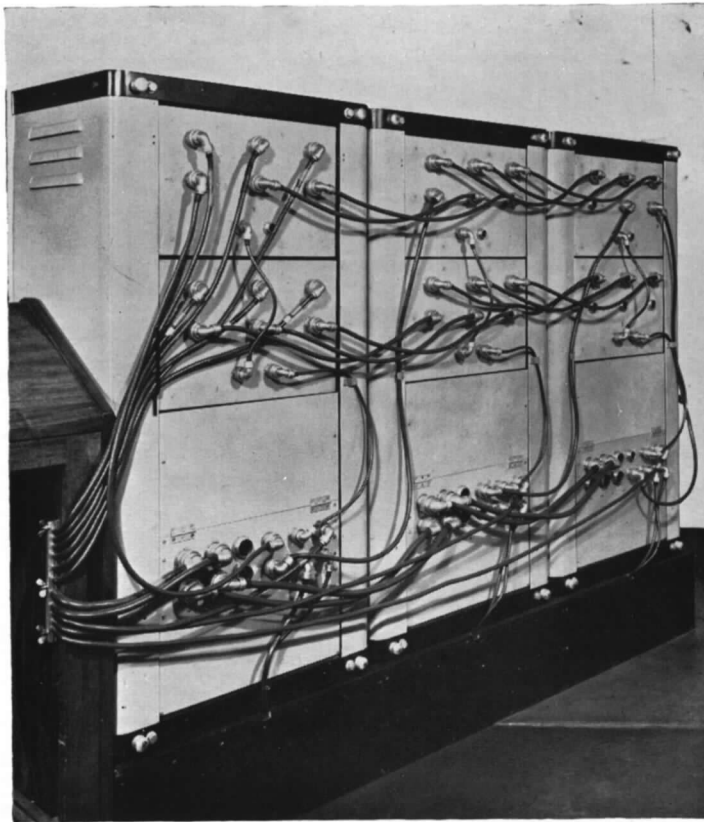


FIG. 16. Rear view of three racks.

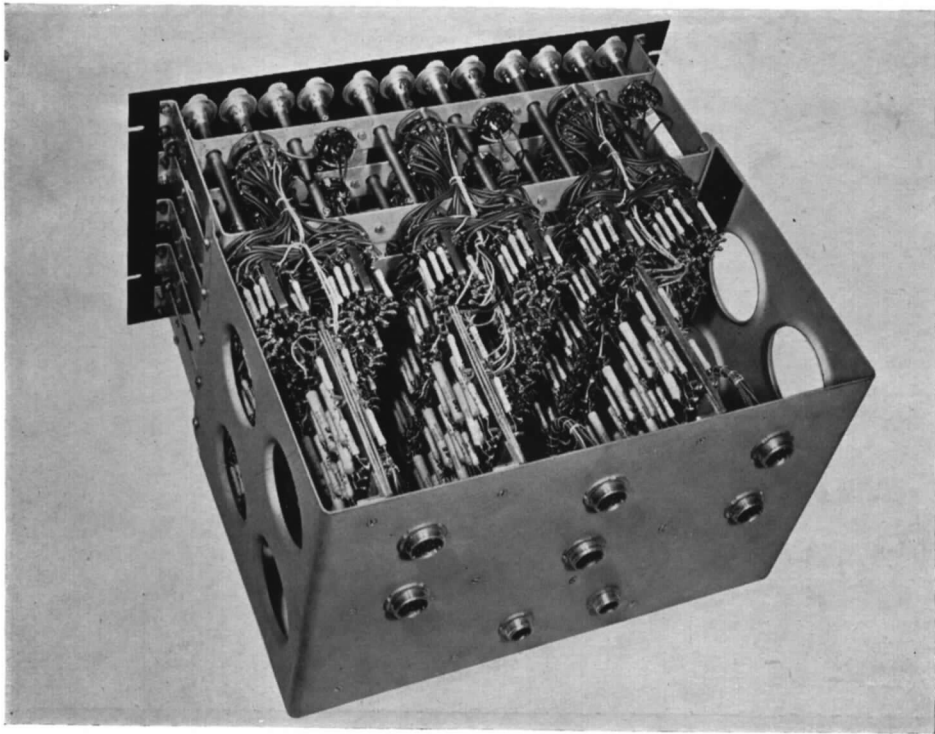


FIG. 17. Coefficient unit.



FIG. 18. Air-speed unit.

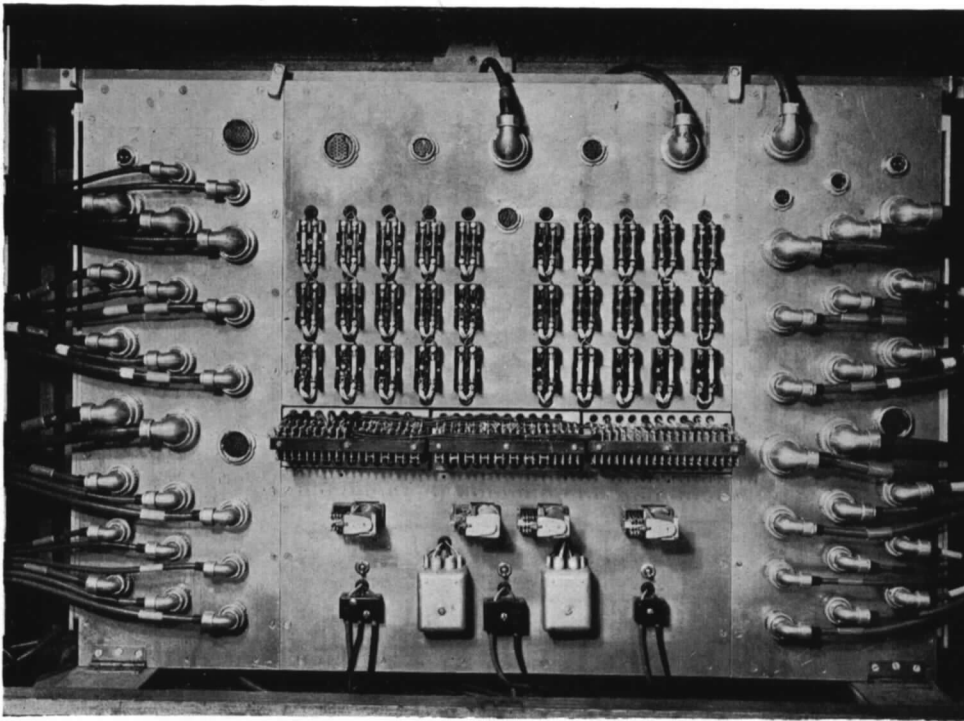


FIG. 19. Main connector panel.

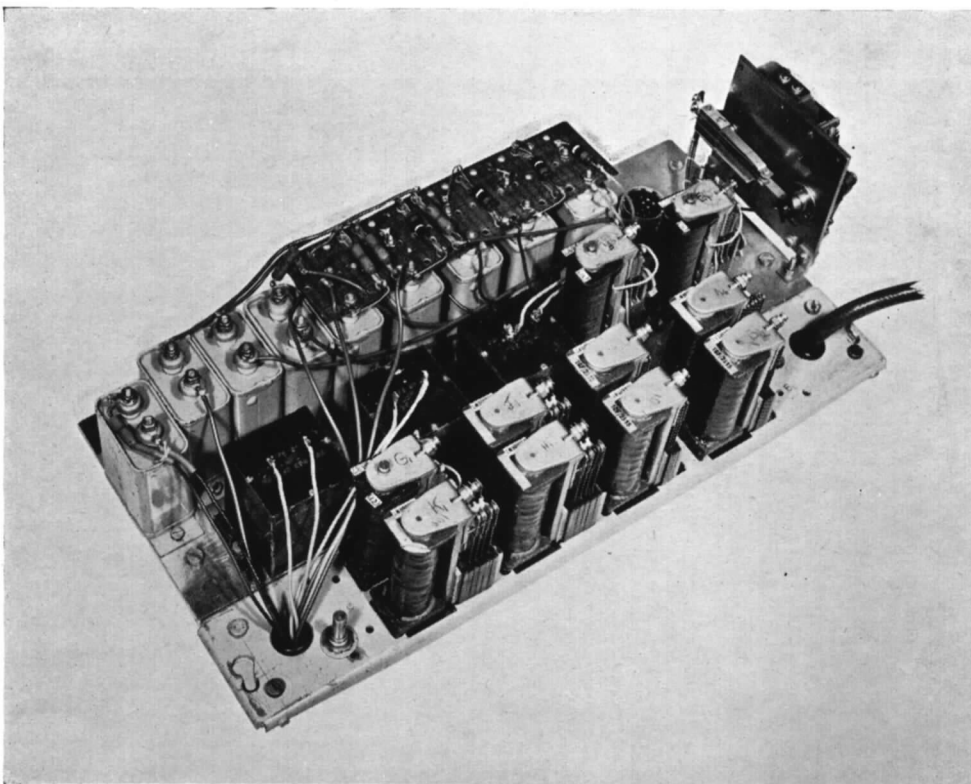


FIG. 20. Air-speed pulsing unit.

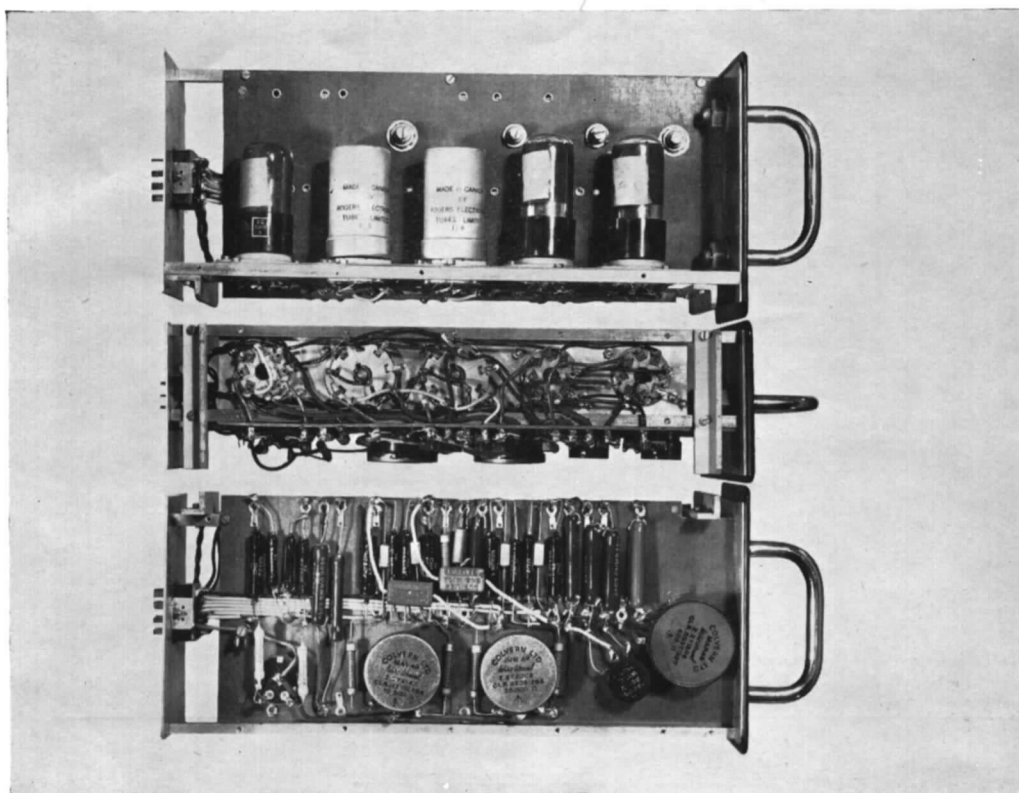


FIG. 21. Amplifier.

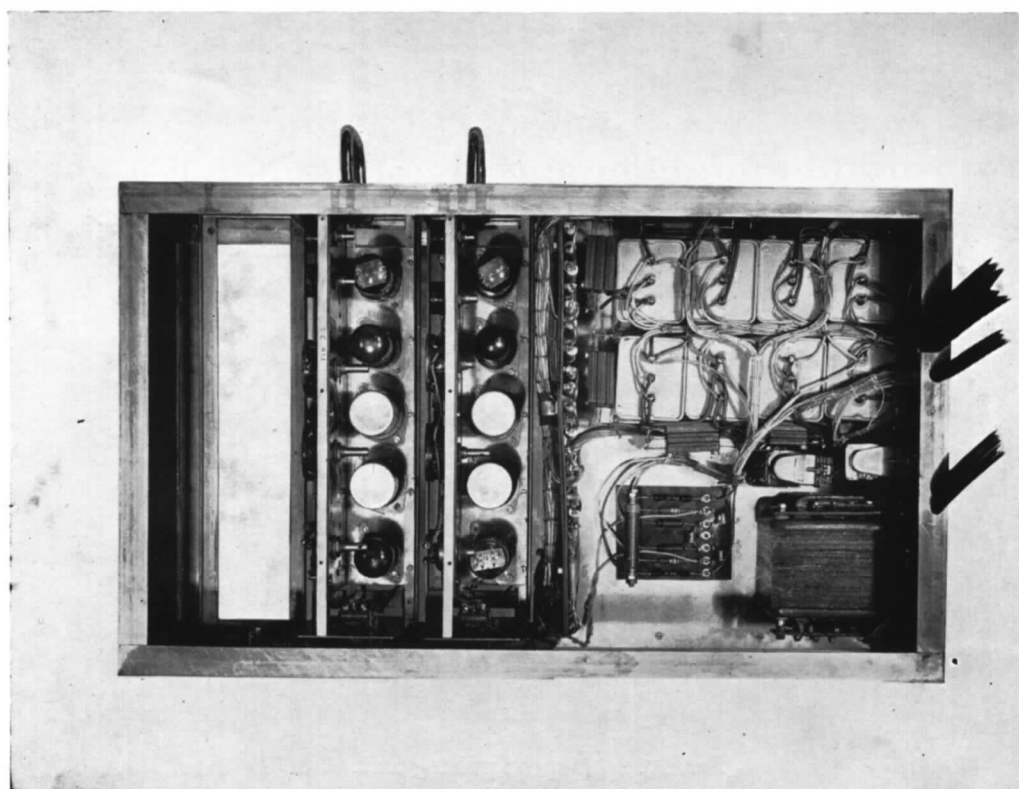


FIG. 22. Resonance unit.

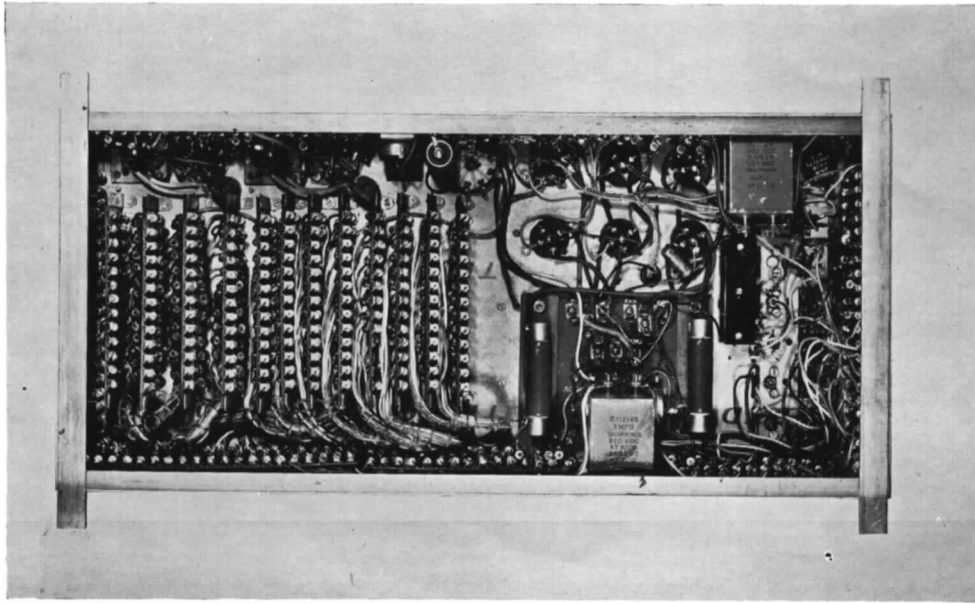


FIG. 23. Top cover removed.

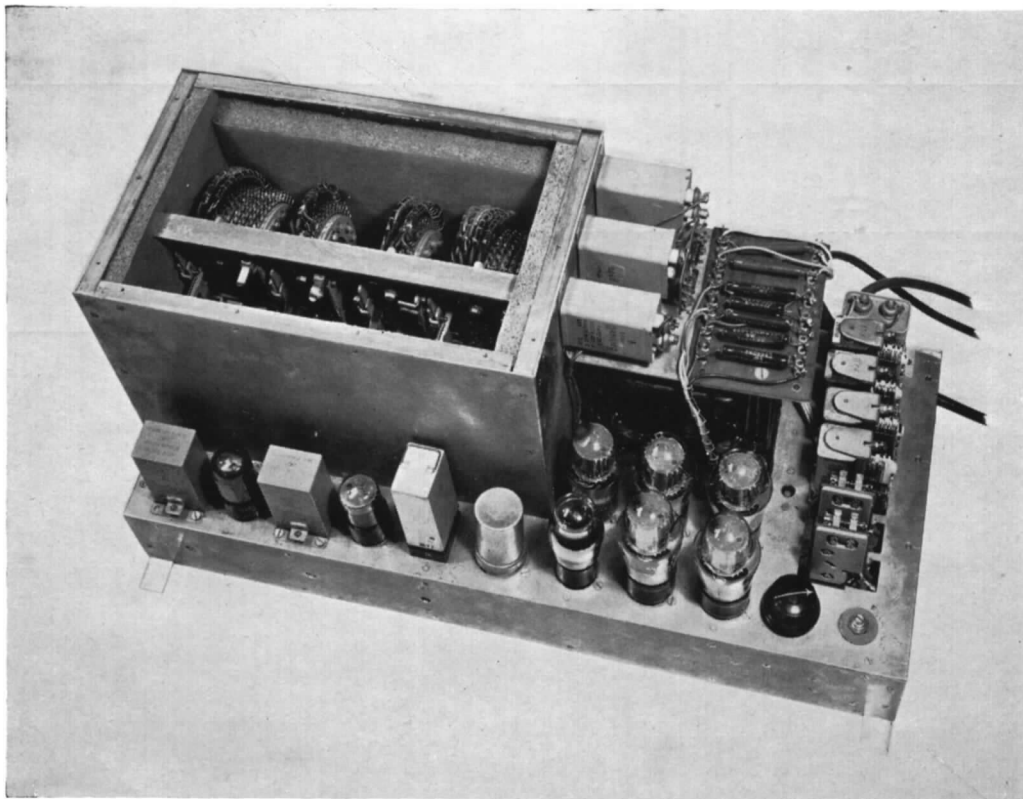


FIG. 24. Inside Chassis.

FIGS. 23 and 24. Phase and frequency unit.

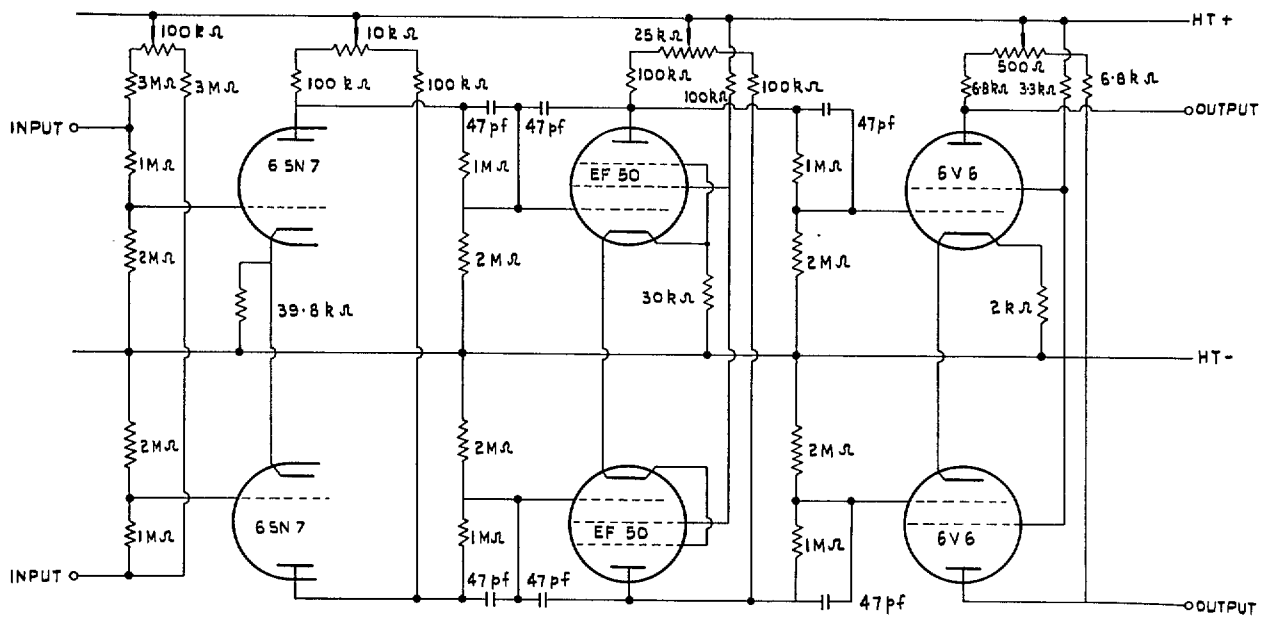
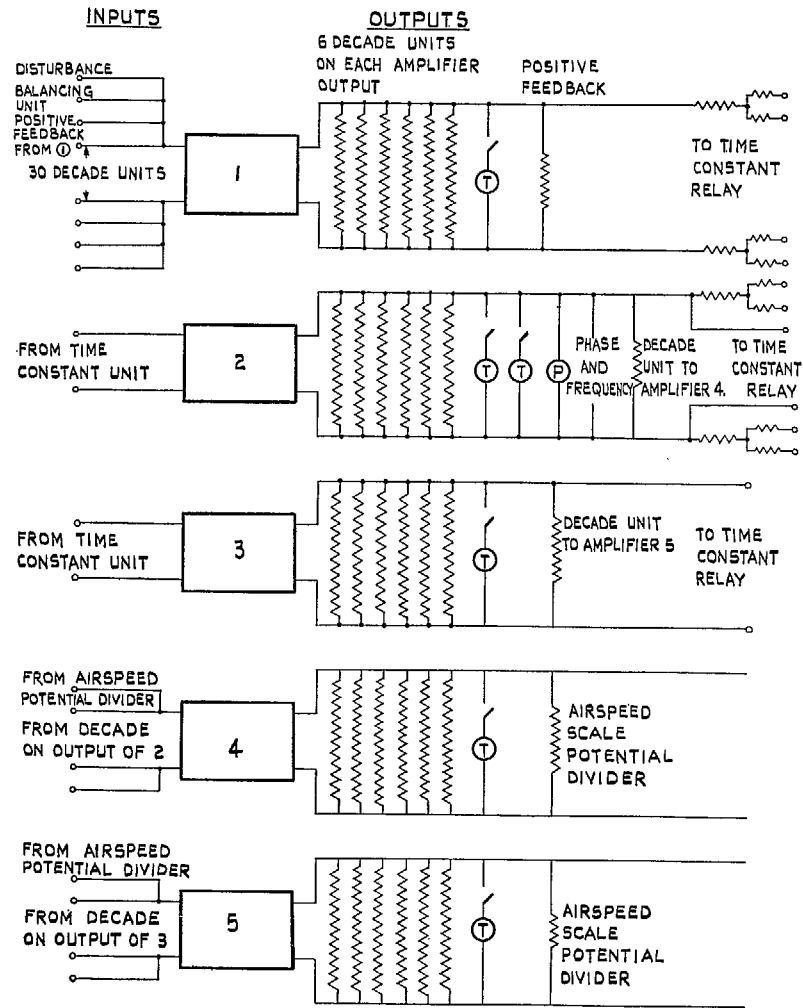


FIG. 25. D.C. Amplifier circuit diagram.



(T) = TEST METER (P) = FRONT PANEL METER.
 LOCKING CONNECTIONS NOT SHOWN.

ON FREQUENCY SCALE "TIMES 100" THE FRONT PANEL METERS ARE OPEN CIRCUIT, WHILE THE TWO TEST METERS ARE CONNECTED TO THE AMPLIFIER OUTPUTS THROUGH A RECTIFIER. THESE CHANGES ARE RELAY OPERATED.

FIG. 26. Amplifier input and output circuits.

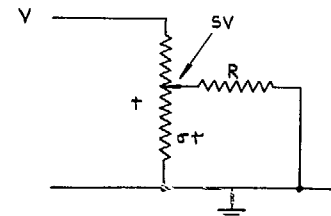
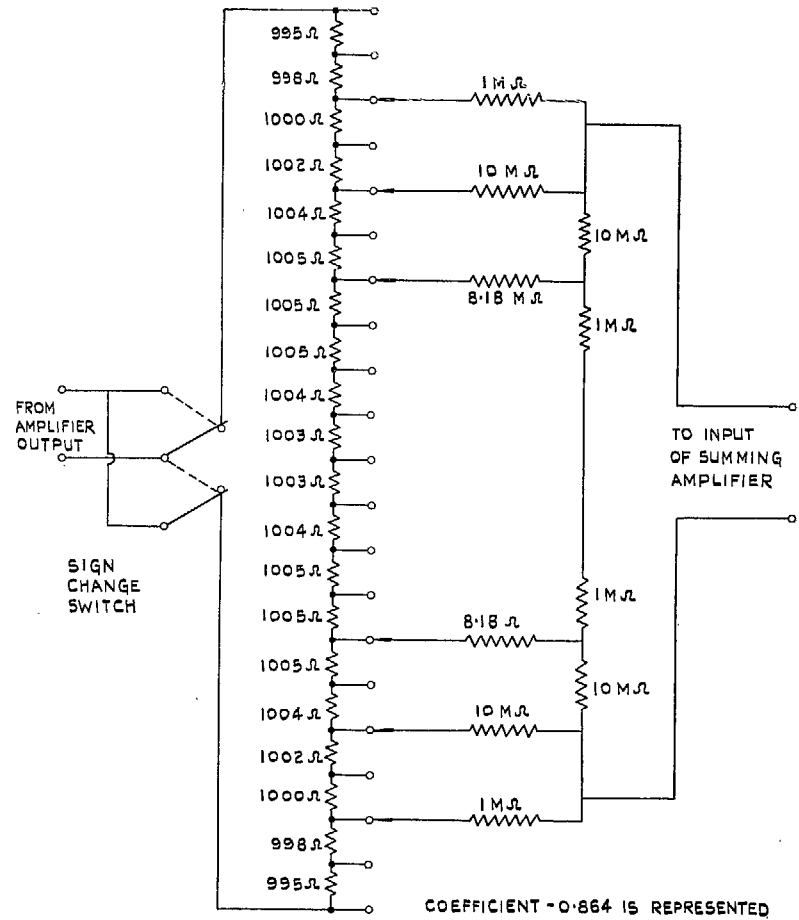


FIG. 27. Decade switch unit.

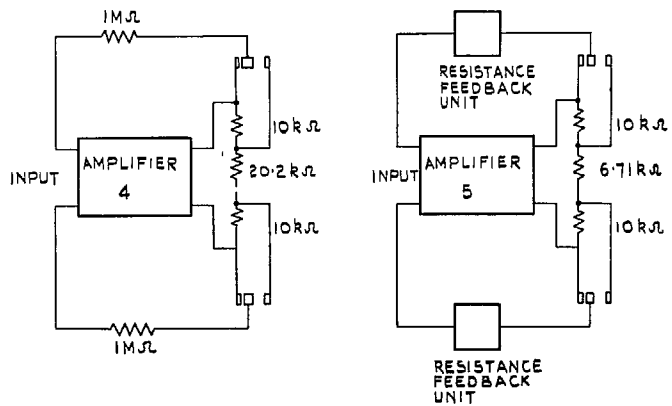
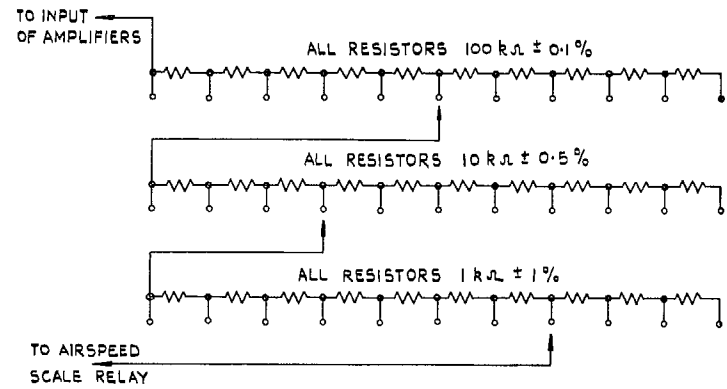


FIG. 28. Air-speed scale change circuit.



TWO UNITS REQUIRED FOR PUSH-PULL FEEDBACK

FIG. 29. Air-speed resistance feed-back unit.

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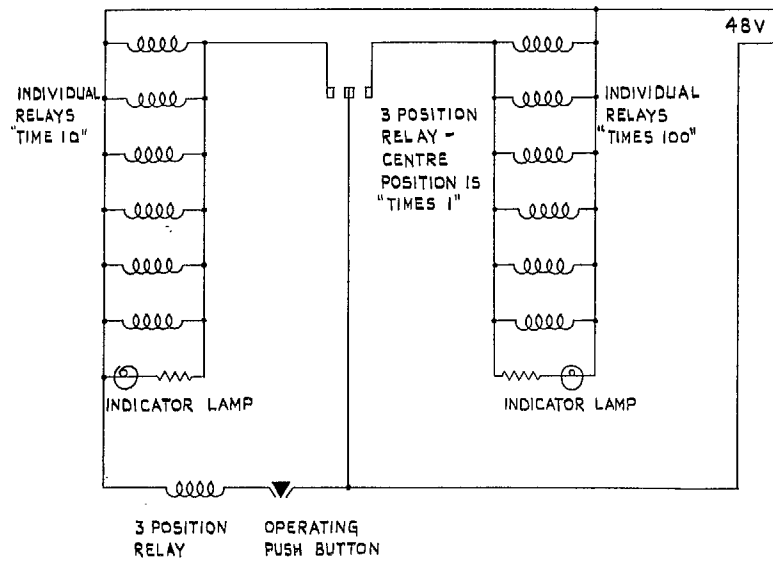


FIG. 30. Main frequency-change circuit.

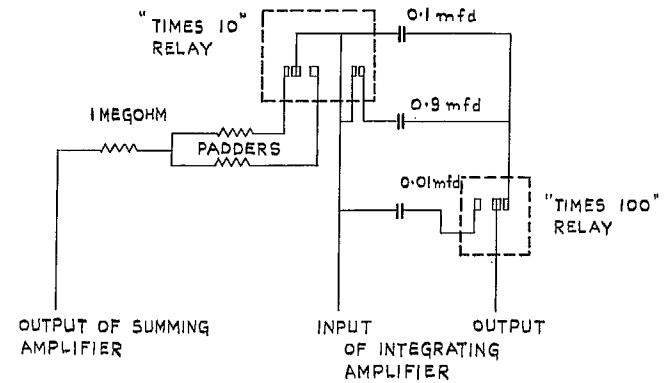


FIG. 31. Typical frequency-change operation.

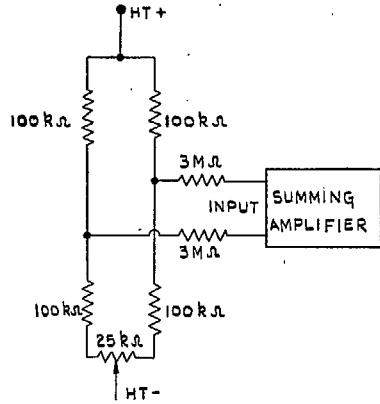


FIG. 32. Balancing circuit to input of amplifier 1.

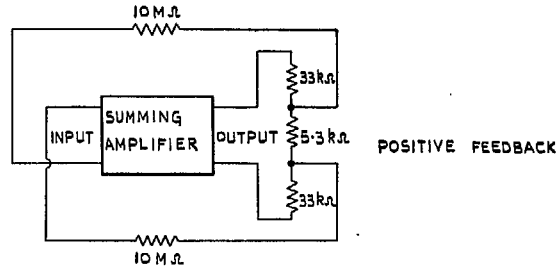


FIG. 33. Corrector circuit on amplifier 1.

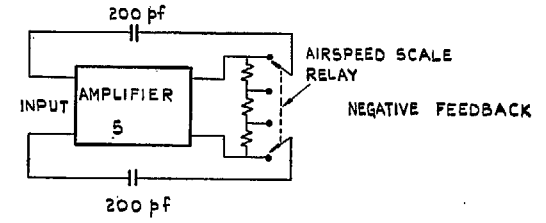


FIG. 34. Oscillation suppressor circuit on amplifier 5.

55

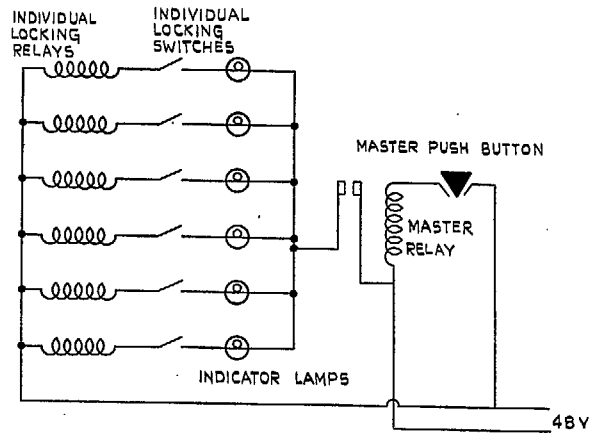


FIG. 35. Main locking circuit.

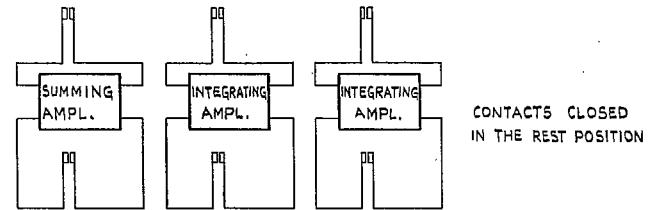


FIG. 36. Locking operation in each rack.

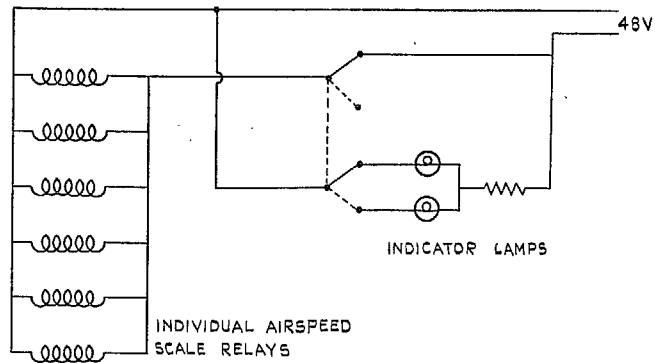


FIG. 37. Circuit to change air-speed scale.

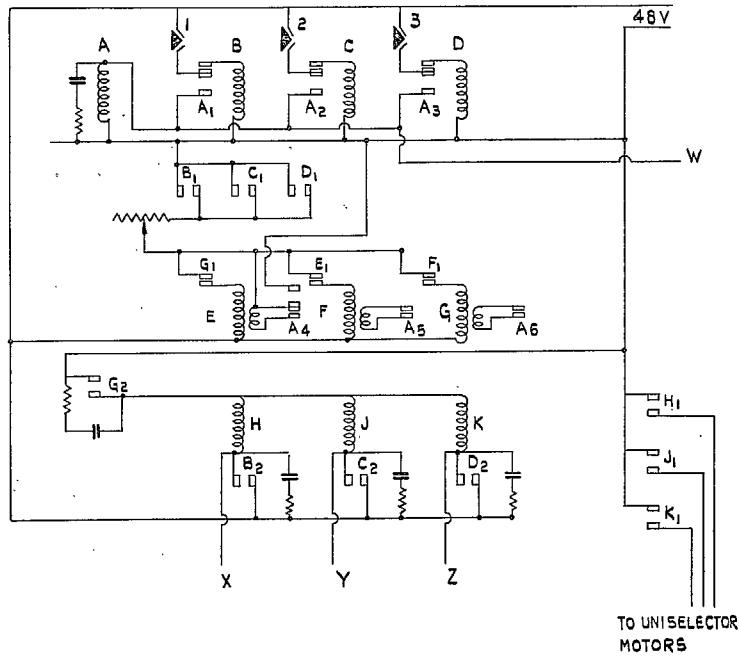


FIG. 38. Air-speed pulsing unit.

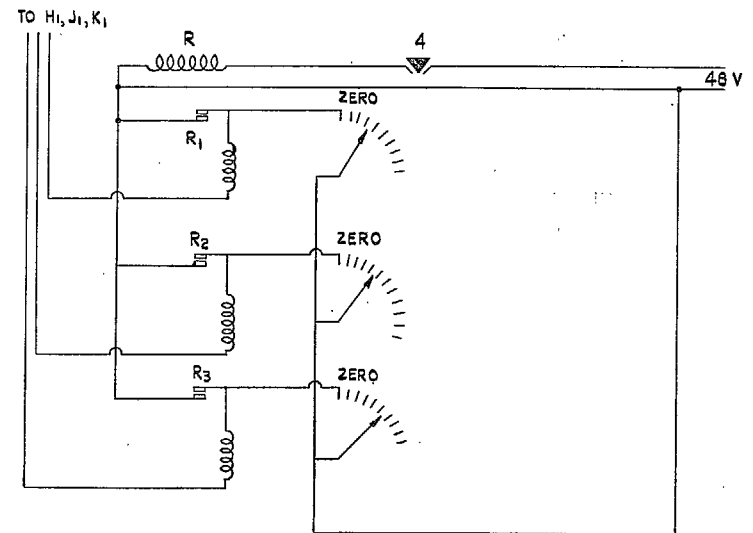
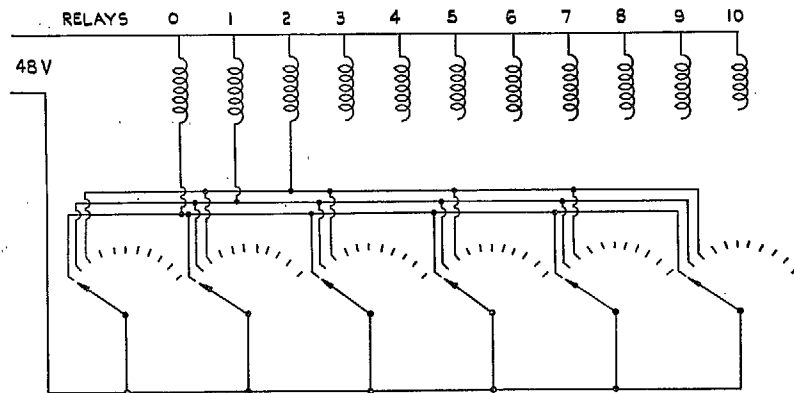
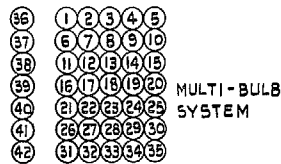


FIG. 39. Air-speed corrector circuit.



ALL 6 UNISELECTORS WIRED IN PARALLEL AS INDICATED ON THE FIRST THREE CONTACTS, EACH CIRCUIT BEING CONNECTED TO THE APPROPRIATELY NUMBERED RELAY. EACH UNISELECTOR WILL OPERATE THE RELAY APPROPRIATE TO ITS SETTING.

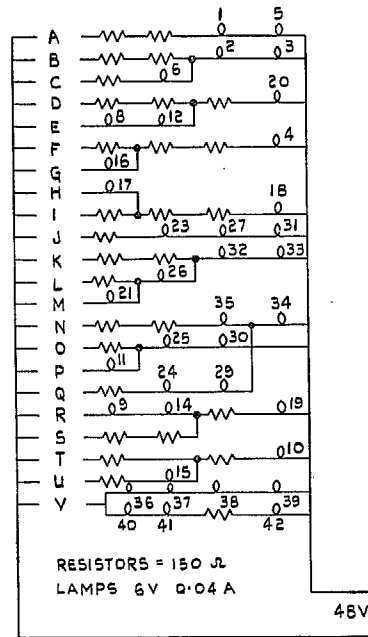


MULTI-BULB SYSTEM

RELAYS OPERATE CIRCUITS	
0	CDG MPU
1	FQR
2	CFJKNSU
3	CFILOSU
4	EGHQR
5	ACGHLPS
6	CGHMPST
7	ABFJSU
8	CFHMPSU
9	CDFHLPS
10	CDGMPUV

EXAMPLE
6

CCGT
P
GHHS
M P
MMP



RESISTORS = 150 Ω
LAMPS 6V 0.04 A

MULTI-BULB CIRCUIT

FIG. 40. Air-speed display unit.

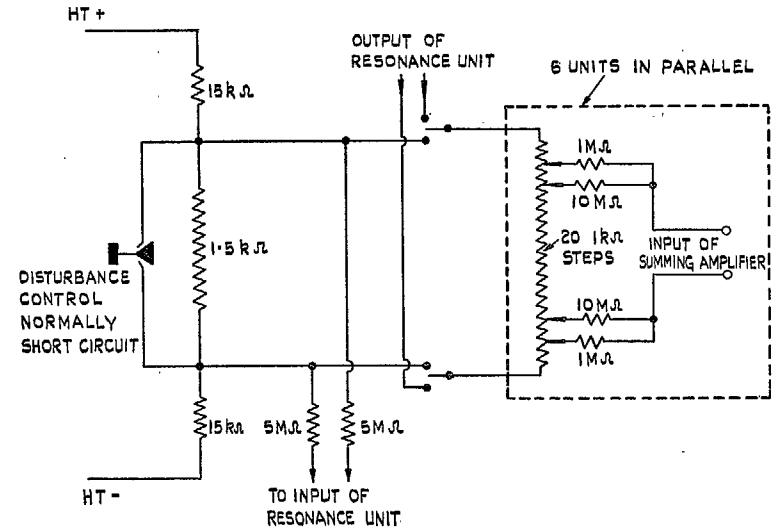


FIG. 41. Disturbance and resonance input.

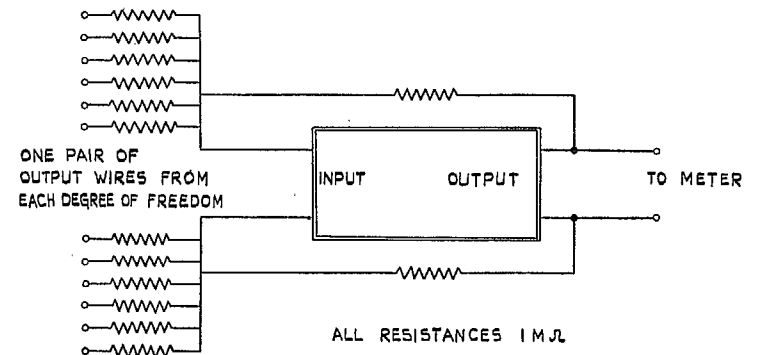
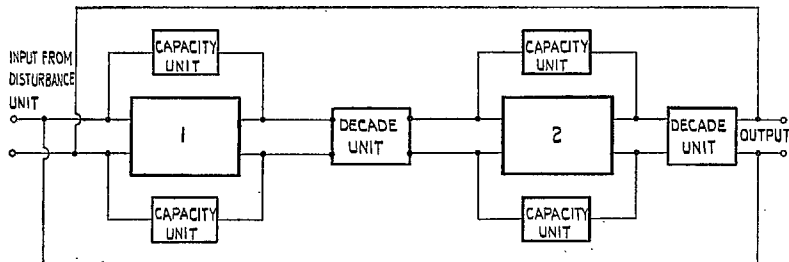
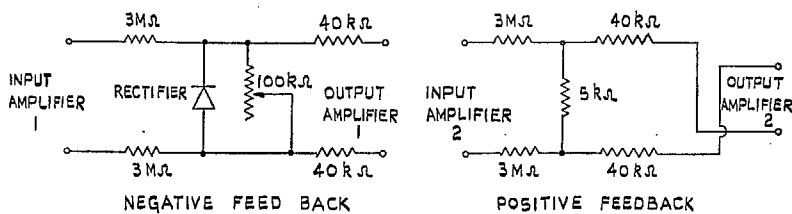
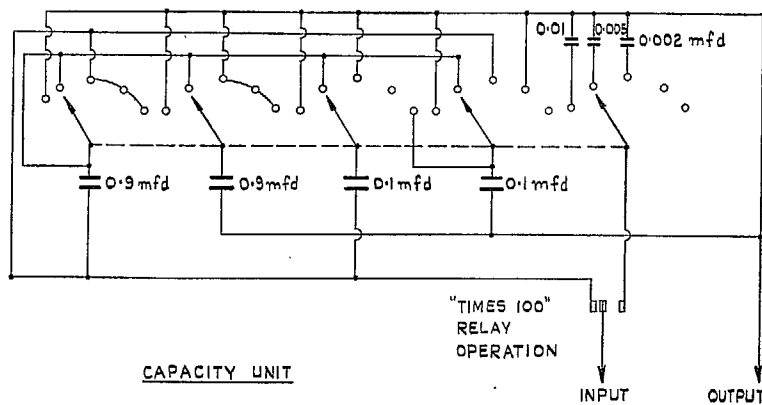


FIG. 42. Output addition unit.

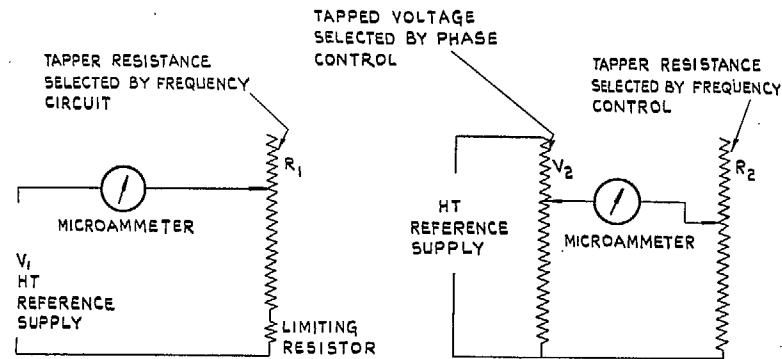


DECADE UNIT IS SIMILAR TO THAT SHOWN IN FIG. 27 EXCEPT THAT THE SIGN CHANGE SWITCH IS OMITTED.



AMPLITUDE CONTROL

FIG. 43. Resonance unit.



(a) FREQUENCY METER.

(b) PHASE METER.

Figs. 44a and 44b. Phase and frequency meter circuits.

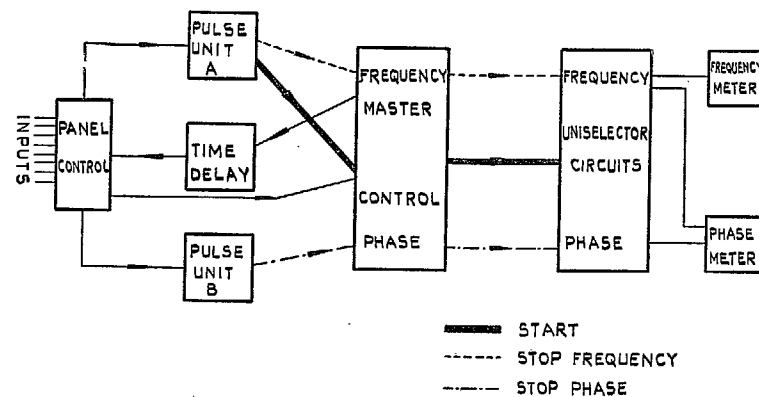


FIG. 45. Block diagram of phase and frequency unit.

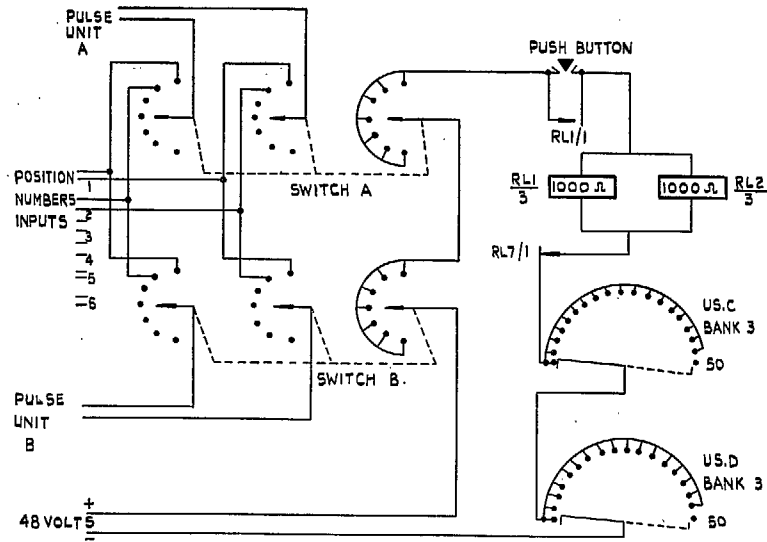
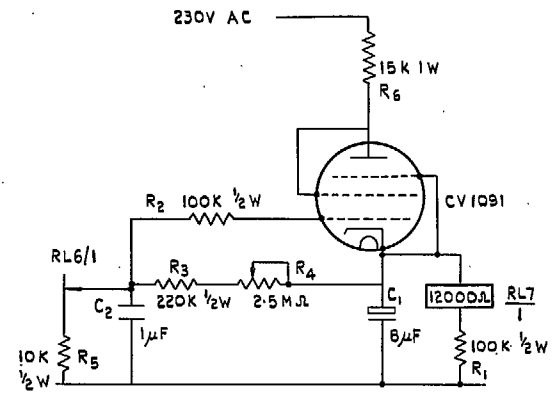


FIG. 46. Panel control and uniselector overriding circuits of phase and frequency unit.



VARIABLE BETWEEN 8 - 100 SECONDS

FIG. 47. Time delay.—Phase and frequency unit.

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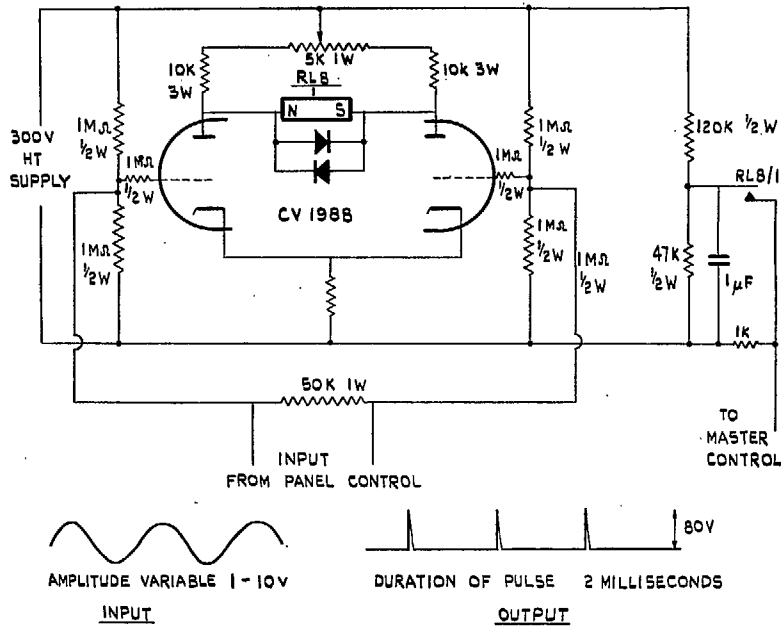


FIG. 48. Pulse circuit.—Phase and frequency unit.

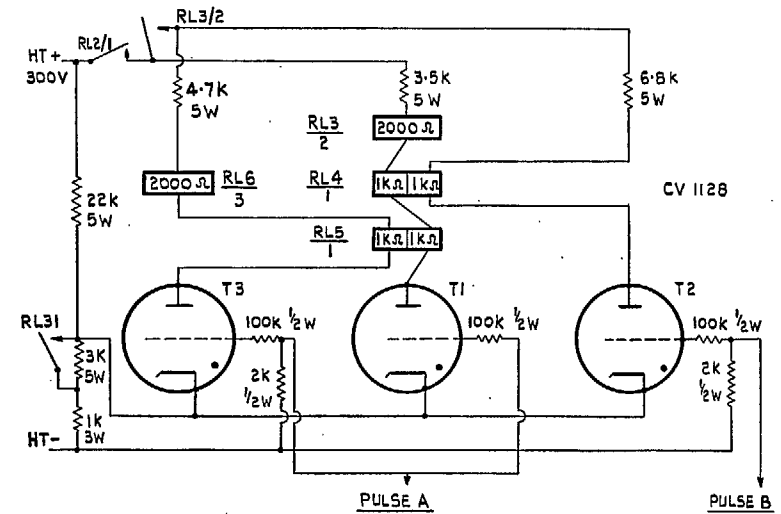
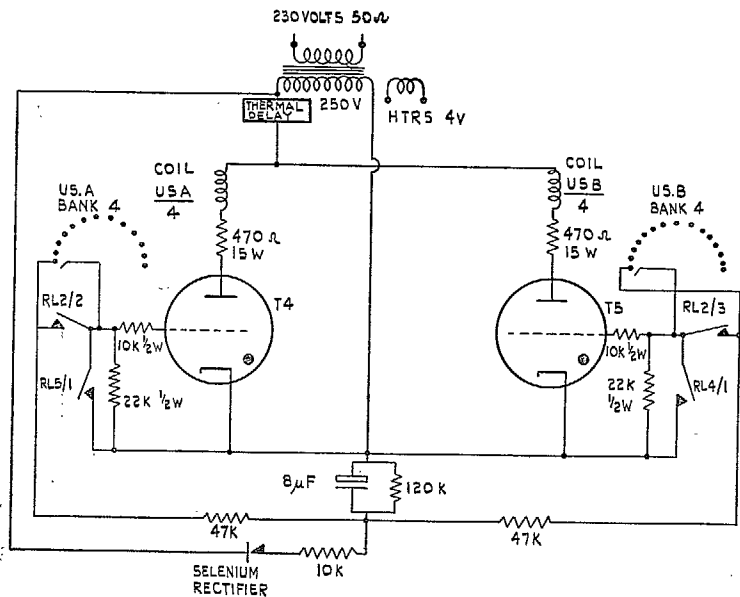
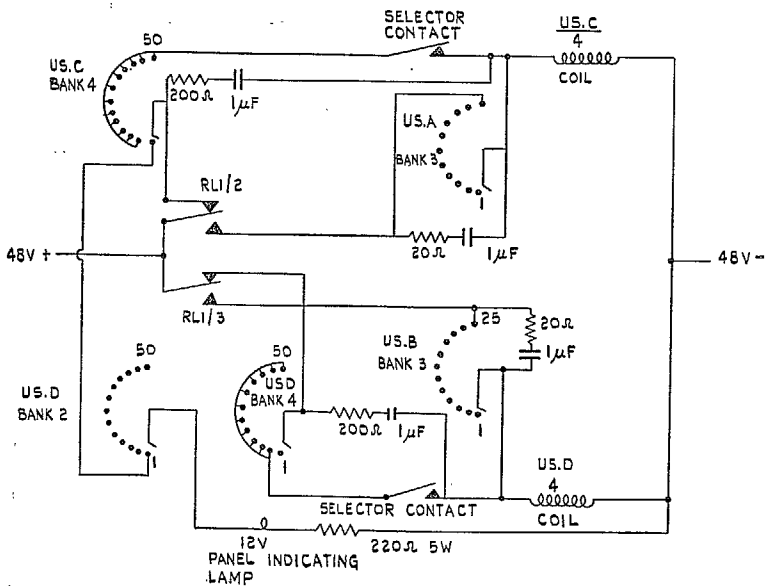


FIG. 49. Master control circuit.—Phase and frequency unit.



(a) THYRATRON DRIVEN UNISELECTORS SYNCHRONISED WITH 50 CYCLE MAINS SUPPLY.



(b) FOLLOWER UNISELECTORS.

Figs. 50a and 50b. Uniselector circuits.—Phase and frequency unit.

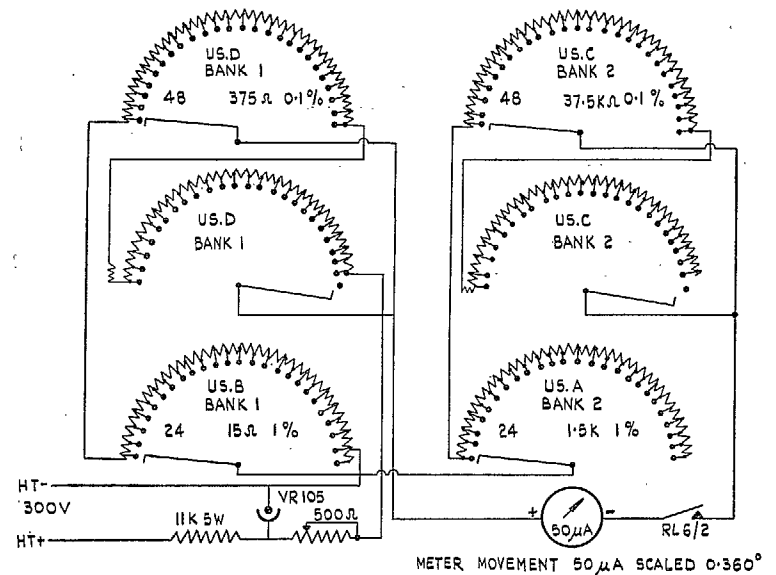


Fig. 51. Phase-indicating circuit.

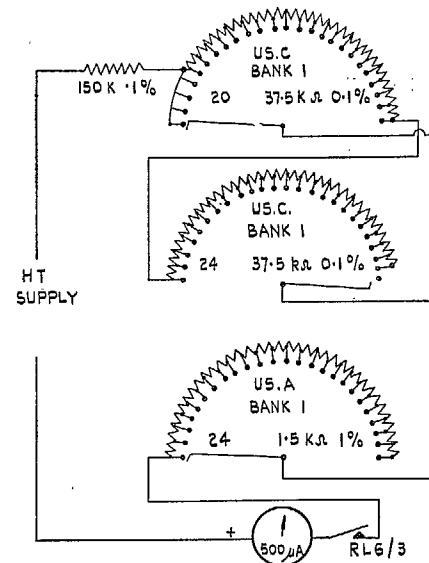


FIG. 52. Frequency-indicating circuit.

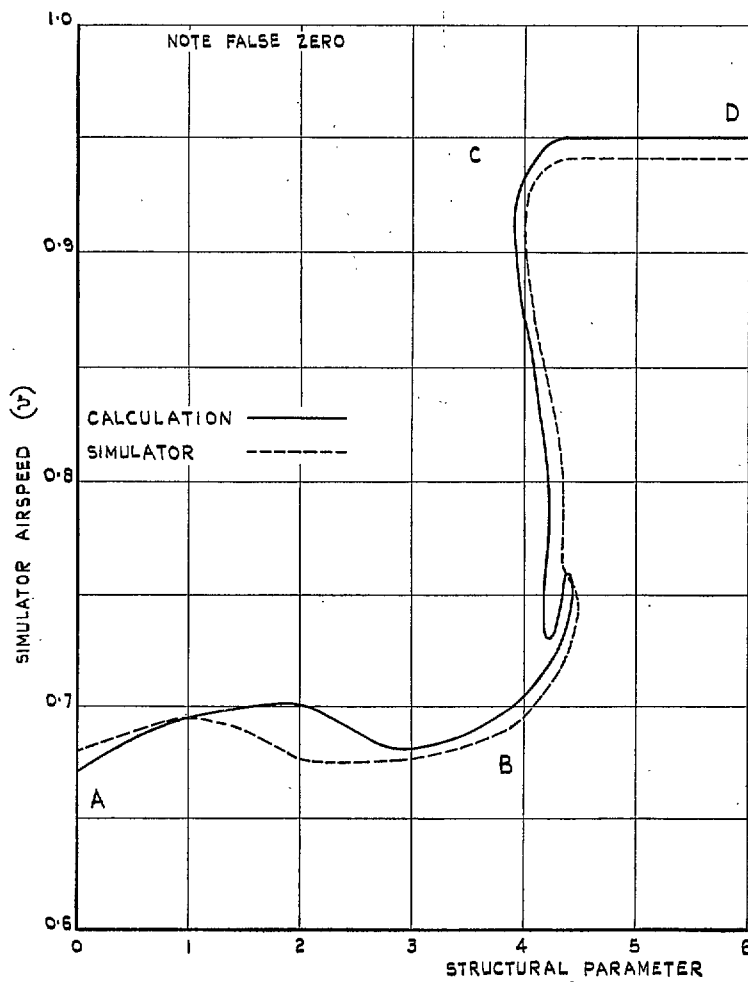


FIG. 53. Comparison between simulator values and calculated values for a four-degree-of-freedom problem.

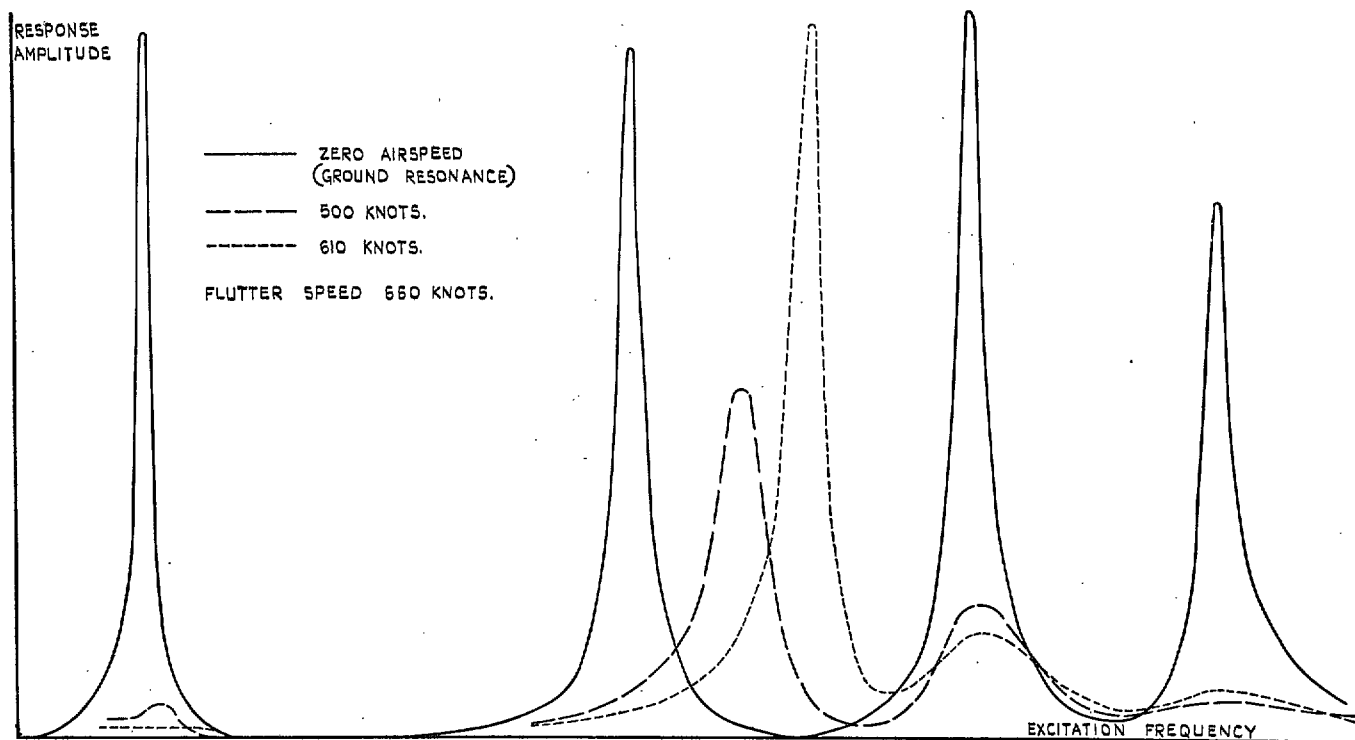


FIG. 54. Flight resonance results for a four-degree-of-freedom problem.

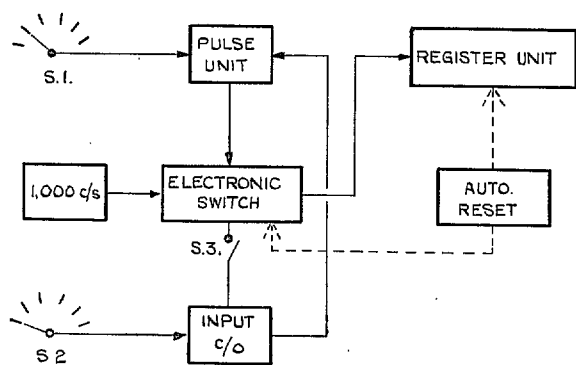


FIG. 55. Block diagram.

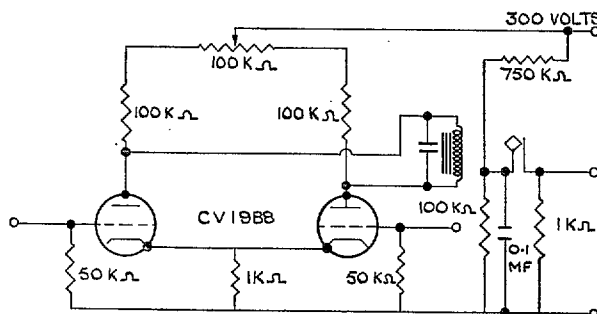


FIG. 56. Pulse unit.

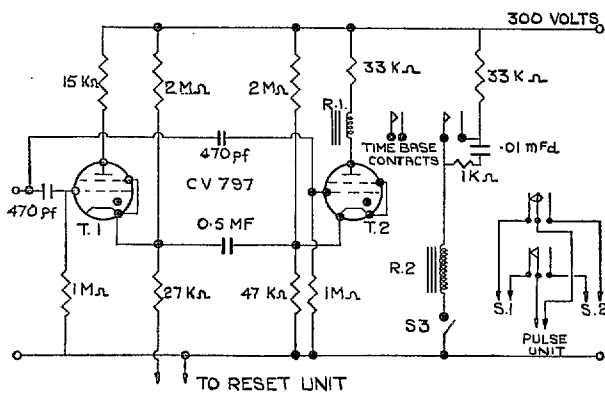


FIG. 57. Electronic switch.

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