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The Sampling Errors of Atmospheric Turbulence Measurements

By

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Summary.—Estimates of atmospheric turbulence from counting accelerometer records show a large scatter. The simple assumption of a random distribution of gusts is inconsistent with this scatter. A formula which takes account of the variations in gust density is given and the calculated sampling errors for 10 ft/sec gusts are found to be about four times those calculated on the basis of a random distribution.

1. *Introduction.*—Information on the frequency and magnitude of gusts in the atmosphere is obtained from counting accelerometers which are fitted in aircraft and which count automatically the number of accelerations at a given series of magnitudes. The total counts are photographed at fixed intervals of time, together with the height and speed of the aircraft[1]. The information so far obtained extends over a few thousand flying hours only and is primarily of value in the estimation of fatigue life of aircraft. This paper examines the sampling errors of counts of gusts of 10 ft/sec, which is in the region in which most of the fatigue damage occurs[2]. The assumption that gusts occur at random is first discussed and shown to be inadequate. The assumption that the average gust density varies in a particular way leads to a better agreement.

An indication of the experimental scatter is given in Table 1, and Fig. 1, which show the relationship between gust frequency and height; these data were obtained from the *Comet* operational records used in the preparation of Ref. 3.

2. *The Distribution of Gusts.*—If the distribution of gusts is completely random, then the numbers occurring in constant time intervals will be a Poisson distribution: out of a large number of intervals n, the expected numbers of intervals containing no gusts, one gust, two gusts, and so on, are given by the terms in the series

$$n\,e^{-m}\left(1, m, \frac{m^2}{2!}, \frac{m^3}{3!}\ldots\right),$$

where m is the mean number of gusts per interval. The expected total number of gusts in the n intervals is nm with sampling variance nm, that is, a standard deviation of $\sqrt{(nm)}$. Thus for any total N an estimate of the standard deviation is given by $\sqrt{N}$ and the duration of the time interval does not influence the standard deviation. On this basis the scatter of experimental points in Fig. 1 would be significant, but we shall in fact find that the assumption of a random occurrence of gusts is untenable. The experimental data are recorded as counts at discrete levels of acceleration and the counts of gusts at discrete levels as given in Table 1 are estimated

* R.A.E. Report Struct. 208, received 22nd October, 1956.

from the acceleration counts. For this reason, this investigation of the distribution of turbulence is based on the acceleration counts. In testing the fit of a Poisson distribution there is a theoretical disadvantage in this as, even if the underlying gust occurrences were random, variations in height and speed of the aircraft would in this case lead to some departure from Poisson for the derived distribution. However, the effect of these variations on the observed distribution is considered to be small.

Table 2 gives acceleration counts obtained from the *Comet* above 27,500 ft for intervals of 10 minutes and increments of $\pm 0 \cdot 23g$ (Ref. 3.) A comparison with the Poisson distribution is made in the following table:

Number of gusts	*Number of intervals*	
	Poisson	*Actual*
0	4,813·6	7,522
1	2,493·2	223
2	645·7	88
3	111·5	43
4	14·4	22
5	1·5	19
6 or more	0·1	163

It is immediately obvious that the distribution is very far from Poisson and the assumption that gusts occur at random is thus totally inadequate to describe the observed distribution. The gusts do in fact occur in groups in regions of turbulence, and for this reason one of the so-called 'contagious' distributions is more appropriate than that of Poisson. These can arise either when there is true contagion such as in the case of an epidemic or when the expected number of occurrences varies from trial to trial ('heterogeneous Poisson sampling'). The latter is the case here, the expected number of gusts varying from interval to interval, and while the major part of the variation is undoubtedly due to atmospheric conditions there is also a contribution from the variation in height and speed of the aircraft. This fact brings within the scope of this investigation data[4] obtained from *Hermes* aircraft at all heights (Table 3). The wide differences in conditions encountered at different altitudes merely contribute to the assumed variation in the expected number of gusts.

Anscombe[10] discusses eight two-parameter contagious distributions and by using the criterion he gives, the negative binomial distribution is selected here. This distribution arises when the expected number of gusts per interval varies in a certain way* and the distribution of intervals in each class is then given by the successive terms in the expansion of

$$n(1 + p - pt)^{-k},$$

$$i.e., \quad n(1 + p)^{-k}\left(1 - \frac{p}{1 + p}\,t\right)^{-k}$$

This distribution has two parameters p and k, compared with one parameter m for the Poisson, in addition to the total number of intervals n. The expected total number of gusts in the n intervals is nkp with sampling variance $nkp(1 + p)$. Thus for an expected total N the standard deviation is $\sqrt{\{(1 + p)N\}}$ compared with $\sqrt{N}$ for the Poisson distribution.

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* Let the expected number per interval be m, and let the distribution of m be given by

$$df = \frac{1}{(k-1)!}\, p^{-k} m^{k-1}\, e^{-m/p}\, dm$$

For a value m, the probability of x occurrences is $e^{-m}m^x/x!$ and the total probability of x is found by integrating over all values of m, and is thus

$$\int_0^\infty \frac{1}{(k-1)!}\, p^{-k} m^{k-1}\, e^{-m/p}\, e^{-m}\, \frac{m^x}{x!}\, dm = \frac{(k + x - 1)!}{x!(k-1)!}\, \frac{p^x}{(1 + p)^{k+x}}$$

The parameter k depends on the length of the time interval. If, for example, the length of interval is increased by a factor M, the new distribution is given by:

$$\{(1 + p - pt)^{-k}\}^M,$$
$$i.e., \quad (1 + p - pt)^{-Mk},$$

so that k is merely replaced by Mk. This is obviously equivalent to increasing the overall gust density by a factor M, while keeping the interval unchanged. The new mean is Mpk so that the value of p is unchanged and is thus seen to be a parameter depending on the degree of clustering of the gusts. As $p \to 0$ and $kp \to m$ the distribution tends to Poisson.

Methods of fitting are discussed in the appendix. Fitting by the first two moments is, for these distributions, very inefficient and utilizes only about 20 per cent of the available information. On the other hand, fitting by the method of maximum likelihood, which is always fully efficient, is very tedious. Anscombe[10] has considered the estimation of the two parameters from the mean and the first term of the distribution. This has the advantage of simplicity and also achieves efficiencies better than 98 per cent in the present cases. The equations of estimation are:

$$kp = \bar{x},$$
$$(1 + p)^{-k} = a_0,$$

where $\bar{x}$ is the observed mean and a_0 is the observed fraction in the zero class. Eliminating k gives

$$p/\log(1 + p) = -\bar{x}/\log a_0$$

and the solution of this equation for p is easily obtained by making use of a table given by Fisher[9]. It is a particularly convenient way of estimating p and k since all the information that is required is:

(*a*) total number of intervals
(*b*) total number of intervals with no counts
(*c*) total number of counts.

3. *Comparison of Calculated and Observed Frequencies*.—The parameters estimated as described above are:

(*a*) *Comet*	$k = 0\cdot022{,}516$	$p = 23\cdot004$
(*b*) *Hermes*	$k = 0\cdot023{,}485$	$p = 16\cdot118$

Anscombe[10] gives a test to indicate significant departures from the negative binomial distribution due to variations in the parameter p when, in effect, the distribution becomes a sum of negative binomial distributions with the same k but differing values of p. It is found that the *Hermes* data show no such tendency, but there is a significant departure in the case of the *Comet* data. Calculation of the class frequencies and application of the χ^2 test confirm this conclusion. The figures are given in Tables 4 and 5 and the comparison is also made graphically in Figs. 3 and 4. For the *Hermes* data the value of χ^2 is $21\cdot07$ which for 17 deg of freedom gives a value for the probability of $0\cdot22$; for the *Comet* data χ^2 is $60\cdot02$ which for 17 deg of freedom gives a probability well outside the tabulated range extending to $0\cdot001$. In spite of this high value of χ^2 the actual fit appears reasonably successful in graduating the data (Fig. 3). On this question, some remarks of Elderton[5] are of interest. Referring to the χ^2 test he says:

> 'I have found, in applying the test, that when the numbers dealt with are large, the probability is often small, even though the curve appears to fit the statistics very closely. The explanation may be that the statistics with which we deal in practice nearly always contain a certain amount of extraneous matter and the heterogeneity is concealed in a small experience by the roughness of the data. The increase in the number of cases observed removes the roughness, but the heterogeneity remains. The meaning from the curve fitting point of view is that the experience is really made up of more than one frequency curve, but a certain curve, approximating to the one calculated, predominates.'

Factors which may influence the *Comet* data are, firstly, that part of the *Comet* cruise takes place in the tropopause, and secondly, that at cruising altitudes the dynamic response of the *Comet* may tend to distort the distribution. It is unlikely that the bias introduced by the tendency to fly slower in rough air than in calm, thus giving fewer miles in rough intervals, has much effect on the fit, as the bias presumably has little effect in the case of the *Hermes*.

As the negative binomial distribution has been found to be the best of the available two-parameter distributions, a better agreement can only be obtained by fitting an additional parameter. This is not considered to be necessary since the main object is to estimate the sampling errors of the mean, and it is considered that the present method is adequate for this purpose.

4. *Magnitude of Sampling Errors.*—The parameters of the distributions give for the standard deviation of a count N the value of $4 \cdot 9\sqrt{N}$ for the *Comet* data and $4 \cdot 1\sqrt{N}$ for the *Hermes* data. The level of acceleration considered corresponds to a gust velocity of about 6 ft/sec for the *Comet* cruise and to 10 ft/sec for the *Hermes* cruise. It is to be expected that as the gust velocity rises the value of p decreases, and for high velocity gusts of very rare occurrence the distribution tends to that of Poisson. The trend shown by the above values confirms this.

To estimate the sampling errors of Table 1, which relates to gusts of 10 ft/sec a rounded off value for $\sqrt{(p + 1)}$ of 4 is assumed as the *Hermes* data is also considered to give a good average value for the range of altitudes under consideration.

In Fig. 2 the data of Table 1 have been replotted, showing a range about each experimental point, the upper and lower limits of which, N_u and N_l, being given by

$$N_u - 8\sqrt{N_u} = N = N_l + 8\sqrt{N_l} ,$$

where N is the actual count. The range shown is thus that for which the observed count is within two standard deviations of the assumed count. The scatter of the experimental points about the smoothed curve, which is linear up to 25,000 ft, is seen to be reasonably small, all points lying well within two standard deviations of the line. The figures also gives a good indication of the sampling errors to be expected in estimates of turbulence.

5. *Conclusions.*—An examination has been made of the sampling errors in estimating the occurrence of the small but frequently occurring gusts which cause the major fatigue damage to aircraft structures.

The assumption that these gusts occur completely at random is untenable since the numbers of 10 minute intervals observed containing no gusts, one gust, two gusts and so on do not approximate to those given by the Poisson distribution. A better representation is given by the negative binomial distribution and this is considered adequate for estimating the sampling errors of gust counts. The estimated frequency of 10-ft/sec gusts has a standard deviation of the order of 4 times that calculated by assuming a random occurrence of gusts. There is no evidence from the *Comet* data considered that the relation between altitude and log (miles per gust) up to about 25,000 ft is other than linear.

6. *Further Developments.*—As further information becomes available the analysis can be extended to the gust velocity corresponding to the static design condition. At the same time a more general knowledge of atmospheric turbulence could be obtained by examining the records within a narrower range of height.

LIST OF SYMBOLS

a_0, a_1, a_2, etc.	The fraction of the total number of intervals in the sample with 0, 1, 2, etc., gusts respectively
k, p	Parameters of the assumed negative binomial distribution $(1 + p - pt)^{-k}$
m	Mean number of gusts per interval
m_2	The sample variance
N	Total number of gusts observed in the sample
N_u, N_l	Limits of a range such that N is two standard deviations below N_u and two standard deviations above N_l
n	Number of intervals in the sample
p	See k
x	Number of gusts in an interval
$\bar{x}$	The sample mean

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Ref. 8 and the relevant part of Ref. 9 are also available in :

R. A. Fisher .. .. .. *Contributions to Mathematical Statistics.* Wiley. 1950.

APPENDIX

The Fitting of Negative Binomial Distributions

1. The negative binomial distribution is given by the expansion of

$$(\overline{1+p}-pt)^{-k},$$

i.e.,
$$(1+p)^{-k}\left(1-\frac{p}{1+p}t\right)^{-k}.$$

The mean and the variance are respectively kp and $kp(1+p)$ so that the distribution is conveniently fitted to a sample by the first two moments. Jeffreys[6] has pointed out, however, that this method is not efficient, in the sense that it utilizes only a fraction of the available information. Fitting by the method of maximum likelihood is fully efficient and Haldane[7] has discussed this method. Fisher[8] has compared numerically the efficiency of fitting by moments with the maximum likelihood method. More recently Anscombe[10] has discussed a general method of fitting from which the method of moments and fitting by the mean and first term emerge as particular cases. For all these methods estimates of the mean and the parameter k are independent. Since the mean is always efficiently estimated by the sample mean, it follows that the efficiencies of the various methods can conveniently be compared by determining the sampling variances of k. Anscombe gives an expression for this variance in each case. For the distributions considered in the main text, fitting by maximum likelihood, though straightforward is tedious, and fitting by moments is very inefficient. On the other hand, fitting by the mean and first term is simple and is found to give efficiencies of over 98 per cent. The three methods are briefly compared.

2. (*a*) *Fitting by Maximum Likelihood.*—Haldane[7] gives as the equations of estimation

$$kp=\bar{x},$$

$$\log(1+p)=\frac{1}{k}(a_1+a_2+a_3\ldots)+\frac{1}{1+k}(a_2+a_3+a_4\ldots)$$
$$+\frac{1}{2+k}(a_3+a_4+a_5\ldots)+\ldots,$$

where a_0, a_1, a_2, etc., are the fractions of the sample in the respective classes, and $\bar{x}$ is the sample mean, so that

$$\bar{x}=a_1+2a_2+3a_3+\ldots.$$

For the sampling variance of k, Anscombe[10] gives

$$\mathrm{Var}(k)=\frac{k}{n}\left\{\frac{1}{2(1+k)}\left(\frac{p}{1+p}\right)^2+\frac{1.2}{3(1+k)(2+k)}\left(\frac{p}{1+p}\right)^3+\ldots\right\}^{-1}.$$

We note that for small values of k

$$\mathrm{Var}(k)\simeq\frac{k}{n}\left\{\log(1+p)-\frac{p}{1+p}\right\}^{-1}$$

and that in general

$$\mathrm{Var}(k)>\frac{k(1+k)}{n}\left\{\log(1+p)-\frac{p}{1+p}\right\}^{-1}$$

(b) *Fitting by moments.*—The equations of estimation are:

$$kp = \bar{x} \, ,$$

$$kp(1 + p) = m_2 \, ,$$

m_2 being the second moment as estimated from the sample.

$$\mathrm{Var}\,(k) = \frac{2k(1 + k)(1 + p)^2}{np^2}$$

(c) *Fitting by the Mean and First Term.*—The equations of estimation are:

$$kp = \bar{x} \, ,$$

$$(1 + p)^{-k} = a_0 \, .$$

Eliminating k gives

$$\frac{p}{\log\,(1 + p)} = \frac{-\,\bar{x}}{\log a_0} \, ,$$

from which p can be determined. The computation is facilitated by the use of a table given by Fisher (Ref. 9, Table 9) where his N/S and N/α correspond to our $-\,\bar{x}/\log a_0$ and p respectively.

$$\mathrm{Var}\,(k) = \frac{\left\{(1 + p)^k - 1 - \dfrac{pk}{1 + p}\right\}}{n\left\{\log\,(1 + p) - \dfrac{p}{1 + p}\right\}^2} \, .$$

For small values of k

$$\mathrm{Var}\,(k) \simeq \frac{k}{n}\left\{\log\,(1 + p) - \frac{p}{1 + p}\right\}^{-1}$$

and the efficiency of the method approaches that of the fully efficient maximum likelihood method.

3. *Numerical Examples.*—The efficiencies of the methods when applied to the distributions in the main text are determined, the values of p and k used being obtained by fitting the mean and first term. In the case of the method of maximum likelihood the expression giving a lower limit for Var (k) is used. This leads to a lower limit for the efficiency of fitting by the mean and first term and places the efficiency of the method of moments within a narrow range.

For the *Comet* data the efficiency of fitting by the mean and first term is greater than 97·1 per cent and that of fitting by moments between 20·7 per cent and 21·3 per cent. For the *Hermes* data the efficiency of fitting by the mean and first term is greater than 97·4 per cent and the efficiency of fitting by moments between 23·3 per cent and 24·0 per cent.

Both sets of values of the parameters lie outside the range of the chart given by Anscombe[10] but it is easy to see that in each case the efficiency of fitting by the mean and first term is in fact greater than 98 per cent.

4. It is concluded that for the distributions considered in the main text, fitting by the mean and first term is justified, since it results only in a loss of efficiency of less than 2 per cent. Fitting by moments, on the other hand, gives efficiencies of between 20 and 25 per cent.

TABLE 1

Turbulence Encountered by Comet

Height (ft)	Miles recorded	Number of gusts*	Miles per gust
0– 2,500	860	193	$4 \cdot 46$
2,500– 7,500	13,620	1,320	$1 \cdot 03 \times 10$
7,500–12,500	28,280	1,235	$2 \cdot 29 \times 10$
12,500–17,500	28,760	240	$1 \cdot 20 \times 10^2$
17,500–22,500	36,240	218	$1 \cdot 66 \times 10^2$
22,500–27,500	49,020	58	$8 \cdot 45 \times 10^2$
27,500–32,500	128,800	140	$9 \cdot 20 \times 10^2$
32,500–37,500	341,200	487	$7 \cdot 01 \times 10^2$
37,500–42,500	57,090	43	$1 \cdot 33 \times 10^3$

* The number of gusts of magnitude greater than 10 ft/sec either up or down is given in this column.

TABLE 2

Data for Comet above 27,500 ft

Number of gusts	Number of intervals
0	7,522
1	223
2	88
3	43
4	22
5	19
6	18
7	16
8	9
9	8
10	9
11	5
12	7
13	6
14	8
15	5
16	4
17	3
18	4
19	4
20	5
21	2
22	2
23	2
24	3
25	1
26	2
27	1
28	—
29	1
30	3
31	—
32	2
33	3
34	—
35	2
—	—
38	2
39	2
40	1
—	—
43	4
44	3
45	1
46	—
47	1
48	—
49	1
50	1
51	1
52	1
—	—
55	1
—	—
61	1
—	—
67	1
—	—
72	1
73	1
—	—
89	1
—	—
93	1
94	1
95	1
—	—
102	1

TABLE 3

Data for Hermes

Number of gusts	Number of intervals	Number of gusts	Number of intervals
0	10,424	27	3
1	232	28	1
2	119	29	3
3	69	30	2
4	56	31	1
5	30	32	—
6	18	33	2
7	21	34	—
8	24	35	3
9	12	—	—
10	20	38	1
11	10	39	1
12	17	40	1
13	7	41	1
14	5	42	1
15	4	43	2
16	8	—	—
17	7	46	1
18	8	—	—
19	5	50	1
20	4	—	—
21	4	62	1
22	5		
23	4		
24	3		
25	1		
26	1		

TABLE 4

Comparison of Calculated and Observed Frequencies for Comet Data

Number of gusts	Number of intervals		χ^2
	Calculated	Observed	
0	7,522	7,522	—
1	162·3	223	22·70
2	79·5	88	0·91
3	51·4	43	1·37
4	37·2	22	6·21
5	28·7	19	3·28
6	23·0	18	1·09
7	19·0	16	0·47
8	16·0	9	3·06
9	13·6	8	2·31
10	11·8	9	0·66
11	10·3	5	2·73
12–13	17·1	13	0·98
14–15	13·6	13	0·03
16–17	11·0	7	1·45
18–20	12·9	13	0·00
21–24	12·7	9	1·08
25–29	11·0	5	3·27
30–37	10·5	10	0·02
38 and over	16·3	28	8·40
		Total ..	60·02

TABLE 5

Comparison of Calculated and Observed Frequencies for Hermes Data

Number of gusts	Number of intervals		χ^2
	Calculated	Observed	
0	10,424	10,424	—
1	230·5	232	0·01
2	111·1	119	0·56
3	70·5	69	0·03
4	50·2	56	0·67
5	38·0	30	1·68
6	30·0	18	4·80
7	24·3	21	0·45
8	20·1	24	0·76
9	16·9	12	1·42
10	14·3	20	2·27
11	12·3	10	0·43
12	10·6	17	3·86
13–14	17·4	12	1·68
15–16	13·4	12	0·15
17–18	10·6	15	1·83
19–21	12·0	13	0·08
22–25	11·1	13	0·33
26–31	10·3	11	0·05
32 and over	15·3	15	0·01
		Total ..	21·07

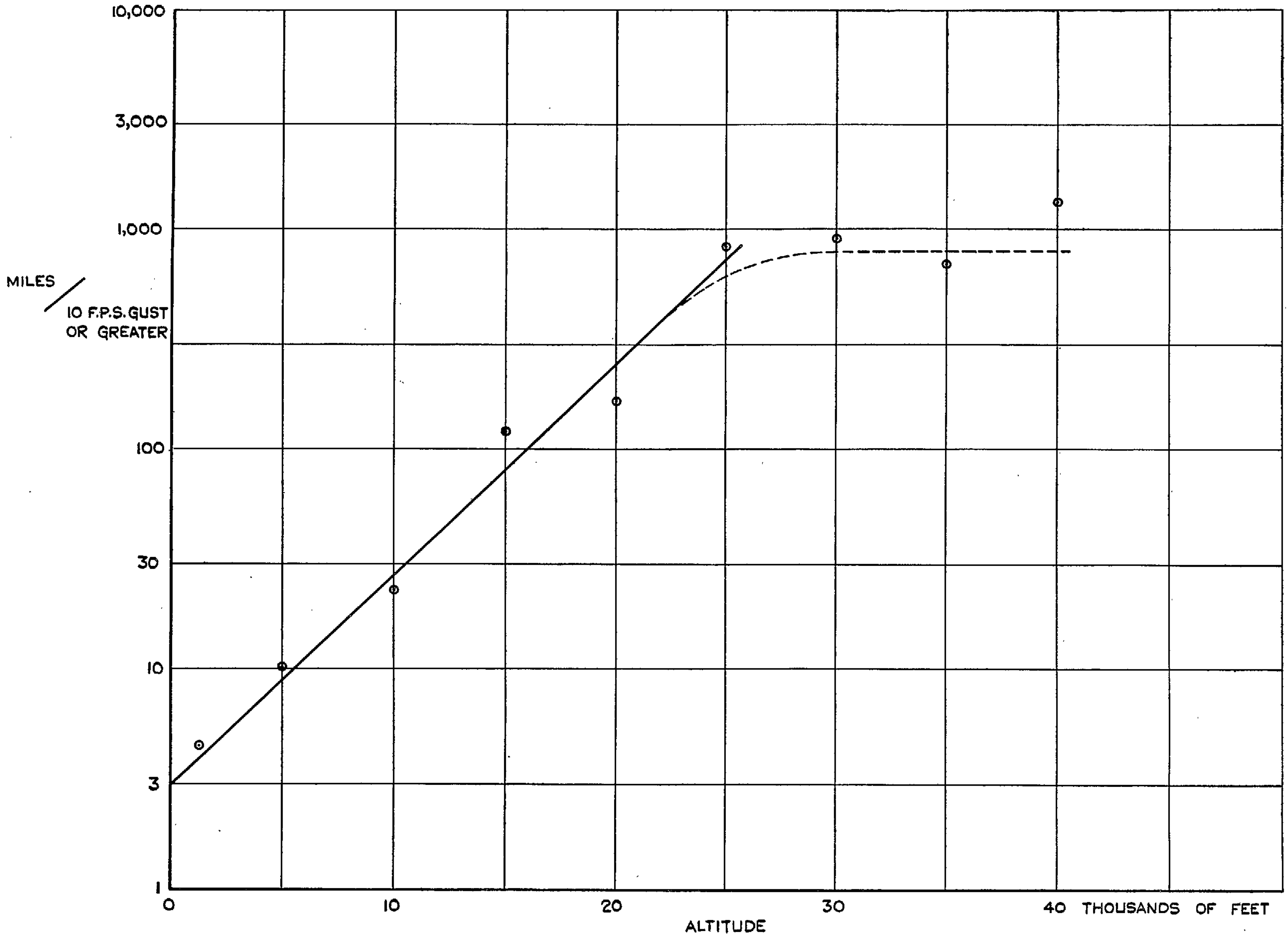

FIG. 1. Turbulence encountered by *Comet*.

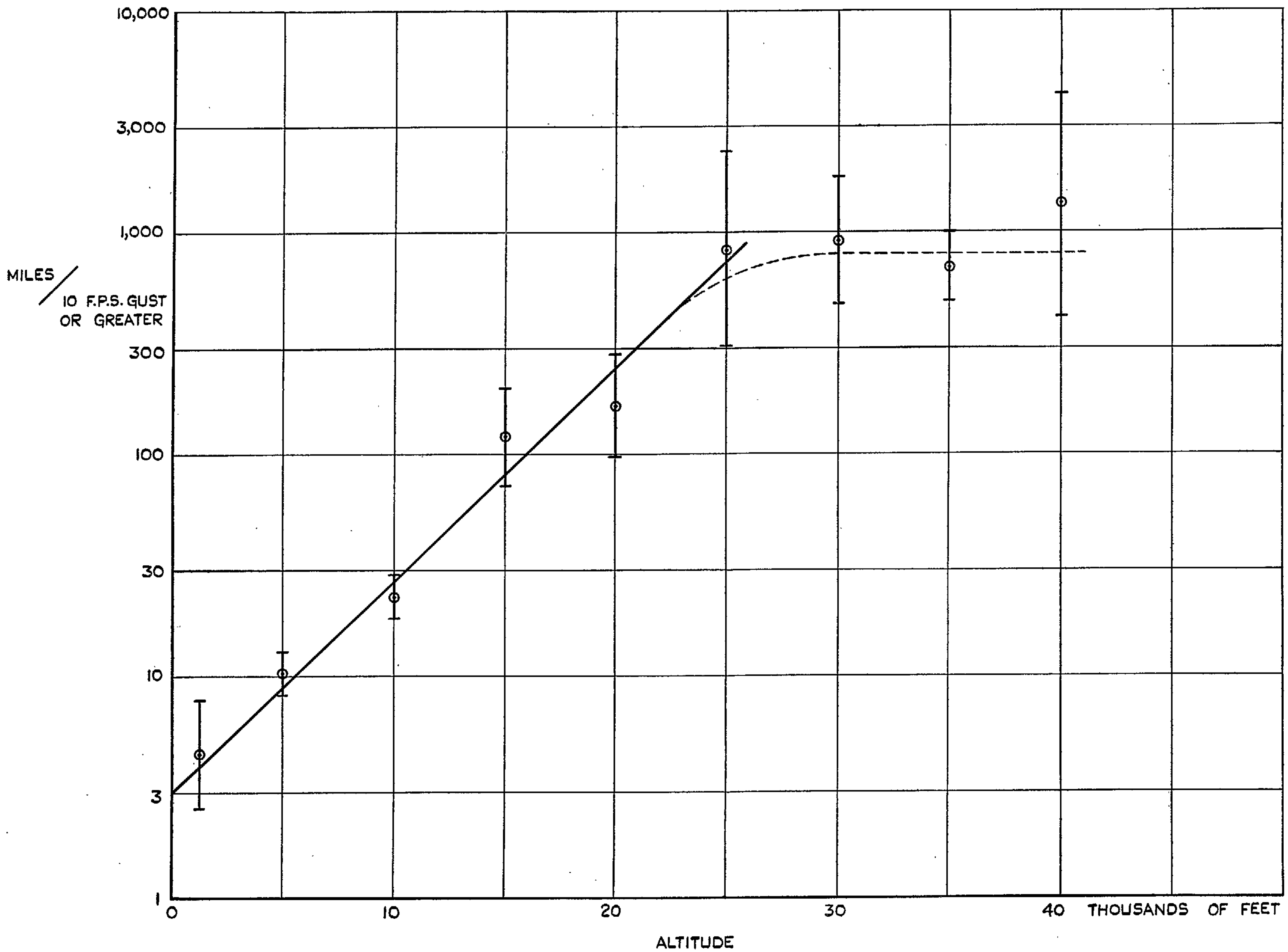

FIG. 2. Turbulence encountered by *Comet*, showing range of two standard deviations up or down.

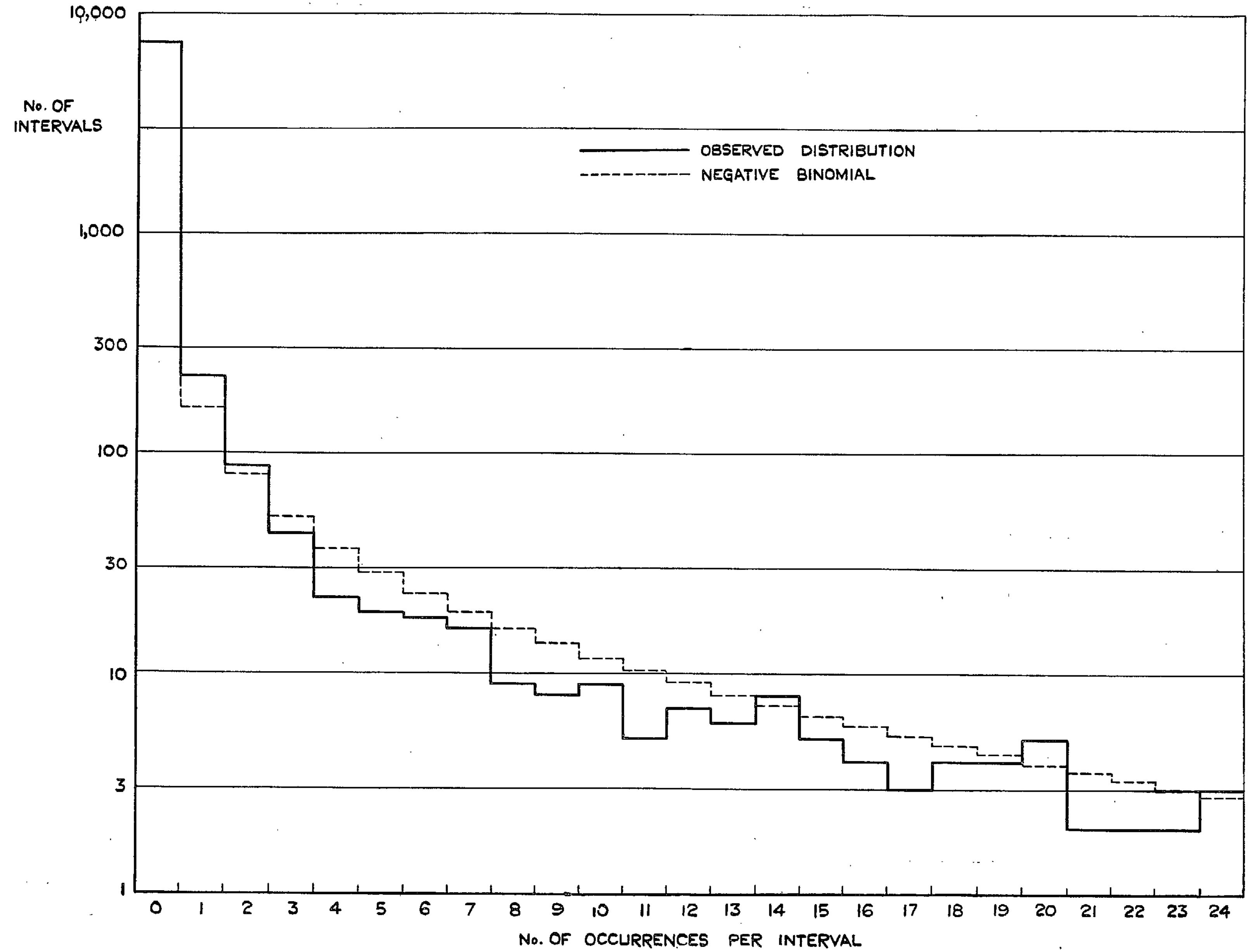

FIG. 3. Comparison of calculated and observed frequencies for *Comet* data.

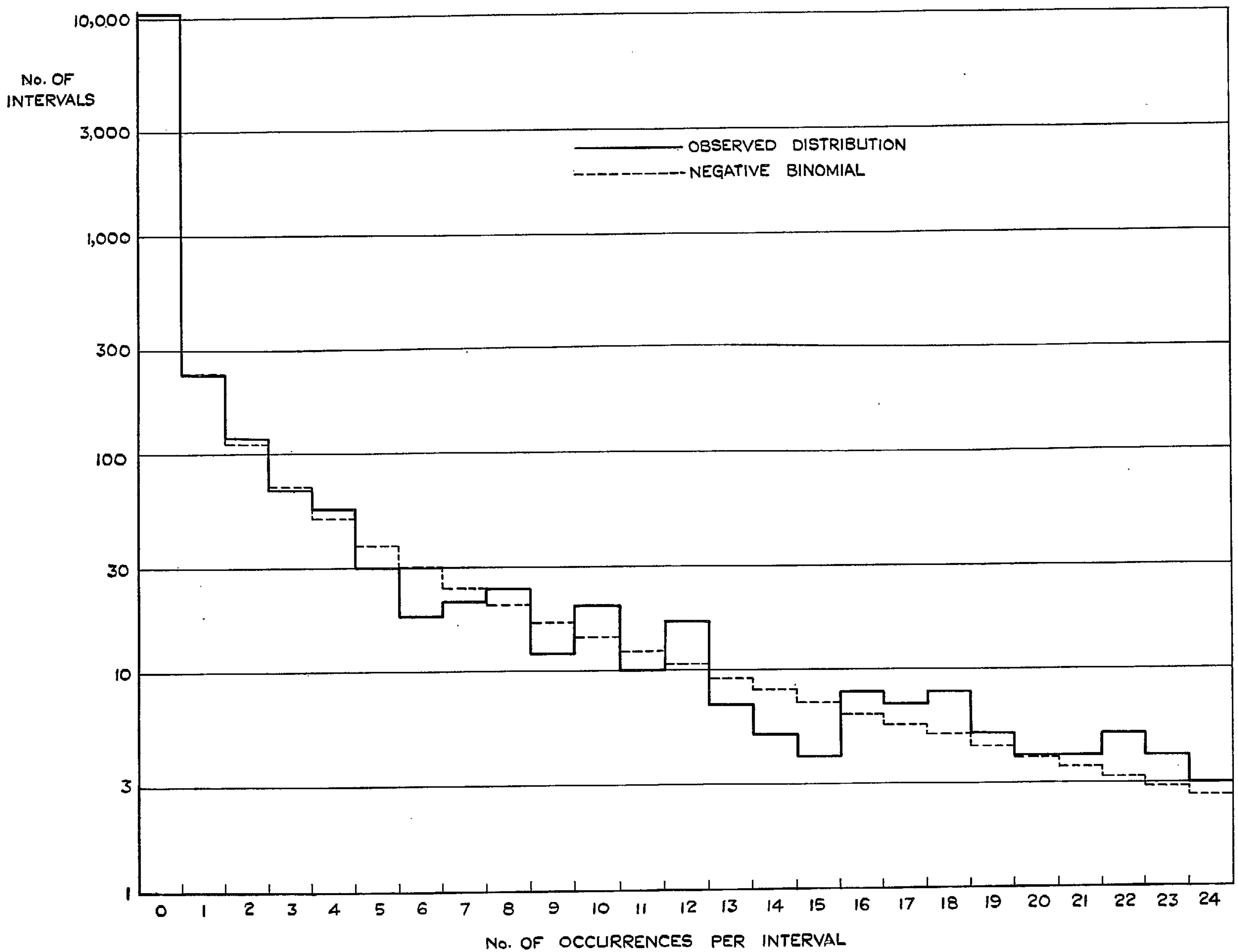

FIG. 4. Comparison of calculated and observed frequencies for *Hermes* data.

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