C.P. No. 337
(18.944)
A.R.C. Technical Report

C.P. No. 337
(18.944)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL

CURRENT PAPERS

Tables of Characteristics Slopes for use in the Design of Nozzles for Supersonic Wind Tunnels

By

K.G.Winter

LONDON: HER MAJESTY'S STATIONERY OFFICE

1957

THREE SHILLINGS NET

C.P. No. 337

U.D.C. No. 533.6.011.5 : 533.6.071.1 : 533.697.4

Technical Note No. Aero.2444

May, 1956.

ROYAL AIRCRAFT ESTABLISHMENT

Tables of characteristics slopes for use in the design of nozzles for supersonic wind tunnels

by

K. G. Winter

SUMMARY

Tables are given which are of use in the calculation of characteristic networks appropriate to two-dimensional nozzles for Mach numbers up to 4.5. The tabulated characteristics slopes enable the co-ordinates of a characteristics intersection each to be evaluated in one operation on a desk calculating machine.

LIST OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

---

1 Introduction

In the design of two-dimensional nozzles for supersonic wind tunnels by the characteristics method the calculations have in many cases been made graphically (for example Harrop and Bright[1]). The accuracy can be improved by calculating the characteristics intersections numerically. If the slopes of the characteristics are available the co-ordinates of intersections may each be evaluated in one operation on a desk calculating machine. In designing nozzles for the R.A.E. intermittent wind tunnels the characteristic slopes have been tabulated for characteristic numbers appropriate to Mach numbers up to 4.5 for given characteristic intervals. Even though there should be enough nozzle ordinates available by now to limit further calculations to special cases, and though the characteristics methods is becoming unfashionable, it is felt worth while to make the tables of slopes available generally.

2 List of Symbols

The flow is taken as being from left to right.

A right up-going characteristic values (in degrees)

B right down-going characteristic values (in degrees)

D flow direction

M Mach number

m_1 slope of A characteristics

$-m_2$ slope of B characteristics

x, y Cartesian co-ordinates

μ Mach angle

ν Prandtl-Meyer angle

3 Calculation of characteristics slopes

The characteristics method adopted is that of 'points' rather than 'regions'. That is to say values are attributed to the intersections of characteristics of opposite families, rather than to regions enclosed by two pairs of characteristics of opposite families. Consider the intersections of two such pairs as in Fig.1. The 'A' characteristics are taken to be right up-going and the 'B' characteristics right down-going with the flow direction from left to right. It is assumed that the ordinates (x_1, y_1 and x_2, y_2) of the intersections of A_1 and B_1 and A_2 and B_2 are known and it is required to find x_3 and y_3. Then if the mean slopes of the characteristics approaching x_3 y_3 are m_1 and $-m_2$ we have

$$x_3 = \frac{y_2 - y_1 + m_1 x_1 + m_2 x_2}{m_1 + m_2} \qquad (1)$$

$$y_3 = \frac{x_2 - x_1 + \frac{y_1}{m_1} + \frac{y_2}{m_2}}{\frac{1}{m_1} + \frac{1}{m_2}} \qquad (2)$$

Thus if the values m_1, m_2, $m_1 + m_2$ and $\frac{1}{m_1}$, $\frac{1}{m_2}$, $\frac{1}{m_1} + \frac{1}{m_2}$ are tabulated the co-ordinates x_3 y_3 can be obtained immediately with the use of a desk calculating machine.

The slopes m_1 and m_2 are given by

$$m_1 = \tan(\mu_1 + D_1) \qquad (3)$$

$$m_2 = \tan(\mu_2 - D_2) \qquad (4)$$

where μ_1, D_1 and μ_2, D_2 are the mean values of the Mach angle and flow direction respectively along the lines x_1, y_1 : x_3, y_3, and x_2, y_2 : x_3, y_3.

The values of μ and D are derived from the characteristics numbers in the usual way, i.e.

$$\nu_1 = A_1 + \tfrac{1}{2}(B_1 + B_2)$$

where for $\gamma = 1.4$ the Prandtl-Meyer angle*

$$\nu = \mu + \sqrt{6} \text{ arc cot } (\sqrt{6} \tan \mu) - \tfrac{1}{2}\pi$$

and

$$D_1 = \tfrac{1}{2}(B_1 + B_2) - A_1$$

$$\nu_2 = B_2 + \tfrac{1}{2}(A_1 + A_2)$$

$$D_2 = B_2 - \tfrac{1}{2}(A_1 + A_2)$$

The values of ν were taken from the tables of Herbert and Older[2] and the tangents of the angles from Milne-Thomson and Comrie[3].

The errors in the tables should not exceed 2 units in the last decimal place quoted.

* The subtraction of ν from 1000 in order to obtain the so-called pressure number appears to be a fruitless operation.

4 Description of Tables

The tables are set out with given values of A along rows and given values of B down columns. Each block of six numbers gives values of the two slopes and their sum and similarly for the reciprocals. The values refer to the slopes of characteristics approaching a given point A, B, with the appropriate interval. As A, B increase the value of the interval is increased progressively. The intervals in degrees are

Table I	0.2	$M < 1.22$
Table II	0.4	$1.2 < M < 1.43$
Table III	0.8	$1.43 < M < 1.81$
Tables IV and V	1.6	$1.81 < M < 4.53$

M being the Mach number appropriate to the characteristic numbers on the centre-line of a nozzle.

The selection of the values of A in Table I perhaps needs some explanation. It will be noted that the values have a minimum in the middle of the range. The minimum arises because the values are designed to be used in the throat region of a nozzle, where constant Mach number lines are convex downstream (see e.g. Fig.1 of Ref.1). A right down-going characteristic is thus tangential to some constant Mach number line. The right up-going characteristic through the point of tangency will therefore have a minimum value.

REFERENCES

No.	Author	Title, etc.
1	R. Harrop and P.I. Bright	Design and testing of supersonic nozzles Part I R & M 2712 October, 1948.
2	P.J. Herbert and S.J. Older	Tables for use in the investigation of supersonic fields of flow by the method of characteristics R.A.E. Tech. Note No. CW1 November, 1946. ARC 10.261
3	L.M. Milne-Thomson and L.J. Comrie	Standard four-figure Mathematical Tables Macmillan 1948.

Slopes refer to characteristics approaching point (A, B)

Table I - Characteristic Slopes Interval 0.2°

Key

m_1	m_2	$m_1 + m_2$
$1/m_1$	$1/m_2$	$1/m_1 + 1/m_2$

Flow from left to right A Right up-going characteristic (in degrees) B Right down-going characteristic (in degrees)

A \ B	0.9			1.1			1.3			1.5			1.7			1.9			2.1		
2.1	1.5527 0.6441	1.7056 0.5863	3.2583 1.2304	1.5255 0.6556	1.6481 0.6067	3.1736 1.2623	1.5005 0.6664	1.5953 0.6268	3.0958 1.2932	1.4778 0.6767	1.5466 0.6466	3.0244 1.3233	1.4569 0.6864	1.5013 0.6661	2.9582 1.3525	1.4375 0.6956	1.4592 0.6853	2.8967 1.3809	1.4197 0.7044	1.4197 0.7044	2.8394 1.4088
1.9	1.6075 0.6221	1.7401 0.5747	3.3476 1.1968	1.5767 0.6342	1.6786 0.5957	3.2553 1.2299	1.5489 0.6456	1.6225 0.6163	3.1714 1.2619	1.5236 0.6564	1.5709 0.6366	3.0945 1.2930	1.5002 0.6665	1.5232 0.6565	3.0234 1.3230	1.4789 0.6762	1.4789 0.6762	2.9578 1.3524	1.4592 0.6853	1.4375 0.6956	2.8967 1.3809
1.7	1.6676 0.5997	1.7783 0.5623	3.4459 1.1620	1.6328 0.6125	1.7123 0.5841	3.3451 1.1966	1.6013 0.6245	1.6522 0.6052	3.2535 1.2297	1.5729 0.6358	1.5974 0.6260	3.1703 1.2618	1.5470 0.6464	1.5470 0.6464	3.0940 1.2928	1.5232 0.6565	1.5002 0.6665	3.0234 1.3230	1.5013 0.6661	1.4569 0.6864	2.9582 1.3525
1.5	1.7342 0.5766	1.8212 0.5491	3.5554 1.1257	1.6943 0.5902	1.7496 0.5716	3.4439 1.1618	1.6586 0.6029	1.6851 0.5935	3.3437 1.1964	1.6265 0.6148	1.6265 0.6148	3.2530 1.2296	1.5974 0.6260	1.5729 0.6358	3.1703 1.2618	1.5709 0.6366	1.5236 0.6564	3.0945 1.2930	1.5466 0.6466	1.4778 0.6767	3.0244 1.3233
1.3	1.8085 0.5529	1.8697 0.5349	3.6782 1.0878	1.7625 0.5674	1.7915 0.5582	3.5540 1.1256	1.7216 0.5808	1.7216 0.5808	3.4432 1.1616	1.6851 0.5935	1.6586 0.6029	3.3437 1.1964	1.6522 0.6052	1.6013 0.6245	3.2535 1.2297	1.6225 0.6163	1.5489 0.6456	3.1714 1.2619	1.5953 0.6268	1.5005 0.6664	3.0958 1.2932
1.1	1.8926 0.5283	1.9250 0.5195	3.8176 1.0478	1.8387 0.5439	1.8387 0.5439	3.6774 1.0878	1.7915 0.5582	1.7625 0.5674	3.5540 1.1256	1.7496 0.5716	1.6943 0.5902	3.4439 1.1618	1.7123 0.5841	1.6328 0.6125	3.3451 1.1966	1.6786 0.5957	1.5767 0.6342	3.2553 1.2299	1.6481 0.6067	1.5255 0.6556	3.1736 1.2623
0.9	1.9890 0.5027	1.9890 0.5027	3.9780 1.0054	1.9250 0.5195	1.8926 0.5283	3.8176 1.0478	1.8697 0.5348	1.8085 0.5529	3.6782 1.0877	1.8212 0.5491	1.7342 0.5766	3.5554 1.1257	1.7783 0.5623	1.6676 0.5997	3.4459 1.1620	1.7401 0.5747	1.6075 0.6221	3.3476 1.1968	1.7056 0.5863	1.5527 0.6441	3.2583 1.2304
0.7	2.1012 0.4759	2.0640 0.4845	4.1652 0.9604	2.0242 0.4940	1.9550 0.5115	3.9792 1.0055	1.9584 0.5106	1.8611 0.5373	3.8195 1.0479	1.9015 0.5259	1.7791 0.5621	3.6806 1.0880	1.8518 0.5400	1.7065 0.5860	3.5583 1.1260	1.8078 0.5531	1.6415 0.6092	3.4493 1.1623	1.7685 0.5655	1.5827 0.6318	3.3512 1.1973
0.5	2.2355 0.4473	2.1543 0.4642	4.3898 0.9115	2.1396 0.4674	2.0278 0.4932	4.1674 0.9606	2.0603 0.4854	1.9218 0.5204	3.9821 1.0058	1.9926 0.5018	1.8303 0.5464	3.8229 1.0482	1.9342 0.5170	1.7504 0.5713	3.6846 1.0883	1.8831 0.5310	1.6795 0.5954	3.5626 1.1264	1.8380 0.5441	1.6160 0.6188	3.4540 1.1629
0.3	2.3988 0.4170	2.2641 0.4417	4.6629 0.8587	2.2781 0.4389	2.1155 0.4727	4.3936 0.9116	2.1792 0.4588	1.9925 0.5018	4.1717 0.9606	2.0974 0.4767	1.8895 0.5293	3.9869 1.0060	2.0278 0.4932	1.8003 0.5554	3.8281 1.0486	1.9677 0.5082	1.7223 0.5806	3.6900 1.0888	1.9153 0.5221	1.6532 0.6049	3.5685 1.1270
0.1	2.6092 0.3833	2.3298 0.4292	4.9390 0.8125	2.4468 0.4087	2.1662 0.4616	4.6130 0.8703	2.3220 0.4307	2.0332 0.4918	4.3552 0.9225	2.2199 0.4504	1.9226 0.5201	4.1425 0.9705	2.1357 0.4682	1.8280 0.5470	3.9637 1.0152	2.0640 0.4845	1.7461 0.5727	3.8101 1.0572	2.0022 0.4994	1.6735 0.5975	3.6757 1.0969
0.1	2.6092 0.3833	2.2641 0.4417	4.8733 0.8250	2.4468 0.4087	2.1155 0.4727	4.5623 0.8814	2.3220 0.4317	1.9925 0.5018	4.3145 0.9325	2.2199 0.4504	1.8895 0.5293	4.1094 0.9797	2.1357 0.4682	1.8003 0.5554	3.9360 1.0236	2.0640 0.4845	1.7223 0.5806	3.7863 1.0651	2.0022 0.4994	1.6532 0.6049	3.6554 1.1043
0.3	2.3988 0.4170	2.1543 0.4642	4.5531 0.8812	2.2781 0.4389	2.0278 0.4932	4.3059 0.9321	2.1792 0.4588	1.9218 0.5204	4.1010 0.9792	2.0974 0.4767	1.8303 0.5464	3.9277 1.0231	2.0278 0.4932	1.7504 0.5713	3.7782 1.0645	1.9677 0.5082	1.6795 0.5954	3.6472 1.1036	1.9153 0.5221	1.6160 0.6188	3.5313 1.1409
0.5	2.2355 0.4473	2.0640 0.4845	4.2995 0.9318	2.1396 0.4674	1.9550 0.5115	4.0946 0.9789	2.0603 0.4854	1.8611 0.5373	3.9214 1.0227	1.9926 0.5018	1.7791 0.5621	3.7717 1.0639	1.9342 0.5170	1.7065 0.5860	3.6407 1.1030	1.8831 0.5310	1.6415 0.6092	3.5246 1.1402	1.8380 0.5441	1.5827 0.6318	3.4207 1.1759
0.7	2.1012 0.4759	1.9890 0.5027	4.0902 0.9786	2.0242 0.4940	1.8926 0.5283	3.9168 1.0223	1.9584 0.5106	1.8085 0.5529	3.7669 1.0635	1.9015 0.5259	1.7342 0.5766	3.6357 1.1025	1.8518 0.5400	1.6676 0.5997	3.5194 1.1397	1.8078 0.5531	1.6075 0.6221	3.4153 1.1752	1.7685 0.5655	1.5527 0.6441	3.3212 1.2096
0.9				1.9250 0.5195	1.8387 0.5439	3.7637 1.0634	1.8697 0.5348	1.7624 0.5674	3.6321 1.1022	1.8212 0.5491	1.6943 0.5902	3.5155 1.1393	1.7783 0.5623	1.6328 0.6125	3.4111 1.1748	1.7401 0.5747	1.5767 0.6342	3.3168 1.2089	1.7056 0.5863	1.5255 0.6556	3.2311 1.2419
1.1							1.7915 0.5582	1.7216 0.5809	3.5131 1.1391	1.7496 0.5716	1.6586 0.6029	3.4082 1.1745	1.7123 0.5841	1.6013 0.6245	3.3136 1.2086	1.6786 0.5957	1.5489 0.6456	3.2275 1.2413	1.6481 0.6067	1.5005 0.6664	3.1486 1.2731
1.3										1.6851 0.5935	1.6265 0.6148	3.3116 1.2083	1.6522 0.6052	1.5729 0.6358	3.2251 1.2410	1.6225 0.6163	1.5236 0.6564	3.1461 1.2727	1.5953 0.6268	1.4778 0.6767	3.0731 1.3035
1.5													1.5974 0.6260	1.5470 0.6464	3.1444 1.2724	1.5709 0.6366	1.5002 0.6665	3.0711 1.3031	1.5466 0.6466	1.4569 0.6864	3.0035 1.3330
1.7																1.5232 0.6565	1.4789 0.6762	3.0021 1.3327	1.5013 0.6661	1.4375 0.6956	2.9388 1.3617
1.9																			1.4592 0.6853	1.4197 0.7044	2.8789 1.3897

TABLE IA

Characteristic Slopes Interval 0.4°

Slopes refer to characteristics approaching point (A, B)
Flow from left to right
A Right up-going characteristic (in degrees)
B Right down-going characteristics (in degrees)

Key

m_1	m_2	$m_1 + m_2$
$1/m_1$	$1/m_2$	$1/m_1 + 1/m_2$

A \ B	1.1			1.5			1.9			2.5		
3.7	1.2295 0.8134	1.4868 0.6726	2.7163 1.4860	1.2061 0.8291	1.4144 0.7070	2.6205 1.5361						
3.3	1.2927 0.7735	1.5211 0.6575	2.8138 1.4310	1.2651 0.7905	1.4429 0.6930	2.7080 1.4835	1.2410 0.8058	1.3733 0.7282	2.6143 1.5340	1.2149 0.8231	1.2814 0.7804	2.4963 1.6035
2.9	1.3639 0.7332	1.5608 0.6407	2.9247 1.3739	1.3308 0.7514	1.4757 0.6777	2.8065 1.4291	1.3021 0.7680	1.4007 0.7139	2.7028 1.4819	1.2714 0.7866	1.3027 0.7676	2.5741 1.5542
2.5	1.4450 0.6921	1.6075 0.6221	3.0525 1.3142	1.4046 0.7119	1.5138 0.6606	2.9184 1.3725	1.3701 0.7298	1.4323 0.6982	2.8024 1.4280	1.3336 0.7498	1.3270 0.7536	2.6606 1.5034

Table II - Characteristic Slopes Interval 0.4°

Flow from left to right
A Right up-going characteristic (in degrees)
B Right down-going characteristic (in degrees)

Slopes refer to characteristics approaching point (A, B)

Key		
m_1	m_2	$m_1 + m_2$
$1/m_1$	$1/m_2$	$1/m_1 + 1/m_2$

A \ B	2.5			2.9			3.3			3.7			4.1			4.5			4.9		
2.9				1.245 0.803	1.245 0.803	2.490 1.606															
2.5				1.303 0.768	1.266 0.790	2.569 1.558	1.281 0.780	1.210 0.826	2.491 1.606	1.263 0.792	1.159 0.863	2.422 1.655	1.246 0.803	1.113 0.899	2.359 1.702	1.231 0.812	1.069 0.935	2.300 1.747	1.217 0.821	1.028 0.972	2.245 1.793
2.1	1.395 0.717	1.355 0.738	2.750 1.455	1.366 0.732	1.289 0.776	2.655 1.508	1.341 0.746	1.230 0.813	2.571 1.559	1.319 0.758	1.176 0.850	2.495 1.608	1.299 0.770	1.127 0.887	2.426 1.657	1.282 0.780	1.082 0.924	2.364 1.704	1.267 0.789	1.040 0.962	2.307 1.751
1.7	1.471 0.680	1.387 0.721	2.858 1.401	1.437 0.696	1.316 0.760	2.753 1.456	1.407 0.711	1.253 0.798	2.660 1.509	1.381 0.724	1.196 0.836	2.577 1.560	1.358 0.737	1.144 0.875	2.502 1.612	1.338 0.748	1.096 0.913	2.434 1.661	1.320 0.758	1.052 0.951	2.372 1.709
1.3	1.559 0.642	1.424 0.702	2.983 1.344	1.517 0.659	1.346 0.743	2.863 1.402	1.481 0.675	1.279 0.782	2.760 1.457	1.450 0.690	1.217 0.821	2.667 1.511	1.422 0.703	1.162 0.860	2.584 1.563	1.398 0.715	1.112 0.899	2.510 1.614	1.377 0.726	1.066 0.938	2.443 1.664
0.9	1.660 0.603	1.468 0.681	3.128 1.284	1.608 0.622	1.382 0.723	2.990 1.345	1.564 0.640	1.308 0.765	2.872 1.405	1.526 0.655	1.242 0.805	2.768 1.460	1.494 0.669	1.183 0.845	2.677 1.514	1.465 0.682	1.130 0.885	2.595 1.567	1.440 0.694	1.081 0.925	2.521 1.619
0.5	1.779 0.562	1.520 0.658	3.299 1.220	1.713 0.584	1.424 0.702	3.137 1.286	1.659 0.603	1.343 0.745	3.002 1.348	1.613 0.620	1.271 0.787	2.884 1.407	1.573 0.635	1.208 0.828	2.781 1.463	1.540 0.649	1.150 0.869	2.690 1.518	1.510 0.662	1.099 0.910	2.609 1.572
0.1	1.923 0.520	1.551 0.645	3.474 1.165	1.839 0.544	1.448 0.690	3.287 1.234	1.770 0.565	1.362 0.734	3.132 1.299	1.713 0.584	1.287 0.777	3.000 1.361	1.664 0.601	1.221 0.819	2.885 1.420	1.624 0.616	1.162 0.861	2.786 1.477	1.588 0.630	1.108 0.902	2.696 1.532
0.1	1.923 0.520	1.520 0.658	3.443 1.178	1.839 0.544	1.424 0.702	3.263 1.246	1.770 0.565	1.343 0.745	3.113 1.310	1.713 0.584	1.271 0.787	2.984 1.371	1.664 0.601	1.208 0.828	2.872 1.429	1.624 0.616	1.150 0.869	2.774 1.485	1.588 0.630	1.099 0.910	2.687 1.540
0.5	1.779 0.562	1.468 0.681	3.247 1.243	1.713 0.584	1.382 0.723	3.095 1.307	1.659 0.603	1.308 0.765	2.967 1.368	1.613 0.620	1.242 0.805	2.855 1.425	1.573 0.635	1.183 0.845	2.756 1.480	1.540 0.649	1.130 0.885	2.670 1.534	1.510 0.662	1.081 0.925	2.591 1.587
0.9	1.660 0.603	1.424 0.702	3.084 1.305	1.608 0.622	1.346 0.743	2.954 1.365	1.564 0.640	1.279 0.782	2.843 1.422	1.526 0.655	1.217 0.821	2.743 1.476	1.494 0.669	1.162 0.860	2.656 1.529	1.465 0.682	1.112 0.899	2.577 1.581	1.440 0.694	1.066 0.938	2.506 1.632
1.3	1.559 0.642	1.387 0.721	2.946 1.363	1.517 0.659	1.316 0.760	2.833 1.419	1.481 0.675	1.253 0.798	2.734 1.473	1.450 0.690	1.196 0.836	2.646 1.526	1.422 0.703	1.144 0.875	2.566 1.578	1.398 0.715	1.096 0.913	2.494 1.628	1.377 0.726	1.052 0.951	2.429 1.677
1.7	1.471 0.680	1.355 0.738	2.826 1.418	1.437 0.696	1.289 0.776	2.726 1.472	1.407 0.711	1.230 0.813	2.637 1.524	1.381 0.724	1.176 0.850	2.557 1.574	1.358 0.737	1.127 0.887	2.485 1.624	1.338 0.748	1.082 0.924	2.420 1.672	1.320 0.758	1.040 0.962	2.360 1.720
2.1	1.395 0.717	1.327 0.754	2.722 1.471	1.366 0.732	1.266 0.790	2.632 1.522	1.341 0.746	1.210 0.826	2.551 1.572	1.319 0.758	1.159 0.863	2.478 1.621	1.299 0.770	1.113 0.899	2.412 1.669	1.282 0.780	1.069 0.935	2.351 1.715	1.267 0.789	1.028 0.972	2.295 1.761
2.5				1.303 0.768	1.245 0.803	2.548 1.571	1.281 0.780	1.193 0.838	2.474 1.618	1.263 0.792	1.144 0.874	2.407 1.666	1.246 0.803	1.099 0.910	2.345 1.713	1.231 0.812	1.057 0.946	2.288 1.758	1.217 0.821	1.019 0.982	2.236 1.803
2.9							1.227 0.815	1.177 0.850	2.404 1.665	1.211 0.826	1.131 0.884	2.342 1.710	1.196 0.836	1.088 0.920	2.284 1.756	1.183 0.845	1.048 0.955	2.231 1.800	1.172 0.853	1.010 0.990	2.182 1.843
3.3										1.163 0.860	1.118 0.894	2.281 1.754	1.150 0.869	1.077 0.928	2.227 1.797	1.139 0.878	1.038 0.963	2.177 1.841	1.130 0.885	1.002 0.998	2.132 1.883
3.7													1.108 0.903	1.068 0.936	2.176 1.839	1.098 0.911	1.030 0.970	2.128 1.881	1.090 0.918	0.995 1.005	2.085 1.923
4.1																1.060 0.944	1.023 0.977	2.083 1.921	1.052 0.950	0.989 1.011	2.041 1.961
4.5																			1.017 0.983	0.983 1.017	2.000 2.000

Table III - Characteristic Slopes Interval 0.8°

Flow from left to right
A Right up-going characteristic (in degrees)
B Right down-going characteristic (in degrees)

Slopes refer to characteristics approaching point (A, B)

Key		
m_1	m_2	$m_1 + m_2$
$1/m_1$	$1/m_2$	$1/m_1 + 1/m_2$

A \ B	5.7			6.5			7.3			8.1			8.9			9.7			10.5		
2.5	1.200 0.833	0.959 1.043	2.159 1.876																		
1.7	1.296 0.772	0.978 1.023	2.274 1.795	1.271 0.787	0.907 1.103	2.178 1.890	1.251 0.799	0.844 1.185	2.095 1.984	1.235 0.810	0.787 1.271	2.022 2.081									
0.9	1.409 0.710	1.001 0.999	2.410 1.709	1.374 0.728	0.924 1.082	2.298 1.810	1.346 0.743	0.857 1.167	2.203 1.910	1.325 0.755	0.797 1.255	2.122 2.010	1.308 0.765	0.743 1.347	2.051 2.112	1.294 0.773	0.693 1.444	1.987 2.217	1.284 0.779	0.647 1.546	1.931 2.325
0.1	1.544 0.648	1.015 0.985	2.559 1.633	1.496 0.669	0.935 1.070	2.431 1.739	1.458 0.686	0.865 1.156	2.323 1.842	1.428 0.700	0.803 1.245	2.231 1.945	1.405 0.712	0.747 1.338	2.152 2.050	1.387 0.721	0.696 1.436	2.083 2.157	1.372 0.729	0.656 1.525	2.028 2.254
0.1	1.544 0.648	1.001 0.999	2.545 1.647	1.496 0.669	0.924 1.082	2.420 1.751	1.458 0.686	0.857 1.167	2.315 1.853	1.428 0.700	0.797 1.255	2.225 1.955	1.405 0.712	0.743 1.346	2.148 2.058	1.387 0.721	0.693 1.444	2.080 2.165	1.372 0.729	0.647 1.546	2.019 2.275
0.9	1.409 0.710	0.978 1.023	2.387 1.733	1.374 0.728	0.907 1.103	2.281 1.831	1.346 0.743	0.844 1.185	2.190 1.928	1.325 0.755	0.787 1.271	2.112 2.026	1.308 0.765	0.735 1.361	2.043 2.126	1.294 0.773	0.687 1.456	1.981 2.229	1.284 0.779	0.643 1.556	1.927 2.335
1.7	1.296 0.772	0.959 1.043	2.255 1.815	1.271 0.787	0.893 1.120	2.164 1.907	1.251 0.799	0.833 1.200	2.084 1.999	1.235 0.810	0.779 1.284	2.014 2.094	1.222 0.818	0.729 1.372	1.951 2.190	1.213 0.825	0.683 1.464	1.896 2.289	1.206 0.829	0.640 1.562	1.846 2.391
2.5	1.200 0.833	0.944 1.059	2.144 1.892	1.182 0.846	0.882 1.134	2.064 1.980	1.167 0.857	0.825 1.213	1.992 2.070	1.155 0.866	0.773 1.294	1.928 2.160	1.146 0.873	0.725 1.380	1.871 2.253	1.140 0.878	0.680 1.470	1.820 2.348	1.135 0.881	0.638 1.566	1.773 2.447
3.3	1.117 0.895	0.933 1.072	2.050 1.967	1.103 0.907	0.873 1.146	1.976 2.053	1.092 0.916	0.818 1.222	1.910 2.138	1.083 0.923	0.768 1.302	1.851 2.225	1.077 0.928	0.722 1.385	1.799 2.313	1.073 0.932	0.679 1.474	1.752 2.406	1.071 0.934	0.638 1.568	1.709 2.502
4.1	1.043 0.959	0.923 1.083	1.966 2.042	1.032 0.969	0.866 1.155	1.898 2.124	1.024 0.976	0.814 1.229	1.838 2.205	1.019 0.982	0.765 1.307	1.784 2.289	1.015 0.985	0.720 1.388	1.735 2.373	1.013 0.987	0.678 1.475	1.691 2.462	1.012 0.989	0.638 1.567	1.650 2.556
4.9	0.976 1.024	0.916 1.092	1.892 2.116	0.969 1.032	0.861 1.161	1.830 2.193	0.963 1.038	0.810 1.234	1.773 2.272	0.960 1.042	0.763 1.310	1.723 2.352	0.958 1.044	0.719 1.390	1.677 2.434	0.957 1.045	0.678 1.474	1.635 2.519	0.957 1.045	0.639 1.564	1.596 2.609
5.7				0.911 1.098	0.858 1.166	1.769 2.264	0.907 1.102	0.809 1.237	1.716 2.339	0.905 1.104	0.763 1.310	1.668 2.414	0.905 1.106	0.720 1.388	1.625 2.494	0.905 1.105	0.679 1.472	1.584 2.577	0.906 1.103	0.641 1.559	1.547 2.662
6.5							0.856 1.168	0.808 1.238	1.664 2.406	0.855 1.170	0.763 1.310	1.618 2.480	0.855 1.169	0.721 1.387	1.576 2.556	0.857 1.167	0.681 1.468	1.538 2.635	0.859 1.164	0.644 1.553	1.503 2.717
7.3										0.808 1.237	0.764 1.308	1.572 2.545	0.809 1.235	0.723 1.383	1.532 2.618	0.812 1.232	0.684 1.462	1.496 2.694	0.815 1.227	0.647 1.545	1.462 2.772
8.1													0.766 1.305	0.726 1.377	1.492 2.682	0.770 1.299	0.688 1.454	1.458 2.753	0.773 1.293	0.651 1.536	1.424 2.829
8.9																0.729 1.371	0.691 1.446	1.420 2.817	0.734 1.363	0.655 1.526	1.389 2.889
9.7																			0.696 1.437	0.660 1.514	1.356 2.951

Table IV - Characteristic Slopes Interval 1.6°

Flow from left to right
A Right up-going characteristic (in degrees)
B Right down-going characteristic (in degrees)

Slopes refer to characteristics approaching point (A, B)

Key:

m_1	m_2	$m_1 + m_2$
$1/m_1$	$1/m_2$	$1/m_1 + 1/m_2$

A \ B	12.1			13.7			15.3			16.9			18.5			20.1			21.7		
0.9	1.274 0.785	0.564 1.773	1.838 2.558	1.268 0.789	0.491 2.038	1.759 2.827															
0.1	1.357 0.737	0.565 1.770	1.922 2.507	1.346 0.743	0.491 2.038	1.837 2.781	1.343 0.744	0.424 2.360	1.767 3.104	1.348 0.742	0.362 2.759	1.710 3.501	1.358 0.736	0.305 3.274	1.663 4.010	1.374 0.728	0.2519 3.969	1.626 4.697			
0.1	1.357 0.737	0.564 1.773	1.921 2.510	1.346 0.743	0.491 2.038	1.837 2.781	1.343 0.744	0.424 2.360	1.767 3.104	1.348 0.742	0.364 2.750	1.712 3.492	1.358 0.736	0.307 3.258	1.665 3.994	1.374 0.728	0.254 3.940	1.628 4.668	1.394 0.717	0.203 4.917	1.597 5.634
0.9	1.274 0.785	0.562 1.779	1.836 2.564	1.268 0.789	0.491 2.035	1.759 2.824	1.269 0.788	0.427 2.342	1.696 3.130	1.276 0.784	0.368 2.720	1.644 3.504	1.288 0.776	0.312 3.203	1.600 3.979	1.301 0.766	0.260 3.846	1.561 4.612	1.326 0.754	0.210 4.753	1.536 5.507
2.5	1.132 0.883	0.563 1.777	1.695 2.660	1.133 0.883	0.495 2.022	1.628 2.905	1.139 0.878	0.432 2.313	1.571 3.191	1.150 0.870	0.375 2.669	1.525 3.539	1.164 0.859	0.321 3.120	1.485 3.979	1.182 0.846	0.269 3.711	1.451 4.557	1.204 0.831	0.221 4.531	1.425 5.362
4.1	1.013 0.987	0.566 1.776	1.579 2.753	1.018 0.982	0.500 1.998	1.518 2.980	1.028 0.973	0.440 2.274	1.468 3.247	1.041 0.961	0.383 2.608	1.424 3.569	1.057 0.946	0.330 3.027	1.387 3.973	1.075 0.930	0.280 3.571	1.355 4.501	1.097 0.912	0.232 4.308	1.329 5.220
5.7	0.911 1.098	0.572 1.747	1.483 2.845	0.919 1.088	0.508 1.968	1.427 3.056	0.931 1.075	0.449 2.228	1.380 3.303	0.945 1.058	0.394 2.541	1.339 3.599	0.962 1.040	0.341 2.929	1.303 3.969	0.981 1.020	0.292 3.427	1.273 4.447	1.002 0.998	0.244 4.090	1.246 5.088
7.3	0.821 1.218	0.581 1.722	1.402 2.940	0.831 1.203	0.518 1.931	1.349 3.134	0.844 1.184	0.460 2.176	1.304 3.360	0.860 1.163	0.405 2.469	1.265 3.632	0.877 1.140	0.354 2.829	1.231 3.969	0.896 1.116	0.305 3.283	1.201 4.399	0.918 1.090	0.258 3.880	1.176 4.970
8.9	0.741 1.349	0.591 1.692	1.332 3.041	0.753 1.328	0.529 1.890	1.282 3.218	0.767 1.304	0.472 2.121	1.239 3.425	0.783 1.278	0.418 2.394	1.201 3.672	0.800 1.250	0.367 2.727	1.167 3.977	0.820 1.220	0.318 3.142	1.138 4.362	0.841 1.190	0.272 3.679	1.113 4.869
10.5	0.669 1.495	0.603 1.659	1.272 3.154	0.682 1.467	0.542 1.846	1.224 3.313	0.696 1.436	0.485 2.062	1.181 3.498	0.712 1.404	0.431 2.318	1.143 3.722	0.730 1.369	0.381 2.625	1.111 3.994	0.750 1.334	0.333 3.005	1.083 4.339	0.770 1.298	0.287 3.488	1.057 4.786
12.1				0.616 1.622	0.556 1.799	1.172 3.421	0.631 1.584	0.499 2.002	1.130 3.586	0.648 1.543	0.446 2.240	1.094 3.783	0.666 1.502	0.396 2.525	1.062 4.027	0.685 1.460	0.348 2.872	1.033 4.332	0.705 1.418	0.302 3.308	1.007 4.726
13.7							0.571 1.750	0.515 1.941	1.086 3.691	0.588 1.701	0.462 2.163	1.050 3.864	0.606 1.651	0.412 2.426	1.018 4.077	0.625 1.600	0.364 2.744	0.989 4.344	0.645 1.551	0.319 3.138	0.964 4.689
15.3										0.532 1.880	0.479 2.088	1.011 3.968	0.550 1.819	0.429 2.331	0.979 4.150	0.568 1.759	0.381 2.622	0.949 4.381	0.588 1.700	0.336 2.979	0.924 4.679
16.9													0.497 2.013	0.447 2.238	0.944 4.251	0.515 1.940	0.399 2.506	0.914 4.446	0.535 1.869	0.353 2.830	0.888 4.699
18.5																0.465 2.149	0.417 2.395	0.882 4.544	0.485 2.063	0.372 2.690	0.857 4.753
20.1																			0.437 2.290	0.391 2.559	0.828 4.849

Flow from left to right
A Right up-going characteristic (in degrees)
B Right down-going characteristic (in degrees)

Table V - Characteristic Slopes Interval 1.6°

Slopes refer to characteristics approaching point (A, B)

Key

m_1	m_2	$m_1 + m_2$
$1/m_1$	$1/m_2$	$1/m_1 + 1/m_2$

A \ B	23.3			24.9			26.5		
0.1	1.4186 0.705	0.1551 6.448	1.5737 7.153	1.4472 0.691	0.1085 9.210	1.5557 9.901	1.4799 0.676	0.0635 15.75	1.5434 16.43
0.9	1.3504 0.740	0.1628 6.143	1.5132 6.883	1.3787 0.725	0.1169 8.554	1.4956 9.279	1.4107 0.709	0.0723 13.82	1.4830 14.53
2.5	1.2282 0.814	0.1739 5.749	1.4021 6.563	1.2557 0.796	0.1288 7.766	1.3845 8.562	1.2864 0.777	0.0849 11.78	1.3713 12.56
4.1	1.1211 0.892	0.1860 5.375	1.3071 6.267	1.1479 0.871	0.1415 7.068	1.2894 7.939	1.1772 0.850	0.0982 10.186	1.2754 11.036
5.7	1.0261 0.975	0.1990 5.026	1.2251 6.001	1.0521 0.951	0.1549 6.455	1.2070 7.406	1.0802 0.926	0.1121 8.919	1.1923 9.845
7.3	0.9408 1.063	0.2127 4.702	1.1535 5.765	0.9659 1.035	0.1690 5.914	1.1349 6.949	0.9929 1.007	0.1267 7.894	1.1196 8.901
8.9	0.8634 1.158	0.2271 4.403	1.0905 5.561	0.8877 1.126	0.1839 5.438	1.0716 6.564	0.9138 1.094	0.1418 7.050	1.0556 8.144
10.5	0.7925 1.262	0.2423 4.128	1.0348 5.390	0.8162 1.225	0.1993 5.017	1.0155 6.242	0.8413 1.189	0.1576 6.347	0.9989 7.536
12.1	0.7270 1.375	0.2581 3.874	0.9851 5.249	0.7501 1.333	0.2154 4.643	0.9655 5.976	0.7744 1.291	0.1738 5.754	0.9482 7.045
13.7	0.6662 1.501	0.2746 3.641	0.9408 5.142	0.6887 1.452	0.2320 4.311	0.9207 5.763	0.7122 1.404	0.1906 5.247	0.9028 6.651
15.3	0.6093 1.641	0.2916 3.429	0.9009 5.070	0.6312 1.584	0.2491 4.014	0.8803 5.598	0.6541 1.529	0.2079 4.811	0.8620 6.340
16.9	0.5557 1.800	0.3094 3.232	0.8651 5.032	0.5772 1.733	0.2668 3.747	0.8440 5.480	0.5995 1.668	0.2256 4.432	0.8251 6.100
18.5	0.5049 1.981	0.3277 3.052	0.8326 5.033	0.5259 1.901	0.2851 3.508	0.8110 5.409	0.5478 1.825	0.2439 4.101	0.7917 5.926
20.1	0.4566 2.190	0.3466 2.886	0.8032 5.076	0.4773 2.095	0.3038 3.290	0.7811 5.385	0.4987 2.005	0.2626 3.808	0.7613 5.813
21.7	0.4104 2.437	0.3660 2.732	0.7764 5.169	0.4308 2.322	0.3232 3.094	0.7540 5.416	0.4518 2.213	0.2818 3.549	0.7336 5.762
23.3				0.3862 2.590	0.3431 2.914	0.7293 5.504	0.4068 2.458	0.3015 3.317	0.7083 5.775
24.9							0.3636 2.750	0.3217 3.109	0.6853 5.659
26.5									
28.1									
29.7									
31.3									
32.9									
34.5									

A \ B	28.1			29.7			31.3		
0.1	1.5165 0.659	0.0194 51.47	1.5359 52.13	1.5572 0.642	-0.0238 -42.06	1.5334 -41.42	1.6019 0.624	-0.0664 -15.06	1.5355 -14.44
0.9	1.4463 0.691	0.0289 34.66	1.4752 35.35	1.4856 0.673	-0.0139 -72.25	1.4717 -71.58	1.5286 0 654	-0.0560 -17.87	1.4726 -17.22
2.5	1.3200 0.758	0.0420 23.79	1.3620 24.55	1.3567 0.737	-0.0001 -19,099	1.3566 -19,098	1.3967 0.716	-0.0415 -24.06	1.3552 23.34
4.1	1.2091 0.827	0.0558 17.91	1.2649 18.74	1.2436 0.804	0.0143 69.87	1.2579 70.67	1.2809 0.781	-0.0267 -37.51	1.2542 -36.73
5.7	1.1105 0.900	0.0703 14.23	1.1808 15.13	1.1431 0.875	0.0292 34.24	1.1723 35.12	1.1781 0.849	-0.0113 -88.69	1.1668 -87.84
7.3	1.0219 0.979	0.0852 11.73	1.1071 12.71	1.0528 0.950	0.0446 22.42	1.0974 23.37	1.0858 0.921	0.0045 220.4	1.0903 221.3
8.9	0.9415 1.062	0.1008 9.925	1.0423 10.987	0.9709 1.030	0.0605 16.54	1.0314 17.57	1.0021 0.998	0.0207 48.14	1.0228 49.14
10.5	0.8679 1.152	0.1168 8.565	0.9847 9.717	0.8960 1.116	0.0767 13.03	0.9727 14.15	0.9257 1.080	0.0374 26.74	0.9631 27.82
12.1	0.8000 1.250	0.1332 7.505	0.9332 8.755	0.8270 1.209	0.0935 10.695	0.9205 11.904	0.8553 1.169	0.0544 18.38	0.9097 19.55
13.7	0.7370 1.357	0.1502 6.658	0.8872 8.015	0.7630 1.311	0.1107 9.038	0.8737 10.349	0.7902 1.266	0.0717 13.94	0.8619 15.21
15.3	0.6781 1.475	0.1676 5.967	0.8457 7.442	0.7032 1.422	0.1282 7.802	0.8314 9.224	0.7293 1.371	0.0895 11.177	0.8188 12.548
16.9	0.6228 1.606	0.1854 5.394	0.8082 7.000	0.6470 1.546	0.1461 6.844	0.7931 8.390	0.6723 1.487	0.1075 9.299	0.7798 10.786
18.5	0.5705 1.753	0.2037 4.909	0.7742 6.662	0.5941 1.683	0.1644 6.082	0.7585 7.765	0.6185 1.617	0.1259 7.941	0.7444 9.558
20.1	0.5209 1.920	0.2224 4.497	0.7433 6.417	0.5438 1.839	0.1831 5.461	0.7269 7.300	0.5676 1.762	0.1447 6.912	0.7123 8.674
21.7	0.4735 2.112	0.2415 4.141	0.7150 6.253	0.4959 2.016	0.2022 4.945	0.6981 6.961	0.5191 1.926	0.1638 6.106	0.6829 8.032
23.3	0.4282 2.336	0.2611 3.830	0.6893 6.166	0.4501 2.222	0.2217 4.511	0.6718 6.733	0.4727 2.115	0.1832 5.458	0.6559 7.573
24.9	0.3845 2.601	0.2811 3.558	0.6656 6.159	0.4061 2.463	0.2416 4.139	0.6477 6.602	0.4283 2.335	0.2031 4.926	0.6314 7.261
26.5	0.3424 2.921	0.3016 3.316	0.6440 6.237	0.3637 2.750	0.2618 3.818	0.6255 6.568	0.3854 2.595	0.2231 4.481	0.6085 7.076
28.1				0.3225 3.100	0.2826 3.539	0.6051 6.639	0.3439 2.907	0.2437 4.103	0.5876 7.010
29.7							0.3037 3.292	0.2646 3.780	0.5683 7.072
31.3									
32.9									
34.5									

A \ B	32.9			34.5			36.1		
0.1	1.6511 0.606	-0.1086 -9.205	1.5425 -8.599						
0.9	1.5755 0.635	-0.0977 -10.236	1.4778 -9.601						
2.5	1.4399 0.694	-0.0827 -12.09	1.3572 -11.40						
4.1	1.3209 0.757	-0.0672 -14.88	1.2537 -14.12	1.3640 0.7331	-0.1076 -9.296	1.2564 -8.563	1.4102 0.7091	-0.1479 -6.764	1.2623 -6.055
5.7	1.2154 0.823	-0.0514 -19.47	1.1640 -18.65	1.2554 0.7966	-0.0912 -10.967	1.1642 -10.170	1.3125 0.7621	-0.1309 -7.640	1.1816 -6.878
7.3	1.1208 0.892	-0.0351 -28.49	1.0857 -27.60	1.1581 0.8635	-0.0744 -13.43	1.0737 -12.57	1.1978 0.8349	-0.1137 -8.795	1.0841 -7.960
8.9	1.0352 0.966	-0.0185 -54.15	1.0167 -53.18	1.0702 0.9344	-0.0574 -17.42	1.0128 -16.49	1.1072 0.9032	-0.0962 -10.393	1.0110 -9.490
10.5	0.9570 1.045	-0.0015 -668.4	0.9555 -667.4	0.9901 1.0100	-0.0401 -24.94	0.9500 -23.93	1.0248 0.9758	-0.0785 -12.75	0.9463 -11.77
12.1	0.8852 1.130	0.0158 63.38	0.9010 64.51	0.9164 1.0912	-0.0225 -44.51	0.8939 -43.42	0.9494 1.0533	-0.0633 -16.53	0.8861 -15.48
13.7	0.8186 1.222	0.0334 29.94	0.8520 31.16	0.8485 1.1786	-0.0046 -218.8	0.8439 -217.6	0.8797 1.1368	-0.0423 -23.64	0.8374 -22.50
15.3	0.7566 1.322	0.0513 19.48	0.8079 20.80	0.7852 1.2736	0.0136 73.64	0.7988 74.81	0.8150 1.2271	-0.0238 -41.91	0.7912 -40.68
16.9	0.6986 1.431	0.0695 14.38	0.7681 15.81	0.7259 1.3775	0.0320 31.25	0.7579 32.63	0.7545 1.3254	-0.0052 -191.7	0.7493 -190.4
18.5	0.6440 1.553	0.0881 11.356	0.7321 12.909	0.6703 1.4919	0.0507 19.73	0.7210 21.22	0.6976 1.4334	0.0137 73.17	0.7113 74.60
20.1	0.5922 1.689	0.1069 9.356	0.6991 11.045	0.6176 1.6190	0.0696 14.36	0.6872 15.98	0.6441 1.5527	0.0326 30.53	0.6768 32.08
21.7	0.5430 1.842	0.1260 7.934	0.6690 9.776	0.5677 1.7615	0.0888 11.259	0.6565 13.020	0.5932 1.6858	0.0521 19.21	0.6453 20.90
23.3	0.4960 2.016	0.1455 6.875	0.6415 8.891	0.5201 1.9230	0.1083 9.235	0.6284 11.158	0.5448 1.8357	0.0716 13.97	0.6164 15.80
24.9	0.4510 2.217	0.1651 6.054	0.6161 8.271	0.4744 2.1079	0.1280 7.811	0.6024 9.919	0.4985 2.0061	0.0914 10.946	0.5899 12.952
26.5	0.4077 2.453	0.1852 5.398	0.5929 7.851	0.4306 2.3226	0.1480 6.756	0.5786 9.079	0.4540 2.2025	0.1114 8.979	0.5654 11.182
28.1	0.3658 2.733	0.2056 4.863	0.5714 7.596	0.3883 2.5757	0.1683 5.941	0.5566 8.517	0.4113 2.4316	0.1317 7.598	0.5430 10.030
29.7	0.3253 3.074	0.2264 4.418	0.5517 7.492	0.3473 2.879	0.1890 5.293	0.5363 8.172	0.3699 2.7037	0.1521 6.573	0.5220 9.276
31.3	0.2859 3.498	0.2475 4.042	0.5334 7.540	0.3076 3.251	0.2098 4.766	0.5174 8.017	0.3298 3.033	0.1729 5.783	0.5027 8.816
32.9				0.2689 3.719	0.2311 4.327	0.5000 8.046	0.2907 3.440	0.1940 5.155	0.4847 8.595
34.5							0.2526 3.958	0.2141 4.671	0.4667 8.629

W.T.2078.CP337.K3 - Printed in Great Britain.

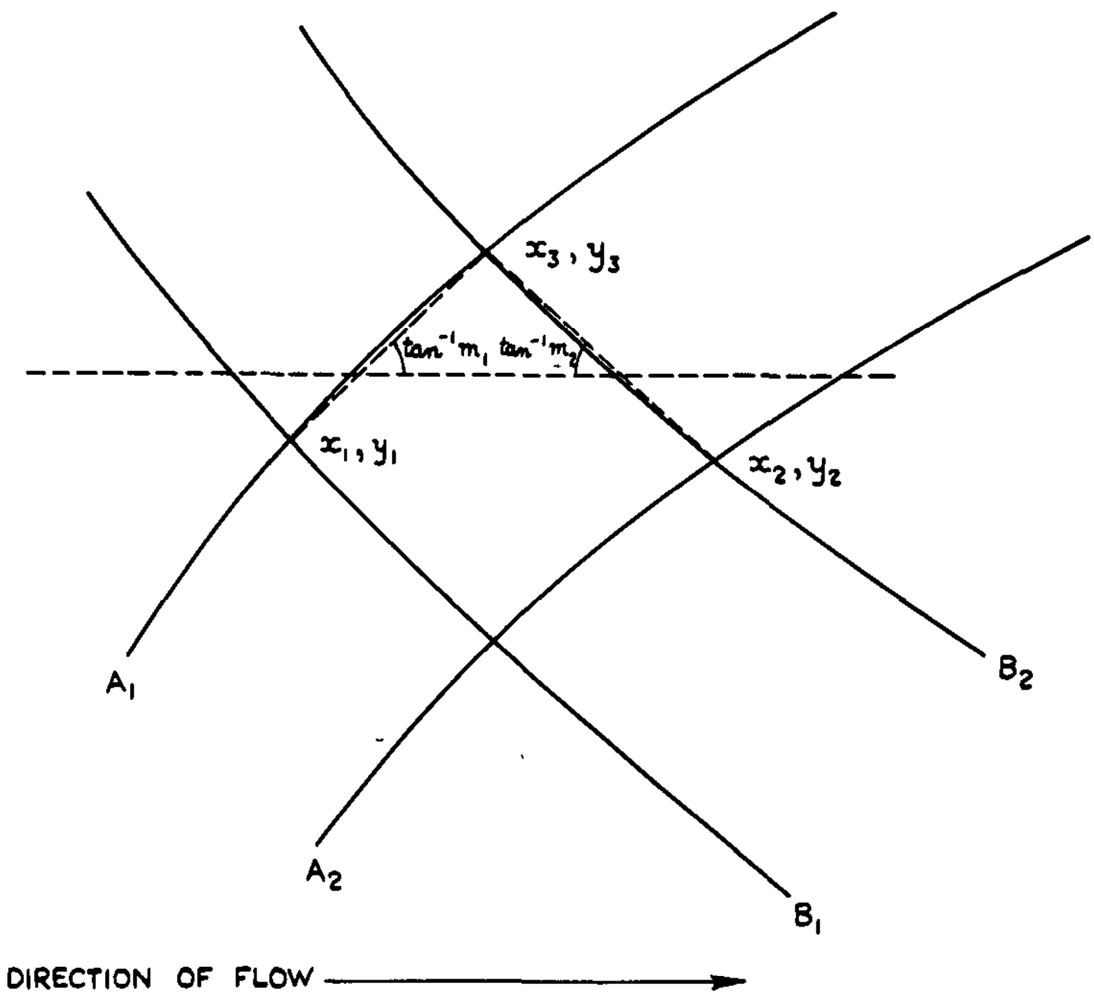

FIG. 1. SKETCH OF CHARACTERISTICS INTERSECTIONS.

C.P. No. 337
(18,944)
A.R.C. Technical Report

Published by
HER MAJESTY'S STATIONERY OFFICE
To be purchased from
York House, Kingsway, London w.c 2
423 Oxford Street, London w 1
13A Castle Street, Edinburgh 2
109 St. Mary Street, Cardiff
39 King Street, Manchester 2
Tower Lane, Bristol 1
2 Edmund Street, Birmingham 3
80 Chichester Street, Belfast
or through any bookseller

PRINTED IN GREAT BRITAIN

S.O. Code No. 23-9010-37

C.P. No. 337