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On a Type of Air Lubricated Journal Bearing

By

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On a Type of Air Lubricated Journal Bearing

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SUMMARY

Experimental journal bearings have been constructed which will support a radial load when supplied with air at high pressure. The principles of this type of bearing are discussed, and some of the available experimental data analysed. The results are collated in terms of a non-dimensional parameter based on the theory of viscid flow between two adjacent surfaces and by this means are extrapolated to give performance figures for bearings outside the range of the experiments. The estimated performance is then compared with that of conventional bearings, and conclusions are drawn regarding possible applications of this type of air lubrication.

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1.0 Introduction

It is a fundamental principle of lubrication that if two surfaces are separated by a fluid film the resistance to relative motion is reduced, and the lower the viscosity of the fluid the smaller the resisting force. If a bearing is constructed in which the lubricant is a gas - having a very low viscosity - and in which the distribution of static pressure supplies the supporting force, it will thus be virtually frictionless. Such bearings have been made to operate with air and have been appropriately called "air lubricated" or "floating" bearings.

The earlier examples were thrust bearings consisting simply of two flat, semi-cylindrical, or hemispherical surfaces separated by a thin film of air introduced at a high pressure through rings or rows of small holes. Bearings of this kind were developed at the Royal Aircraft Establishment and were subsequently incorporated there in a 2 ton wind tunnel balance. Later, an air lubricated journal bearing was constructed on the same principle with two peripheral rings of inlet holes and was used in a device for measuring the torque in a rotating propeller shaft. This work was discontinued after 1932, and until quite recently little further research has apparently been done in the field. In the last few years, however, there has been some interest concerning the application of gas lubricated bearings to the gas turbine, particularly to plant in which the weight of the rotor is so great as to preclude the use of ball or roller bearings because of the Brinelling of the races when stationary. The conventional alternative in the latter case - the oil journal bearing - is especially subject to large frictional losses at high rotational speeds, and it has been suggested that a comparatively frictionless bearing like the air lubricated journal bearing might be an economical substitute; the chief disadvantage is, of course, the power consumption of the necessary air compressor. This renewed interest in air lubrication revealed a scarcity of experimental data and of means of estimating bearing performances. An investigation into the properties of air lubricated journal bearings was therefore begun at the N.G.T.E. with the aim of finding a means of predicting approximately the performance of any given bearing or, alternatively, of designing a bearing to specification.

An attempt was first made to obtain a theoretical solution based on viscid flow theory. Experiments were carried out to determine the properties of air flowing in a narrow slot (Ref. 1), and fundamental relationships for the flow between adjacent surfaces were verified. These expressions were then applied to the case of a journal bearing, and some estimates of performance based on the assumption of axial flow were made. However, tests performed with a 2" diameter bearing showed that the theoretical values of load capacity were optimistic, and the problem was therefore considered in greater detail. Unfortunately, the conditions at the inlet holes are so complicated that a general solution could not be obtained and the theoretical approach was rejected in favour of the experimental. Further tests (Ref. 2) were then performed with the 2" diameter bearing, using various values of clearance and various hole sizes and positions. The results of these experiments are analysed in this report and correlated in terms of a non-dimensional parameter derived from viscid flow theory. Approximate general agreement is obtained, and the empirical factors deduced are then used in the estimation of the performance of bearings outside the range of sizes and pressures investigated. From consideration of these predicted values it is possible to draw general conclusions regarding the scope of application of the air lubricated journal bearafter passing from the inlet holes, flows outwards towards the ends of the bearing which are open to atmosphere. When the bearing is loaded the path of the air is curved, because there is a circumferential component of velocity from the smaller to the larger clearance. However, for the purpose of this qualitative description the fluid can be imagined as flowing axially away from each inlet hole and along an equivalent slot (Fig. 2) of width, a, equal to the circumferential distance between adjacent inlet holes; of length, ℓ, equal to the distance from the ring of the inlet holes to the end; and of depth, h, equal to the local radial clearance.

If the mass flow through each hole is assumed to be independent of h, the principle of the bearing is revealed in the equations (Ref. 1) relating the pressure, p_1, just outside the hole to the exhaust pressure, p_2. These are

$$p_1^2 - p_2^2 = 24\,\mu \cdot \frac{RT}{g} \cdot \frac{G_1 \cdot \ell}{a \cdot h^3} \qquad \text{.............................(1)}$$

for laminar flow, and

$$p_1^2 - p_2^2 = 0.133 \cdot \mu^{\frac{1}{4}} \cdot \frac{RT}{g} \cdot G_1^{7/4} \cdot \frac{\ell}{a^{7/4} \cdot h^3} \qquad \text{...................(2)}$$

for turbulent flow; where G_1 is the mass flow from the hole, μ is the absolute viscosity and T the absolute temperature of the air and g is the acceleration due to gravity. Obviously, when h is small p_1 is large, and when h is large p_1 is small. Hence, when a radial load is applied the shaft becomes eccentric within the shell and a distribution of pressure is produced which tends to oppose the applied load. If the load is within the capacity of the bearing, an equilibrium eccentric position is attained in which the summation of the components of pressure in the direction of the load is equal to the load.

This description has, of course, been simplified, since the mass flow is not constant nor are the flow paths purely axial. These two effects combine to make the actual bearing load capacity less than that estimated according to this simple theory. The mass flow, for instance, is often affected by the value of the local clearance, the degree of dependence being a function of size and shape of the inlet holes. If the holes are very large, their resistance is negligible and the mass flow is governed almost entirely by the local clearance; a decrease in h reduces the mass flow considerably and thus reduces or eliminates the local pressure rise. With small inlet holes, however, the total resistance of the flow path is not greatly affected by variations in clearance, and if the local value of $\frac{p_1}{p_o}$ is less than 0.528 it is entirely independent. If this latter condition holds at all the holes, the form of the peripheral pressure distribution is an optimum, although the maximum value of p_1 is then less than 0.528 p_o. Further, experiments have shown that, as two bearing surfaces approach, the local value of p_1 does not reach the value p_o but drops away to zero as contact occurs. This is due to the effect of a reduction of G_1 overriding the effect of a decrease in h, and occurs whenever local pressures are much in excess of 0.528 p_o, in other words, when the holes are unchoked. A similar detrimental effect occurs at the low pressure side of the bearing, where the ideal low pressures are only attained when the mass flow is limited by choking at the holes. The conclusion is, therefore, that for the maximum value of the load capacity the bearing must be operated with most of its inlet holes in a choked condition.

So far, the drop in pressure $(p_o - p_1)$ has been assumed to take place within the inlet hole whereas, in fact, it may take place in the hole and/or between the bearing surfaces at the commencement of the radial flow away from the hole. If, for instance, plain holes (Fig. 3a) are used, the cross sectional area of radial flow, $2\pi rh$, may be less than the area of the hole, πr^2, in which case choking occurs between the bearing surfaces and the mass flow is then proportional to h. This reduces the rate of change of p_1 with h and hence the bearing load capacity as a whole. In the experiments this effect was experienced, but was overcome by the use of flared holes (Fig. 3b) or holes joined by a peripheral groove (Fig. 3c). Both methods increase the load capacity, the latter more than the former, probably because the flow near the grooves is almost purely axial. However, the cross sectional area of the groove is critical, and if too large has the unwanted effect of equalising the pressure round the periphery. Most of the results discussed in the following section were obtained with bearings having well flared unconnected inlet holes, no information being available regarding the optimum dimensions of connecting grooves.

Air lubricated bearings have been made with other systems of inlet holes, grooves, or recesses (Ref. 3), but for the purpose of this note only the type described above is considered.

3.0 The Experimental Data

The data upon which the empirical expressions for bearing performance are based were obtained from tests previously reported (Ref. 2) and from further experiments with the same apparatus, different sizes of inlet hole being used. These N.G.T.E. results are also compared with those which were obtained with similar bearings by the R.A.E. and by Messrs. Gilkes and Gordon of Kendal. All the figures for performance, load and mass flow are expressed as non-dimensional coefficients, and the correlation is obtained by plotting these values against the ratio of the actual clearance to the inlet hole choking clearance based on the "equivalent slot theory".

3.1 The Equivalent Slot Theory

In section 2.0 the principle of the bearing was described with reference to the equivalent slot. Although the simple theory does not provide a direct means of estimating performance it serves as a basis for the empirical method outlined below. The theoretical choking clearance is based on the assumption of the ideal condition deduced in the previous section, namely that all of the inlet holes are choked. The number of holes which are choked in an actual bearing depends on its eccentricity, but for the purpose of comparison the simple case is considered of that bearing, with a co-axial shaft and shell, which corresponds to the axial mass flow assumed in the slot theory.

The two basic expressions for the pressure drop along a narrow slot are given in equations (1) and (2) as

$$p_1^2 - p_2^2 = 24 \cdot \mu \cdot \frac{RT}{g} \cdot \frac{G_1 \cdot \ell}{a \cdot h^3} \quad \text{for laminar flow} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and

$$p_1^2 - p_2^2 = 0.133 \cdot \mu^{\frac{1}{4}} \cdot \frac{RT}{g} \cdot \frac{G_1^{7/4} \cdot \ell}{a^{7/4} \cdot h^3} \quad \text{for turbulent flow} \ldots\ldots\ldots\ldots (2)$$

If each hole is considered as a nozzle in which the velocity head at exit is lost, the relationship between the reservoir pressure p_o and the pressure just outside the hole, p_1 is

$$\frac{p_o}{p_1} = \left\{1 + \frac{\gamma - 1}{2} \cdot M^2\right\}^{\frac{\gamma}{\gamma-1}}$$

where M is the Mach number at exit from the hole and γ is the ratio of the specific heats. Since for the ideal condition considered M = 1.0,

$$\frac{p_o}{p_1} = \left\{\frac{\gamma + 1}{2}\right\}^{\frac{\gamma}{\gamma-1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Also, the mass flow, G_1, along the equivalent slot can be expressed as

$$G_1 = s \cdot \frac{(\gamma g R T_o)^{\frac{1}{2}}}{R T_o} \cdot p_o \cdot \frac{M}{\left\{1 + \frac{\gamma - 1}{2} M^2\right\}^{\frac{1}{2}\frac{(\gamma+1)}{(\gamma-1)}}}$$

where s is the equivalent inlet hole area and T_o is the air temperature in the reservoir. When $M = 1$, this reduces to

$$G_1 = s \cdot \frac{(\gamma g R T_o)^{\frac{1}{2}}}{R T_o} \cdot p_o \cdot \left\{\frac{\gamma + 1}{2}\right\}^{-\frac{1}{2}\frac{(\gamma+1)}{\gamma-1}} \quad \text{.....................(4)}$$

Now for the laminar case the relationship between p_1 and p_2 is that given by equation (1), which can be transformed by substituting the values of $\frac{p_o}{p_1}$ and G_1 above to give

$$p_o \left[1 - \left\{\frac{\gamma + 1}{2}\right\}^{\frac{2\gamma - 1}{\gamma - 1}} \cdot \left\{\frac{p_2}{p_o}\right\}^2\right] = 24 \cdot \left\{\frac{\gamma + 1}{2}\right\}^{\frac{1}{2}\frac{(3\gamma - 1)}{(\gamma - 1)}} \cdot \frac{\mu}{g} \cdot (\gamma g R T_o)^{\frac{1}{2}} \cdot s \cdot \frac{\ell}{a \cdot h^3}$$

since, because of the large rate of heat flow within the bearing shell, the air temperature in the slot, T, and that in the reservoir, T_o, are substantially equal. Hence

$$h^3 = \frac{24 \cdot \left\{\frac{\gamma + 1}{2}\right\}^{\frac{1}{2}\frac{(3\gamma - 1)}{(\gamma - 1)}} \cdot \frac{\mu}{g} \cdot (\gamma g R T_o)^{\frac{1}{2}} \cdot s \cdot \frac{\ell}{a}}{p_o \cdot \left[1 - \left\{\frac{\gamma + 1}{2}\right\}^{\frac{2\gamma}{\gamma - 1}} \cdot \left\{\frac{p_2}{p_o}\right\}^2\right]}$$

In the complete bearing the total inlet area is

$$S = n \cdot \left(\frac{\pi d^2}{4}\right) = n \cdot \frac{2s}{\text{number of rings of inlet holes}} ,$$

where n is the total number of inlet holes and d is the diameter of each. When one central ring of inlet holes is used the fluid flows away from each hole towards both ends of the bearing so that the equivalent hole area, s, is then half the actual hole area, i.e. $\frac{1}{2}\left(\frac{\pi d^2}{4}\right)$, and the width of the equivalent slot, a, is $\left(\frac{\pi D}{n}\right)$; whereas when two rings of holes are used the corresponding values are $\left(\frac{\pi d^2}{4}\right)$ and $2\left(\frac{\pi D}{n}\right)$. In either case

$$\frac{s}{a} = \frac{n \cdot \left(\frac{\pi d^2}{4}\right)}{2\pi D} = \frac{S}{2\pi D}$$

Hence, for the ideal conditions considered the choking diametral clearance is given as

$$c_o^3 = 8h^3 = \frac{30.5 \cdot \left\{\frac{\gamma + 1}{2}\right\}^{\frac{1}{2}\frac{(3\gamma - 1)}{(\gamma - 1)}} \cdot \frac{\mu}{g} \cdot (\gamma g R T_o)^{\frac{1}{2}} \cdot \ell \cdot S}{p_o \cdot \left[1 - \left\{\frac{\gamma + 1}{2}\right\}^{\frac{2\gamma}{\gamma - 1}} \cdot \left\{\frac{p_2}{p_o}\right\}^2\right] \cdot D} \quad \text{..........(5)}$$

For turbulent flow the corresponding expression is obtained by making the same substitutions in equation (2), which gives

$$c_o^3 = \frac{4.28 \times 10^{-2} \cdot \left\{\frac{\Upsilon+1}{2}\right\}^{\frac{1}{8}\cdot\left(\frac{9\Upsilon-7}{\Upsilon-1}\right)} \cdot \frac{\mu}{g}^{\frac{1}{4}} \cdot (\Upsilon g R T_o)^{7/8} \cdot \ell \cdot s^{7/4}}{p_o^{\frac{1}{4}} \cdot \left[1 - \left\{\frac{\Upsilon+1}{2}\right\}^{\frac{2\Upsilon}{\Upsilon-1}} \cdot \left\{\frac{p_2}{p_o}\right\}^2\right] \cdot (RT_o)^{3/4} \cdot D^{7/4}} \qquad \ldots(6)$$

When air at N.T.P. is the working fluid (T_o = 288°C abs., $\mu = 1.20 \times 10^{-5}$ lb/ft. sec., $\Upsilon = 1.40$) we have for laminar flow

$$c_o^3 = \frac{2.19 \times 10^{-3} \cdot \ell \cdot s}{p_o \cdot \left[1 - 3.59 \left\{\frac{p_2}{p_o}\right\}^2\right] \cdot D} \quad \text{in}^3 \qquad \ldots\ldots(7)$$

and for turbulent flow

$$c_o^3 = \frac{5.82 \times 10^{-3} \cdot \ell \cdot s^{7/4}}{p_o^{\frac{1}{4}} \cdot \left[1 - 3.59 \left\{\frac{p_2}{p_o}\right\}^2\right] \cdot D^{7/4}} \quad \text{in}^3 \qquad \ldots\ldots(8)$$

the units in both cases being those indicated in Appendix I.

Theoretically, when the clearance has the value c_o all the inlet holes are just choked, and according to section 2.0 the load capacity should be a maximum. The experimental results show that the optimum clearance is less than c_o, the discrepancy being due to the assumptions of concentricity and axial flow made in the theory.

3.2 The Analysis of the Results

A series of experiments was performed with a 2" diameter bearing of 3" length (Ref. 2), and some of the results are shown in Fig. 4 and Fig. 5 expressed in terms of the non-dimensional load coefficient $\frac{W_{max}}{A'.(p_o - p_1)}$ and plotted against the difference between the supply and the exhaust pressure. The effective area, A', is given by

$$A' = D. \left\{(L - 2\ell) + \frac{4\ell}{3}\right\} = D. \left\{L - \frac{2\ell}{3}\right\} \qquad \ldots\ldots(9)$$

where L is the total bearing length and ℓ the distance from the inlet holes to the end. Its use is based on the fact that the axial pressure distribution is approximately parabolic from the rings of inlet holes to the end and is approximately uniform between them, this being confirmed by the good correlation which is obtained when the experimental values of load coefficient for two different positions of the inlet holes are derived with the aid of A' as a parameter. The load coefficient is effectively a measure of the bearing load capacity.

Apparently, therefore equation (7) is valid in form and so should provide a means of correlating the experimental results. This is done in Fig. 6 by plotting the load coefficient against the clearance ratio $\frac{c}{c_o}$, where c is the actual clearance and c_o the theoretical choking clearance obtained by substituting the appropriate values of p_o, ℓ, and S in equation 7. The chosen values of p_o were 54.7, 74.7 and 94.7 lbs/in^{2}abs., and the values of $\frac{W_{max}}{A'.(p_o-p_2)}$ were read from the curves in Fig. 4 and Fig. 5. The results for the four cases are in approximate agreement when plotted in this way and a mean curve is drawn. The maximum value of 0.29 for the load coefficient, $\frac{W_{max}}{A'.(p_o-p_2)}$, then corresponds to a clearance ratio of $\frac{c}{c_o} = 0.86$.

Since all these results were obtained with bearings of the same overall dimensions, it is interesting to compare them with the results for a larger bearing expressed in the same non-dimensional form. At Messrs. Gilkes and Gordon a 3.5" diameter bearing has been made with unconnected inlet holes (Appendix II), and some test results were made available to the N.G.T.E. These are represented in Fig. 7 as a point which lies very close to the mean curve obtained with the 2" diameter bearings and so indicates that the overall size of the bearing apparently has little effect on its performance. Nevertheless other factors may have an effect, the most important of these being the ratio $\frac{L}{D}$ and the type of inlet hole. The value of $\frac{L}{D}$ is significant in that, above a certain value between 1.5 and 2.4, the deterioration of performance due to the circumferential components of flow becomes appreciable. This is illustrated by the case of an overlong bearing - the static rig (Ref. 2) - which is also represented in Fig. 7.

In Appendix II some details of bearings with connecting grooves are also given, and the results are denoted in Fig. 7 by three points joined by a broken curve. The improved performance is probably partly due to the more even distribution of air in these cases, and a similar result might well be obtained with unconnected inlet holes if a sufficiently larger number were used. The value for $\frac{W_{max}}{A'.(p_o-p_2)}$ of 0.29 which is applied in the subsequent calculation of bearing performance is not, therefore, the ultimate attainable, values as high as 0.42 being apparently possible with correctly designed connecting grooves.

In estimating performance the air consumption is obviously important, and Fig. 8 shows typical curves of the mass flow coefficient, $\frac{G}{G_o}$, plotted against (p_o-p_2) where G is the total mass flow through the bearing and G_o is the theoretical choking mass flow obtained by multiplying G_1 in equation (4) by the number of equivalent holes. The shape of the curve is dependent upon the load, and for the purpose of the analysis light loads are assumed. In correlating the various curves the same parameter, $\frac{c}{c_o}$, is again used, and Fig. 9 shows the results. The same specific values of p_o were chosen, namely 54.7, 74.7, and 94.7 lb/in^{2}abs., the values of c_o being calculated from equation (7) as before and the corresponding values of $\frac{G}{G_o}$ read off graphs similar to those of Fig. 8. The degree of correlation is not so good as that obtained when comparing values of the load coefficient, but is sufficient to justify the drawing of a mean curve. This curve is analogous to the normal mass flow against pressure curve for an orifice, and shows the choking effect with values of $\frac{c}{c_o}$ greater than unity. When $\frac{c}{c_o} = 0.86$, corresponding to the maximum load capacity, $\frac{G}{G_o}$ is 0.60 as compared with the maximum value of approximately 0.7, the latter being in effect a discharge coefficient.

3.3 The Effects of Rotation

The previous section has been devoted to experimental results obtained with static bearings, that is to say with no relative movement between the shaft and the shell, but attempts have of course been made to determine the effects of rotation. At the N.G.T.E. a 2" diameter bearing was run at speeds up to 12,000 R.P.M., equivalent to a peripheral speed of more than 100 ft./sec., and no change in the pressure distribution or mass flow was detected. As has been shown (Fig. 8) a change in eccentricity (caused in that case by a change of load) produces a change in mass flow, the supply pressure being constant. Hence, the constancy of the mass flow in these tests indicates that the eccentricity and therefore the load capacity is unaffected by relative motion at least up to 100 ft. /sec. However, if the peripheral speed of the rotor is of the same order as the axial velocity of the air (about 500 ft./sec. in the N.G.T.E. experiments) some change in performance might be expected. Nevertheless, the effect cannot be very serious, since at Messrs. Gilkes and Gordon a 2" diameter bearing was run at 60,000 R.P.M., i.e. at a peripheral speed of 524 ft./sec. The empirical expressions describing the performance of static bearings can therefore be applied in the case of rotating bearings, certainly when the peripheral speed is less than 100 ft./sec. and probably at much higher values also.

On several occasions air lubricated journal bearings have failed while running under load. The two chief causes of failure appear to have been

(a) out of balance of the rotating parts

(b) the introduction with the air of solid matter, such as dust or rust.

At the N.G.T.E. a single bearing was used to support a small overhung turbine rotor. There was considerable out of balance in the turbine disc, and at 12,000 R.P.M. failure occurred owing to the excessive asymmetrical dynamic load on the bearing. This weakness to asymmetrical load will in general necessitate the use of the bearings in pairs, so that good alignment will be essential. The effect of rapidly fluctuating loads and impulsive forces have not been fully investigated, and no quantitative information regarding the limiting values of out of balance forces is available. However, experience with air lubricated journal bearings indicates that failure is often caused by out of balance centrifugal forces, but it is not known whether this is due to the load exceeding instantaneously the static load capacity or whether the bearing is inherently sensitive to rapidly fluctuating or impulsive loads. With steady radial loads, however, the bearing behaves perfectly and no form of instability has been experienced.

The other cause of failure - the introduction of solid matter - can be prevented by the installation of an air cleaner or filter, which is an essential ancillary to any air lubricated bearing system. Another similar danger is the chemical deterioration of the bearing surfaces, and in most applications the use of a stainless material would be essential.

4.0 Performance Estimation

The values of the mass flow and load coefficients based on the experimental results are non-dimensional and can be used as a means of predicting the performance of any size of bearing. Further, since the generalised curves show these values as functions of $\frac{e}{c_o}$, they are independent of the type of flow and, although obtained from data relating to bearings with laminar flow only, may also reasonably be used in determining the performance of bearings with turbulent flow. This statement may be clarified if we examine again the general principle of the bearing. Since the axial pressure distribution is approximately parabolic both for laminar and for turbulent flow, the load coefficient $\frac{W_{max}}{A' \cdot (p_o - p_2)}$ is dependent only on the peripheral pressure distribution just outside the inlet holes. Now if the pressure near a given hole is denoted by p_1, equations (1) and (2) give

$$p_1^2 - p_2^2 \;\alpha\; \frac{G_1 \cdot \ell}{d \cdot h^3} \qquad \text{for laminar flow,}$$

and

$$p_1^2 - p_2^2 \;\alpha\; \frac{G_1^{7/4} \cdot \ell}{d^{7/4} \cdot h^3} \qquad \text{for turbulent flow,}$$

where in this case h is the local radial clearance at any point in an eccentric bearing.

Hence, if we assume that G_1 is proportional to s and to p_o and substitute $\frac{S}{D}$ in place of $\frac{s}{d}$,

$$\left\{\frac{p_1}{p_o}\right\}^2 - \left\{\frac{p_2}{p_o}\right\}^2 \;\alpha\; \frac{S \cdot \ell}{p_o \cdot D \cdot h^3} \quad \text{i.e., from equation (7), } \alpha\,\frac{c_o^3}{h^3}$$

for laminar flow,

and $$\left\{\frac{p_1}{p_o}\right\}^2 - \left\{\frac{p_2}{p_o}\right\}^2 \;\alpha\; \frac{S^{7/4} \cdot \ell}{p_o^{\frac{1}{4}} \cdot D^{7/4} \cdot h^3} \quad \text{i.e., from equation (8), } \alpha\,\frac{c_o^3}{h^3}$$

for turbulent flow.

The local value of p_1, is therefore theoretically a function only of p_o, p_2 and $\frac{h}{c_o}$, which implies that the peripheral pressure distribution is dependent only on p_o, p_2, $\frac{c}{c_o}$ and the eccentricity of the bearing. This indicates that the relationship between the load coefficient, which corresponds always to an eccentricity of unity, and the clearance ratio is independent of the type of flow prevailing.

The assumption that G_1 is directly proportional to $p_o \cdot S$ and independent of h is not correct for all values of $\frac{c}{c_o}$, the variation of $\frac{G}{G_o}$ being evidence of this. With laminar flow $\frac{G}{G_o}$ varies very little when the values of $\frac{c}{c_o}$ are between 0.8 and 1.0, because then $\frac{p_1}{p_o}$ is less than 0.528 over a large part of the periphery and most of the holes are choked. Similar reasoning applies in the turbulent case, and if $\frac{p_1}{p_o}$, which is again a function of $\frac{c}{c_o}$, is less than this value the holes will be choked. The value of $\frac{G}{G_o}$ will then depend only on the coefficient of discharge of the holes, which is independent of the type of flow between the bearing surfaces. Hence, for values of $\frac{c}{c_o}$ near the design value of 0.86, the values of $\frac{G}{G_o}$ will probably be approximately the same for both turbulent and laminar flow, and the reasoning of the previous paragraph is valid. However, with values of $\frac{c}{c_o}$ much smaller than 0.86 some of the holes will be unchoked and the corresponding values of $\frac{G}{G_o}$ may be different in the two cases. For this reason the prediction of the performance of bearings with turbulent flow is restricted to cases operating at or near the design point.

4.1 Performance at the Design Point

In bearing design the ratio, $\frac{\text{load supported}}{\text{compressor power}}$, is the criterion and for most purposes its value should be a maximum. However, the shape of the curves of $\frac{W_{max}}{A' \cdot (p_o - p_2)}$ and $\frac{G}{G_o}$ against $\frac{c}{c_o}$ are such that this condition is approximately satisfied when the load coefficient is also a maximum. Hence, the corresponding values $\frac{W_{max}}{A' \cdot (p_o - p_2)} = 0.29$, $\frac{G}{G_o} = 0.60$, and $\frac{c}{c_o} = 0.86$ are used as the basis of the design and of the performance estimation.

The first stage of the design procedure (Appendix III) is the calculation of the necessary overall dimensions from the equation

$$W_{max} = 0.29\, D^2 \cdot \left\{ \frac{L}{D} - \frac{2}{3} \cdot \frac{\ell}{D} \right\} \cdot (p_o - p_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

on the assumption of values for the safety factor $\frac{W_{max}}{W}$ and the ratios $\frac{L}{D}$ and $\frac{\ell}{D}$. Of these $\frac{W_{max}}{W}$ will be determined by the consideration of any instantaneous overload which might occur and of the possible variations of the supply pressure; the value of $\frac{L}{D}$ must be less than 2.0 owing to the deterioration in performance caused by the circumferential components of flow in long bearings but is otherwise apparently arbitrary; and $\frac{\ell}{L}$ for economic operation should be 0.5, as is shown below. Having fixed the values of D, L, and ℓ, we must then decide upon the clearance. The value chosen will be the smallest practicable, and the limits will usually be set by the manufacturing process and by consideration of possible thermal or centrifugal expansions. The necessary total inlet area corresponding to the chosen dimensions is given by

$$S = \frac{718 \cdot c^3 \cdot D \cdot p_o \cdot \left[1 - 3.59 \left\{ \frac{p_2}{p_o} \right\}^2 \right]}{\ell} \text{ in}^2 \quad \ldots\ldots\ldots\ldots\ldots(11)$$

for laminar flow, and

$$S = \frac{24.6 \cdot c^{12/7} \cdot D \cdot p_o^{1/7} \cdot \left[1 - 3.59 \left\{ \frac{p_2}{p_o} \right\}^2 \right]^{4/7}}{\ell^{4/7}} \text{ in}^2 \quad \ldots(12)$$

for turbulent flow (see Appendix III).

The hole spacing has an appreciable effect on load capacity, since local, and therefore useless, pressure losses occur near the holes because of the high velocity of the radial flow from them. As the hole size is reduced relative to the clearance so the local velocity decreases and the performance improves. This explains the 25% increase in load capacity obtained with connecting grooves by which the radial flow is completely eliminated. The ideal arrangement, therefore, is probably a very large number of very small holes.

and, substituting in this equation the appropriate values of S, we have

$$P = \frac{1768}{\eta_c} \cdot \frac{D \cdot c^3 \cdot p_o^2}{\ell} \cdot \left[1 - 3.59 \left\{\frac{p_2}{p_o}\right\}^2\right] \cdot \left[\left\{\frac{p_o}{p_2}\right\}^{0.286} - 1\right] \text{H.P.} \quad ..(14)$$

for a bearing designed for, and operating under, laminar conditions, and

$$P = \frac{60.6}{\eta_c} \cdot \frac{D \cdot c^{12/7} \cdot p_o^{8/7}}{\ell^{4/7}} \cdot \left[1 - 3.59 \left\{\frac{p_2}{p_o}\right\}^2\right]^{4/7} \cdot \left[\left\{\frac{p_o}{p_2}\right\}^{0.286} - 1\right] \text{H.P.} \quad ..(15)$$

for a bearing designed for, and operating under, turbulent conditions. P is the power absorbed by the compressor supplying the bearing and η_c its efficiency.

The ratio $\frac{W}{P}$ is effectively a measure of the bearing efficiency, and a comparison of equations (10), (14), and (15) shows that

$$\frac{W}{P} \propto \frac{\ell}{L} \cdot \left\{1 - \frac{2}{3}\,\frac{\ell}{L}\right\} \qquad \text{for laminar flow} \quad ....(16)$$

and

$$\frac{W}{P} \propto \left\{\frac{\ell}{L}\right\}^{4/7} \cdot \left\{1 - \frac{2}{3}\,\frac{\ell}{L}\right\} \qquad \text{for turbulent flow} \quad ....(17)$$

The greatest possible value of $\frac{W}{P}$ will therefore correspond to $\frac{\ell}{L} = 0.5$, i.e. one centrally disposed ring of inlet holes, in both cases.

The type of flow between the bearing surfaces is governed by the value of the Reynolds number, R_e, appertaining. In Appendix IV it is shown that the mean value for a bearing is given by

$$R_e = \frac{12 \cdot G}{\pi \cdot D \cdot \mu} \quad ....(18)$$

Experiments (Ref. 1) have shown that laminar flow breaks down when $R_e \simeq 2040$, that there is then a range of transition, and that fully turbulent flow is finally established when $R_e \simeq 3810$.

By substituting the value of G from equation (13) and the values of S from equations (11) and (12), we obtain the limiting conditions. These are

$$\frac{c^3 \cdot p_o^2 \left[1 - 3.59 \left\{\frac{p_2}{p_o}\right\}^2\right]}{\ell} \ngtr 0.635 \times 10^{-3} \quad ....(19)$$

for laminar flow in a bearing designed for laminar flow, and

$$\frac{c^3 \cdot p_o^2 \left[1 - 3.59 \left\{\frac{p_2}{p_o}\right\}^2\right]}{\ell} \nless 2.776 \times 10^{-3} \quad ....(20)$$

for turbulent flow in a bearing designed for turbulent flow.

These limits are not exact, since they do not take into account the eccentricity of the bearing. In actual bearings (Ref. 2) the flow lines from each hole diverge where the clearance is small and converge where it is large, so that although a bearing may be operating under generally laminar conditions the flow where the clearance is maximum may be turbulent, and similarly under generally turbulent conditions it may be laminar where the clearance is minimum. In either case the peripheral pressure distribution would be adversely affected and the load capacity reduced. Hence, it would be unreasonable to design for conditions proximate to the limiting ones, and the effective gap between the laminar and the turbulent design ranges is probably considerably greater than that indicated by the above values.

The theoretical limits to the design ranges are illustrated in Fig. 10, which shows the effect of clearance upon power consumption. It is clear that c is a major determinent of P, and that its value must be kept as small as possible if the bearing is to be economical. Unfortunately inaccuracies of manufacture may be appreciable with values of $\frac{c}{D}$ less than 0.5×10^{-3}, and for this and other reasons discussed below in connection with thermal and centrifugal expansion it is unlikely that the ideally small clearances would be practicable.

4.2 Performance away from the Design Point

In the previous section the bearing design was discussed and the performance estimated under design conditions. Now let us consider the operation of a bearing away from the design point, that is to say with a value of $\frac{c}{c_o}$ not equal to 0.86. The expressions for the theoretical choking clearance, equations (7) and (8), contain the bearing dimensions and the terms p_o and p_2. Of these, the former are fixed according to the previous calculations, while the latter are variables. Hence, if the supply pressure to a bearing is changed, the values of $\frac{c}{c_o}$ will change also and the load coefficient will be affected. Obviously, too, any absolute change in c will have an appreciable effect on the performance. The change of $\frac{c}{c_o}$ due to an alteration of the supply pressure is not so serious in the case of turbulent flow as in the case of laminar, since c_o is approximately proportional to $p_o^{1/3}$ in the laminar and to $p_o^{1/12}$ in the turbulent case. In either case the change in load coefficient is only appreciable when the change in pressure is quite large or when a change in the type of flow is involved, from laminar to turbulent for instance.

Some of these effects are shown in Fig. 11 which represents the case of a 5" square bearing designed for laminar flow at a supply pressure of 138.2 lb/in^2 absolute. As the pressure increases $\frac{W_{max}}{A' \cdot (p_o - p_2)}$ decreases only very slowly owing to the form of the relationship between p_o and c_o and the flatness of the curve of $\frac{W_{max}}{A' \cdot (p_o - p_2)}$ against $\frac{c}{c_o}$. When the critical value of p_o is reached the transition from laminar to turbulent flow commences and the theoretical choking clearance is no longer that calculated from equation (7), but is replaced eventually by that from equation (8). Its value is greater and so $\frac{c}{c_o}$ is reduced to what is almost the design value. Here we have a case in which turbulence causes an increase in load capacity. That $\frac{c}{c_o}$ is hardly affected by p_o in the turbulent range is clearly illustrated by the flatness of the curve of $\frac{W_{max}}{A' \cdot (p_o - p_2)}$ at pressures above the transition values.

The two possible causes of change in the clearance c are relative thermal expansion due to temperature difference between the shaft and the shell and enlargement of the shaft diameter due to centrifugal strain. The shape of the load coefficient-clearance ratio curve (Fig. 6) is such that, while a fairly large increase in clearance is allowable, a small decrease may produce a sufficient reduction in load capacity to cause failure. Quantitatively expressed, a reduction in $\frac{W_{max}}{A' \cdot (p_o - p_2)}$ of 22% is produced either by an increase in $\frac{c}{c_o}$ of 86% or a decrease of 23%. A further decrease in $\frac{c}{c_o}$ reduces the load coefficient rapidly to zero.

In Appendix V the phenomenon of thermal expansion is considered. It is shown that the maximum allowable temperature difference between the shaft and the shell is proportional to $\frac{c}{D}$, and that for a steel shaft with $\frac{c}{D} = 0.5 \times 10^{-3}$ the safe temperature of the shell may exceed that of the shaft by 72°F whereas the temperature of the shaft may exceed that of the shell by only 17°F. Hence, in applications to machinery with temperature gradients it would be essential to design the bearing so that the shell would always be at the higher temperature, although this might not be easy since the shell will be cooled by the passage through it of the lubricating air. The chief danger is that from such temperature fluctuations as might be due to a change of load in a gas turbine; an increase in the temperature of the turbine disc might affect the bearing shaft temperature considerably while hardly influencing that of the shell. The unfortunate fact is that safety in this respect can only be obtained at the expense of power consumption, and whereas the maximum safe temperature difference is proportional to c the compressor power involved is proportional to c^3 so that the cost is very great.

The effect of centrifugal expansion (Appendix V) is somewhat similar except that the clearance is always reduced by rotation of the shaft. Allowing a 20% reduction in clearance the maximum safe peripheral speed, which is proportional to $(\frac{c}{D})^{\frac{1}{2}}$, is 386 ft./sec. for a steel shaft with a value of $\frac{c}{D} = 0.5 \times 10^{3}$. There is, therefore, a top limit to the rotational speed of any bearing above which the load capacity is seriously reduced owing to the reduction in clearance. This effect, which is more serious with large diameters, can be compensated for by designing for a larger static clearance; either so that the optimum clearance occurs at the design speed or so that the changes of clearance are relatively insignificant. Designing for a specific speed has the disadvantage that it entails an increased pressure supply to maintain the optimum value of $\frac{c}{c_o}$ while running up to speed. With turbulent flow this form of compensation may not be practicable owing to the fact that p_o appears in equation (6) to the power $\frac{1}{4}$. These facts must be considered when deciding on the value of the clearance and considerably affect the value of the compressor power associated with the bearing.

4.3 The Compressor Power Consumption

In an air lubricated journal bearing the concomitant of a large load is either a higher supply pressure or a large diameter. The choice depends upon the space available, the limiting peripheral speed, and on the desired economy of power consumption. In Fig. 12 bearings with a clearance to diameter ratio of $\frac{c}{D} = 0.5 \times 10^{-3}$ are considered without reference to the limiting peripheral speed; the obvious conclusion is that the larger the diameter the more economical the bearing. However, quite apart from the fact that the diameters involved are in themselves excessive, the peripheral speeds at normal rotational speeds would involve appreciable centrifugal expansion of the shaft which would, in fact, necessitate much larger clearances than those corresponding to $\frac{c}{D} = 0.5 \times 10^{-3}$. As was noted in the previous section there are two alternative ways of compensating for the centrifugal expansion, the first entailing a high pressure supply for running at other than design speed and the second involving large compressor power losses, which in most cases would preclude its application. With turbulence the value of c_o is hardly affected by a change in p_o, and the second method may be the only one available. Hence, with turbulent flow the peripheral speed would be of special importance, and large diameter bearings would be impracticable at high speeds.

An example of the effect of the peripheral speed limitation is illustrated by the curves of power loss (Fig. 13) for laminar and turbulent bearings designed to run at 3,000 R.P.M., 9,000 R.P.M. and 18,000 R.P.M. At 3,000 R.P.M. the maximum diameter corresponding to the limiting peripheral speed is 29.4", and with a supply pressure of 100 lb/in^2 no compensation or reduction in diameter would be necessary. At 9,000 R.P.M. the maximum diameter is 8.90", and at 18,000 R.P.M. it is 4.45". These smaller diameters would necessitate higher values of p_o and would result in increased values of the compressor power as shown.

An improved efficiency, that is to say a smaller power consumption for the same load, could be obtained by designing the bearings with one central ring of inlet holes. It can be calculated from equations (16) and (17) that the power consumption per unit of load is then 0.625 times that shown in Fig. 13 for the laminar range and 0.844 times that in turbulent range. Further, it can be shown from equations (19) and (10) that the pressure corresponding to the critical Reynolds number is increased in the ratio 1.414 and that the load range for laminar flow is correspondingly increased by 13%. However, even with the maximum efficiency the compressor power loss would be higher than the frictional loss in a corresponding oil journal bearing, which is represented by the broken lines in Fig. 13. In the turbulent region the discrepancy would be considerable, the loss in the air lubricated bearing being more than 80% greater than that in the oil journal bearing; in the laminar region the difference would be approximately 20%.

With special care in the manufacture smaller values of $\frac{c}{D}$ might be used, but the limiting peripheral speed would then be less, the maximum allowable diameter smaller, and the power loss only slightly reduced. The only cases outside this vicious circle are the laminar ones, in which the clearance can be designed for a specific speed and extra power supplied while running up to speed. However, the range of this latter type of bearing could only be extended to cover loads up to 5,000 lb. and then only if very low pressures were used. It appears, therefore, that for large loads turbulent flow bearings would compare unfavourably with conventional oil bearings, and that economic laminar flow bearings could only be constructed with very large diameter and low supply pressures. The latter would entail an additional mechanism for linking the pressures of the supply with the rotational speed.

4.4 The Frictional Power Loss

When two surfaces separated by a fluid film are in relative motion the shearing of the fluid will produce a resisting force which is proportional to the viscosity. While the viscosity of lubricating oil varies from 1.04×10^{-4} lb.sec./ft.^{2} (5 centipoises) to 4.17×10^{-3} lb.sec./ft.^{2} (200 centipoises), the viscosity of air at room temperature is only 3.72×10^{-7} lb.sec./ft.^{2}., and the resisting torque in an oil lubricated journal bearing is therefore at least 280 times that in a geometrically similar air lubricated journal bearing. If two bearings supporting the same load are considered, the ratio of the frictional loss is less than 280 owing to the larger diameter of the air bearing but is nevertheless quite large.

In Appendix VI the formula for the frictional loss in an air lubricated bearing is derived and compared with an empirical formula due to Linn and Irons (Ref. 4) for the losses in high speed oil journal bearings. Values of the power consumption calculated from these formulae are plotted against R.P.M. in Fig. 14 for bearings supporting a load of 200 lb. When subjected to a light load the diameter of the air lubricated bearing is small and high rotational speeds can be attained without the necessity of large clearances to take up the expansion of the rotor. In this case the compressor power is not excessive, and the total power loss at high speeds is less than the frictional loss in the corresponding oil lubricated bearing. With larger loads the necessary compressor power is considerably greater, and if similar curves of loss against speed are drawn for oil and air lubrication they will cross at a higher value of the rotational speed. However, an important fact is that in all cases the frictional resisting torque and the power loss at the shaft are much less in the air lubricated bearing than in the oil.

5.0 Some Possible Applications

When considered in direct comparison with the standard bearings as used today, the air lubricated bearing has certain disadvantages as regards robustness and power consumption. The conventional types of bearing will sustain for short periods loads in excess of their design values, whereas in the air lubricated bearing a complete air film is essential and metallic contact during rotation would soon cause scoring, overheating, and complete seizure. Owing to the fact that there is no adsorbed lubricant as in an oil journal bearing, this process of failure would occur almost instantaneously and the effects, unless special precautions were taken, would be serious. For this reason an air lubricated bearing cannot support a load in excess of its design capacity even for an instant, and should not be subjected to fluctuating forces which might supplement the steady load and produce a total load exceeding this value. Vibration, either external or due to out of balance of the rotor, has apparently been the chief cause of failure to date. Hence, the scope of application of air lubrication is at present limited to cases where the fluctuating loads are either small or can be accurately predetermined. However, although in their present state air lubricated bearings are not suitable as a general substitute for the more robust rolling bearings or oil lubricated journal bearings, there are several particular fields of application which are worth consideration.

One form of machine in which the fluctuating loads are small or calculable is the stationary industrial steam or gas turbine unit. Whether or not air lubrication in these cases is feasible depends chiefly upon the compressor power required as compared with the frictional losses in the conventional type of bearing. For light rotors reliable rolling bearings have been developed which satisfy the general design requirements up to quite high speeds, and since the frictional loss with this type of bearing is comparatively low the air lubricated bearing could not compete economically. However, there are cases in which oil lubricated journal bearings with a high frictional loss have to be used, particularly when the weight of the rotor is too great to be supported by rolling bearings. It is in these cases that the application of air lubrication first seemed feasible, but as has been shown even here the compressor power requirement is greater than the possible gain due to the reduction of the frictional loss, and the general application of air lubrication in this field is not considered practicable.

There are however, special cases in which power consumption is not the most important factor, and in which conventional bearings are inherently unsuitable. It is conceivable that such a situation may occur in manufacturing processes involving bearings running in a gaseous atmosphere which would be contaminated by contact with oil. In these cases the bearing could be lubricated by the gas itself, the mechanical design being considerably simplified by the elimination of the necessity for elaborate sealing devices.

One of the unique properties of the air lubricated bearing is the relative lack of friction, and this suggests applications to test rigs and apparatus for making very accurate measurements of torque or power consumption. The frictional loss in conventional bearings at high speeds is considerable, and corrections have always to be made when measuring power. These are rarely very accurate owing to uncertainty of the existing formulae and the difficulty of determining, in the case of an oil journal bearing, the oil film temperature. With air lubricated bearings on the other hand the frictional loss is so small that the overall effect of a small percentage error in its calculation would be negligible. Clearly, the value of air lubrication in this type of test rig is greatest when the total power being measured is small and the losses in conventional bearings of the same order. This same property of low friction would enable high speeds to be attained with a small driving torque, and in the case of small lightly loaded bearings would make air lubrication the most economical form at very high speeds. This is again of considerable advantage in experimental work where high speeds are desirable but where bulky high power driving units would be inconvenient.

Finally, there are several possible applications for an almost frictionless bearing in instruments and in some of the cases, as in the measurement of the out of balance of rotors, where knife edges are now used.

6.0 Conclusions

Experiments have shown that journal bearings can be constructed which, when supplied with air under pressure, will support radial loads, and that the functioning of these bearings is dependent upon the choking action of their inlet holes. Analysis of the available experimental data reveals that the maximum load which a bearing will support is a function of the supply pressure, its dimensions, and the ratio of the actual clearance to the theoretical choking clearance, c_o. The value of the latter is derived from viscid flow theory and is given by the equations,

$$c_o^3 = \frac{30.5 \cdot \left\{\frac{\gamma+1}{2}\right\}^{\frac{1}{2}\frac{(3\gamma-1)}{(\gamma-1)}} \cdot \frac{\mu}{g} \cdot (\gamma gRT)^{\frac{1}{2}} \cdot \ell \cdot S}{p_o \left[1 - \left\{\frac{\gamma+1}{2}\right\}^{\frac{2\gamma}{\gamma-1}} \cdot \left\{\frac{p_2}{p_o}\right\}^2\right] \cdot D} \quad \text{for laminar flow}$$

and

$$c_o^3 = \frac{4.28 \times 10^{-2} \cdot \left\{\frac{\gamma+1}{2}\right\}^{\frac{1}{8}\frac{(9\gamma-7)}{(\gamma-1)}} \cdot \frac{\mu}{g} \cdot (\gamma gRT_o)^{7/8} \cdot \ell \cdot S^{7/4}}{p_o^{\frac{1}{4}} \cdot \left[1 - \left\{\frac{\gamma+1}{2}\right\}^{\frac{2\gamma}{\gamma-1}} \left\{\frac{p_2}{p_o}\right\}^2\right] \cdot (RT_o)^{3/4} \cdot D^{7/4}}$$

for turbulent flow. It is found that the maximum value of the load coefficient is given by

$$\frac{W_{max}}{A' \cdot (p_o - p_2)} = 0.29,$$

that this occurs when $\frac{c}{c_o} = 0.86$, and that the corresponding value of the mass flow coefficient is

$$\frac{G}{G_o} = 0.60,$$

where the symbols have the meaning defined in Appendix I.

The effects of rotation on load capacity and upon mass flow are apparently negligible, certainly at peripheral speeds up to 100 ft./sec. and probably at greater speeds also.

Failure of experimental bearings have occurred because of asymmetrical loading, excessive out of balance of the rotating parts, and the introduction of particulate matter with the air. In application the first would not occur because the bearings would be used in pairs, and the last would be prevented by use of an air filter or other cleaning device. The centrifugal forces appear to be potentially troublesome, and further mechanical tests to determine quantitatively the effects of rapidly fluctuating and impulsive loads are essential before the bearing can be generally applied.

Under steady loads the air lubricated bearing has proved reliable and no form of inherent instability has been detected but the field of application is limited at present by the loading conditions and the power consumption. The latter, as calculated from empirical formulae which are in effect a means of extrapolating from the existing experimental data, is high when compared with the frictional losses in the equivalent conventional bearings. For this reason the application of air lubrication to gas turbine power plant is not considered feasible when the standard oil journal or rolling bearings would suffice. However, in special cases when an alternative lubricant to oil is required, air lubricated bearings would probably fill the role satisfactorily.

Finally, the fact that the bearings are almost frictionless suggests their application in experimental rigs where accurate measurements of torque and power are required or where small rotors are to be run at high speeds with only a small driving torque.

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APPENDIX I

Nomenclature

List of Symbols Used			Units

Equivalent slot dimensions:-

ℓ = length of slot .... in.

a = width of slot .... in.

h = depth of slot .... in.

Bearing dimensions:-

L = length of the bearing .... in.

D = diameter of the bearing .... in.

ℓ = distance from the inlet holes to the end .... in.

A' = effective bearing area.. in^2

$$= D. \left\{(L - 2\ell) + \frac{4\ell}{3}\right\} = D. \left\{L - \frac{2\ell}{3}\right\}$$

d = diameter of the inlet holes .... in.

s = effective area of one inlet hole .... in^2

$$= \frac{\pi d^2}{4} \cdot \frac{\text{number of inlet rings}}{2}$$

n = total number of inlet holes.

S = total inlet area .... in^2

$$= n \cdot \frac{\pi d^2}{4}$$

c = diametral clearance .... in.

c_o = theoretical choking clearance .... in.

$$= \left(\frac{2.54 \cdot \left\{\frac{\gamma+1}{2}\right\}^{\frac{1}{2}\frac{(3\gamma-1)}{(\gamma-1)}} \cdot \frac{\mu}{g} \cdot (\gamma g R T_o)^{\frac{1}{2}} \cdot \ell \cdot S}{p_o \left[1 - \left\{\frac{\gamma+1}{2}\right\}^{\frac{2\gamma}{\gamma-1}} \cdot \left\{\frac{p2}{p_o}\right\}^2\right] \cdot D} \right)^{\frac{1}{3}} \text{..... in.}$$

for laminar flow

$$= \left(\frac{2.30 \times 10^{-2} \cdot \left\{\frac{\gamma+1}{2}\right\}^{\frac{1}{6}\frac{(9\gamma-7)}{(\gamma-1)}} \cdot \frac{\mu}{g}^{\frac{1}{4}} \cdot (\gamma g R T_o)^{7/8} \cdot \ell \cdot S^{7/4}}{p_o^{\frac{1}{4}} \cdot \left[1 - \left\{\frac{\gamma+1}{2}\right\}^{\frac{2\gamma}{\gamma-1}} \cdot \left\{\frac{p2}{p_o}\right\}^2\right] \cdot (RT_o)^{3/4} \cdot D^{7/4}} \right)^{\frac{1}{3}} \text{... in.}$$

for turbulent flow.

Gas conditions.-

p = absolute pressure .... lb./in^2

ρ = gas density .... lb./ft^3

T = absolute temperature .... $^\circ$C abs.

μ = absolute viscosity .... lb./ft.sec.

ν = kinematic viscosity .... ft.^{2}/sec.

γ = ratio of specific heats.

R = gas constant .... ft./$^\circ$C.

M = Mach number.

R_e = Reynolds number.

p_o = supply pressure .... lb/in^2 abs.

p_2 = exhaust pressure .... lb/in^2 abs.

T_o = temperature of the supply .... $^\circ$K.

Bearing performance:-

W = bearing load .... lb.

W_{max} = maximum bearing load .... lb.

G_1 = mass flow from an equivalent hole .... lb./sec.

G = total mass flow .... lb./sec.

G_o = total choking mass flow .... lb./sec.

$$= S \cdot \frac{(\gamma g R T_o)^{\frac{1}{2}}}{R T_o} \cdot p_o \cdot \left\{\frac{\gamma+1}{2}\right\}^{-\frac{1}{2}\frac{(\gamma+1)}{(\gamma-1)}} \text{ .... lb./sec.}$$

P = compressor power consumption .... H.P.

η_c = compressor efficiency.

F = frictional power loss .... H.P.

N = rotational speed .... R.P.M.

V_s = peripheral speed .... ft./sec.

Non-dimensional parameters:-

$\frac{c}{c_o}$ = clearance ratio.

$\frac{W_{max}}{A'(p_o - p_2)}$ = load coefficient.

$\frac{G}{G_o}$ = mass flow coefficient.

APPENDIX II

Further Experimental Data

Establishment	R.A.E.	R.A.E.	N.G.T.E.	Gilkes & Gordon	N.G.T.E. (Static Rig)
Details of admission.	*C.G.	*C.G.	*C.G.		
Number of rings of inlet holes.	2	2	2	1	2
Bearing diameter, D..in.	4.0	4.0	2.0	3.5	4.0
Bearing length, L....in.	2.5	2.5	3.0	1.375	9.7
Distance from inlet to end, ℓ............in.	0.625	0.625	0.75	0.688	4.85
Effective bearing area $A' = D.\left\{\frac{4}{3}\ell + (L-2\ell)\right\}$ in.^{2}	8.33	8.33	5.00	3.21	25.9
Diametral clearance, c .in.	3.0×10^{-3}	2.0×10^{-3}	3.5×10^{-3}	2.0×10^{-3}	6.0×10^{-3}
Total number of inlet holes, n.	24	24	16	6	24
Diameter of inlet holes, d................in.	20×10^{-3}	20×10^{-3}	16×10^{-3}	29×10^{-3}	20×10^{-3}
Equivalent hole area, **s.... ....in.^{2}	3.14×10^{-4}	3.14×10^{-4}	2.02×10^{-4}	3.31×10^{-4}	3.14×10^{-4}
Total inlet area, S...in.^{2}	7.55×10^{-3}	7.55×10^{-3}	3.23×10^{-3}	3.97×10^{-3}	7.55×10^{-3}
Theoretical choking clearance, c_0.....in. $c_0=\left(\dfrac{2.19\times10^{-3}.\ell.S}{p_0.\left[1-3.59\left\{\frac{p_2}{p_0}\right\}^2\right].D}\right)^{\frac{1}{3}}$	3.94×10^{-3}	3.83×10^{-3}	3.13×10^{-3}	2.93×10^{-3}	6.26×10^{-3}
—— corresponding to —— Supply pressure, p_0.lb./in.^{2} abs.	56	59	94.7	77.5	89.5
—— which gives a —— Maximum load, W_{max} - lb.	140	130	126	52	223
Load coefficient, $\dfrac{W_{max}}{A'(p_0-p_2)}$.	0.41	0.36	0.32	0.26	0.115
Clearance ratio, $\dfrac{c}{c_0}$.	0.762	0.522	1.118	0.682	0.959
Ratio, L/D.	0.625	0.625	1.50	0.393	2.425

*C.G. Inlet holes joined by peripheral connecting grooves.

$$**s = \left(\frac{\pi d^2}{4}\right) \times \frac{\text{number of inlet rings}}{2}$$

APPENDIX III

Performance Estimation

For simplicity only bearings operating with air at room temperature are considered, although, since the performance is dependent only on the value of $\frac{c}{c_o}$, predictions for other gases and for air at other temperatures are possible. The general expressions for the theoretical choking clearance, c_o, are given in equations (5) and (6), and substituting in these $\gamma = 1.40$, $T = 288°C.Abs.$, and $\mu = 1.20 \times 10^{-5}$ lb./ft.sec. we obtain the following:-

$$c_o^3 = \frac{2.19 \times 10^{-3} \cdot \ell \cdot S}{p_o \cdot \left[1 - 3.59 \left\{\frac{p_2}{p_o}\right\}^2\right] \cdot D} \quad in^3 \quad \text{for laminar flow, } ..(7)$$

and

$$c_o^3 = \frac{5.83 \times 10^{-3} \cdot \ell \cdot S^{7/4}}{p_o^{\frac{1}{4}} \cdot \left[1 - 3.59 \left\{\frac{p_2}{p_o}\right\}^2\right] \cdot D^{7/4}} \quad in^3 \quad \text{for turbulent flow,} ..(8)$$

Assuming that the bearings are to be design with the optimum ratio, $\frac{c}{c_o} = 0.86$, we may calculate the necessary inlet hole area and the associated compressor power consumption in the following way.

Case 1. Laminar Flow

Substituting the value $\frac{c}{c_o} = 0.86$ in equation (7) and rearranging the terms, we have

$$S = \frac{718 \cdot c^3 \cdot D \cdot p_o}{\ell} \cdot \left[1 - 3.59 \left\{\frac{p_2}{p_o}\right\}^2\right] in^2 \quad ....(11)$$

Hence for the maximum load capacity the bearing should be designed with a total inlet area equal to the value of S obtained by substituting in this expression the design values p_o, p_2, C, D and ℓ.

The choking mass flow in the bearing is given by

$$G_o = 2.34 \times 10^{-2} \cdot S \cdot p_o,$$

and since $\frac{G}{G_o} = 0.60$,

$$G = 1.404 \times 10^{-2} \cdot S \cdot p_o \quad ....(13)$$

$$= \frac{10.06 \cdot c^3 \cdot D \cdot p_o^2}{\ell} \cdot \left[1 - 3.59 \left\{\frac{p_2}{p_o}\right\}^2\right] \text{lb./sec.}$$

The power required by the compressor is then

$$P = \frac{1 \cdot \gamma \cdot RT_1}{\eta_c \cdot (\gamma-1) \cdot 550} \cdot \left[\left\{\frac{p_o}{p_2}\right\}^{\frac{\gamma-1}{\gamma}} - 1\right] \cdot G \quad H.P.$$

where T_1 is the temperature of the air entering the compressor.

When $T_1 = T_o = 288°C$,

$$P = \frac{176}{\eta_c} \cdot \left[\left\{\frac{p_o}{p_2}\right\}^{0.286} - 1 \right] \cdot G$$

Hence, in H.P., $P = \frac{1,768}{\eta_c} \cdot \frac{c^3 \cdot D \cdot p_o^2}{\ell} \cdot \left[1 - 3.59\left\{\frac{p_2}{p_o}\right\}^2\right] \cdot \left[\left\{\frac{p_o}{p_2}\right\}^{0.286} - 1\right]$..(14)

Case 2. Turbulent Flow

Substituting $\frac{c}{c_o} = 0.86$ in equation (8) and rearranging the terms

$$S = \left(\frac{271 \cdot c^3 \cdot D^{7/4} \cdot p_o^{\frac{1}{4}}}{\ell} \cdot \left[1 - 3.59\left\{\frac{p_2}{p_o}\right\}^2\right] \right)^{4/7} \quad in^2$$

$$= \frac{24.6 \cdot c^{12/7} \cdot D \cdot p_o^{1/7}}{\ell^{4/7}} \cdot \left[1 - 3.59\left\{\frac{p_2}{p_o}\right\}^2\right]^{4/7} \quad in^2 \; .....(12)$$

If we now replace S in the equation (13) by this expression, we have

$$G = \frac{0.345 \cdot c^{12/7} \cdot D \cdot p_o^{8/7}}{\ell^{4/7}} \cdot \left[1 - 3.59\left\{\frac{p_2}{p_o}\right\}^2\right]^{4/7} \quad lb./sec.,$$

and the corresponding compressor power in H.P. is given by

$$P = \frac{60.6}{\eta_c} \cdot \frac{c^{12/7} \cdot D \cdot p_o^{8/7}}{\ell^{4/7}} \cdot \left[1 - 3.59\left\{\frac{p_2}{p_o}\right\}^2\right]^{4/7} \cdot \left[\left\{\frac{p_o}{p_2}\right\}^{0.286} - 1\right] ..(15)$$

APPENDIX IV

The Effect of Reynolds Number

The expression for Reynolds number is

$$R_e = \frac{4m \cdot \bar{u}}{\nu}$$

where m = the mean hydraulic depth

$\bar{u}$ = the mean velocity

and ν = the kinematic viscosity.

For a concentric bearing, in which the flow is purely axial,

$$m = \frac{\pi D \cdot \frac{c}{2}}{2\pi D} = \frac{c}{4}$$

hence $R_e = \frac{c \cdot \bar{u}}{\nu} = \frac{c \cdot \bar{u} \cdot \rho}{\mu}$

Now the total mass flow, G, through the bearing is given by

$$G = 2 \cdot \frac{\pi Dc}{2} \cdot \bar{u} \cdot \rho$$

Thus, we have

$$R_e = \frac{G}{\pi \cdot D \cdot \mu} \qquad \text{..............................(21)}$$

It has been shown that for the optimum design conditions $G = 0.60\ G_o$, or

$$G = 0.60 \cdot S \cdot \frac{(\gamma g R T_o)^{\frac{1}{2}}}{RT_o} \cdot p_o \cdot \left\{\frac{\gamma+1}{2}\right\}^{-\frac{1}{2}\frac{(\gamma+1)}{(\gamma-1)}}$$

which, for air at 288°C, gives

$$G = 1.404 \times 10^{-2} \cdot S \cdot p_o, \qquad \text{..............................(13)}$$

and hence

$$R_e = 4.47 \times 10^3 \cdot \frac{S \cdot p_o}{D}$$

When a bearing is designed to operate at a supply pressure p_o, the values of S will be those given in equations (11) and (12). Substituting these values we obtain

$$R_e = 3.21 \times 10^6 \cdot \frac{c^3 \cdot p_o^2}{\ell} \left[1 - 3.59\left\{\frac{p_2}{p_o}\right\}^2\right] \qquad \text{..............(22)}$$

for a bearing designed for laminar flow, and

$$R_e = 1.099 \times 10^5 \cdot \frac{c^{12/7} \cdot p_o^{8/7}}{\ell^{4/7}} \cdot \left[1 - 3.59\left\{\frac{p_2}{p_o}\right\}^2\right]^{4/7} \qquad \text{.......(23)}$$

for a bearing designed for turbulent flow.

The critical value of R_e obtained by equating the r.h.s. of equation (1) to that of equation (2), namely 2040, is representative of the experimental values (Ref. 1) for the beginning of transition. However, there is actually a range of transition and fully turbulent flow is not established unless R_e is greater than approximately 3810.

Equating the expression for R_e to 2040 in the laminar case and to 3810 in the turbulent case we obtain the limiting conditions. For laminar flow in a bearing designed for laminar flow,

$$\frac{c^3 \cdot p_o^2}{\ell} \cdot \left[1 - 3.59 \left\{ \frac{p_2}{p_o} \right\}^2 \right] \ngtr 0.635 \times 10^{-3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

and for turbulent flow in the corresponding bearing,

$$\frac{c^3 \cdot \dot{p}_o^2}{\ell} \cdot \left[1 - 3.59 \left\{ \frac{p_2}{p_o} \right\}^2 \right] \nless 2.776 \times 10^{-3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

APPENDIX V

The Effects of Thermal and Centrifugal Strain

Fig. 6 shows the decrease in load coefficient produced by changes in $\frac{c}{c_o}$ from the design value of 0.86. If a generous safety factor is assumed in the design a large increase in clearance is allowable, but owing to the steep slope of the curve at values of $\frac{c}{c_o}$ less than 0.7 a decrease of 20% is the maximum permissible.

$\frac{c}{c_o}$	$\Delta\frac{c}{c_o}$	$\frac{W_{max}}{A' .(p_o-p_2)}$	$\Delta\frac{W_{max}}{A' .(p_o-p_2)}$
0.86	-	0.29	-
0.59	-31.4%	0.225	-22.4%
1.60	+86.1%	0.225	-22.4%

1) Thermal Expansion

If a temperature difference occurs between the shaft and the shell the actual clearance, c, and the clearance ratio, $\frac{c}{c_o}$, will change.

Assuming a steel shaft and a steel shell having a coefficient of thermal expansion of 6×10^{-6} per °F, the change of clearance produced by an increase in the temperature of the shell above that of the shaft by ΔT is

$$\Delta c = 6 \times 10^{-6} . D . \Delta T$$

$$\therefore \frac{\Delta c}{c} . \frac{c}{D} = 6 \times 10^{-6} . \Delta T$$

Now for safety, allowing an increase of 86% and a decrease in 20% of the clearance,

$$0.86 > \frac{\Delta c}{c} > -0.20$$

Hence, when $\frac{c}{D} = 10^{-3}$

$$143^{\circ}F > \Delta T > -33^{\circ}F ;$$

and when $\frac{c}{D} = 0.5 \times 10^{-3}$,

$$72^{\circ}F > \Delta T > -17^{\circ}F.$$

2) Centrifugal Expansion

Neglecting axial stress, the formula for the increase in diameter of a plain cylinder rotating about its axis at ω rad./sec. is

$$\frac{\Delta D}{D} = \frac{m-1}{m} . \frac{D^3}{E} . \frac{\rho_s \omega^2}{16 g}$$

where D = the diameter of the cylinder

m = Poisson's ratio

ρ_s = density of the cylinder

E = Young's modulus for the material.

For steel, $n = 4$, $\rho_s = 0.29$ lb./in^3, and $E = 3 \times 10^7$ lb./in^2.

Hence $\dfrac{\Delta D}{D} = \dfrac{3}{4} \times \dfrac{12}{3 \times 10^7} \times \dfrac{0.29}{4 \times 32.2} \;.\; V_s^{\,2}$

where V_s, the peripheral speed, $= \dfrac{\omega . D}{2}$

$$\therefore \quad \frac{\Delta D}{D} = 6.75 \times 10^{-10} \;.\; V_s^{\,2}$$

When the shaft is rotating the actual clearance is less than the static clearance, the difference being given approximately by

$$\frac{\Delta c}{D} = 6.75 \times 10^{-10} \;.\; V_s^{\,2}$$

Now $\dfrac{\Delta c}{c}$ must be less than 0.20.

Hence $V_s^{\,2} \ngtr \dfrac{0.20}{6.75 \times 10^{-10}} \;.\; \dfrac{c}{D}$, i.e. $\ngtr 2.96 \times 10^8 \;.\; \dfrac{c}{D}$

When $\dfrac{c}{D} = 10^{-3}$, $\qquad V_s \ngtr 545$ ft./sec.

and when $\dfrac{c}{D} = 0.5 \times 10^{-3}$, $\qquad V_s \ngtr 386$ ft./sec.

APPENDIX VI

Estimation of the Frictional Losses

1) Air Lubrication

A simple formula may be derived for the frictional torque in an air lubricated bearing on the assumption that the shaft and the shell are coaxial. Experiments have shown that this formula is quite accurate for light loads.

The shear strain in the fluid film between the two bearing surfaces is given by

$$f = \frac{\mu}{g} \cdot \frac{dv}{dr},$$

where v is the circumferential velocity of the fluid. Hence, if there are no other tangential forces in the plane perpendicular to the axis, f will be independent of r and the velocity gradient $\frac{dv}{dr}$ will be a constant.

$$\therefore \frac{dv}{dr} = \frac{2V_s}{c} = \frac{\omega.D}{c}$$

The forces, Q, acting tangentially on the bearing surface is then

$$Q = \frac{\pi.D.L.f}{(12)^2} \quad \text{lb.}$$

$$Q = \frac{1}{(12)^2} \frac{\pi.D^2.L}{c} \cdot \frac{\mu}{g} \cdot \omega \quad \text{lb.,}$$

and the resisting torque is

$$T = \frac{1}{(12)^3} \frac{\pi.D^3.L}{2c} \cdot \frac{\mu}{g} \cdot \omega \quad \text{lb.ft.} \qquad \text{............(24)}$$

The frictional loss, therefore, is given by

$$F = \frac{1}{(12)^3} \frac{\pi.D^3.L}{2c} \cdot \frac{\mu}{g} \cdot \omega^2 \quad \text{ft.lb./sec.}$$

$$= \frac{1}{550\ (12)^3} \frac{\pi.D^3.L}{2c} \cdot \frac{\mu}{g} \left\{\frac{2\pi.N}{60}\right\}^2 \quad \text{H.P.}$$

$$= 1.832 \times 10^{-2} \cdot \frac{D^3.L}{c} \cdot \frac{\mu}{g} \cdot \left\{\frac{N}{10^3}\right\}^2 \quad \text{H.P.} \qquad \text{....(25)}$$

For air at room temperature,

$$F = 6.81 \times 10^{-9} \cdot \frac{D^3.L}{c} \cdot \left\{\frac{N}{10^3}\right\}^2 \quad \text{H.P.} \qquad \text{......(26)}$$

2) Oil Lubrication

Equation (25) could be used to obtain an approximate figure for the frictional loss in an oil journal bearing. Alternatively, an empirical formula, such as the following due to F.C. Linn and D.E. Irons (Ref. 4), may be used.

$$F = 3.77 \times 10^{-3} \cdot D^{1.55} \cdot L^{0.55} \cdot \left(\frac{N}{10^3}\right)^{1.43} \cdot Z^{0.43} \cdot Q^{0.43}$$

where Z = viscosity of the oil - centipoises

and Q = oil flow - gal./min.

When bearing loads of up to 5000 lb. are being considered (Fig. 13) the following values are chosen as representative of standard turbine practice.

Unit bearing load, $\frac{W}{D.L}$ = 150 lb./in^2

Viscosity of the oil, Z = 15 centipoises

Oil Flow, Q, when W = 5000 lb. = 5 gal./min.

For comparison purposes the oil flow is assumed directly proportional to the load, and the pump power which is less than 1 H.P. is neglected.

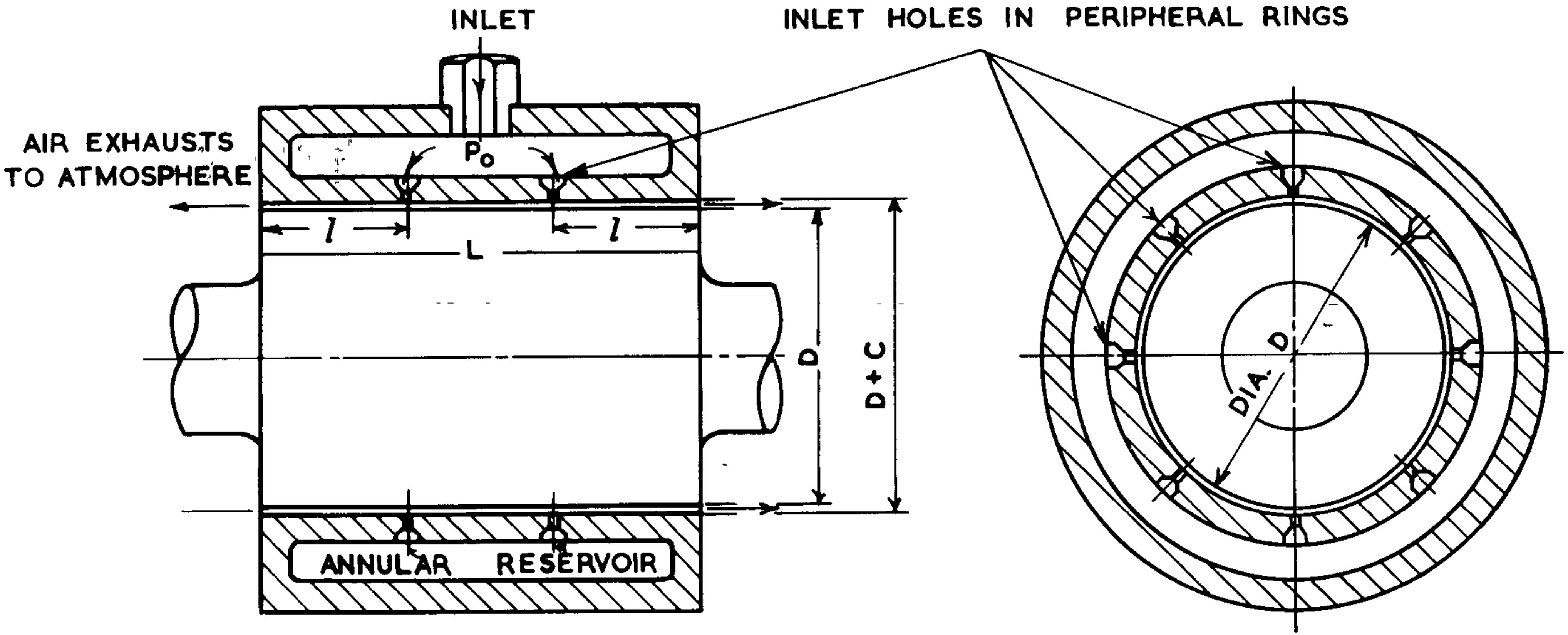
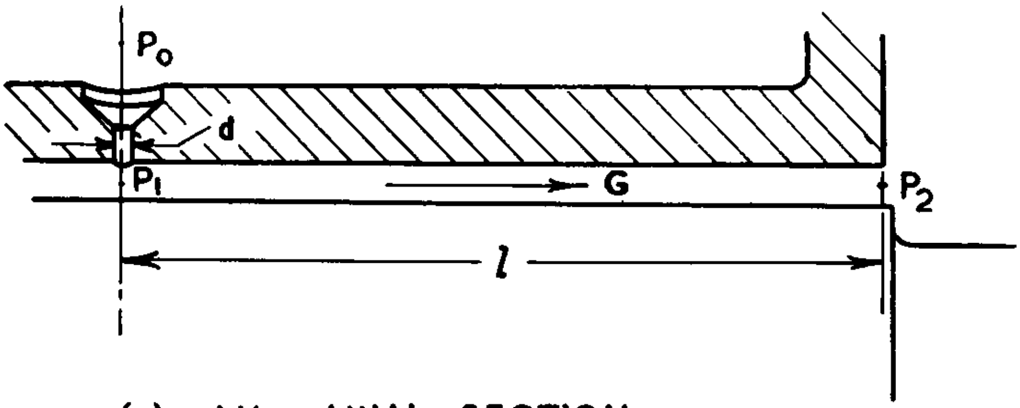
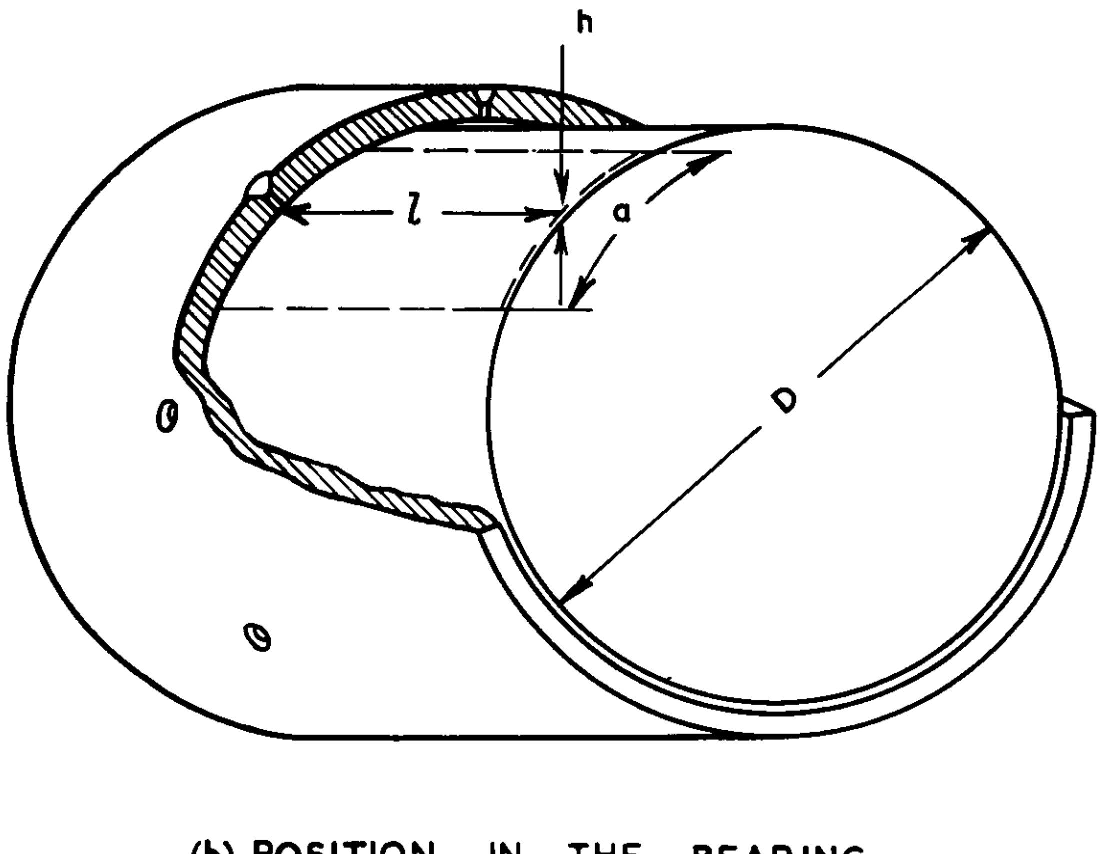


Fig.1. An Air Lubricated Journal Bearing.



(a) AN AXIAL SECTION



(b) POSITION IN THE BEARING

Fig.2 The Equivalent Slot.

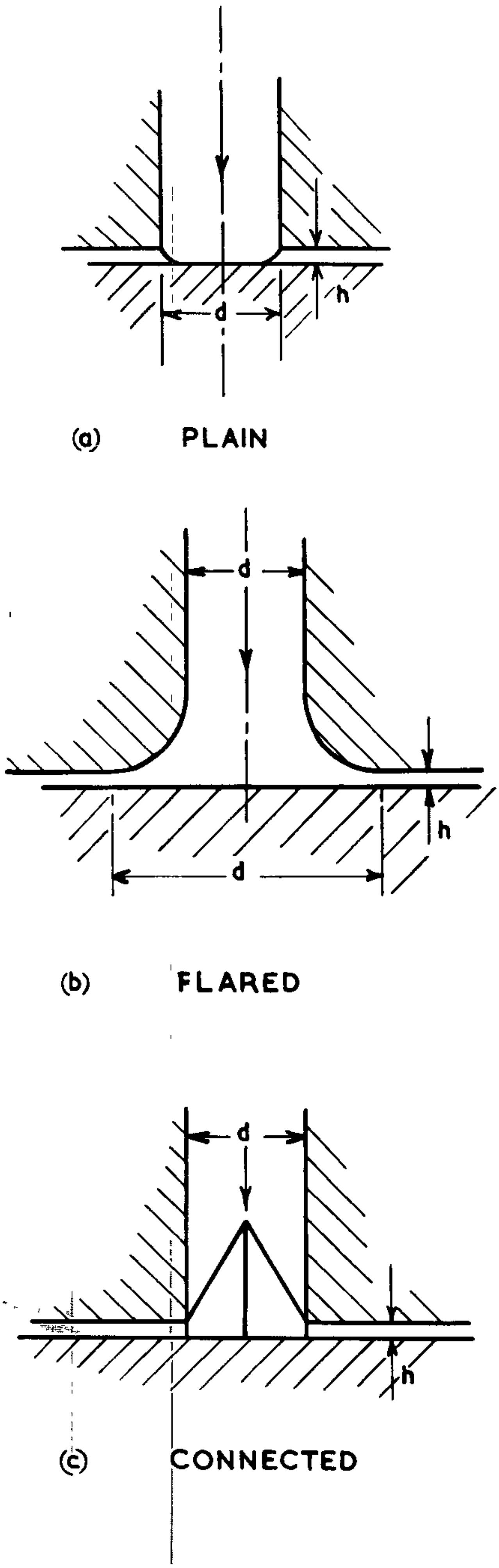
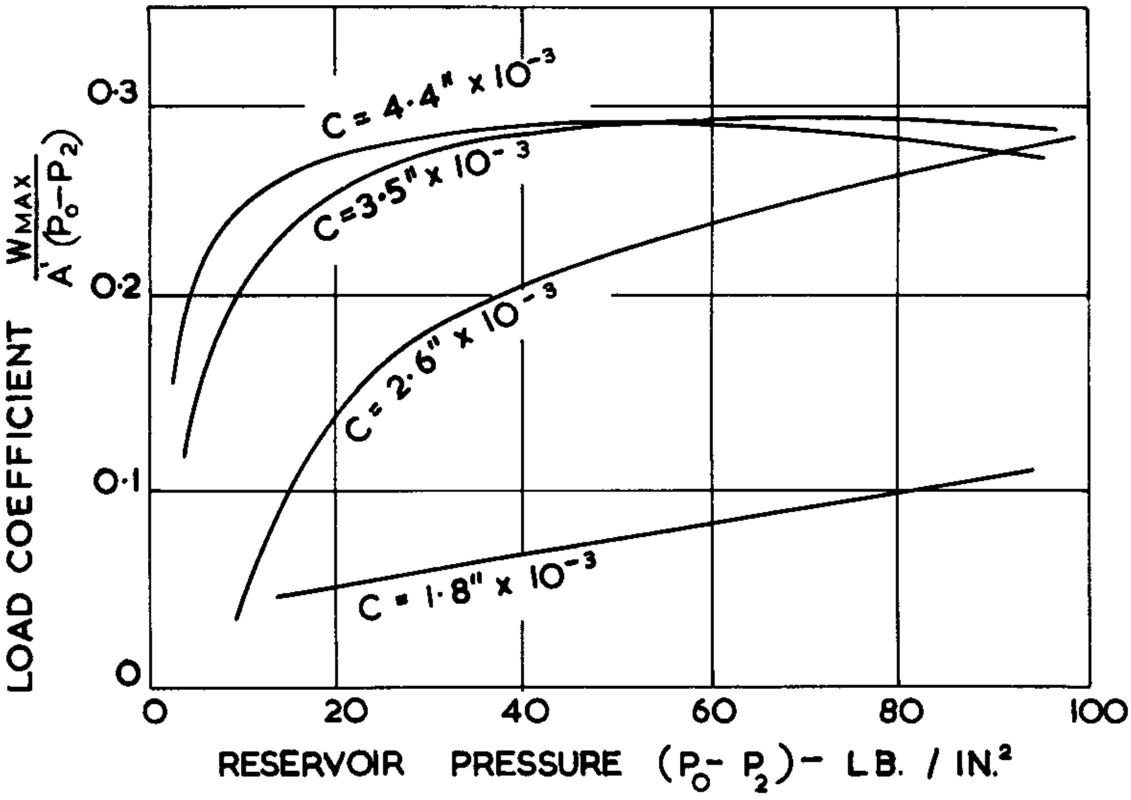
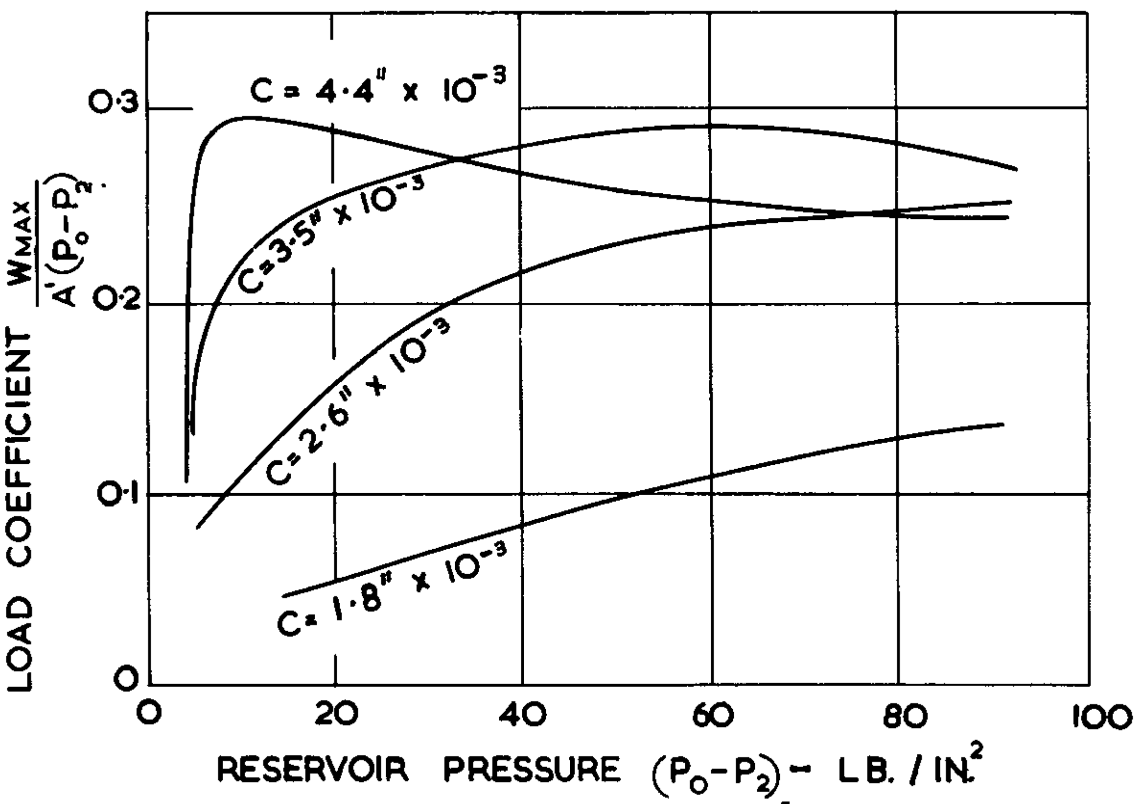


Fig. 3 Three Types of Inlet Hole.



(a) CENTRAL ADMISSION $\frac{l}{L} = 0{\cdot}5$



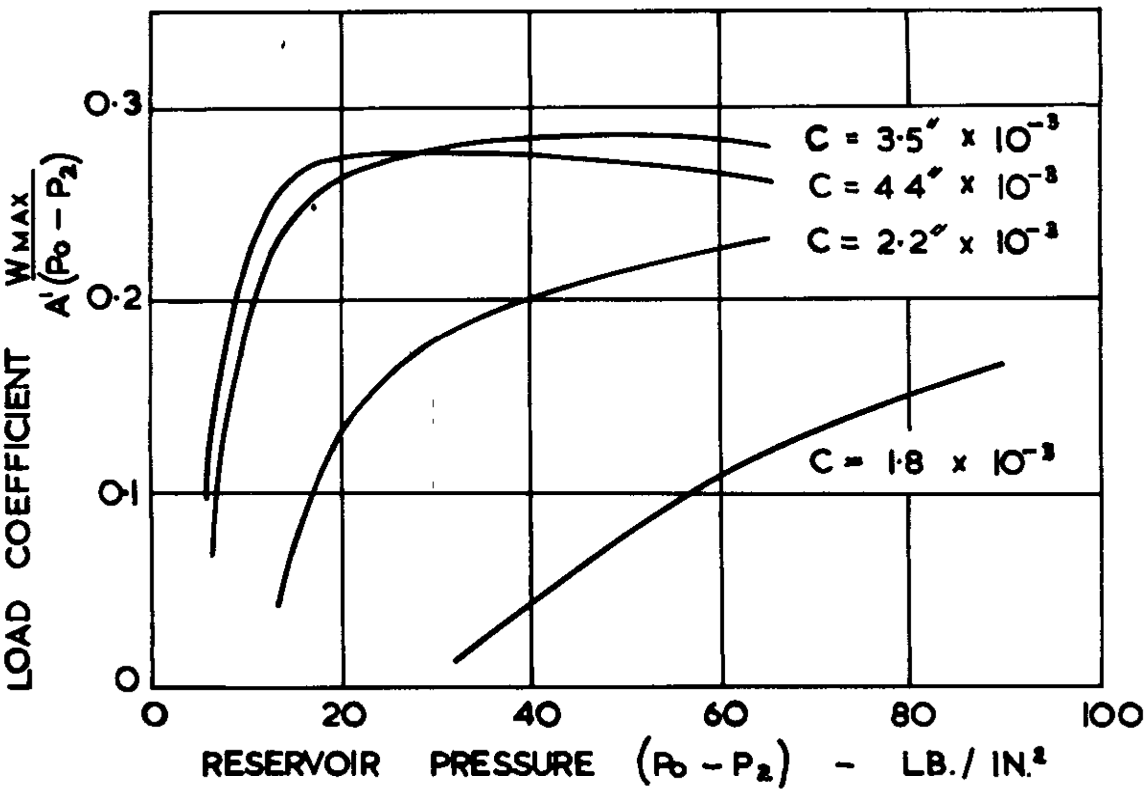
(b) QUARTER ADMISSION $\frac{l}{L} = 0{\cdot}25$

(EQUIVALENT INLET HOLE AREA $S \simeq 2{\cdot}0 \times 10^{-4}$ IN²; $a = 0{\cdot}796$ IN.)

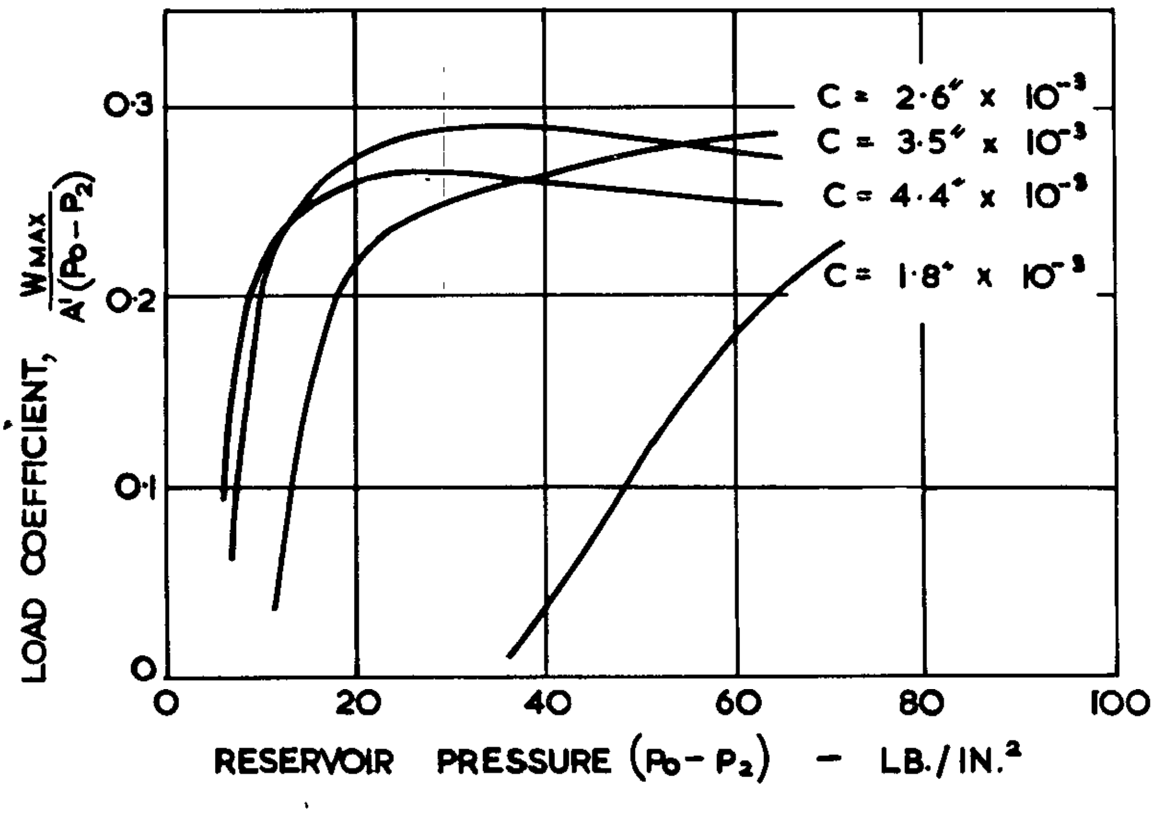
Fig 4. Effect of Inlet Position on Load Capacity

Fig.5. Effect of Inlet Hole Area on Load Capacity.

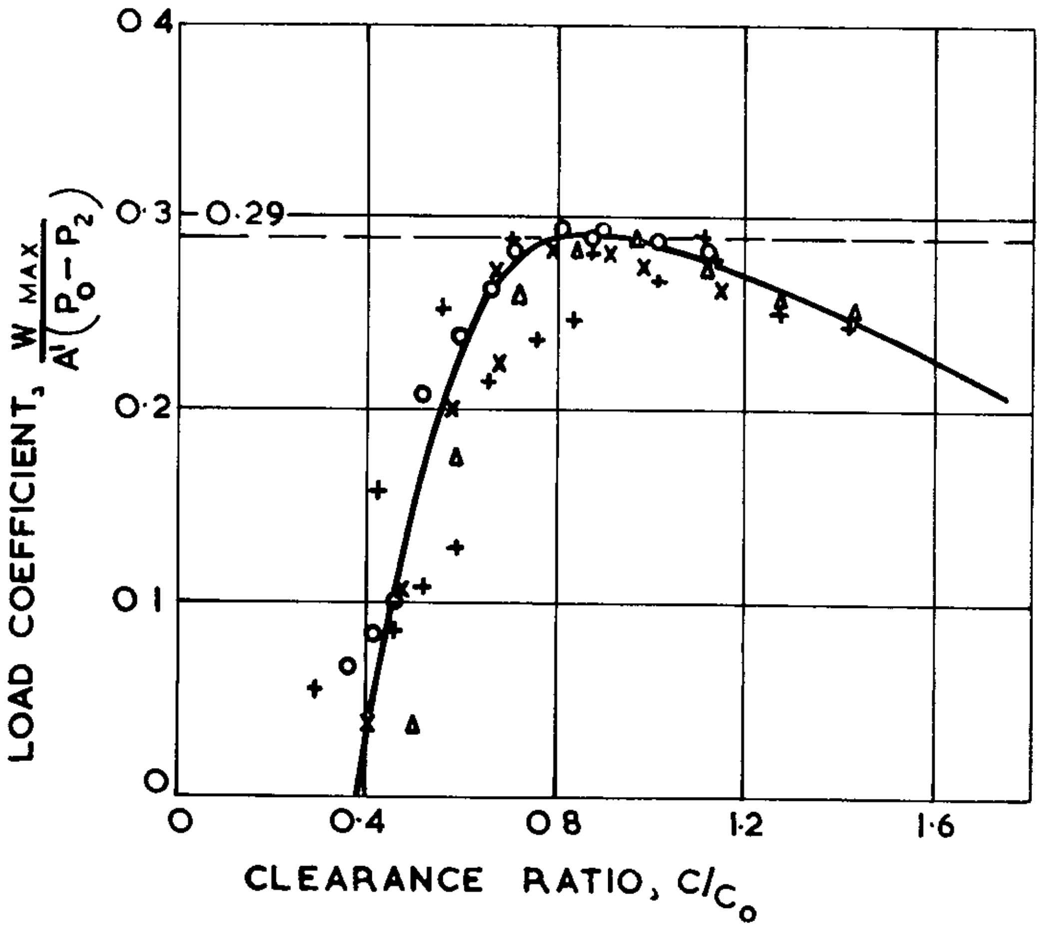
(CENTRAL ADMISSION, $\frac{l}{L} = 0.5$; $a = 0.796$ IN.)



(a) EQUIVALENT INLET HOLE AREA, $S \simeq 1{\cdot}38 \times 10^{-4}$ IN.^{2}



(b) EQUIVALENT INLET HOLE AREA $S \simeq 0{\cdot}71 \times 10^{-4}$ IN.^{2}



EXPERIMENTAL POINTS

o CENTRAL ADMISSION, $S = 1{\cdot}99 \times 10^{-4}$ IN^2

+ QUARTER ADMISSION, $S = 2{\cdot}02 \times 10^{-4}$ IN^2

x CENTRAL ADMISSION, $S = 1{\cdot}38 \times 10^{-4}$ IN^2

Δ CENTRAL ADMISSION, $S = 0{\cdot}71 \times 10^{-4}$ IN^2

$\left(D = 2{\cdot}0 \text{ IN}; \ a = 0{\cdot}796, \ \frac{L}{D} = 1{\cdot}50\right)$

Fig.6. Effect of Clearance Ratio on Load Capacity

Fig 7. Other Experimental Data

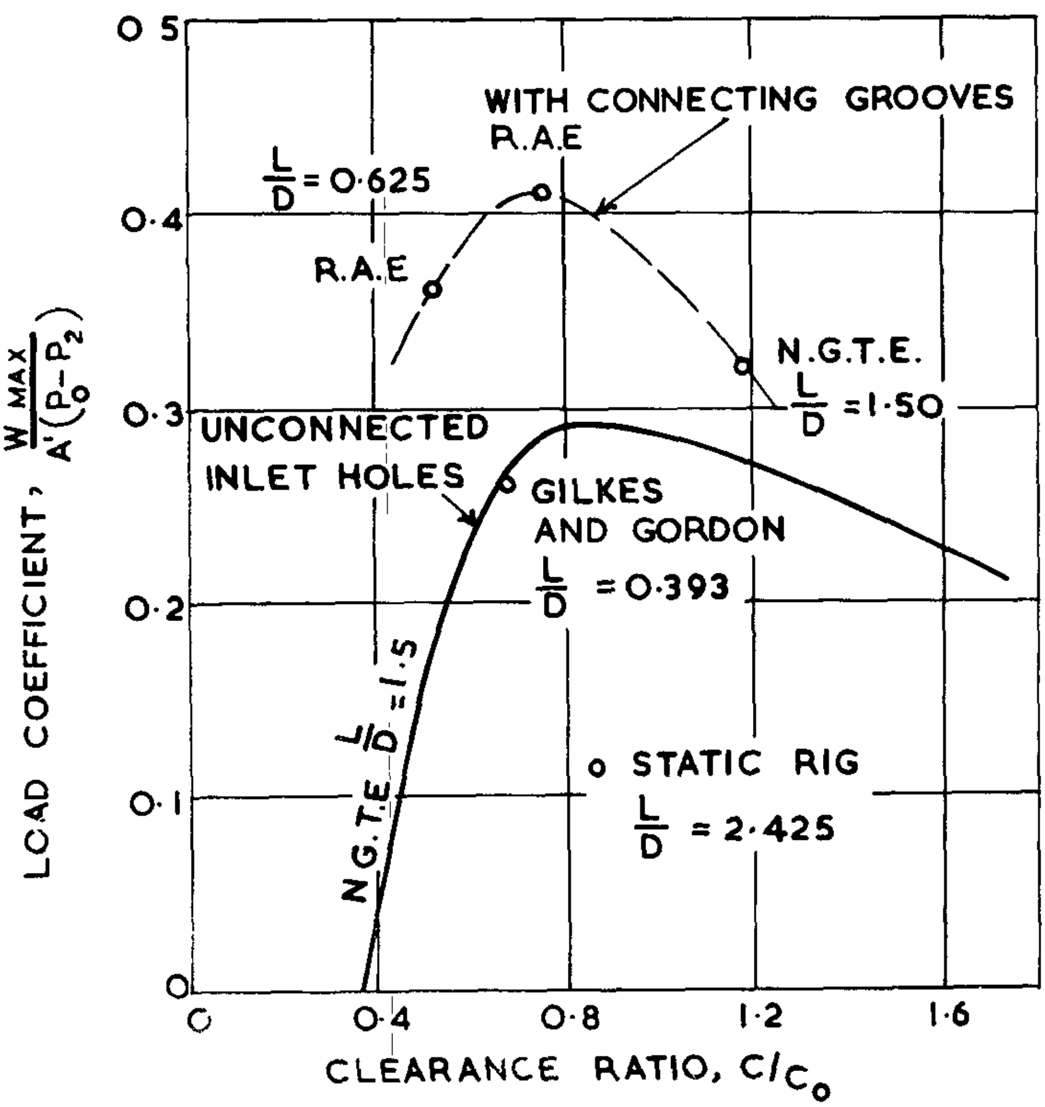
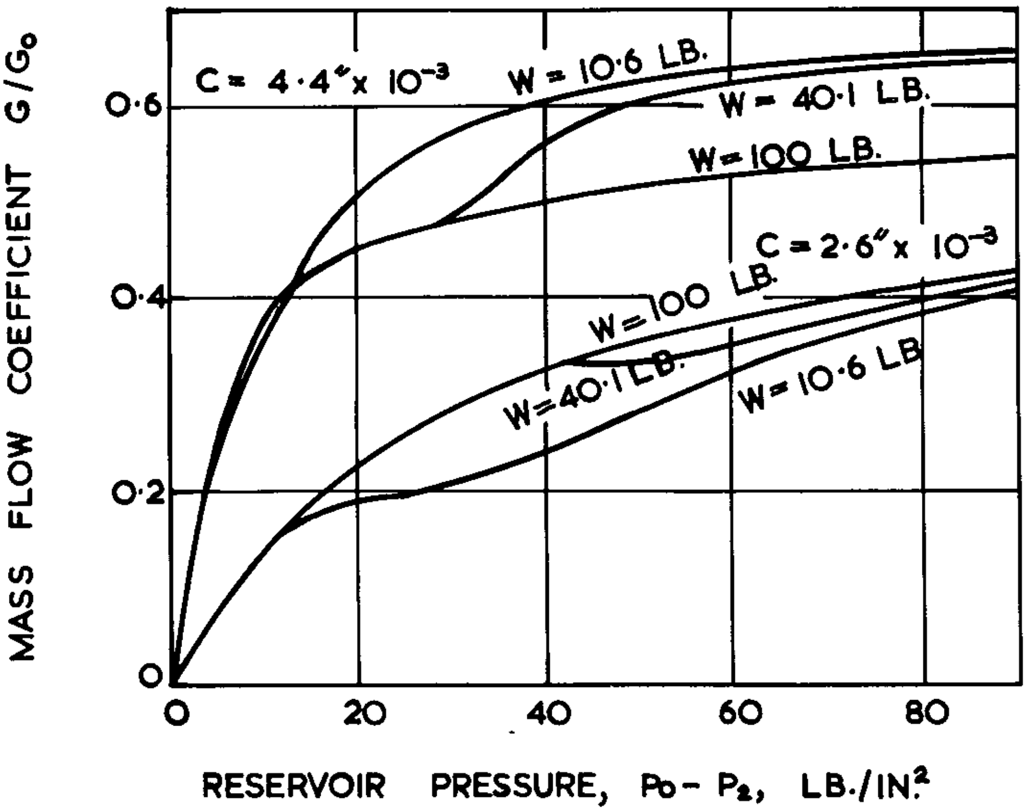
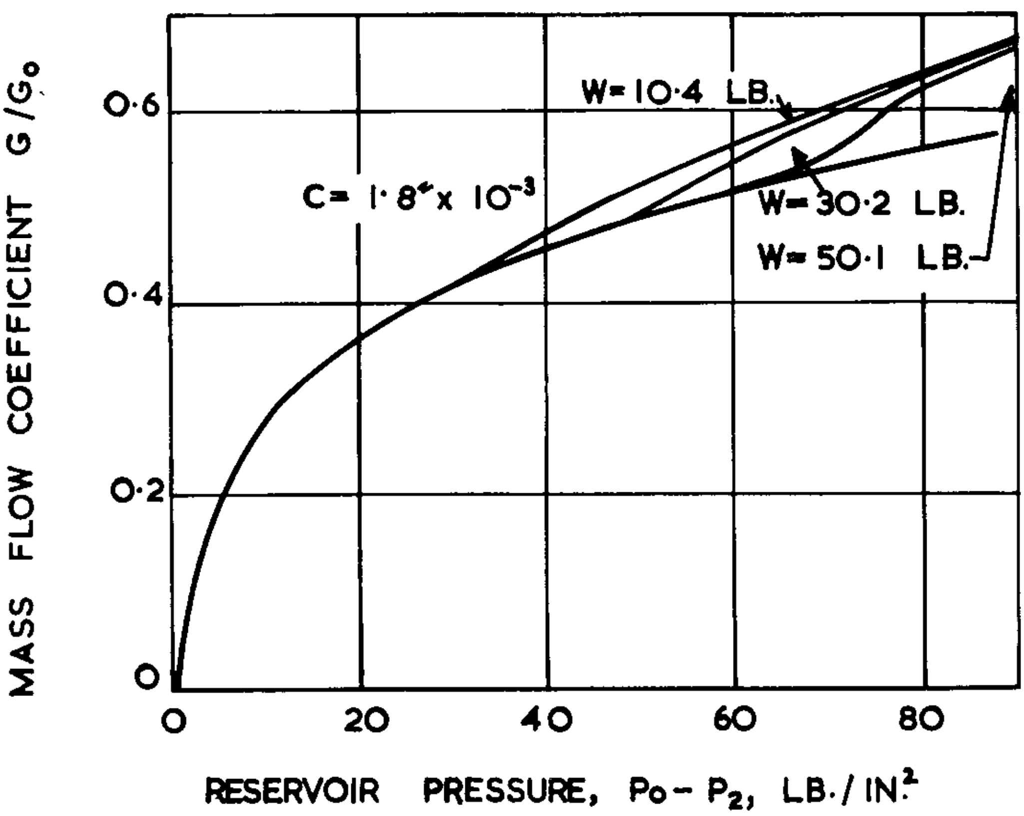


Fig. 8. Mass Flow

(CENTRAL ADMISSION, $\frac{l}{L} = 0.5$; $a = 0.796$ IN.)



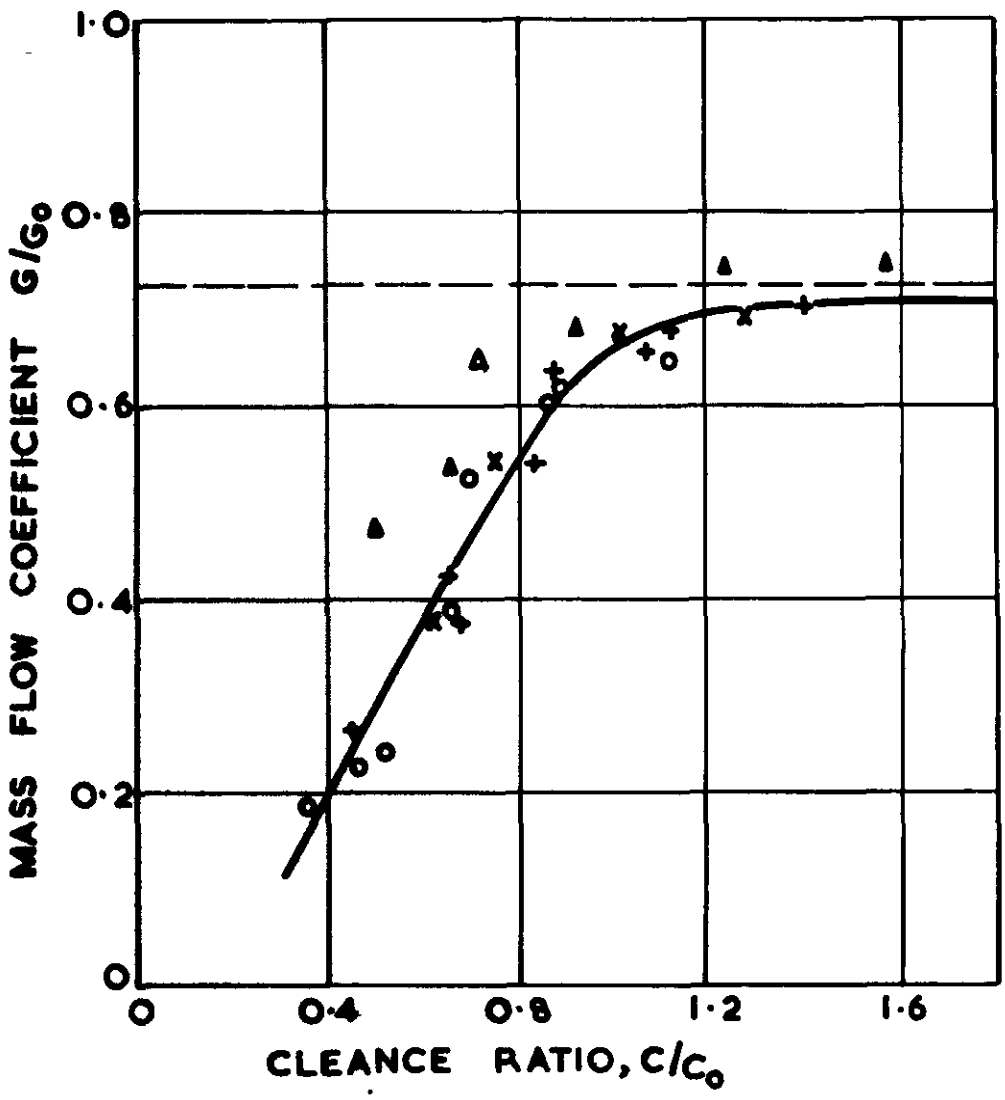
(a) EQUIVALENT INLET HOLE AREA, $S = 1{\cdot}99 \times 10^{-4}$ IN.^{2}



(b) EQUIVALENT INLET HOLE AREA, $S = 0{\cdot}71 \times 10^{-4}$ IN.^{2}

Fig. 9. Effect of Clearance Ratio on Mass Flow

$(D = 2{\cdot}0 \text{ IN}; \ a = 0{\cdot}796; \ \frac{L}{D} = 1{\cdot}50)$



EXPERIMENTAL POINTS

o CENTRAL ADMISSION, $S = 1{\cdot}99 \times 10^{-4}$ IN^2

+ QUARTER ADMISSION, $S = 2{\cdot}02 \times 10^{-4}$ IN^2

x CENTRAL ADMISSION, $S = 1{\cdot}38 \times 10^{-4}$ IN^2

▲ CENTRAL ADMISSION $S = 0{\cdot}71 \times 10^{-4}$ IN^2

Fig.10 Effect of Reynolds Number

(A SQUARE BEARING, $\frac{L}{D} = 1{\cdot}0$, $\frac{l}{L} = 0{\cdot}25$; $D = 8{\cdot}54''$; $\eta_c = 0{\cdot}75$)

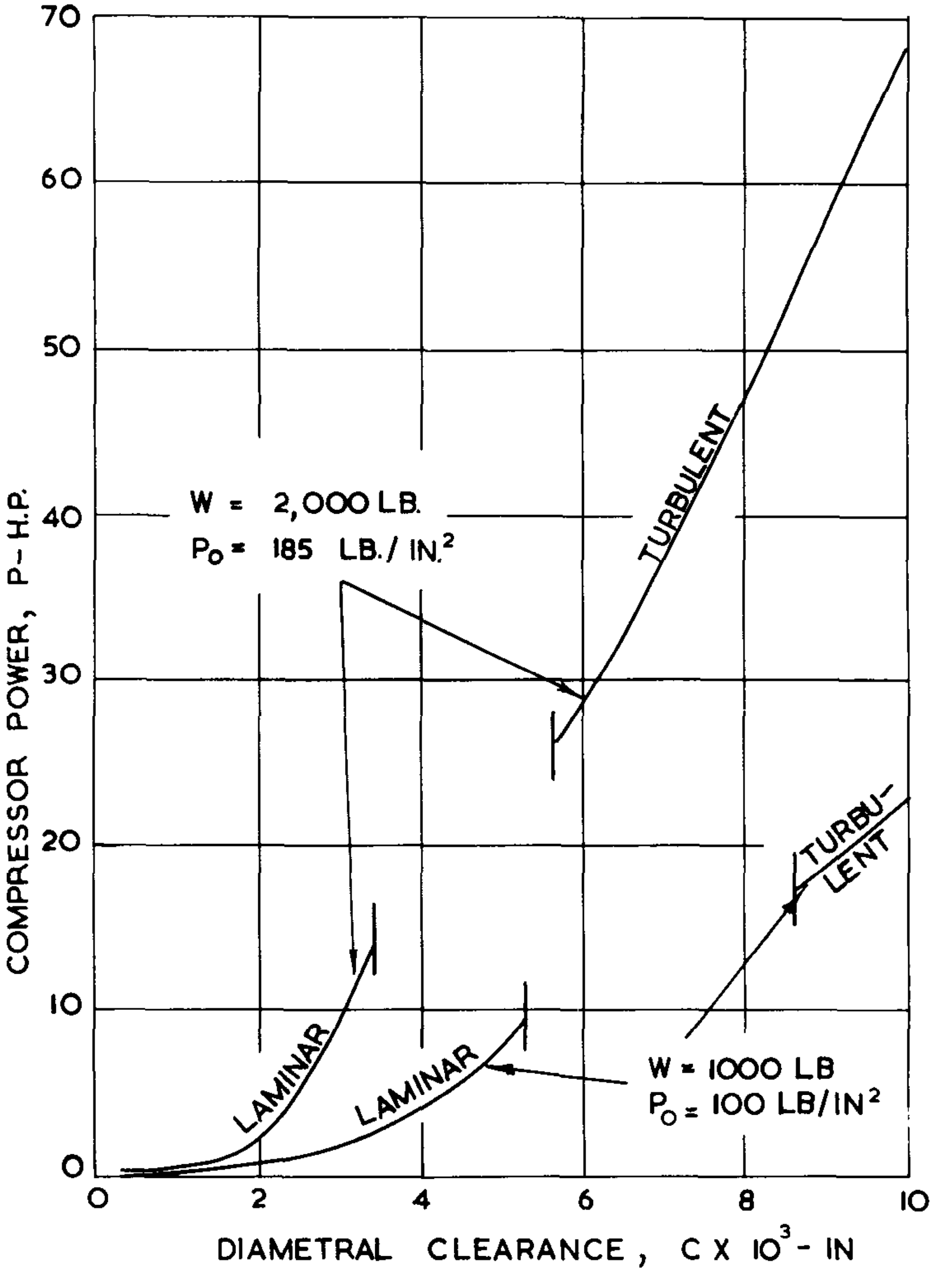


Fig.11. Performance away from the Design Point

$$\left(\frac{L}{D} = 1\cdot0;\quad \frac{l}{L} = 0\cdot25;\quad D = 5'';\quad C = 3''\times10^{-3}\right)$$

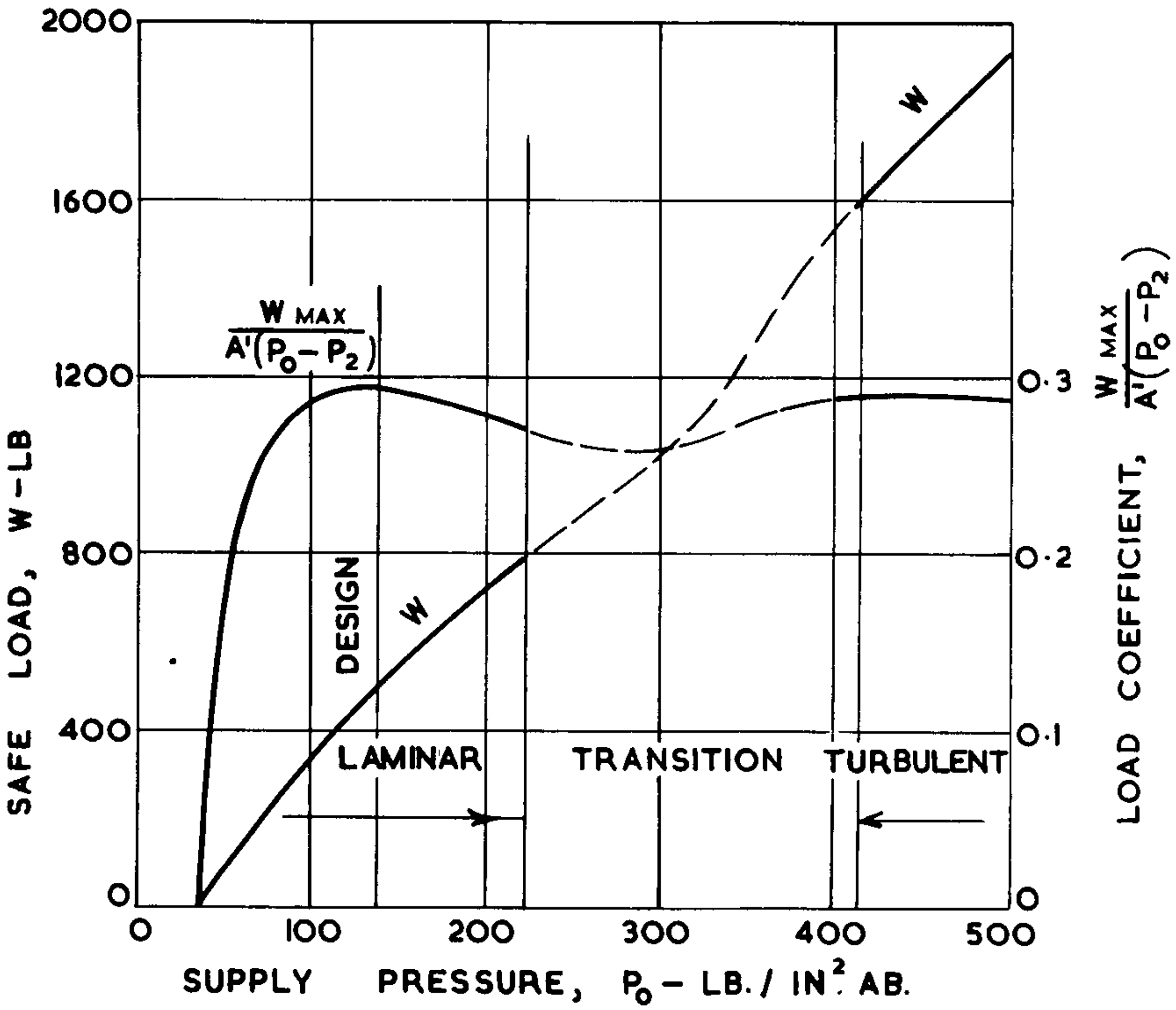


Fig.12 Compressor Power Consumption

$\left(\frac{L}{D} = 1{\cdot}0\ ;\ \frac{l}{L} = 0{\cdot}25\ ;\ \frac{C}{D} = 0{\cdot}5 \times 10^{-3}\ ;\ \eta_c = 0{\cdot}75\right)$

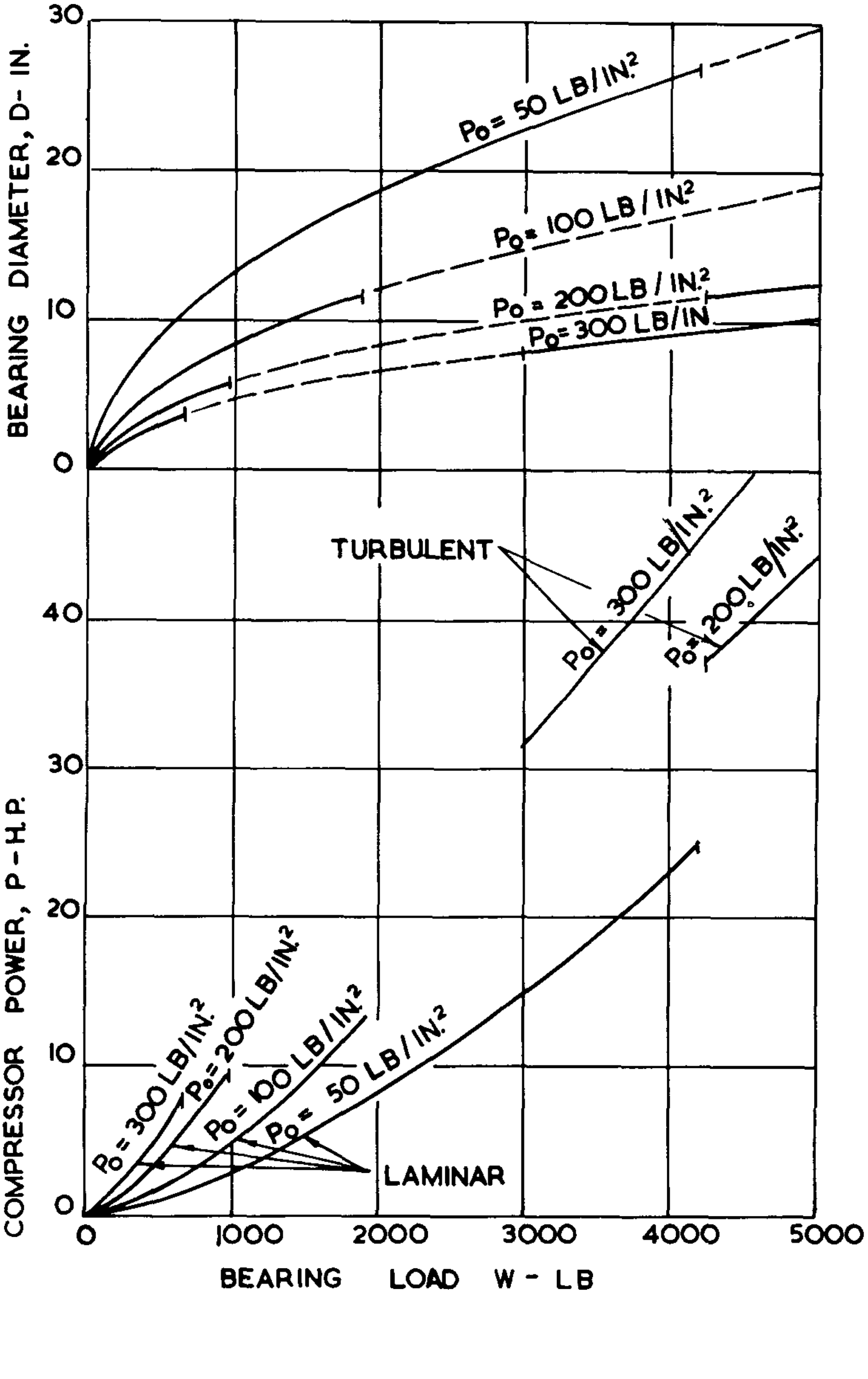


Fig. 13. A Comparison of Bearing Losses

$$\left(\frac{L}{D} = 1{\cdot}0 \;;\; \frac{l}{L} = 0{\cdot}25 \;;\; \frac{C}{D} = 0{\cdot}5 \times 10^{-3} \;;\; \eta_c = 0{\cdot}75\right)$$

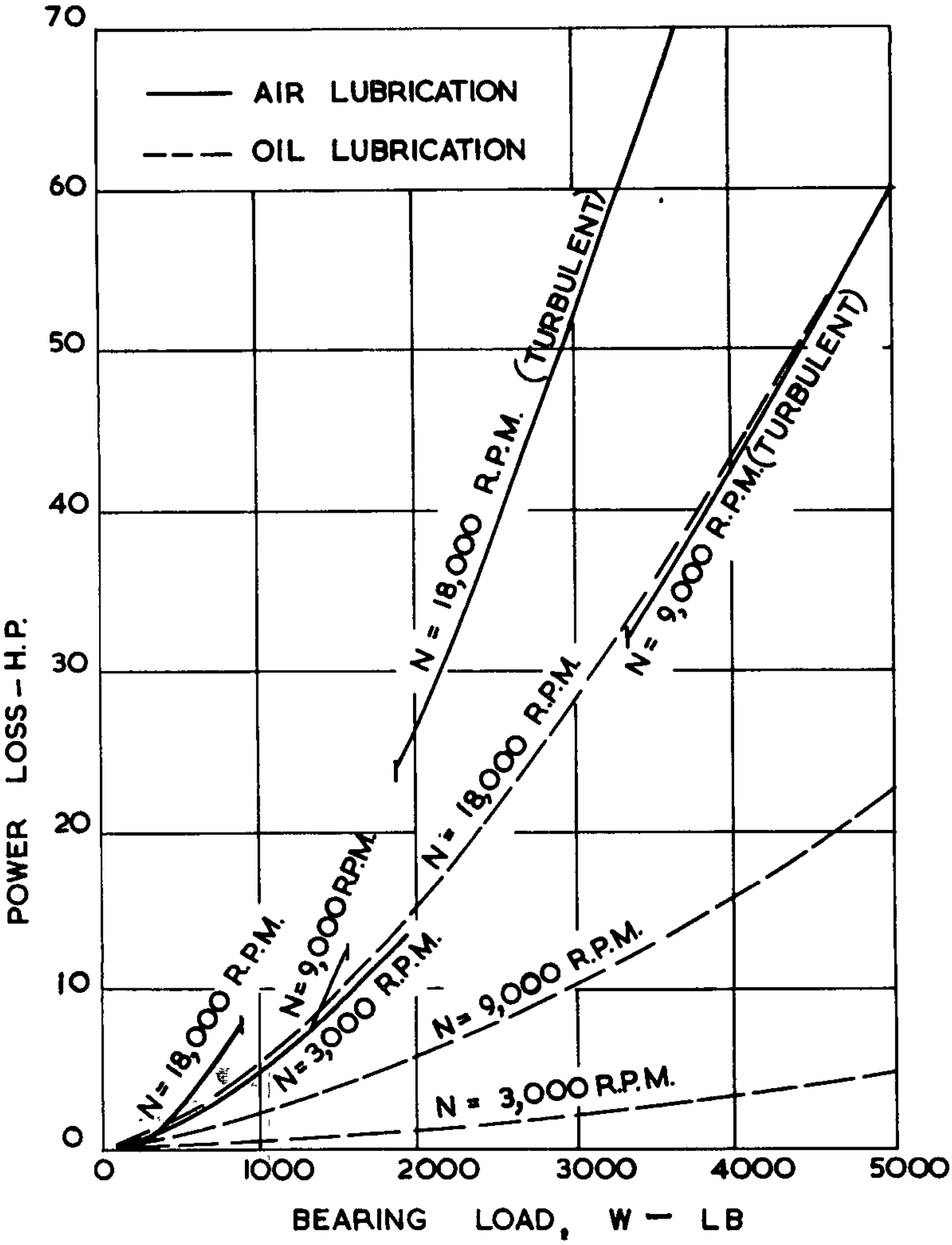
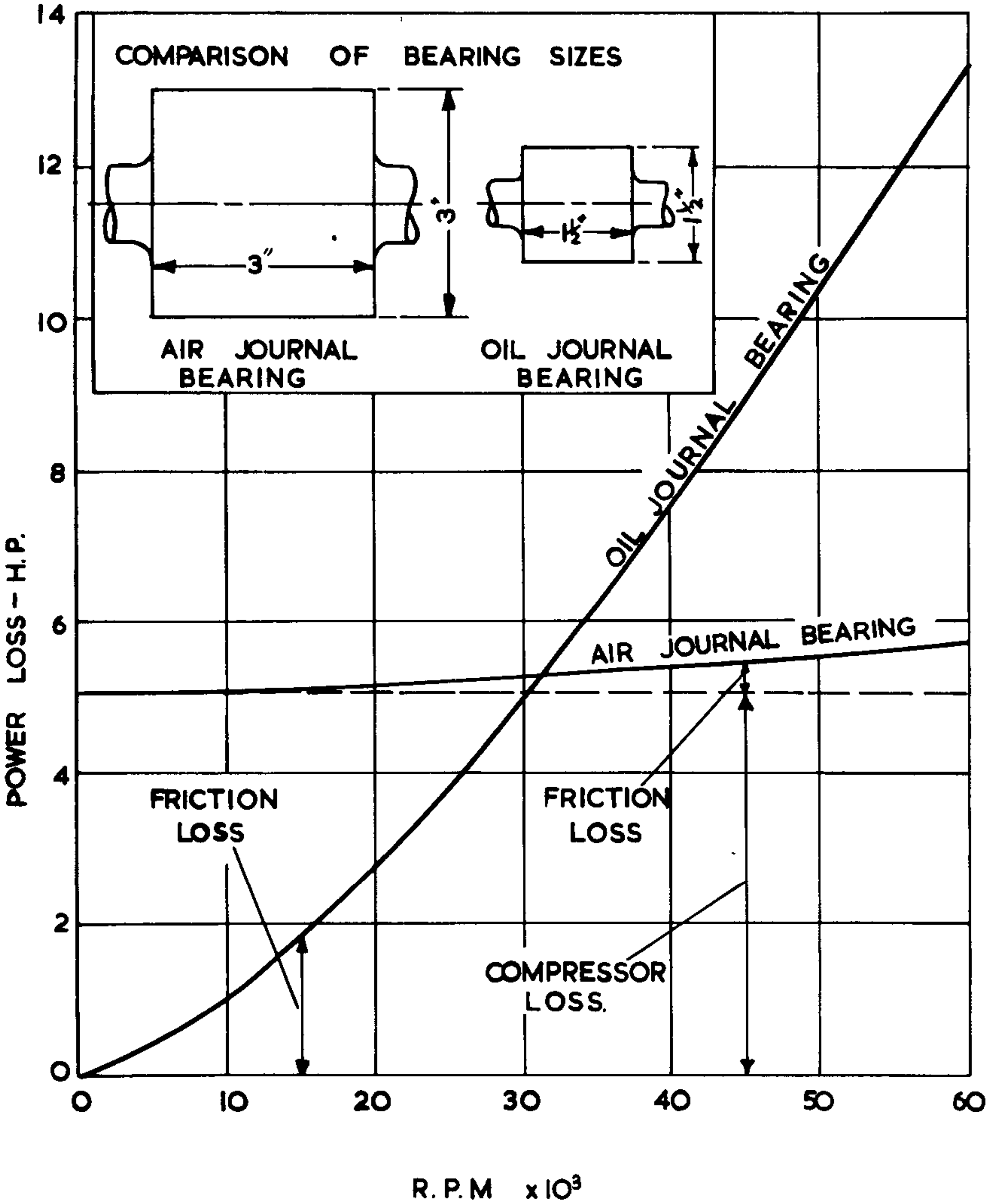


Fig. 14. Bearing Losses at High Speeds.

(LOAD, W = 200 LB.; $\frac{l}{L}$ = 0·25; $\frac{C}{D} = 10^{-3}$; η_c = 0·75)



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