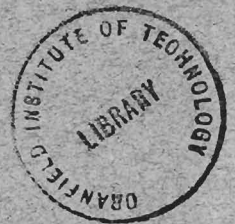R 55212

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THE THEORY OF THE DESIGN OF AEROFOILS, WITH AN ANALYSIS OF THE EXPERIMENTAL RESULTS FOR THE AEROFOILS R.A.F. 25, 26, 30 TO 33.—BY H. GLAUERT, M.A.—PRESENTED BY THE DIRECTOR OF SCIENTIFIC RESEARCH.

NOVEMBER, 1924.

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THE THEORY OF THE DESIGN OF AEROFOILS, WITH AN ANALYSIS OF THE EXPERIMENTAL RESULTS FOR THE AEROFOILS R.A.F. 25, 26, 30 TO 33.

By H. Glauert, M.A.

Presented by The Director of Scientific Research.

Summary.—(a) *Introductory.*—Recently a number of aerofoils have been designed with the object of obtaining (1) a good thick wing, and (2) a racing wing.   Experimental results for these aerofoils are contained in reports R. & M. 915, R. & M. 928, and R. & M. 943.

(b) *Range of Investigation.*—An account is given of the theory on which the aerofoils were designed, the essential feature being to curve the centre line of a good symmetrical section into a circular arc of suitable camber.   In the case of high camber, a cubic curve was also tried for the centre line in order to reduce the movement of the centre of pressure. The experimental results are analysed for comparison with the theoretical predictions, and curves are drawn showing the relative merits of the aerofoils.

(c) *Conclusions.*—The theoretical basis of the method of design has been fully confirmed by the experimental results.   In addition, it appears that the method leads to aerofoil shapes which compare very favourably with previous aerofoils.

(d) *Further developments.*—Further progress may be obtained by seeking for the best possible symmetrical sections of suitable thickness, and further experimental investigation is also required on the effect of reflex curvature in thin and in thick aerofoils.

1. *Theory of Design.*—The wings of an aeroplane serve the purpose of producing a lift force to balance the weight of the aeroplane.   This lift force is closely related to the circulation of the flow round the wings, and the magnitude of the circulation is usually determined by the shape and attitude of the aerofoil section.   It would be possible, however, to produce a similar effect by replacing the wing by a circular cylinder rotating rapidly about its axis, and in this case the circulation would be caused by the viscous drag of the surface of the cylinder.   The primary object of the aerofoil is therefore to produce the necessary circulation.

The lift of a wing is associated with a drag force, consisting of two parts—the induced drag and the profile drag.   The induced drag depends on the type of wing structure and on the load distribution across the span, but is independent of the shape of the aerofoil.   The profile drag, on the other hand, depends mainly on the shape and attitude of the aerofoil and is the only part of the drag which occurs in two dimensional motion.

The aim of aerofoil design is therefore to obtain an aerofoil shape which will give the required circulation or lift force with as low a profile drag as possible. Limitations are also imposed by structural considerations, since the aerofoil must have sufficient thickness to enclose suitable wing spars, and since it is also desirable that the movement of the centre of pressure with changing angle of incidence shall not be too rapid.

Now in two dimensional motion a circular arc of camber γ, set at zero angle of incidence, gives rise theoretically to a lift coefficient $k_L = 2\pi\gamma$ when the flow enters the leading edge and leaves the trailing edge smoothly. The drag under these conditions would be of the same order of magnitude as the skin frictional drag of a flat plate. From aerodynamic considerations alone, a circular arc of suitable camber would therefore appear to be the ideal method of producing the required lift. In practice the thin circular arc would be unsatisfactory for two reasons, firstly that a certain thickness is necessary for structural reasons, and secondly that at any other angle of incidence the flow would not enter the leading edge smoothly. Both these objections are removed if a suitable symmetrical fairing is placed round the circular arc, so that the aerofoil is essentially a symmetrical section whose centre line has been curved into a circular arc of the requisite camber. Actually also the minimum drag of a good symmetrical section is less than the frictional drag of a flat plate. An aerofoil of this type would be expected to have its minimum profile drag approximately equal to that of the symmetrical section, associated with the lift coefficient $k_L = 2\pi\gamma$. This conclusion applies to two dimensional motion and will also apply to a finite wing of elliptic plan form, where all the elements work at the same effective incidence. It should also be approximately true for a wing of rectangular plan form.

The choice of the symmetrical fairing is still open, and the main consideration is that it should have a low minimum drag. Its thickness must be sufficient to enclose suitable spars, but for different purposes it may be desirable to have a relatively thick section or a relatively thin one. The practical range is probably covered by a maximum thickness from 5 per cent. to 15 per cent. of the chord. The shape may be calculated by some theoretical rule or may be drawn by eye, but the former method is probably preferable as it will give a steady variation in the curvature of the surface.

Finally there is the question of the rate of movement of the centre of pressure. The critical quantity can suitably be taken as k_{m_o}, the value of k_m at zero lift. This quantity is almost invariably negative and a large numerical value implies rapid movement of centre of pressure. It is not possible to lay down a definite limiting value, but perhaps $k_{m_o} = -0.030$ may be taken as marking roughly the division between suitable and unsuitable aerofoils.

In the case of circular arc aerofoils, the theoretical value is $k_{m_o} = -\dfrac{\pi}{2}\gamma$, and so it appears that the centre of pressure movement will become excessive when the camber exceeds 2 per cent. A method of avoiding this difficulty is to replace the circular arc centre line of the aerofoil by the cubic curve—

$$y = h\,x\,(1-x)\,(1-a\,x)$$

for which*

$$k_{m_o} = -\frac{\pi}{64}\,h\,(8-7a)$$

The case $a = 0$ is that of the circular arc when h and y are small, and $a = 8/7$ should give an aerofoil with a constant centre of pressure. In general the constants a and h can be chosen to give any desired maximum ordinate or camber γ and any desired value of k_{m_o}. The influence of this double curvature of the centre line on the other characteristics of the aerofoil can only be determined experimentally.

The angle of no lift of the aerofoil, measured relative to the base line joining leading and trailing edges is—

$$\alpha = -\frac{h}{8}\,(4-3a)$$

2. *Experimental Aerofoils.*—A number of aerofoils have been designed on the lines of the theory developed above and have recently been tested in the 7-ft. wind tunnel at the Royal Aircraft Establishment.

The main series consists of the aerofoils R.A.F. 30–33, designed with the object of obtaining a good thick aerofoil with a rather higher maximum lift than is obtained from the usual type of thin aerofoil. The basic symmetrical section (R.A.F. 30) was calculated by the method described in R. & M. 911†, using the constants $k = 1\cdot08$, $n = 1\cdot95$, $\beta = 0$. The aerofoil shape so obtained ends in a sharp angle, and so the last 1 per cent. of the chord was cut off in order to avoid a thin trailing edge. The form of the aerofoil was also adjusted slightly towards the trailing edge in order to remove a slight reflex curvature. The aerofoil has a maximum thickness of $0\cdot13$ of the chord at a distance of one third of the chord from the leading edge, and its shape approximates closely to the symmetrical Göttingen section 459 which was known to possess good aerodynamic characteristics.

The aerofoils R.A.F. 31 and R.A.F. 32 were obtained by curving the centre line of the symmetrical section R.A.F. 30 into circular arcs of camber $0\cdot02$ and $0\cdot05$ respectively, in the hope of obtaining aerofoils which would give their minimum profile drag at high speed ($k_L = 0\cdot13$) and at low speed ($k_L = 0\cdot31$). Finally,

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* *See* R. & M. 910 : " A Theory of Thin Aerofoils," by H. Glauert.

† R. & M. 911 : " A generalised type of Joukowski Aerofoil," by H. Glauert.

as R.A.F. 32 has too large a value of k_{m_0}, a fourth aerofoil R.A.F. 33 was designed, using the centre line

$$19 \cdot 36 \, y = x \, (1 - x) \, (7 - 8x)$$

and the same symmetrical fairing. This aerofoil has the same centre line camber $0 \cdot 05$ as R.A.F. 32, but should have constant centre of pressure (zero k_{m_0}).

Two thin aerofoils have also been designed on similar lines. The symmetrical section was calculated as before, using the constants $k = 1 \cdot 04$, $n = 1 \cdot 98$, $\beta = o$, and the last 4 per cent. of the chord was cut off in order to avoid a thin trailing edge. This symmetrical section has a maximum thickness of $0 \cdot 065$ of the chord at a distance of one-third of the chord from the leading edge. The two aerofoils R.A.F. 25 and R.A.F. 26 were obtained by curving the centre line of this symmetrical section into circular arcs of camber $0 \cdot 01$ and $0 \cdot 02$ respectively. The object of the design in this case was to obtain a thin aerofoil suitable for a racing aeroplane.

Rectangular aerofoils, 8 in. $\times$ 48 in. of these six sections, have been tested over a range of wind speed of 40 to 90 f.p.s. and the experimental results are given in the following reports :—

> R. & M. 928. Test of four thick aerofoils, R.A.F. 30 to 33—F. B. Bradfield and A. S. Hartshorn.
> R. & M. 915. Test of a thin low drag aerofoil, R.A.F. 25 —Aerodynamics Staff, R.A.E.
> R. & M. 943. Test of three aerofoils suitable for high speed—F. B. Bradfield and A. S. Hartshorn.

The experimental results provide a good confirmation of the theory of aerofoil design and an analysis of some of the principal features is of considerable interest.

3. *Experimental results.*—The following table gives the maximum lift coefficients obtained at a wind speed of 80 f.p.s. :—

Aerofoil.	R.A.F. 25	26	30	31	32	33
Camber - - -	$0 \cdot 01$	$0 \cdot 02$	0	$0 \cdot 02$	$0 \cdot 05$	$0 \cdot 05$
k_L (max.) -	$0 \cdot 43$	$0 \cdot 47$	$0 \cdot 46$	$0 \cdot 54$	$0 \cdot 66$	$0 \cdot 62$
k_D (min.) -	$0 \cdot 0037$	$0 \cdot 0046$	$0 \cdot 0050$	$0 \cdot 0061$	$0 \cdot 0078$	$0 \cdot 0079$

In most cases there was little or no change in the maximum lift of the aerofoils with increasing wind speed, the only exception being the symmetrical section R.A.F. 30, for which k_L (max). rose from $0 \cdot 415$ at 60 f.p.s. to $0 \cdot 46$ at 80 f.p.s. It is interesting to note that for both the thin and the thick aerofoils with circular arc centre lines, the value of k_L (max.) increases by $4 \cdot 0 \, \gamma$ for an increase of camber γ. The reflex curvature of R.A.F. 33, compared with R.A.F. 32, causes a small reduction in k_L (max.) equivalent to a reduction of camber of $0 \cdot 01$.

The minimum drag coefficient showed a distinct scale effect in every case over the whole speed range (40 to 90 f.p.s.). The results are shown in Fig. 1 on a logarithmic scale, and the average slope of the curves indicates that the minimum drag is varying roughly as $V^{1.7}$. The curve for the skin friction of a flat plate* is given in the figure as a comparative standard of merit. It appears that the thick symmetrical aerofoil R.A.F. 30 and the two thin aerofoils have less drag than this standard, the drag of R.A.F. 25 being roughly 70 per cent. of the frictional drag of a flat plate. Similar low drag coefficients have been obtained with the Göttingen aerofoils 411 and 445, which are of the same thickness as R.A.F. 25.

The values for k_D (min.) in the table correspond to a wind speed of 80 f.p.s. It appears that the value of k_D (min.) rises with the camber and with the thickness of the aerofoil, but is scarcely affected by the reflex curvature of R.A.F. 33. The ratio of k_D (min.) to k_L (max.) shows less variation, but in the same sense.

Finally as regards the moment coefficient at no lift the following table gives a comparison between the observed and predicted values :—

Aerofoil.	R.A.F. 25	26	30	31	32	33
Camber - -	0·01	0·02	0	0·02	0·05	0·05
k_{m_o} {observed -	− 0·016	− 0·028	− 0·003†	− 0·029	− 0·067	− 0·009
{predicted -	− 0·016	− 0·031	0	− 0·031	− 0·078	0

The agreement is good for small camber, while for larger camber the actual observed values are more favourable than the predicted values. The effect of the reflex curvature of R.A.F. 33 is, however, less than anticipated.

4. *Profile Drag.*—The drag of an aerofoil can be separated into the induced drag and the profile drag, and the induced drag coefficient of a rectangular aerofoil is

$$k_{D_1} = N \frac{2}{\pi A} k_L^2$$

where A is the aspect ratio and N is the coefficient given in Table 2 of report R. & M. 824.‡ In the case of the present series of aerofoils the value of N can be taken to be $1 \cdot 053$, and so :—

$$k_{D_1} = 0 \cdot 112 \ k_L^2.$$

This induced drag coefficient has been subtracted from the observed values obtained at a wind speed of 60 f.p.s. to obtain the profile drag coefficient k_{D_o}, and for each aerofoil the best

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* Taken from the Göttingen Ergebnisse, vol. 1.
† Probably due to asymmetry of model.
‡ R. & M. 824 : " A Method of Calculating the Characteristics of a tapered wing," H. Glauert.

parabola has been fitted to the resulting values. Examples of the actual values and the mean curves are shown in Fig. 2. The mean curves for the aerofoils were as follows :—

Aerofoil.	k_{D_0}.
R.A.F. 25	$0 \cdot 0039 + 0 \cdot 095 \ (k_L - 0 \cdot 052)^2$
,, 26	$0 \cdot 0036 + 0 \cdot 110 \ (k_L - 0 \cdot 113)^2$
,, 30	$0 \cdot 0056 + 0 \cdot 030 \ (k_L - 0 \cdot 027)^2$
,, 31	$0 \cdot 0059 + 0 \cdot 027 \ (k_L - 0 \cdot 159)^2$
,, 32	$0 \cdot 0058 + 0 \cdot 028 \ (k_L - 0 \cdot 353)^2$
,, 33	$0 \cdot 0064 + 0 \cdot 032 \ (k_L - 0 \cdot 350)^2$

Taking account of the fact that an observed drag coefficient is liable to an error of $\pm 0 \cdot 0003$, these results are remarkably consistent, and show some very interesting features.

(1) The minimum profile drag coefficient appears to be independent of the camber, but to rise slightly on account of the reflex curvature of R.A.F. 33.

(2) The same conclusion may probably be accepted for the coefficient of k_L^2, since the variation in the experimental values is small and in no way systematic.

(3) The value of k_L at which k_{D_0} has its minimum value agrees fairly well with the theoretical value $2\pi\gamma$, particularly if the effect of a change of camber is considered. The effect of the reflex curvature of R.A.F. 33 has been simply to increase the profile drag of R.A.F. 32 about 10 per cent.

5. *Merit of Aerofoils.*—The drag of an aeroplane can be expressed in the form

$$D = R\rho V^2 + k_{D_0} S\rho V^2 + k_{D_1} S\rho V^2$$

$$= R\rho V^2 + W \frac{k_{D_0}}{k_L \text{ (max.)}} \frac{V^2}{V^2_m} + \frac{NW^2}{2\pi s^2 \rho V^2}$$

where R is the drag of body, bracing, etc., at unit density and speed, and V_m is the stalling speed. Thus, if we consider a change of aerofoil section, keeping the stalling speed and span unaltered, the best result at a chosen speed V is obtained from the aerofoil which gives the lowest value of k_{D_0}/k_L (max.). The values of this quantity must be compared at the same value of k_L/k_L(max.).

Curves of this type have been prepared and are shown in Fig. 3. The important regions of the curves are, roughly :—

Climb - - - - -	k_L/k_L (max.) $= 0 \cdot 5$
Level flight - - - - - -	$0 \cdot 2$
Racing - - - - - -	$0 \cdot 1$

For comparative purposes it may be noted that the lift and drag characteristics of R.A.F. 31 are almost identical with those of the well-known aerofoil R.A.F. 15. Also to obtain numerical results it should be noted that an increase δ in k_{D_0}/k_L (max.) implies an increase of drag $\delta W k_L$ (max.)$/k_L$.

An inspection of the curves leads to the following conclusions :

(1) R.A.F. 26 is always superior to R.A.F. 25.

(2) Thin aerofoils are better than the thick aerofoils for level speed, the difference in drag for R.A.F. 26 and R.A.F. 31 being $0 \cdot 016$ W; but there is little difference between the two types under climbing conditions.

(3) R.A.F. 32 is superior to R.A.F. 31, but if R.A.F. 32 must be replaced by R.A.F. 33 in order to avoid a high value of k_{m_o}, then R.A.F. 31 should be chosen for level speed and R.A.F. 33 for climbing.

6. *Conclusions.*—The theory of aerofoil design has been substantially confirmed by the experimental results. The profile drag of a cambered aerofoil is the same as that of the basic symmetrical section except that the position of the minimum is moved to the point $k_L = 2\pi\gamma$. The value of k_{m_o} agrees with the theoretical value at low cambers, and is slightly smaller at high cambers. The effect of reflex curvature is slightly less than predicted, but this question requires further investigation.

The method of designing an aerofoil by curving a suitable symmetrical section about a circular arc has proved successful. An aerofoil R.A.F. 31 has been produced with the same characteristics as R.A.F. 15, but with double the thickness. Also a thin aerofoil (R.A.F. 26) has been produced which, under racing conditions, would have a drag less than that of R.A.F. 15 by roughly $0 \cdot 016$ W.

Given the characteristics of a symmetrical aerofoil, it appears to be possible to predict with good accuracy the characteristics of a cambered aerofoil using the symmetrical section as basis, and so further progress may be facilitated by concentrating on the design of the best possible symmetrical aerofoil of the desired thickness. The important point is to obtain a good maximum lift, a low minimum drag, and a low rate of increase of profile drag with lift coefficient. The characteristics of a cambered aerofoil can then be predicted with good accuracy, but it would probably be desirable to have further experimental checks on the change of maximum lift with camber and with reflex curvature.

FIG. 1.

SCALE EFFECT ON MINIMUM DRAG.

K2347. H&S.R. G227. 122A. 10176/1085. 900. 3'25.

FIG 2

PROFILE DRAG COEFFICIENTS.

LIFT COEFFICIENT.

FIG 3.
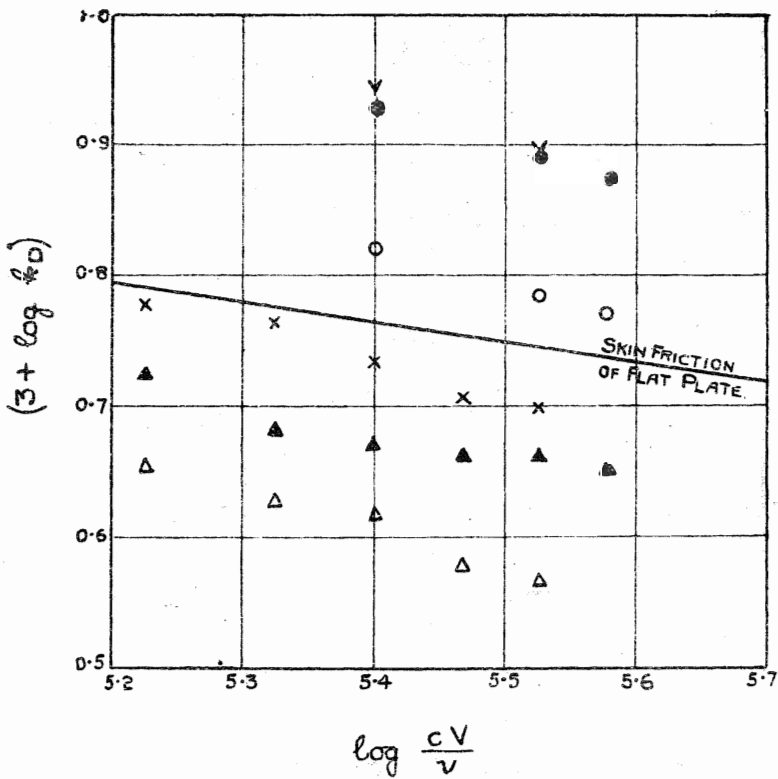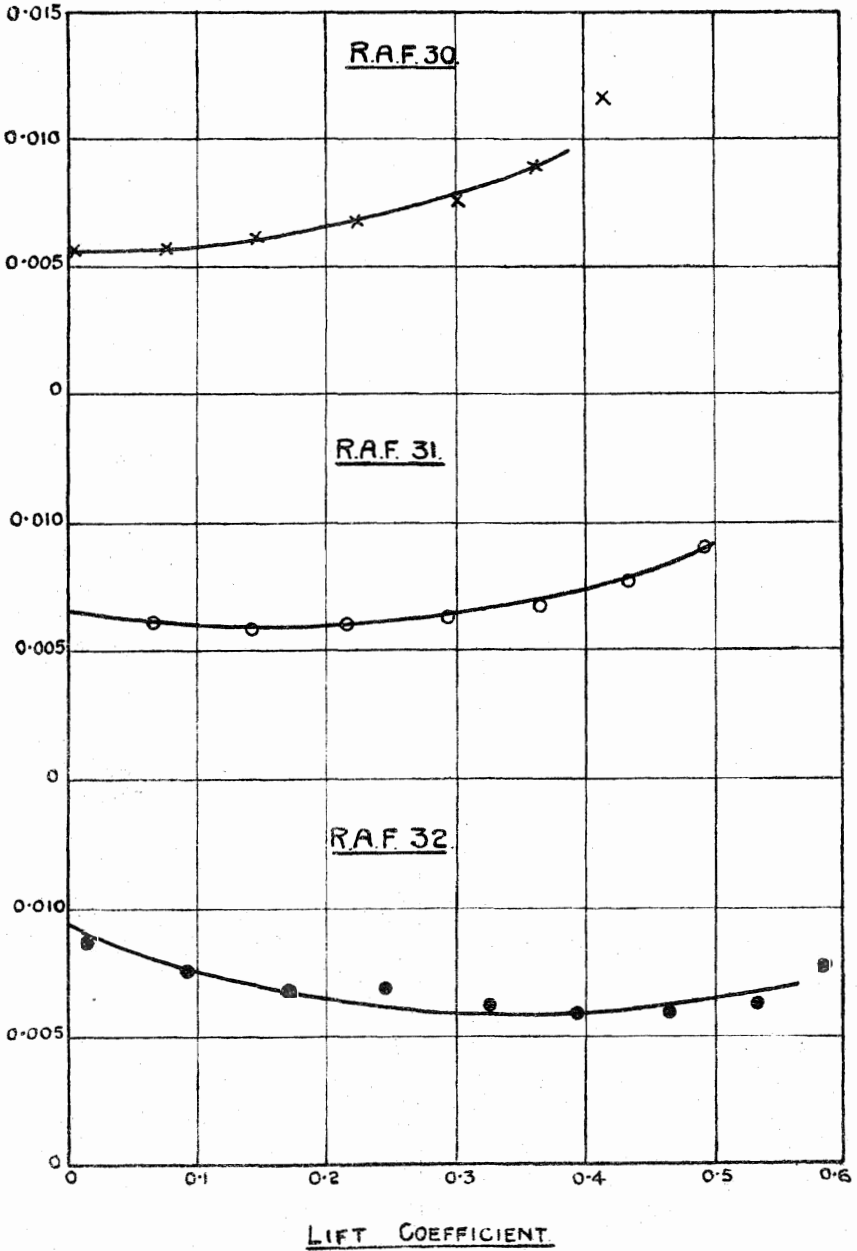
FIGURE OF MERIT.

SYMBOLS :- R.A.F. 25 △ } THIN WINGS.
            R.A.F. 26 ▲
            R.A.F. 30 ×
            R.A.F. 31 ○ } THICK WINGS.
            R.A.F. 32 ●
            R.A.F. 33 ∨

THE FIGURE SHOWS THE MERIT OF THE WING SECTIONS ON
THE BASIS OF THE SAME SPAN AND STALLING SPEED.

$$\frac{k_{DO}}{k_{L\,(MAX)}}$$

R.A.F. 25
R.A.F. 26
R.A.F. 30
R.A.F. 31
R.A.F. 33
R.A.F. 32

$$\frac{k_L}{k_{L\,(MAX)}}$$

x 23721

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