ON THE CONDITIONS AT THE BOUNDARY OF A FLUID IN TURBULENT MOTION.

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Summary. (*a*) *Introductory* (*Reasons for Inquiry*).—The object of the experiments was to determine the nature of the flow in the neighbourhood of the boundary of a fluid flowing in turbulent motion through a channel with parallel walls.

(*b*) *Range of the Investigation.*—The observations were made on air flowing through long pipes of circular cross section at mean rates of flow covering as wide a range as possible below and above the critical speed. The pipes used were 0·269, 0·714 and 12·7 cms. in diameter, and the range in experimental conditions varied from

$$\frac{vl}{\nu} = 460 \text{ to } \frac{vl}{\nu} = 325{,}000.$$

(*c*) *Conclusions.*—The conclusions are that for speeds above the critical value as high as could be obtained, there is a layer of fluid of finite thickness at the boundary which is in laminar motion, and that the boundary condition is

$$\mu \mathrm{Lt} \left(\frac{dv}{dx}\right)_{x \to 0} = \mathrm{R},$$

where the origin is taken in the boundary, and x is measured along the normal; v is the velocity parallel to the boundary, μ is the coefficient of viscosity, R is the intensity of surface friction.

The experiments here described form part of a general research into the phenomena of skin friction of solid surfaces due to the flow over them of fluids whose motion, not in the immediate vicinity of the surface, is eddying or turbulent. Considerable information has been obtained in recent years as to the magnitude of the frictional forces brought into existence in this condition of flow, and the manner of variation of these forces with the relative mean speed of surface and fluid, the roughness of the surface, and the physical characteristics of the fluid is fairly well known. Practically nothing, however, is known about the mechanism by which the resistance to flow is transmitted to the bounding surfaces. For speeds below the critical when the general motion of the fluid throughout is streamline in character, it is generally accepted that the layer of fluid in contact with the boundary is at rest relative to it, as any slipping of finite amount would be detected in a variation from the Poiseuille law of the relationship between the diameter of a pipe and the time of efflux of a given volume of fluid. At speeds above the critical, observations near the walls have shown that the mean velocity falls rapidly as the solid bounding surface is approached, and it has been suggested that at the walls there may exist a thin layer in which the flow is laminar in character, in which case, if there is no slipping, the frictional resistance would be determined from the slope of the velocity curve in the surface layer and the coefficient of viscosity of the fluid. During the last few years several attempts have been made at the National Physical Laboratory to obtain evidence as to the truth of this assumption. The method adopted has been to set up a condition of turbulent fluid motion over a surface of which the frictional resistance to the flow could be accurately determined, and to measure by means of a very fine pitot tube the velocity of the fluid at a point as near the wall as possible. The high order of accuracy attainable by the pitot tube method, in the determination of the mean velocity at a point in a fluid in turbulent motion, has been fully demonstrated during the last ten years. Careful experiments at the N.P.L. have shown that the velocity estimations by this method of a body moving in free air have a limit of accuracy of one-tenth of one per cent. (Report A.C.A. 1912–13, page 39.) This accuracy, however, was obtained when the flow in the neighbourhood of the pitot tube was fairly uniform, and the tube was a considerable distance from the solid boundary, conditions which would be far from being realised in the proposed experiments.

It was expected, therefore, that in the new work the following two difficulties would be encountered: (1) the pressure indications of the pitot tube might be considerably affected by an interference in the flow in the neighbourhood of the boundary due to the presence of the tube itself; and (2) when such a tube was placed in a current of fluid in which the variation of velocity across the mouth of the tube was very great (amounting in the extreme case when the tube touched the walls to 200 per cent. of the mean speed), it was by no means certain that the value of the speed deduced from the pressure in the pitot tube would be the speed at the geometrical centre of the tube. These objections were felt to be formidable, but it was thought that the difficulties in obtaining corrections for the interference and in interpreting the readings of the pitot tube in a current of rapidly varying speed would not prove insuperable. Owing to the extreme difficulty of an accurate estimation of the frictional resistance of a flat surface, these experiments were in all cases limited to the use of pipes of circular cross section. Experiments were made in 1911*

* Proc. Royal Society, A, Vol. 85, page 366.

on the distribution of axial velocity of air flowing in pipes of 4·9 and 7·4 cms. diameter, for which the frictional resistances could be determined for any value of the mean speed. At mean velocities of flow of 1,240 cms. per second in the 4·9 pipe and 1,905 cms. per second in the 7·4 pipe, the measured surface friction was 5·06 and 10·2 dynes per sq. cm. respectively. Taking the value of the coefficient of viscosity of air as 0·000176, this would mean, for the case of laminar flow, velocity slopes at the walls of the pipes of 29,400 and 59,400 in centimetre second units respectively. The maximum slopes actually measured as close to the wall as possible with the appliances available were, however, only 4,700 and 7,500 at distances from the wall of 0·4 mm. It would appear, therefore, that in the above cases the thickness of the layer of fluid which was in a state of laminar motion at the walls, if it existed at all, was certainly less than half a millimetre. As in these experiments the external dimensions of the pitot tube which was used for the velocity determinations was one-third of a millimetre, it was evident that the exploration of the hypothetical region of laminar flow would necessitate methods of considerably higher refinement. On account of the urgency of other work, the investigation was not carried further until, by the appointment of Miss C. N. Jones (Mrs. Bryant) as a research worker in fluid motion in 1916, an opportunity occurred of taking the matter up again, and some consideration was then given to the possibilities of measuring the velocity of flow at distances of the order of a tenth of a millimetre from the boundary. Methods other than those involving the use of a pitot tube were discussed, but before the previously tried method was abandoned it was decided to try if it would not be possible to work with a pitot tube of considerably smaller dimensions than the one previously used. After several attempts, a tube of rectangular section was finally made in the workshop, of which the external dimensions at the orifice were $0 \cdot 1 \times 0 \cdot 8$ mm., and the internal dimensions $0 \cdot 05 \times 0 \cdot 75$ mm. On setting this tube up in the centre of a pipe through which a current of air was passing, and connecting it to a sensitive tilting water gauge, the other side of the gauge being connected to a hole in the wall of the pipe, difficulties were experienced in obtaining steady readings owing to the fact that any disturbance due to a momentary draught in the room was transmitted more rapidly to that arm of the gauge connected to the static pressure hole than to that connected to the pitot tube. By inserting in the static pressure connection a piece of capillary tube of such a length that forced disturbances did not produce any appreciable effect on the gauge reading, this trouble was eliminated.

The first series of experiments were made on a small pipe of 0·714 cms. diameter, through which a current of air was passed by means of a centrifugal fan.

The small pitot tube referred to above was set up in this pipe at a section sufficiently far removed from the inlet end for the velocity distribution, in both streamline and turbulent flow, to have reached steady conditions. The movement of the tube in a radial direction was controlled by a micrometer, so that readings at any desired radius could be obtained. The general arrangement was similar to that in Fig. 1, which shows the same pitot tube attached to a 5″ pipe.

The first set of observations was directed to an estimation of the reliability of the instrument as an indicator of the square of the speed at the geometrical centre of the orifice of the pitot tube, when this was placed close to the wall of the pipe. For this purpose the speed of the flow was kept below the critical value for the pipe, so that the distribution of velocity across any section could be calculated from the formula $V = V_c \left(1 - \frac{r^2}{a^2}\right)$ where V_c is the velocity at the axis of the pipe and a is the radius of the pipe.

The value of the mean velocity of the flow at the critical value for this pipe was 580 cms. per second, so that it was not possible to set up a condition of streamline flow in it with a velocity at the axis exceeding about 1,050 cms. per second. On taking a series of observations with the pitot tube at different positions along the radius, and comparing the values of the velocities calculated from them with the theoretical values corresponding with the known speed at the axis and the distance of the geometrical centre of the pitot tube from the axis, a satisfactory check between observed speeds and theoretical speeds was obtained up to a distance of 0·25 mm. from the walls. It has been inferred by Morrow, from observations on the distribution of velocity of water flowing through a pipe at speeds below the critical, that this theoretical distribution is not that which actually exists.* This deduction, however, appears to be due to (1) an erroneous calibration of the pitot tube used, and (2) the fact that in Morrow's experiments the pitot tube was situated at a section of the tube which was approximately 30 diameters from the inlet end of the tube. It is almost certain that at this distance the radial distribution of velocity has not reached steady conditions. No appreciable departure from the parabolic distribution of flow has been found in the N.P.L. experiments at distances of 100 diameters from the inlet. At distances closer to the walls than 0·25 mm., the sensitiveness of the manometer employed was not sufficient to measure the differences between the pitot and static pressures with sufficient accuracy for the purpose of making the desired comparison. The difficulty of measuring the velocity at distances from the walls of less than 0·25 mm. will be realised from the fact that, with a velocity at the axis of

* Proc. Royal Society. Vol. 76, page 205.

1,050 cms. per second, the velocity at a distance of 0·1 mm. from the wall is only 58 cms. per second, corresponding with a pressure head on the manometer of 0·02 mm. of water. As the readings of the very sensitive manometer employed for the work could not be depended upon to an accuracy greater than $\pm$ 0·005 mm. of water, it is obvious that no reliable comparison between the actual and theoretical speeds at this distance from the wall could be obtained.

The proposed method of calibration of the pitot tube readings when the tube was very close to the walls proved, therefore, to be one of considerable difficulty. Several months were spent in trying to improve the accuracy of the observations, but it was finally concluded that the only satisfactory method of obtaining the comparison was by the use of an experimental pipe of a diameter of the order of 0·25 cm., in which case the maximum speed at a distance of 0·1 mm. from the walls, under streamline conditions of flow, would be about 460 cms. per second. The mechanical difficulties in the fitting of a pitot tube in a pipe of such small dimensions without obstructing the flow were obviously very formidable, and after some consideration it was decided to postpone the attempt at calibration until the observations when the motion was turbulent had been completed.

Accordingly observations of the pressure differences between the pitot and static pressure tubes for a given distance of the geometrical centre of the former from the walls were taken over as wide a range in the mean rate of flow through the pipe as possible. At the same time, the corresponding slopes of the static pressure gradient down the pipe were accurately determined. From the latter measurements the values of the surface friction at the walls for different rates of flow were calculated. The pitot tube observations were then repeated for other distances of the tube from the walls (Table 1). For each distance a curve of speed variation with surface friction was plotted, and from these curves it was possible to scale off, for any given value of the surface friction, the speeds corresponding to the different distances of the pitot tube from the walls, and so plot a curve showing the variation of speed with position of pitot tube. These curves are shown in Fig. 3. It will be realised, of course, that these curves can only be taken as showing the radial distribution of velocity in the neighbourhood of the walls, under the assumption that the speed indicated by the pitot tube is that which exists at the geometrical centre of the orifice, *i.e.*, that there is no interference with the flow near the walls due to the presence of the tube. In the same figure are shown in dotted lines the velocity slopes which would exist at the boundary if the flow were laminar. These are calculated from the known frictional resistances and the value of the coefficient of viscosity for air. Another interesting feature of the velocity distribution is the manner in which the speed at any given distance from the walls varies with the frictional resistance. If the flow were laminar, the velocity at any point would, of course, be simply proportional to the surface friction. For turbulent flow, it is known that the surface friction is proportional to the nth power of the mean rate of flow, where n varies from $1 \cdot 75$ to 2 according to the diameter and roughness of the pipe. This relation is usually obtained by plotting points of which the ordinates and abscissæ are the logarithms of the surface friction and mean flow. These points are found to be on a straight line, of which the slope gives the value of n. Adopting the same method for velocities at a point, the curves shown on Fig. 4 have been obtained. These show that the relation between surface friction and speed at a point near the wall is of the same form as that between surface friction and mean flow, but that the value of n diminishes as the boundary is approached until, when the geometrical centre of the tube is $0 \cdot 075$ mm. from the wall, the value of n is $1 \cdot 16$. It would appear, therefore, that at this distance the eddy motion has nearly disappeared.

As the experiments in this small pipe ($0 \cdot 714$ cms. diameter) were made at speeds not greatly exceeding the critical speed, *i.e.*, at values of $V . d / \nu$ between 2,000 and 5,000 (where ν = kinematic viscosity of air, V = mean rate of flow, and d = diameter of pipe), it was considered desirable to repeat the experiments at values of $V . d / \nu$ very much greater than this, as it is known that the distribution of the velocity across a pipe changes appreciably with the value of this function.

For this purpose a pipe of $12 \cdot 7$ cms. diameter was used. Air was supplied to this by means of a high speed Sturtevant fan which could maintain a constant rate of flow through the pipe of about 2,900 cm. per second, corresponding with a value of $V . d / \nu$ of 250,000. The same pitot tube was used as in the small pipe, the method of attachment being shown in Fig. 1. Experiments similar to those described above were made by Miss Marshall, who took up the work after the transfer of Miss C. N. Jones to the Wind Channel Staff. The observations (Table 2) were reduced in the same way as before, and the curves of variation of indicated speed with mean rate of flow and with position of pitot tube are shown in Fig. 5. On examining the curves in Fig. 3 and 5 and assuming the interference effects to be negligible, the following characteristics will be seen to be common to both:

(1) The slopes of the tangents to the velocity curves at distances of $0 \cdot 075$ mm. and above from the walls are very much smaller than would be the case if the flow were laminar at these distances from the wall. For example, when the surface friction in the $0 \cdot 714$ cm. pipe is 10 dynes per sq. cm., the value of dV / dr at $0 \cdot 075$ mm. from the wall would be 57,200 if the flow were streamline in character. The measured value of

dV/dr was 33,600. In the case of the 12·7 cm. pipe, the corresponding values are 211,200 for streamline motion, and 90,000 actually observed.

(2) The speeds calculated from the observations at distances of 0·075 mm. and less from the walls are appreciably higher than would exist in laminar flow with the measured surface friction. For example, in the 0·714 cm. pipe with a frictional resistance of 10 dynes per sq. cm., the calculated speed at 0·075 mm. from the walls was 795 cms. per second. In laminar flow with this value of the surface friction the speed would be 443 cms. per second. In the case of the 12·7 cm. pipe at a value of the surface friction of 36·1 dynes per sq. cm. and at a distance of 0·5 mm. from the wall, the corresponding values are 1,230 for the calculated speed, and 1,050 for the speed in streamline flow.

To sum up the evidence afforded by the curves of Figs. 3 and 5, it would appear that if it may be assumed that the speed indicated by the pitot tube is the speed of the fluid at its geometrical centre during the experiment, streamline motion if it exists must be confined to a region of less than 0·05 mm. from the boundary, and further, the observations are not inconsistent with a finite amount of slip at the boundary.

It was evident, therefore, that the proof of the existence or otherwise of streamline motion at the boundary would involve a closer exploration of the region near the boundary than had hitherto been possible. This, however, could not be done with the available appliances, owing to the fact that any further reduction in the cross dimensions of the pitot tube would have rendered it unworkable owing to the time fluctuations of pressure in the room. After some consideration it was thought that if the wall of the pitot tube adjacent to the wall of the pipe could be removed and its place taken by the wall of the pipe itself, it would be possible to obtain readings at distances of 0·025 and 0·01 mm. from the wall. The arrangement of tube previously used was accordingly modified, and the final form adopted is shown in Fig. 2. This was set up in the 12·7 cm. pipe and a series of observations (Table 3) similar to those previously described were made and reduced in the same way as before. The final results are shown in the curves of Fig. 6, which give the variation of the velocity calculated from the new pitot tube pressures with the surface friction and with the distance of the geometrical centre of the tube opening from the wall of the pipe.

From an examination of these curves it will be seen that by means of the new device, the nature of the flow in the region of the boundary is revealed to a very much greater extent than was possible with the original form of pitot tube, and that the new evidence afforded by the velocity estimations up to within 0·01 mm.

from the wall strengthens considerably the probability of the existence at the boundary of a layer of fluid of finite thickness which is in streamline motion.

The existence of a slip at the boundary is not, however, decidedly negatived, as it will be seen from Fig. 6 that the values of the speeds at 0·01 mm. from the wall are much higher than those corresponding with the velocity slope at the boundary calculated from the known viscosity of air and the measured friction. In order to obtain more definite evidence on this point, the possibility of a calibration of the pitot tube in a current of air in which the distribution of velocity up to the boundary was known, was re-examined, and in view of the success which had attended the use of the new form of pitot tube, it was decided to attempt the fitting of a tube of this form to a pipe of about 0·25 cms. diameter. The construction of this attachment, it is needless to say, required considerable skill and accuracy, and much credit is due to Mr. H. G. Pincott, of the Mechanical Staff of the Engineering Department, for the success with which it was carried out. The diameter of the pipe used was 0·269 cms., the pitot tube (here referred to as No. 3) was 1·27 mm. wide, and the upper wall had a thickness of 0·05 mm.

In the experiments with this pipe the outlet end was connected by a rubber tube to a gas holder, of which the rate of displacement could be varied by means of the tension in the supporting cord. It was possible, therefore, in these tests to make a series of observations at a constant value of the surface friction, and there was no necessity to reduce the actual observations by preliminary plotting to obtain these results. The first series of tests were made with the pitot tube at a given distance from the wall and over as large a range in the rate of flow as possible (Table 4). These observations were then repeated for another distance of the tube from the wall. The results are shown plotted in Fig. 7, in which the ordinates of the plotted points are the speeds indicated by the pitot tube and the abscissæ the rates of mean flow. In the same figure are also plotted the values of the surface friction obtained from the fall of static pressure in the pipe (Table 5). It will be seen that at rates of flow below the critical, when the flow over the whole pipe was streamline in character, the plotted points in the velocity curves lie on straight lines passing approximately through the origin. It may be concluded, therefore, that these calculated speeds are the values of the speeds existing at a certain point near the boundary, although this point may not be the geometrical centre of the pitot tube orifice. The desired calibration of the pitot tube appeared, therefore, to be a simple matter and was made as follows. For a certain value of the mean rate of flow below the critical value, a series of readings of the pitot tube pressures were made at different distances of the geometrical centre of the pitot tube from the wall (Table 6). From the equation of the distribution of velocity in streamline flow,

$$V = V_c\left(1 - \frac{r^2}{a^2}\right),$$

the values of r were calculated for each value of V obtained from the observations. Points were then plotted whose ordinates were the values of $(a - r)$ and whose abscissæ were the corresponding distances of the geometrical centre of the pitot tube from the wall. This process was repeated for different values of the mean flow, all below the critical value, and the mean curve through all the plotted points was taken to be the calibration curve for the pitot tube. This curve is shown in Fig. 8. It will be seen that for openings of the pitot tube of the order of 0·3 mm. the assumption that the calculated speed is that which exists at the geometrical centre of the orifice is not greatly in error, but that when the opening is of the order of 0·075 mm., the interference with the flow is so considerable that the calculated speed is that which exists at the edge of the pitot tube furthest from the wall.

A series of observations were then made at known values of the surface friction and with the centre of the pitot tube at different distances from the wall (Table 7). The speeds calculated from these observations are shown plotted in two methods in Fig. 9. On the left hand side of the figure the abscissæ of the points are the distances of the centre of the pitot tube from the wall, and on the right hand side the abscissæ are the "effective distances" of the pitot tube from the wall, as scaled off from the calibration curve of Fig. 8. It is realised, of course, that the application of the calibration curve of Fig. 8 to conditions of flow above the critical value may be open to some criticism on the ground that, in the latter case, the pitot tube is placed in fluid which is in turbulent motion, and, therefore, the magnitude of the interference at a given distance from the wall may not be the same as in laminar flow. To this objection it may be pointed out that, from the evidence afforded by the curves in Fig. 4, the amount of turbulence at the distances in question must be very small, from the fact that in this region the surface friction is proportional to V^n where n is very little in excess of unity. Assuming, therefore, that such an assumption is legitimate, the curves on the right hand side of Fig. 9 show the velocity distribution near the wall when the readings of the pitot tube are corrected for interference.

On examination of the two sets of curves, it will be seen that the effect of the correction for interference in the slope of the curves at distances from the wall of the order of 0·3 mm. is practically negligible as would be expected, but that at distances of the order of 0·05 mm. the slopes when corrected for interference are considerably increased, and approximate fairly closely to the values existing in laminar flow. Further, there is no indication of the existence of any slip at the boundary. It was concluded, therefore, that in the turbulent flow of air through this small pipe at speeds of about double the critical speed, the exploration of the flow near the boundary by means of a fine pitot tube showed unmistakably the existence at the boundary of a finite layer of fluid in laminar motion and having zero velocity at the boundary. As the rates of flow in the experiments in the 0·269 cm. pipe were not greatly in excess of the critical value, it was decided to repeat the experiments in the 12·7 cm. pipe, and for this purpose the No. 3 pitot tube was transferred to this pipe and a series of observations made with it at fairly high rates of flow (Table 8). The results are shown plotted in Fig. 10, and the values of the velocities reduced from these curves to show the variation of the speed at constant values of the surface friction are shown in Fig. 11, in the same way as the results illustrated in Fig. 9, *i.e.*, on the left are shown the curves of velocity variation with distance of centre of pitot tube from wall, and on the right the same results corrected for the interference, on the assumption that this is unaffected by the radius of curvature of the walls. This assumption may not be true to a high degree of accuracy, but it is obviously the only one which could be made under the circumstances. It will be seen that when the correction for interference is made, the conditions of flow at the boundary are in complete agreement with those found for the case of the 0·269 cm. pipe in indicating the existence at the boundary of a finite layer of fluid in laminar motion and having zero velocity at the boundary.

TABLE 1.

Diameter of Pipe $= 0{\cdot}714$ cm.

d = Distance of centre of Pitot from Wall. v = Surface speed. R = Friction.

$d = 3{\cdot}568$ mm.		$d = 0{\cdot}508$ mm.		$d = 0{\cdot}279$ mm.		$d = 0{\cdot}179$ mm.		$d = 0{\cdot}127$ mm.		$d = 0{\cdot}076$ mm.	
v (cm. per sec.).	R (dynes per sq. cm.).	v (cm. per sec.).	R (dynes per sq. cm.).	v (cm. per sec.).	R (dynes per sq. cm.).	v (cm. per sec.).	R (dynes per sq. cm.).	v (cm. per sec.).	R (dynes per sq. cm.).	v (cm. per sec.).	R (dynes per sq. cm.)
805	2·00	420	1·68	315	1·35	160	0·84	160	1·22	170	1·68
855	2·60	555	2·29	525	2·87	445	3·46	255	2·27	260	2·72
1,095	4·28	630	2·80	720	4·31	690	5·70	525	5·05	425	4·90
1,315	6·0	765	3·89	780	5·15	880	7·8	695	7·0	660	8·1
1,550	8·3	925	5·35	950	6·95	1,050	10·0	905	9·65	830	10·7
1,730	10·1	1,015	6·3	1,050	7·75	1,090	10·7	1,120	12·6	920	12·2
1,800	10·9	1,110	7·5	1,190	9·55	1,230	12·6	1,125	13·0	—	—
1,817	11·0	1,235	9·0	1,250	10·8	1,325	14·1	—	—	—	—
1,920	12·3	1,560	13·6	1,410	13·0	—	—	—	—	—	—

TABLE 2.

No. 1 PITOT IN 12·7 cm. PIPE.

d = 0·267 mm.		d = 0·216 mm.		d = 0·165 mm.		d = 0·114 mm.		d = 0·076 mm.		d = 0·051 mm.	
v (cm. per sec.).	R (dynes per sq. cm.).	v (cm. per sec.).	R (dynes per sq. cm.).	v (cm. per sec.).	R (dynes per sq. cm.).	v (cm. per sec.).	R (dynes per sq. cm.).	v (cm. per sec.).	R (dynes per sq. cm.).	v (cm. per sec.).	R (dynes per sq. cm.).
530	3·80	500	4·26	420	4·07	340	4·00	260	3·1	300	4·9
925	7·9	845	7·9	710	7·9	465	6·2	455	6·9	405	6·7
1,090	9·9	990	9·8	830	9·6	600	8·05	600	10·6	480	9·0
1,300	12·8	1,185	12·7	1,045	12·5	700	10·1	815	16·0	585	11·9
1,525	16·5	1,435	16·7	1,285	16·8	705	10·2	1,065	23·8	705	16·1
1,690	19·8	1,620	20·2	1,495	20·8	820	12·8	1,275	30·1	810	19·5
1,890	23·6	1,795	23·7	1,645	23·8	1,030	16·4	—	—	935	23·8
—	—	—	—	—	—	1,150	18·8	—	—	1,095	29·9
—	—	—	—	—	—	1,320	22·8	—	—	—	—

TABLE 3.

NO. 2 PITOT IN 12·7 cm. PIPE.

d = 0·127 mm.		d = 0·076 mm.		d = 0·051 mm.		d = 0·038 mm.		d = 0·025 mm.		d = 0·019 mm.		d = 0·013 mm.	
v (cm. per sec.).	R (dynes per sq. cm.).	v (cm. per sec.).	R (dynes per sq. cm.).	v (cm. per sec.).	R (dynes per sq. cm.).	v (cm. per sec.).	R (dynes per sq. cm.).	v (cm. per sec.).	R (dynes per sq.cm.)	v (cm. per sec.).	R (dynes per sq.cm).	v (cm. per sec.).	R (dynes per sq.cm.)
185	2·16	165	2·52	130	2·52	130	3·02	105	3·74	105	4·69	95	5·60
295	3·78	235	4·01	190	4·10	185	4·69	195	6·3	160	6·65	125	7·85
420	5·85	320	6·00	270	6·25	240	6·55	245	8·45	220	8·95	175	10·6
530	8·13	410	8·15	330	7·95	295	8·5	305	11·1	265	11·5	215	14·0
660	10·9	500	10·5	405	10·3	375	11·3	380	14·7	330	14·9	260	17·1
800	14·3	610	13·4	500	13·4	455	14·4	435	18·1	370	17·4	300	20·2
920	17·5	730	17·2	590	17·1	520	17·5	470	19·8	425	21·4	335	22·8
1,025	20·8	825	20·3	675	20·4	595	21·1	525	22·9	470	24·6	360	24·3
1,115	23·3	910	23·3	750	23·3	695	25·9	590	26·1				
1,160	25·0	980	25·8	810	25·8								

Table 4.

No. 3 PITOT IN 0·269 cm. PIPE.

Distance of Effective Centre 0·1118 mm.		Distance of Effective Centre 0·086 mm.	
Mean Speed V (cm. per sec.).	Surface Speed v (cm. per sec.).	Mean Speed V (cm. per sec.).	Surface Speed v (cm. per sec.).
255	95	315	76
480	155	710	160
575	190	1,000	215
780	240	1,345	285
960	285	1,420	300
1,210	350	1,460	295
1,385	390	1,555	310
1,465	410	1,575	320
1,490	450	1,635	415
1,535	525	1,665	510
1,590	645	1,700	560
1,690	730	1,830	630
1,820	815	1,970	695
2,125	1,010	2,105	785
2,415	1,195	2,290	855
2,695	1,370	2,350	940
2,900	1,525	2,400	940
—	—	2,535	1,015
—	—	2,665	1,080
—	—	2,900	1,230

Table 5.

No. 3 PITOT IN 0·269 cm. PIPE.

Mean Speed V (cm. per sec.).	Friction R (dynes per sq. cm.).
255	1·15
595	2·80
790	3·88
955	4·60
1,185	6·15
1,325	6·40
1,390	6·75
1,465	7·15
1,545	7·65
1,570	9·65
1,590	10·9
1,645	14·3
1,745	16·5
1,810	17·6
1,960	20·6
2,100	23·4
2,380	30·1
2,650	36·4
2,925	42·4
—	—

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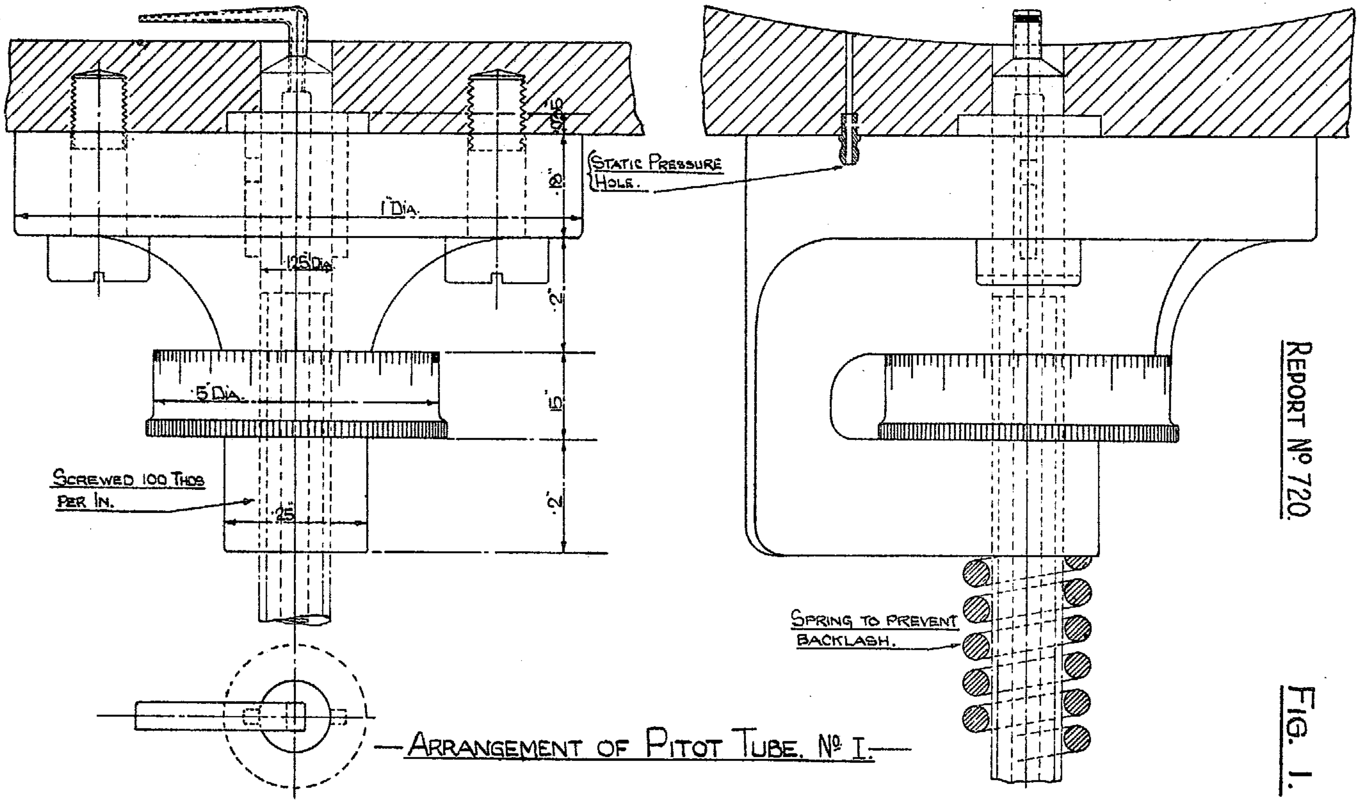

— Arrangement of Pitot Tube. No I. —

Report No 720.

Fig. 1.

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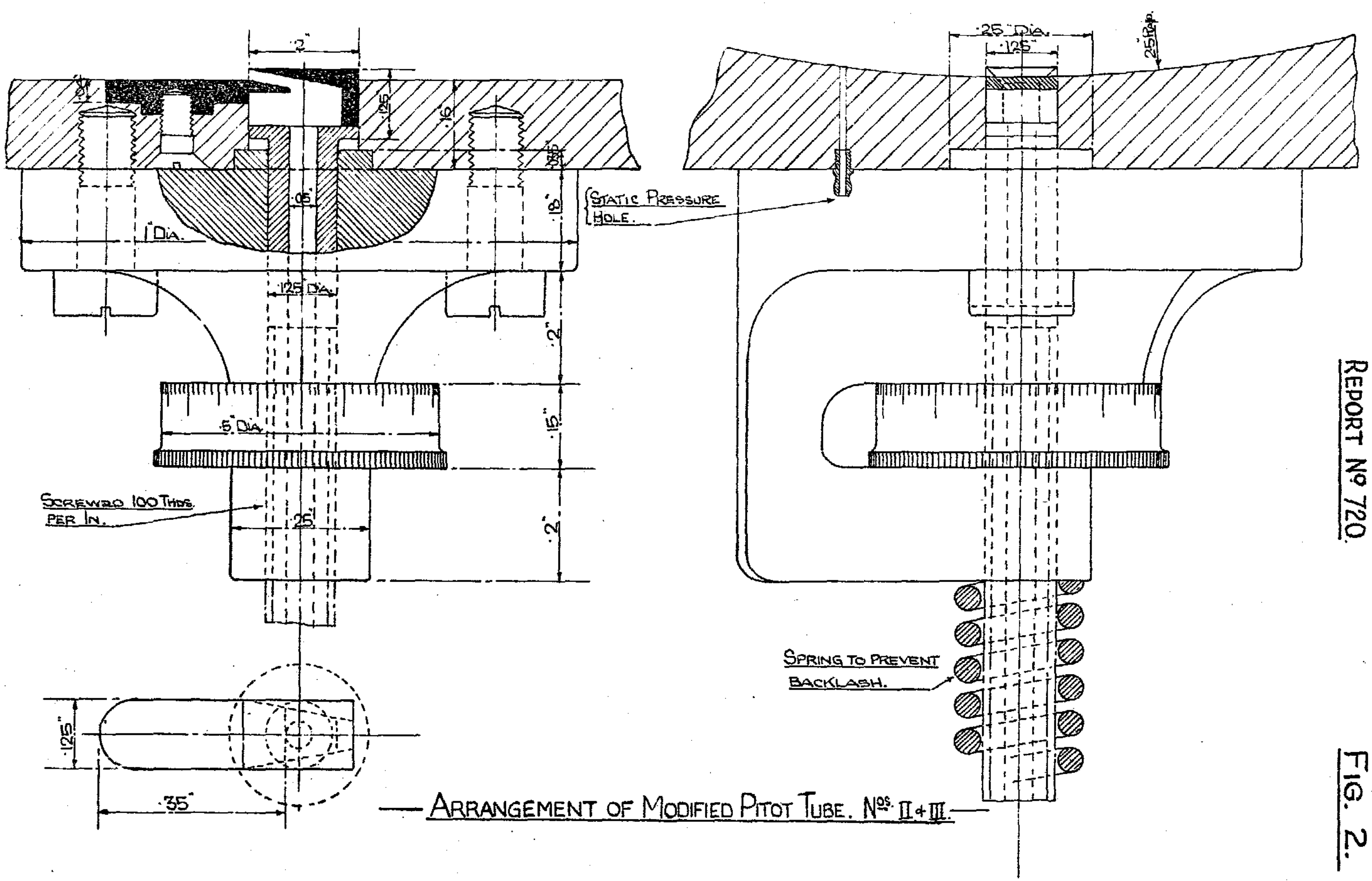


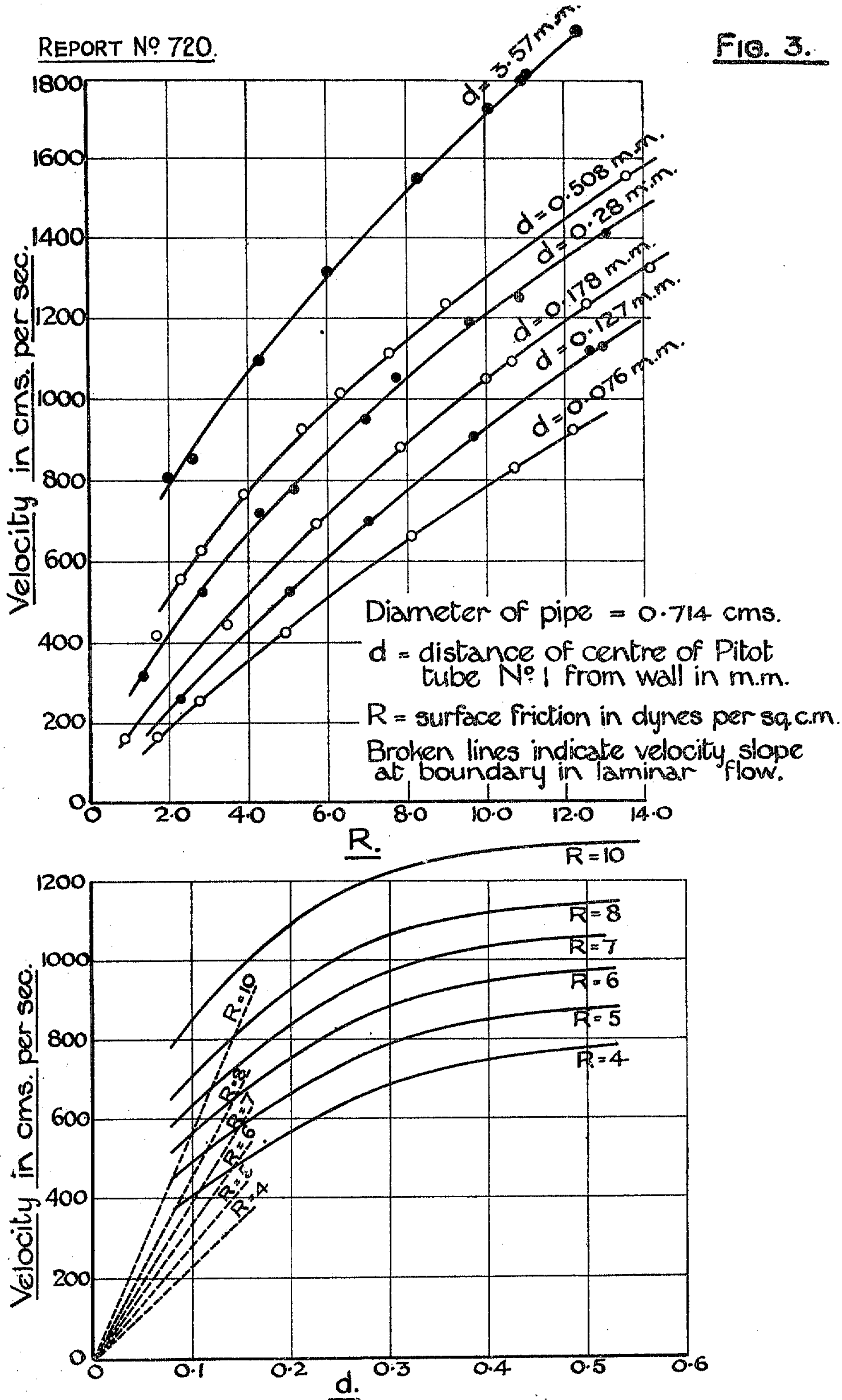
ARRANGEMENT OF MODIFIED PITOT TUBE. Nos. II + III.

REPORT No 720.

FIG. 2.

REPORT Nº 720.

FIG. 3.



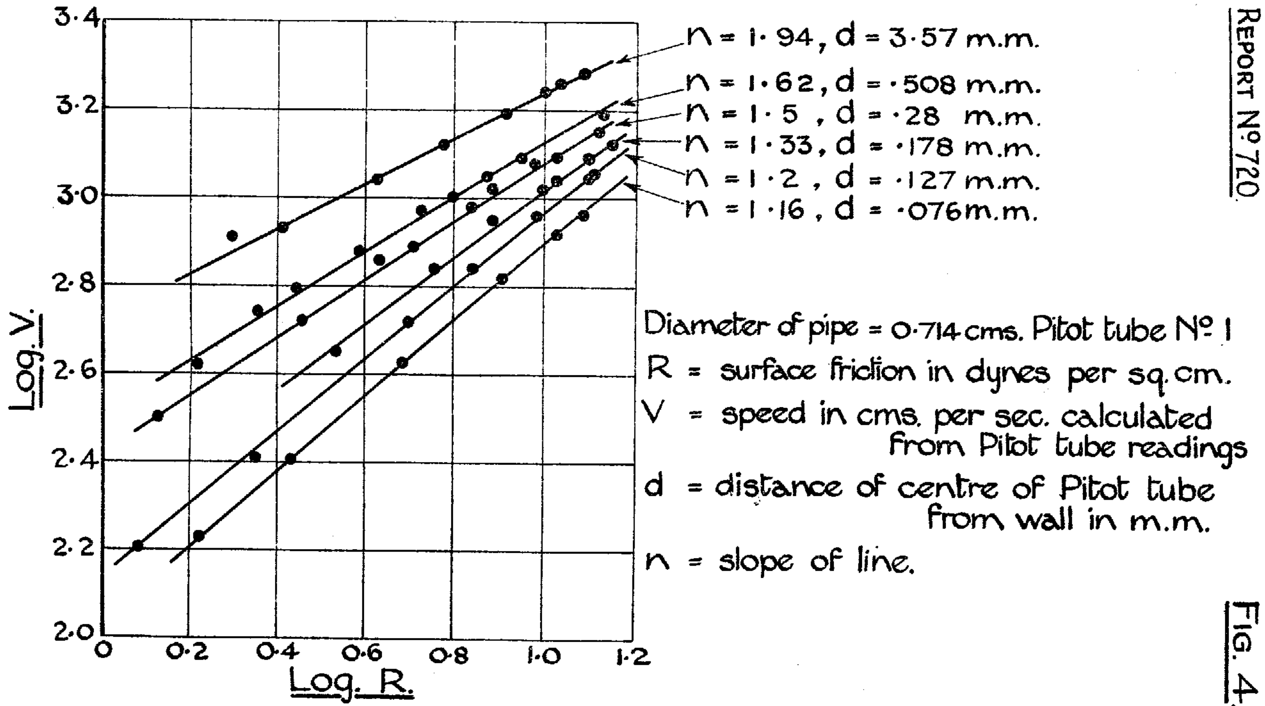

3·4
3·2
3·0
2·8
2·6
2·4
2·2
2·0
Log. V.
0
0·2
0·4
0·6
0·8
1·0
1·2
Log. R.
n = 1·94, d = 3·57 m.m.
n = 1·62, d = ·508 m.m.
n = 1·5 , d = ·28 m.m.
n = 1·33, d = ·178 m.m.
n = 1·2 , d = ·127 m.m.
n = 1·16 , d = ·076 m.m.
Diameter of pipe = 0·714 cms. Pitot tube No 1
R = surface friction in dynes per sq. cm.
V = speed in cms. per sec. calculated from Pitot tube readings
d = distance of centre of Pitot tube from wall in m.m.
n = slope of line.

FIG. 4.

REPORT No 720.

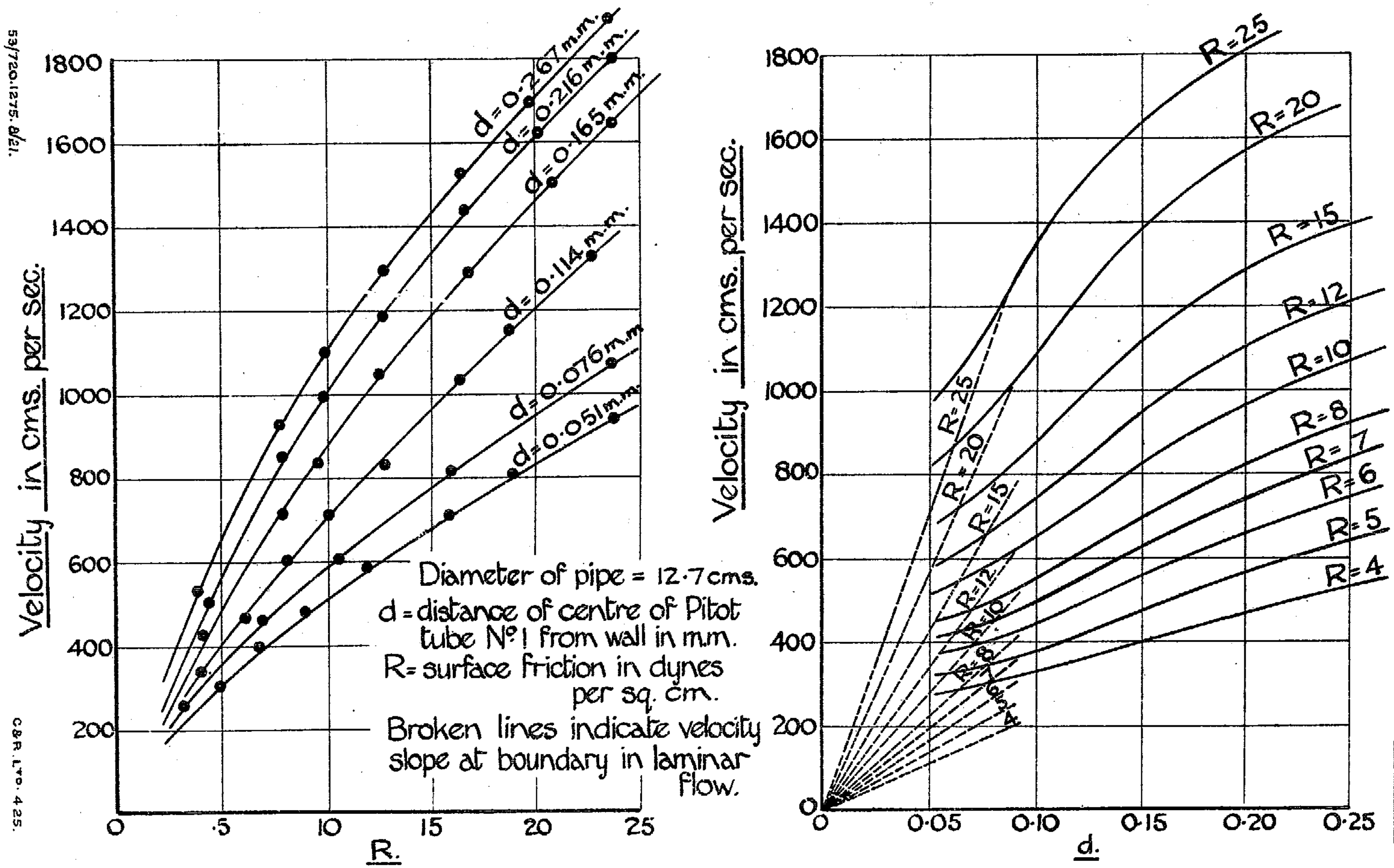


FIG. 5.

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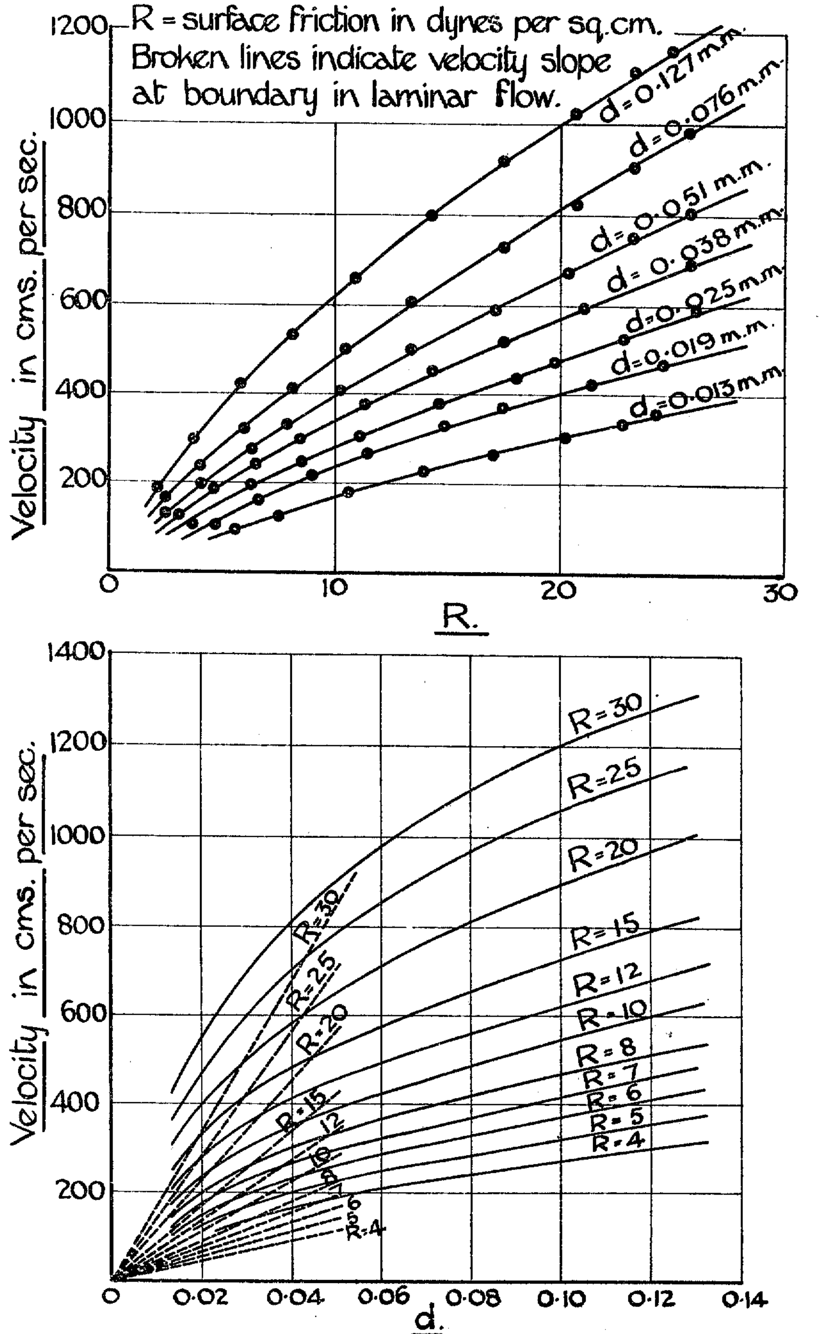
REPORT No 720. Diameter of pipe = 12·7 cms. FIG. 6.

Pitot tube No 2

d = distance of centre of Pitot tube from wall in cms.



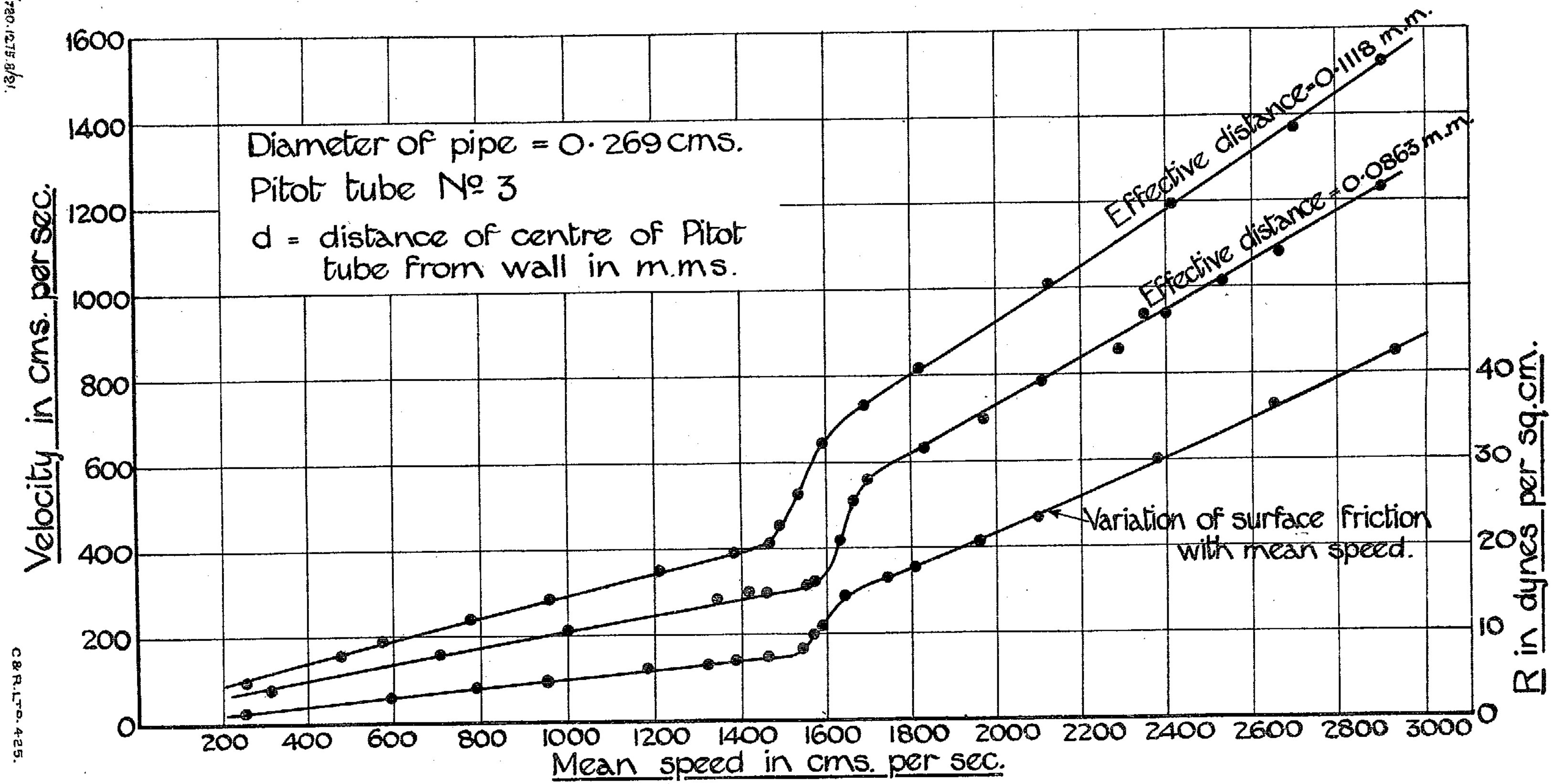


FIG. 7.

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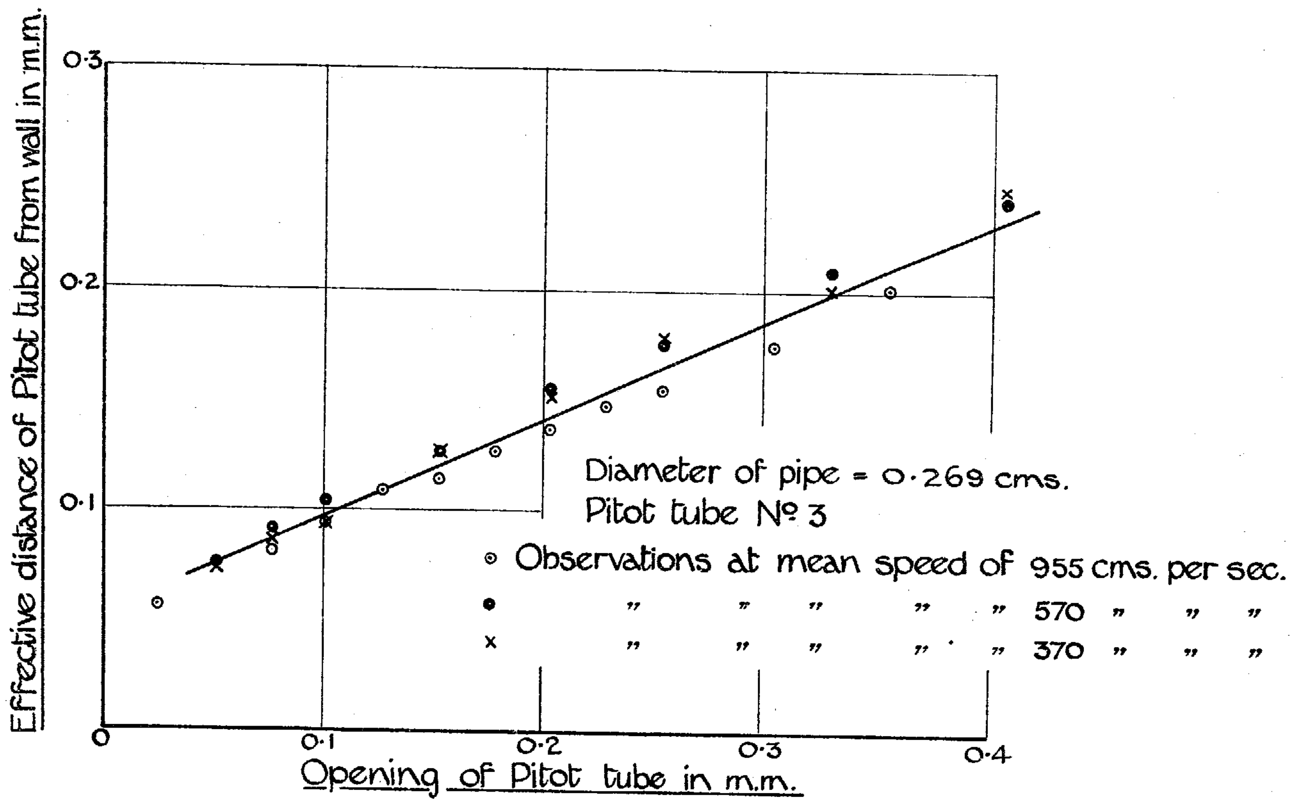

Effective distance of Pitot tube from wall in m.m.
0·3
0·2
0·1
0
0·1
0·2
0·3
0·4
Opening of Pitot tube in m.m.
Diameter of pipe = 0·269 cms.
Pitot tube Nº 3
⊙ Observations at mean speed of 955 cms. per sec.
● " " " " " 570 " " "
x " " " " " 370 " " "

FIG. 8.

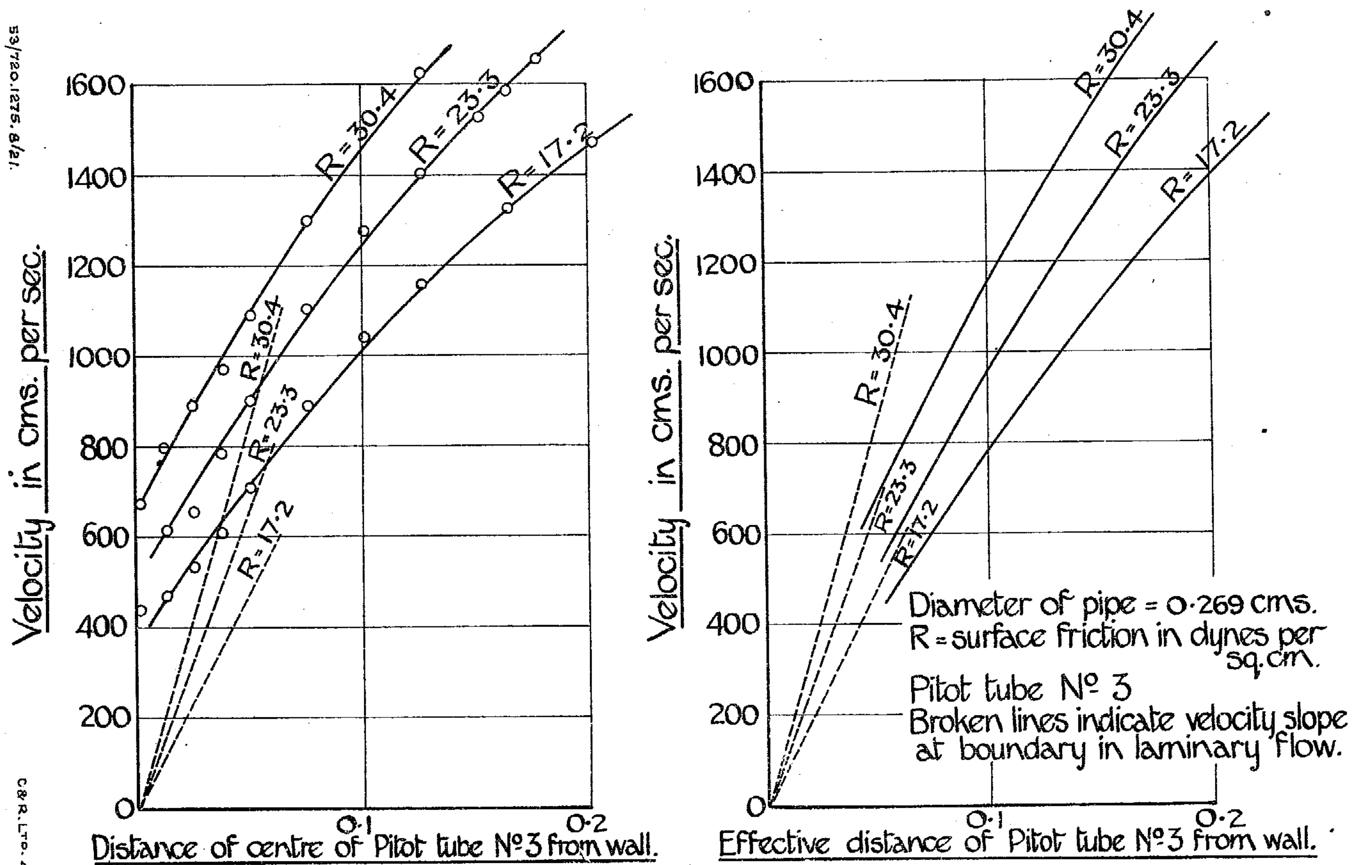
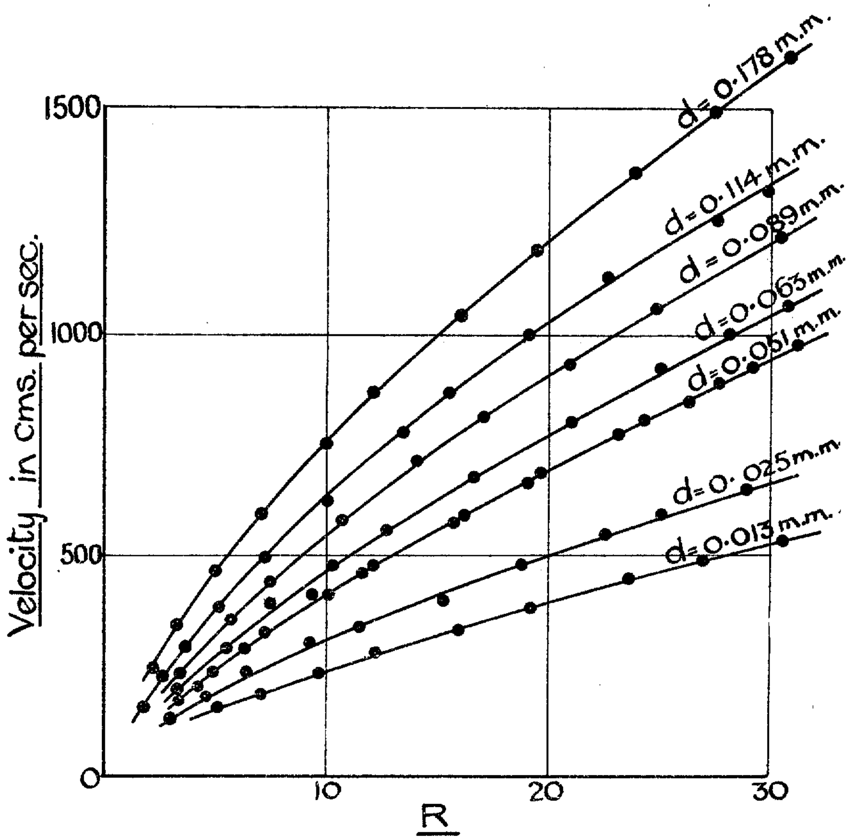

1600
1400
1200
1000
800
600
400
200
0
0·1
0·2
R=30·4
R=23·3
R=17·2
Velocity in cms. per sec.
Distance of centre of Pitot tube No 3 from wall.
Effective distance of Pitot tube No 3 from wall.
Diameter of pipe = 0·269 cms.
R = surface friction in dynes per sq. cm.
Pitot tube No 3
Broken lines indicate velocity slope at boundary in laminary flow.

FIG. 9.

Diameter of pipe = 12·7 cms.

d = distance of centre of Pitot tube Nº 3 from wall.

R = surface friction in dynes per sq. cm.



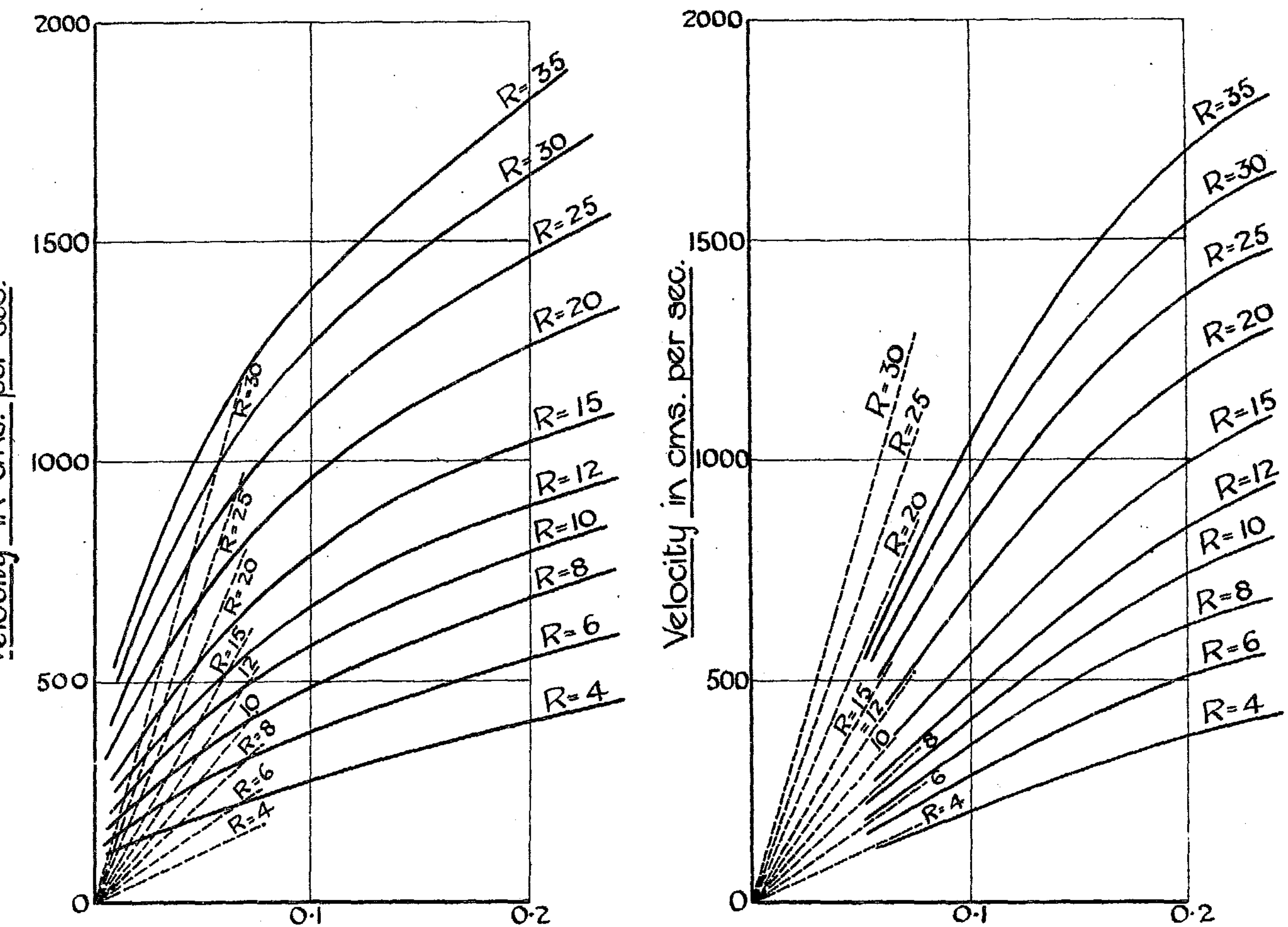


FIG. 11.

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Table 6.

No. 3 PITOT IN 0·269 cm. PIPE.

Pitot Tube Opening (mm.).	Mean Speed = 955 cm. per sec.		Mean Speed = 570 cm. per sec.		Mean Speed = 370 cm. per sec.	
	Surface Speed v (cm. per sec.)	Calculated Effective Distance (mm.).	Surface Speed v (cm. per sec.).	Calculated Effective Distance (mm.).	Surface Speed v (cm. per sec.).	Calculated Effective Distance (mm.).
0·025	160	0·056	—	—	—	—
0·051	200	0·074	125	0·076	80	0·074
0·076	225	0·081	150	0·091	90	0·086
0·102	260	0·094	170	0·104	105	0·094
0·127	300	0·109	—	—	—	—
0·152	310	0·114	205	0·127	135	0·127
0·178	345	0·127	—	—	—	—
0·203	370	0·137	245	0·155	160	0·152
0·229	395	0·147	—	—	—	—
0·254	420	0·155	280	0·175	185	0·178
0·305	465	0·175	—	—	—	—
0·330	—	—	325	0·208	215	0·201
0·356	530	0·201	—	—	—	—
0·406	—	—	370	0·241	250	0·246

TABLE 7.

No. 3 PITOT IN 0·269 cm. PIPE.

Mean Velocity = 2,400 cm. per sec. Mean Friction = 30·4 dynes/sq. cm.			Mean Velocity = 2,095 cm. per sec. Mean Friction = 23·3 dynes/sq. cm.			Mean Velocity = 1,810 cm. per sec. Mean Friction = 17·2 dynes/sq. cm.		
Mean Distance (mm.).	Effective Distance (mm.).	Surface Speed (cm. per sec.).	Mean Distance (mm.).	Effective Distance (mm.).	Surface Speed (cm. per sec.).	Mean Distance (mm.).	Effective Distance (mm.).	Surface Speed (cm. per sec.).
0·127	0·159	1,625	0·178	0·198	1,655	0·203	0·216	1,470
0·076	0·118	1,300	0·165	0·188	1,585	0·165	0·188	1,325
0·051	0·097	1,090	0·152	0·178	1,525	0·127	0·158	1,160
0·038	0·084	970	0·127	0·159	1,405	0·102	0·137	1,040
0·025	0·071	890	0·102	0·140	1,275	0·076	0·117	890
0·013	0·056	795	0·076	0·118	1,105	0·051	0·094	710
0·006	0·046	675	0·051	0·097	900	0·038	0·083	610
—	—	—	0·038	0·084	785	0·025	0·071	535
—	—	—	0·025	0·071	655	0·013	0·055	470
—	—	—	0·013	0·056	615	0·006	0·044	440

TABLE 8.

No. 3 PITOT IN 12·7 cm. PIPE.

d = 0·178 mm.		*d* = 0·114 mm.		*d* = 0·089 mm.		*d* = 0·063 mm.		*d* = 0·051 mm.		*d* = 0·025 mm.		*d* = 0·013 mm.	
v (cm. per sec.).	R (dynes per sq. cm.).	*v* (cm. per sec.).	R (dynes per sq. cm.).	*v* (cm. per sec.).	R (dynes per sq. cm.).	*v* (cm. per sec.).	R (dynes per sq. cm.).	*v* (cm. per sec.).	R (dynes per sq.cm.)	*v* (cm. per sec.).	R (dynes per sq.cm).	*v* (cm. per sec.).	R (dynes per sq.cm.)
								(i)					
245	2·30	155	1·85	235	3·5	195	3·4	170	3·45	130	3·11	155	5·2
340	3·38	225	2·75	355	5·8	290	5·6	240	5·0	180	4·7	185	7·2
465	5·15	290	3·75	440	7·6	370	7·6	325	7·3	235	6·5	235	9·8
595	7·1	385	5·3	580	10·8	475	10·4	410	10·1	300	9·4	280	12·3
755	10·1	495	7·35	715	14·2	555	12·8	475	12·2	335	11·6	330	16·1
870	12·1	625	10·1	815	17·1	680	16·7	595	16·3	395	15·4	385	19·3
1,045	16·1	780	13·5	935	21·0	805	21·1	690	19·7	480	18·9	450	23·7
1,190	19·5	870	15·6	1,060	24·9	925	25·1	810	24·3	550	22·6	490	27·0
1,365	23·9	1,000	19·1	1,220	30·5	1,005	28·2	895	27·7	595	25·1	535	30·7
1,495	27·5	1,130	22·7			1,070	30·8	980	31·3	655	29·0		
1,615	30·9	1,260	27·7										
		1,325	29·9					(ii)					
								205	4·15				
								290	6·4				
								420	9·4				
								460	11·7				
								575	15·8				
								665	19·1				
								780	23·2				
								850	26·4				
								930	29·3				