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Scatter in Fatigue:-  
Elements and Sections  
from Aircraft Structures

by

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SCATTER IN FATIGUE:- ELEMENTS AND SECTIONS FROM AIRCRAFT STRUCTURES

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A. M. Stagg

SUMMARY

The fatigue test results, for elements and sections from the structures of aircraft, obtained by a number of experimenters are analysed in terms of the scatter present in the lives to fatigue failure. Data for structures made of any light alloy material and tested under any form of loading have been included in an attempt to increase the definition of any trends noted in the scatter, which was calculated using a log-normal distribution of fatigue lives as a basis.

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## 1 INTRODUCTION

Whereas the manufacturing and machining processes of laboratory specimens are usually closely controlled and in most cases all specimens forming one set of tests are produced from one batch of material, the assembly of actual aircraft structures is by no means so rigorously governed. The variation in properties caused by corresponding parts in the different structures being made from several batches of material, the interaction between the varying degrees of interference and looseness of fit allowed by the machining tolerances, the different techniques and skills of the various fitters employed on the assembly line, all these factors may be sources of scatter that do not arise in laboratory specimens. Thus, although laboratory specimens may provide an insight into some of the basic parameters that decide the magnitude of the variation that is always apparent in a set of fatigue tests of apparently identical specimens under identical loading conditions<sup>1,2</sup>, the scatter in the fatigue properties of elements or sections from aircraft structures is also of great interest to aircraft designers. A knowledge of the distribution of the probability of fatigue failure of the structure of the aircraft in a fleet makes possible an evaluation of the safety of the fleet and enables estimates to be made of the economics of running and repairing that fleet to comply with the safety regulations.

The aim of this analysis is therefore to present as many data as possible on the variations in the fatigue properties of structural elements and of complete built-up sections from aircraft and to indicate any apparent trends in the scatter observed. However, few data on tests to determine the fatigue properties of such specimens are available, as the expense involved is considerable, and so the conclusions reached are only tentative. It seems likely that the number of elements or sections of aircraft of any one type that are fatigue tested is always going to be small and so if any extra confidence is to be given to assumed values for the scatter in aircraft structures another source of data must be found. Such data are available from the occurrence of cracks in aircraft in service and if rigorously collected could be most valuable in expanding the small numbers of test results which are analysed below.

## 2 A REVIEW OF PREVIOUSLY USED FATIGUE FAILURE DISTRIBUTIONS

The time and the cost involved in the fatigue testing of complete aircraft structures have limited the number of studies that have been made of the characteristics of the scatter in fatigue performance achieved by composite structures. Thus the number of sets of tests which can give any relevant

information as to the form of the distribution of the probability of fatigue failure is also very small. This paucity of results and the fact that in general aircraft are not made throughout from just one material led to the inclusion, in this Report, of data for built-up structures made from any aluminium alloy and tested under any form of fatigue loading. Also several sets of tests have been conducted on wings and elements from wings that have been flown in service either until they were life expired or until for some other reason they were withdrawn from use. The precise fatigue history experienced by such wings is not usually known and so allowance is made for this part of the life of each wing by assuming that each aircraft has been subjected to the average load spectrum for the fleet. This assumption obviously introduces inaccuracies into the final lives, obtained after failure on a test rig, but as the service lives of such wings are usually only 5 to 10% of the final lives the discrepancies introduced are probably small, and so these sets of data are also included in this Report.

Perhaps the largest set of tests on similar built-up structures is that reported in Refs.3 and 4 by Parish and Grove on mainplanes that had been flown in service on Piston Provost aircraft. Forty one half wings were tested under six level block spectrum loading with a high-low sequence of load level applications (see Ref.2), whilst in a second set 11 half wings were tested to the same load spectrum but applied in a low-high sequence of load levels. The cumulative distributions of the probability of fatigue failure obtained from these two sets of tests are plotted in Fig.1 in terms of a normal distribution of the logarithm of the fatigue lives achieved and the agreement with this form of representation can be seen to be good.

Yeomans<sup>5,6</sup> presents the results of constant amplitude fatigue tests of inner wing spar booms taken from Viking aircraft and in Ref.7 he analyses these results in terms of a log-normal distribution of cycles. The spar booms are divided into two separate populations, those from aircraft that had flown mainly on African routes (24 booms) and those that had flown mainly on European routes (44 booms); and the cumulative probability of failure distributions for these two groups are plotted separately in a log-normal cycle representation in Fig.2 again showing good agreement with this form of distribution. Similarly Refs.8 to 13 contain constant amplitude fatigue test results for Dove booms, Viking outer spar booms, Bristol Freighter Mk.21 front spar bottom booms, Valetta mainplanes, Britannia wing joints and Bristol Freighter boom joints

tested by various experimenters. The distribution functions of the probability of failure given by these sets of tests are presented in Figs.2, 3 and 4 of this Report and can be seen to approximate to a log-normal distribution except for the plot for the Viking outer research specimens in Fig.2, which was the result of the pooling of two sets of data. Refs.9 and 10 both contain results for Viking outer specimens but those specimens, for which the tests were reported in Ref.9, were manufactured from one batch of material whilst those in Ref.10 were manufactured from two distinct batches of material. The pooling of the two sets of data from the separate batches may be the cause of the non-linearity of the corresponding plot in Fig.2, for there can be considerable variation in properties between one batch of material and another. This variation is not often noticed in tests of laboratory specimens because in general laboratory specimens are small enough to allow a complete set to be made from just one batch of material. However when much larger specimens, such as a spar boom of which only a few can be made from one batch of material, are considered the batch to batch variation is likely to become more apparent.

In addition to those constant amplitude fatigue tests mentioned above, spectrum fatigue tests on Mustang wings to a random gust spectrum both with and without ground-to-air cycles have been conducted by Mann and Patching<sup>14</sup> and the cumulative probability distributions of failure obtained from their results are plotted in Fig.5. Unfortunately only 10 specimens were tested at each of the two conditions and so the definition of the parent population is not at all precise but the general trend can be seen to agree with that of the log-normal distribution approximately.

As with simple laboratory specimens, some experimenters in attempts to increase the size of the sample and thus to increase the definition of the form of the parent population have used various methods for pooling the fatigue test results obtained from various types of structure under various loading conditions. Examples of this procedure are such reports as Ref.15 by Ford and Payne, which pools 130 constant amplitude fatigue test results on Mustang wings, Ref.16 by Ford, Graff and Payne in which 187 results for structures tested under both constant amplitude and spectrum loading are standardised and Ref.17 by Impellizziri which contains results for 102 specimens normalised to give one population. The final standardised distributions presented in these references are reproduced in Figs.6, 7 and 8 respectively of this Report for ease of reference and from these it can be seen that a log-normal distribution of cycles gives a fairly good representation of the parent population.

Experimenters in the field of helicopter research have made several different approaches to the derivation of scatter factors for the results of fatigue tests on helicopter components. Among these methods is that of Liard<sup>18</sup> who extrapolates constant amplitude fatigue results according to an S-N curve shape and thus obtains a distribution of fatigue endurance limits, which in the case of the Alouette III helicopter blades shown in Ref.18 approximates closely to a log-normal distribution over the central region but involves increasing discrepancies towards the tails of the distribution. Westlands<sup>19</sup> compare the representations of fatigue data for main rotor blades in terms of the normal probability distributions of log-life and plain stress, the latter giving the better agreement with the data, but later a log-normal distribution of stress at an extrapolated constant endurance was assumed. More recently, however, both in this country and in the USA the trend in the helicopter field has been towards the use of an extreme value distribution as in Ref.20 which presents fatigue data for turbine buckets plotted on extreme value paper, giving good approximations to straight lines.

So far in this section the scatter considered has been that between the measured fatigue lives obtained for one particular type of failure in several distinct but nominally identical specimens. However in a large complicated structure often more than one fatigue failure can take place and throughout a fatigue test of one such structure cracks occur at various different positions, leading to a scatter in the fatigue lives at various stations within the structure. Data on this latter form of scatter is presented in Refs.21, 22 and 23 and summarised in Ref.24. The specimens were skin joints from the centre section wing panels of F-27 Friendship aircraft and at the joints the skin is reinforced by three doubler plates, the innermost of which is a finger plate bonded to the skin. Cracks started at the tips of the fingers at 52 of which on each aircraft the stress was approximately equal, being different for the remaining 28 fingers on the aircraft. The cumulative distribution plots obtained from the failures that occurred at any of these 52 finger tips are presented in Appendix F, Figs.F1a to F1e of Ref.24 and reproduced in Fig.9 of this Report showing that a log-normal distribution of life gave a reasonable representation of the observed data for constant amplitude, programme and random load tests for scatter within the structure as well as for the scatter between structures.

### 3 SELECTION OF A DISTRIBUTION

Section 2 reviewed some of the available data that are suitable for analysis in terms of the representation of the probability distribution function of the fatigue failures of elements and complete sections from full scale structures. The quantity of such data is small and so the confidence that can be placed in any deductions from such a review is low. However, the reasons quoted in Refs.1 and 2 in support of the assumption of a log-normal distribution of lives to failure under any type of fatigue loading on laboratory specimens appear to be equally valid for all forms of loading on structural segments from aircraft and so the log-normal form of distribution of lives to fatigue failure was also adopted as the basis of the analysis of full scale structural failures in this Report. The arguments of Refs.1 and 2 in favour of the log-normal distribution of life are reproduced below for ease of reference, namely:-

- (a) The discrepancies between practice and the log-normal distribution appear to be small in the range of probabilities of failure  $0.90 > P > 0.10$ . (These limits are those of Ref.2 only, as the amount of available data is again smaller than in Ref.1.)
- (b) The mean and variance of a log-normal distribution provide the basis for a readily comprehensible comparison of the positions and extents of the distributions obtained under different loading conditions using various forms of specimen.
- (c) The use of a normal form of distribution, rather than any other basic type, considerably eases the analysis of the data without impairing the validity of the conclusions.
- (d) The analysis in terms of a log-normal distribution of life under constant conditions involves no graphical procedures and is thus exactly reproducible.

### 4 DEFINITION AND DERIVATION OF TERMS

The normal distribution is defined by the probability density function

$$p(x) dx = \frac{1}{\sqrt{2\pi\sigma^2}} \exp - \frac{1}{2} \left( \frac{\mu - x}{\sigma} \right)^2 dx$$



where  $p(x)dx$  is the probability of occurrence of the event between conditions  $x$  and  $x + dx$ , and  $\mu$  and  $\sigma$  are the population mean and standard deviation respectively. When a log-normal distribution of cycles to failure is considered the above expression must be replaced by

$$p(\log N) d(\log N) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp - \frac{1}{2} \left( \frac{\mu - \log N}{\sigma} \right)^2 d(\log N)$$

where  $\mu$  and  $\sigma$  are now the mean and standard deviation of  $\log(\text{life})$  and  $N$  is the number of cycles to failure of a randomly selected specimen.

The best estimates of  $\mu$  and  $\sigma$  that can be obtained from a sample of  $n$  test specimens are given by

$$m = \overline{\log(N)} = \frac{1}{n} \sum_{r=1}^n \log N_r$$

and

$$s = \left\{ \frac{\sum_{r=1}^n (\log N_r)^2 - n(\overline{\log N})^2}{(n-1)} \right\}^{\frac{1}{2}}$$

where the unbiased estimate of the standard deviation is used, as the number of specimens tested under one set of condition is generally small. These two parameters are enough by themselves to define the normal distribution, but sometimes another parameter, the coefficient of variation  $v$  is used. Any two of these three parameters  $\mu$ ,  $\sigma$  and  $v$  are sufficient to define the normal distribution, for the coefficient of variation is defined by the expression

$$v = \frac{\sigma}{\mu}$$

while the estimated coefficient of variation as used in this publication is defined as the ratio of the estimated standard deviation to the estimated mean i.e.  $v_{\text{estimate}} = \frac{s}{m}$ .

Comparisons between the scatter, or standard deviation, obtained under different conditions will be required and these will be conducted using the variance ratio  $F$ , where  $F$  is defined as  $\frac{s_1^2}{s_2^2}$ ,  $s_1$  and  $s_2$  being the two sample estimated standard deviations being compared. The null hypothesis tested is that the two samples from which  $s_1$  and  $s_2$  were calculated were drawn at random from normal populations of equal variance. Table 4 of Ref.25 gives values of  $F$  at various levels of significance such that if the appropriate value of  $F$  is exceeded the null hypothesis is contradicted and the two samples of which the standard deviations are being compared can be said at the appropriate level of significance to be taken from different parent populations.

## 5 ANALYSIS OF THE RESULTS OF SOME FATIGUE TESTS OF ELEMENTS AND SECTIONS FROM AIRCRAFT STRUCTURES

### 5.1 Constant amplitude fatigue tests

Fatigue tests on a number of different types of specimens taken from various aircraft have been carried out in the past, under constant amplitude load conditions. The materials from which these aircraft were constructed varied from aircraft type to aircraft type and so only a small amount of information relates directly to the data for 2024 and 7075 materials. However as so few data are available on elements and sections from aircraft structures it seems necessary to present the analysis of all such data together irrespective of the material. The form of presentation adopted for this information is that used in Refs.1 and 2, namely plots of the sample estimated values of the coefficient of variation of the distribution of cycles to failure against the sample estimated values of the mean life to failure, using a log-normal distribution of failure times as a basis.

The relevant results of an analysis of the fatigue data on structural elements from wings, on wings themselves and on tailplanes, tested at the RAE are presented in Fig.10, the data being obtained from Refs.5 to 13 and Refs.26 to 35 and the other references quoted in Ref.7. Some of the data applied to used and some to unused specimens, but when the specimens had been

in service allowance was made for this period of time in the derivation of the total endurance before the results were analysed. It is of interest to note that the coefficient of variation of these results, for mean lives generally greater than  $10^5$  cycles, covers a considerable range of values.

In Australia a series of tests has been carried out under constant and variable amplitude loading on P51D Mustang wings which are constructed from 2024 material<sup>15</sup>. The constant amplitude tests were conducted at several combinations of mean and alternating stress and so no large single population of data is available. However the results for all the small groups of specimens tested are presented in Fig.11 of this Report, which indicates that in these tests the mean life was generally in the range of 1000 to 100 000 cycles and that the coefficients of variation of log life were below 0.04. A superposition of the results from Fig.10 on the results from Fig.11 gives Fig.12, in which data from Refs.12 and 36 have also been plotted. It is noticeable from Fig.12 that the general trend of the results is very much that associated with the constant amplitude results of laboratory specimens, namely an increase in scatter for mean lives of  $10^5$  cycles and upwards.

In Ref.15 Ford and Payne also provide a statistical analysis into four comparisons namely:-

- (a) Tests conducted in a hydraulically operated rig against similar tests in a vibration rig.
- (b) Tests on wings from the port sides of aircraft against wings from the starboard sides (testing for any effects of asymmetric manoeuvres etc.).
- (c) Tests on wings with a previous service history against new wings. Effect on endurance to final failures.
- (d) As (c) but considering the effect on the endurance to the initial failure.

Comparisons (a), (b) and (c) were all found to be not significant statistically, whilst comparison (d) was significant, an effect which might be caused by the problem of assessing the damage suffered in the service life of the aircraft. Ford and Payne pooled all the data to final failure at each stress level and concluded that the standard deviation of log-endurance was independent of loading condition and so independent of the mean life, suggesting a value of 0.116 for the standard deviation of Mustang wings. The data from other sources plotted in Fig.12 however indicate that, at mean lives greater than those

covered in Ref.15, the amount of scatter present increases with an increase in the mean life in the range  $10^5 - 2 \times 10^6$  cycles.

Comparison (a) of Ford and Payne was also substantiated by Haas<sup>12</sup> who compared fatigue data on Britannia wing joints obtained from a hydraulically operated rig with that from a vibration rig and found no significant difference. He also compared the fatigue performance of front spar bottom boom joints from Bristol Freighter aircraft under constant amplitude loads, with and without a 2g pre-stress, and found no apparent effect on the scatter although the pre-stressed joints gave a marked improvement in mean life over the non pre-stressed joints (Fig.3).

In Refs.37 to 39 results are presented for fatigue tests on C-46 Commando airplane wings which are of 24S-T clad material. Several cracks occurred in each specimen before the test was stopped, but at two stations cracks occurred in each aircraft tested under identical conditions. Final failure of the wings never took place as the experimenters feared that it might damage the equipment but by a crack propagation analysis the life to final failure of each crack was calculated and the following table, the values from which are plotted in Fig.12, summarises the results.

Number of specimens	Loading	$\overline{(\log_{10} \text{ life})}$	Standard deviation of $\log_{10}$ (life)	Coefficient of variation of $\log_{10}$ (life)
5	$\pm 14\%$ UDC	5.51	0.065	0.012
4	$\pm 22\%$ UDC	4.94	0.102	0.021

## 5.2 Variable amplitude fatigue tests

The most numerous set of variable amplitude fatigue tests carried out on full scale structures is that reported in Refs.3 and 4 on Piston Provost main-planes to a spectrum of six load levels representing an overall flight spectrum. Three sequences of application of these six load levels were used in distinct sets of tests the results of which are summarized in the table, page 12, and plotted in Fig.13.

Sequence of load level applications	Number of specimens	$\overline{(\log_{10} \text{ life})}$	Standard deviation of $\log_{10} (\text{life})$	Coefficient of variation of $\log_{10} \text{ life}$
H <sub>i</sub> -L <sub>o</sub>	39	5.17	0.077	0.015
L <sub>o</sub> -H <sub>i</sub>	11	5.10	0.084	0.016
L <sub>o</sub> -H <sub>i</sub> -L <sub>o</sub>	4	5.15	0.064	0.013

In Ref.3 as in Ref.15 comparisons were made between the failures of port and starboard wings and again no statistical significance could be attached to the differences noted.

Whilst the spectrum employed by Parish and Grove represented the overall flight spectrum of the Provost aircraft the tests conducted by Mann and Patching<sup>14</sup> and Jost<sup>40</sup> on Mustang wings were to a gust spectrum with and without ground-to-air cycles and to an asymmetric fighter reconnaissance spectrum respectively. The order of selection of the 21 load levels used in the spectrum representation was approximately random in all cases, the exception being the introduction of ground-to-air cycles after every 49 load selections, each ground-to-air cycle being preceded and followed by the smallest level of gust loading. The results from these two references 14 and 40 are tabulated below and plotted in Fig.13 with those of Refs.3 and 4.

Loading	Number of specimens	Reference number	$\overline{(\log_{10} \text{ life})}$	Standard deviation of $\log_{10} \text{ life}$	Coefficient of variation of $\log_{10} \text{ life}$
Gust	10	14	6.467	0.204	0.032
Gust + GAC	10	14	5.849	0.193	0.033
Asymmetric	6	40	5.861	0.133	0.023

The results of fatigue tests on five Britannia wing joints to a spectrum representing gusts, take-offs and landings are presented in Ref.12 by Haas. A sixth specimen was tested under the same spectrum with one load level applied at a lower frequency than in the other tests but this specimen failed at a different position and so was not included in the analysis which provided a mean  $\log_{10}$  life of 5.651, a standard deviation of log life of 0.047 and a coefficient of variation of 0.0082 Fig.13, whilst two sets of tests reported in Ref.41 on an unspecified trainer aircraft made of 2L65 material have also been analysed and the results plotted in Fig.13 as the loading was once again programmed.

### 5.3 Scatter within a structure

Schijve<sup>21-24</sup> reports fatigue test results for centre section wing panels of 7075 material from Fokker F-27 aircraft, in which failures occurred at a finger doubler plate at a skin joint. The analysis of failures from the 52 fingers in each joint at which the stress levels are very nearly equal shows that there is a scatter between the times to failure at the various positions in an aircraft structure where the conditions are alike. The results of this analysis, which is presented in Appendix F of Ref.24, are reproduced below:-

Type of loading	Number of fingers available at which cracks occurred	(log <sub>10</sub> life)	Standard deviation of log <sub>10</sub> (life)	Coefficient of variation of log <sub>10</sub> life
Random + GTAC	9	5.15	0.090	0.017
Programme	15	5.59	0.067	0.012
{ Programme + GTAC	{ 34	{ 5.19	{ 0.125	{ 0.024
{ Programme + GTAC	{ 13	{ 5.12	{ 0.067	{ 0.013
GTAC	9	4.39	0.086	0.020
GTAC	7	4.44	0.061	0.014
Constant amplitude ±14.7% UDL	15	4.61	0.062	0.013
Constant amplitude ±9.35% UDL	14	5.37	0.131	0.024
Constant amplitude ±21.8% UDL	6	4.38	0.060	0.014

The general values of the coefficient of variation are small but for one set in which the loading was programmed including ground-to-air cycles and one under a low constant amplitude stress. If the first 13 cracks from the programme + GTAC test had been analysed on their own the coefficient of variation, as shown in the table would have been 0.013 whilst continuing the test increased this value to 0.024. There is a possibility that this increase is caused by the effect of repairs at the previously cracked finger tips redistributing the stress round other finger tips and thus changing the failure distribution. Other points of interest are that the largest scatter in the constant amplitude tests is associated with the lowest stress level and that the higher constant amplitude stress levels give the same amount of

scatter approximately as the programme tests. Also from Fig.9 it can be seen that the two specimens tested under equal constant amplitude loadings of a magnitude corresponding to GTAC give quite distinct distributions for the failures of the finger tips indicating a scatter between structures as well as a scatter within a structure.

## 6 DISCUSSION

The scarcity of fatigue data on the scatter in elements and sections of aircraft structures is clearly illustrated by the lack of points in Figs.12 and 13 of this Report. The data on variable amplitude tests are very limited and so any conclusions drawn from these data must be only tentative. There is general agreement between the trend of an increase in scatter for aircraft parts tested under constant amplitude loading with an increase in mean life above  $10^5$  cycles shown in Fig.12 and the trend shown in Ref.1 by the scatter from small laboratory specimens, but the number of sets of results for variable amplitude tests is too small to say whether these also follow the trend of variable amplitude data for laboratory specimens indicated in Ref.2. It appeared in that reference that the scatter observed in variable amplitude fatigue tests was dependent to a large extent on the magnitude of the highest, reasonably frequently occurring, stress level applied in the tests. There is no direct evidence to suggest that this effect also exists in the testing of built-up structures although it is interesting to note that in the two investigations reported in Refs.37 to 39 and 42 more cracks occurred in each aircraft at the higher constant amplitude stress level than at the lower stress levels. Also in the former investigation the number of cracks occurring per wing in the variable amplitude tests agreed more with the number occurring on the tests at a higher constant amplitude stress level than those occurring in the tests at a lower constant amplitude stress level. Thus variable amplitude tests of elements or sections from aircraft structures might be expected to give low scatter generally, if less scatter is associated with the higher constant amplitude stress levels as seems likely.

In Ref.43 results are given for the number of fatigue crack nuclei present, revealed by opening each crack statically, at some of the finger tips of the skin joints from the F-27 aircraft wing sections of which the fatigue tests are reported in Refs.21 to 24. These tests have already been mentioned in section 5.3 of this Report, but the total numbers of crack nuclei present at the finger tips studied, which were the same for every specimen, are tabulated below and so give an idea of the average number of nuclei per finger tip.

Specimen number	Type of loading	Level of loading as percentage of ultimate	Total number of nuclei present at 48 finger tips
1	Random	-	138
2	Programme	-	166
4	Random	-	93
5	Programme	-	196
6	Random + GTAC	-	114
7	Programme + GTAC	-	64
9	Constant amplitude	16.6% $\pm$ 23.5%	65
10	Constant amplitude	16.6% $\pm$ 23.6%	71
11	Constant amplitude	25.6% $\pm$ 14.7%	63
12	Constant amplitude	25.4% $\pm$ 9.35%	43
13	Constant amplitude	25.0% $\pm$ 21.8%	82

From this table it can be seen that the smallest number of nuclei was produced by the lowest constant amplitude level and that the higher constant amplitude levels all gave much the same number of nuclei, whilst programmed and random tests invariably gave at least as many nuclei as, and often more than, the number of nuclei given by the high level constant amplitude tests. This corroborates the trends found in Refs.1 and 2 whereby an increase in constant amplitude stress level increased the number of nuclei formed and a spectrum test produced a number of nuclei corresponding to the higher constant amplitude stress levels. As the amount of scatter is considered to be dependent on the number of nuclei, it seems likely that the scatter observed in elements and sections of aircraft structures follows much the same trends as the scatter in laboratory specimens.

## 7 CONCLUSIONS

(1) The scatter in constant amplitude fatigue tests of elements and sections from aircraft structures seems to be of the same magnitude and to follow the same trend, namely the increase in scatter for values of mean life greater than  $10^5$  cycles, as the scatter in constant amplitude fatigue tests of laboratory specimens.

(2) Few sets of variable amplitude results for fatigue tests of elements and sections from aircraft structures are available, but those that can be found



indicate that possibly, as for laboratory specimens, the scatter in fatigue lives is more independent of the mean life than is the case for constant amplitude tests.

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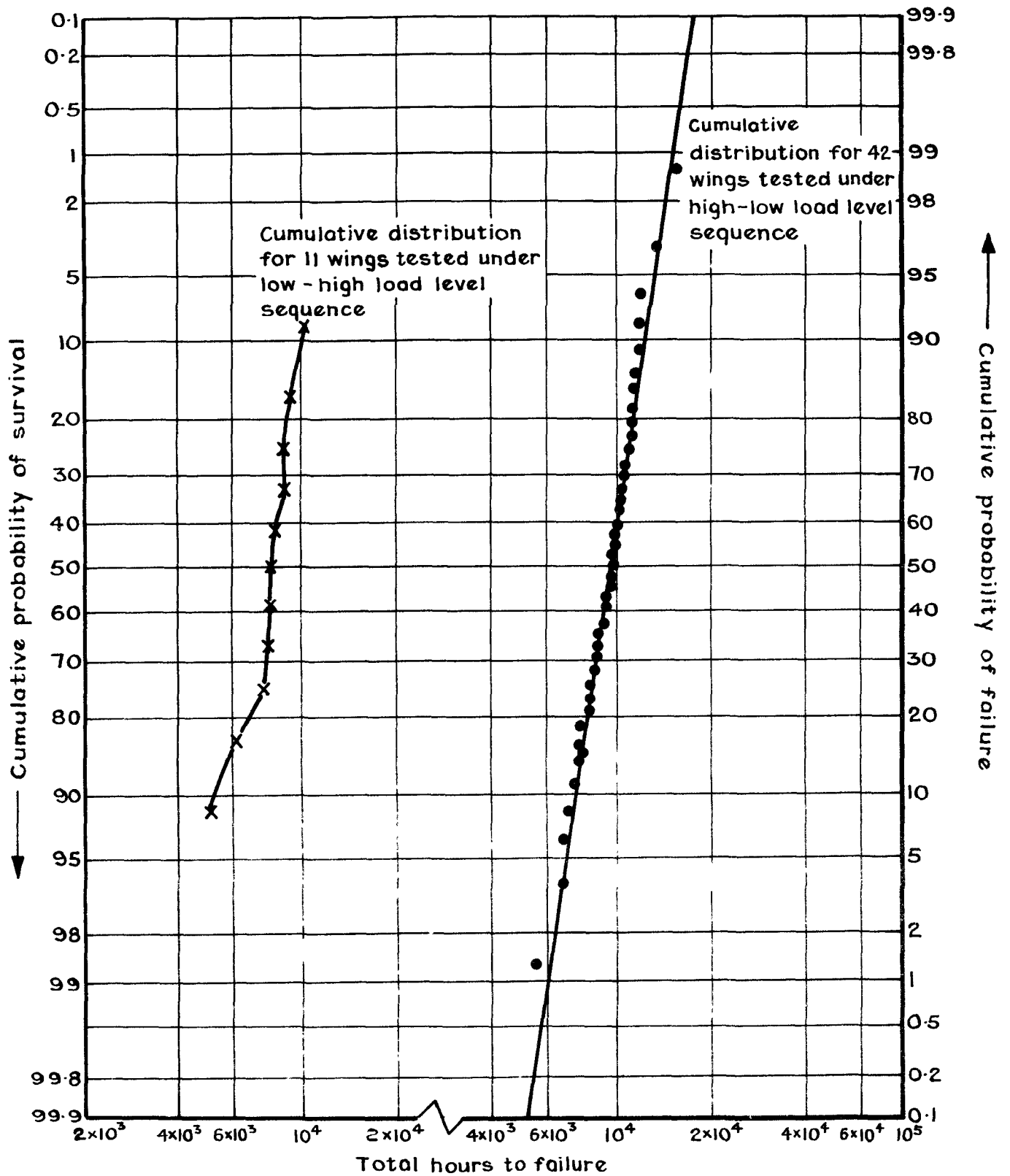


Fig.1 Cumulative distribution of variable amplitude fatigue tests of Piston Provost wings

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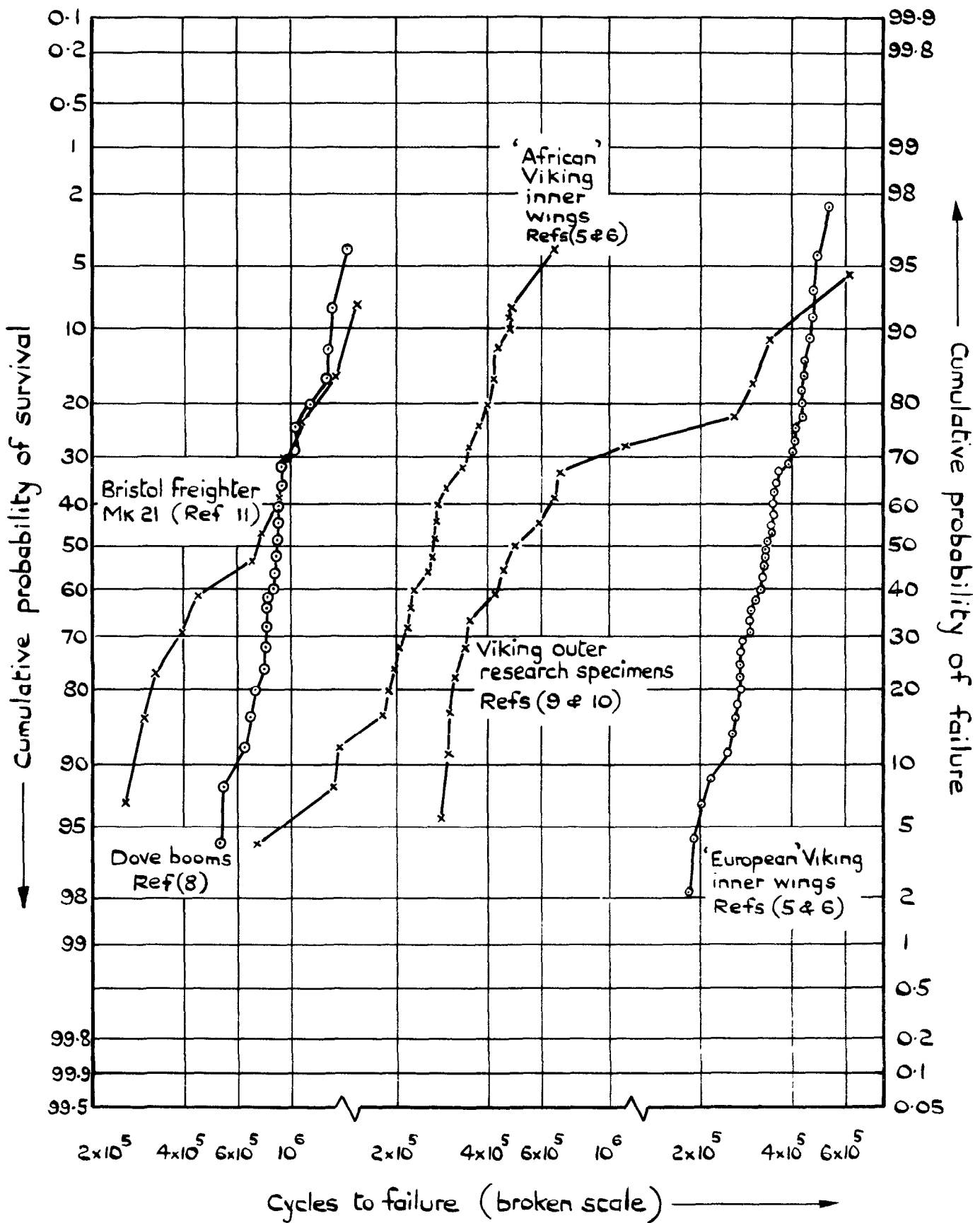


Fig.2 Cumulative distributions of constant amplitude fatigue tests of various elements from aircraft structures



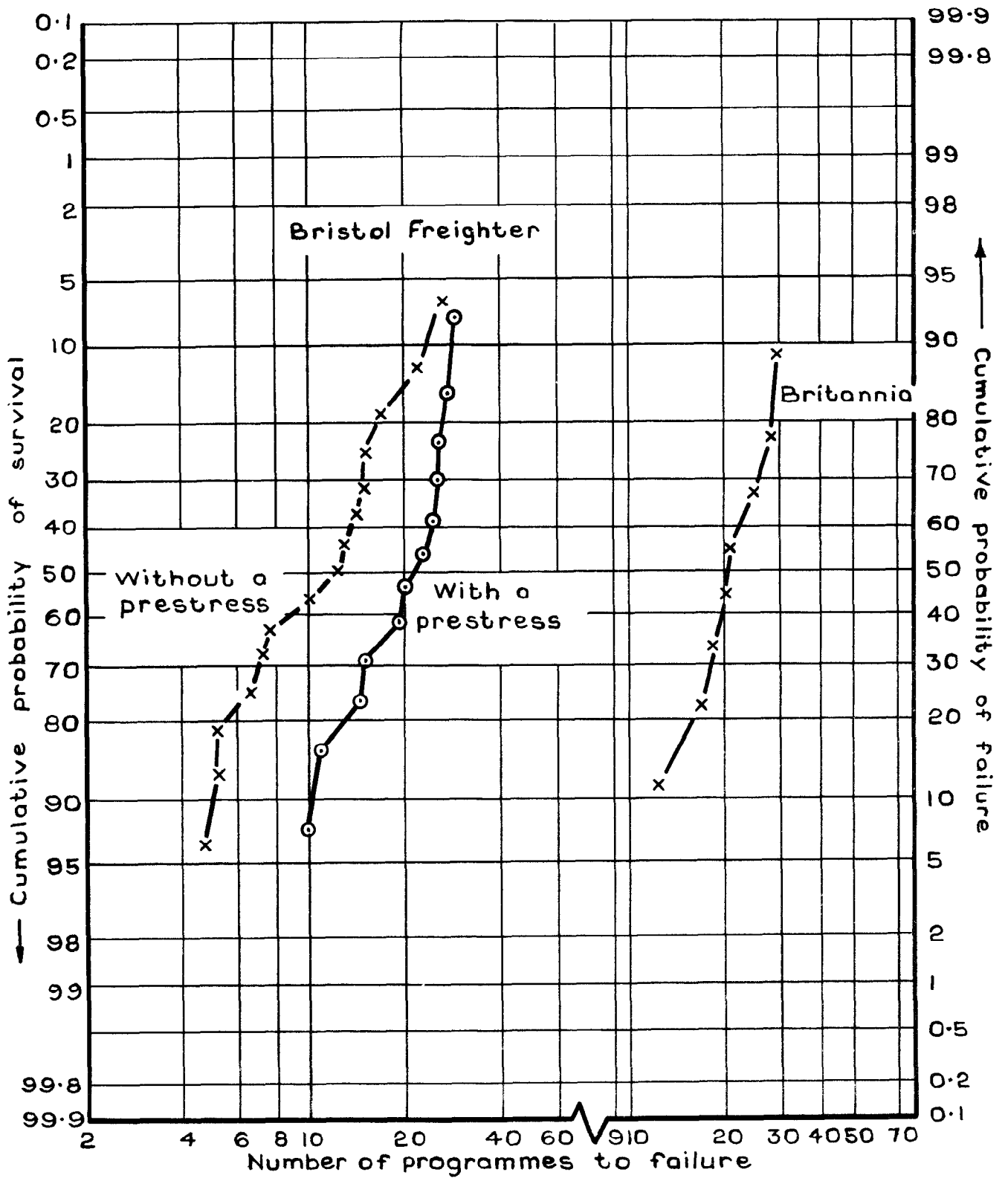


Fig.3 Cumulative distributions of constant amplitude fatigue tests on Britannia and Bristol Freighter wing joints (Ref 12)

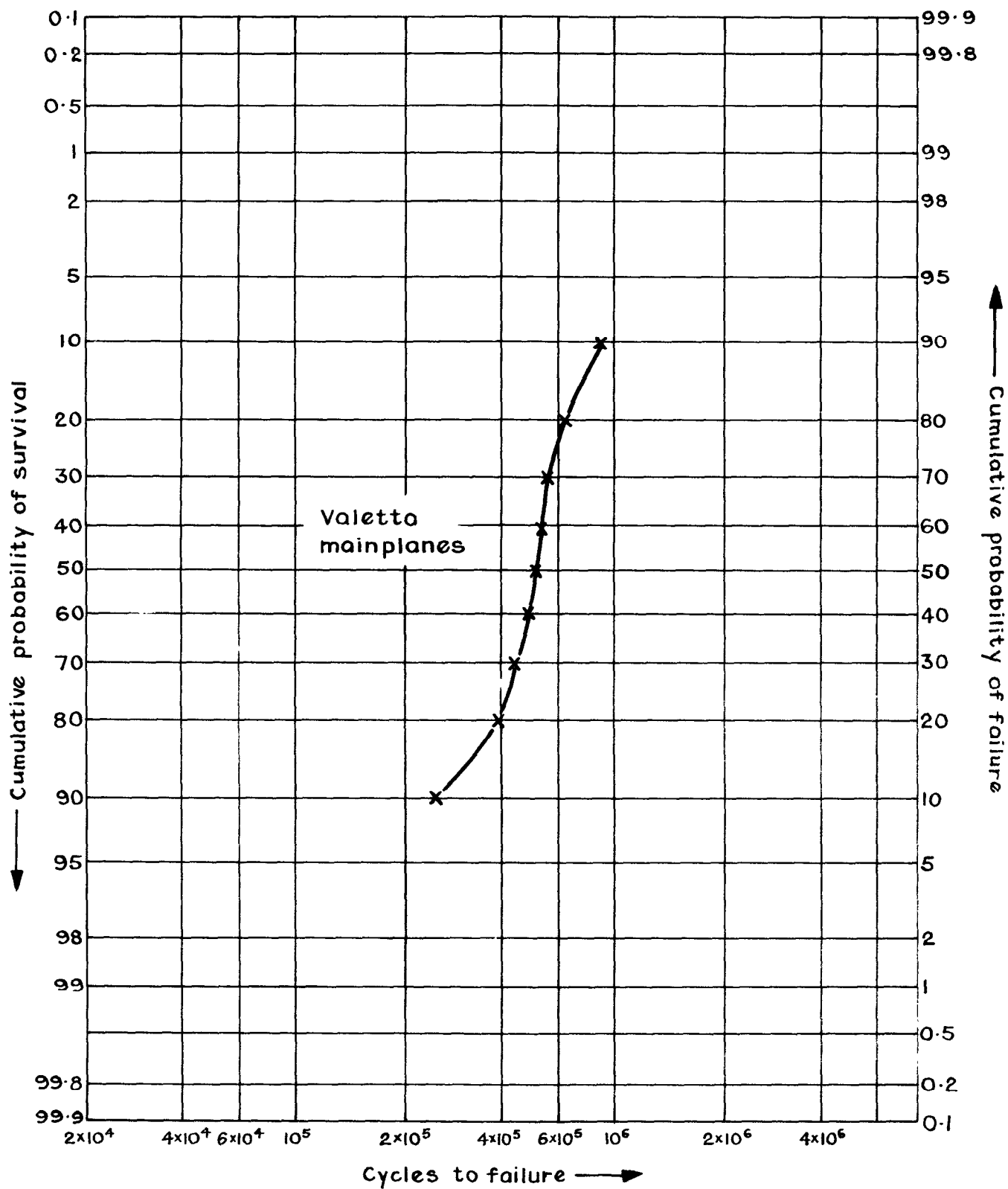


Fig.4 Cumulative distribution of constant amplitude fatigue tests on Valetta mainplanes (Ref 13)

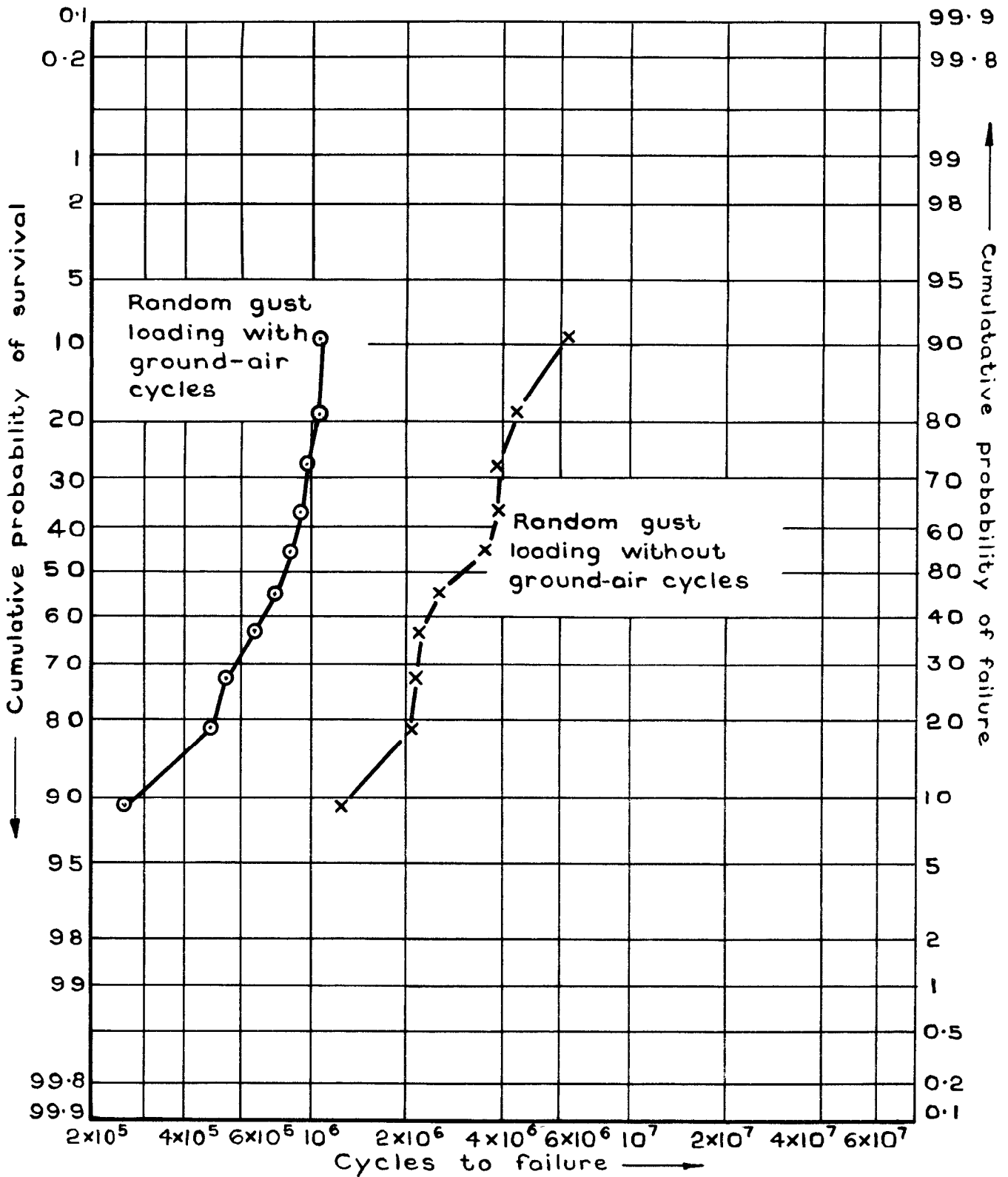
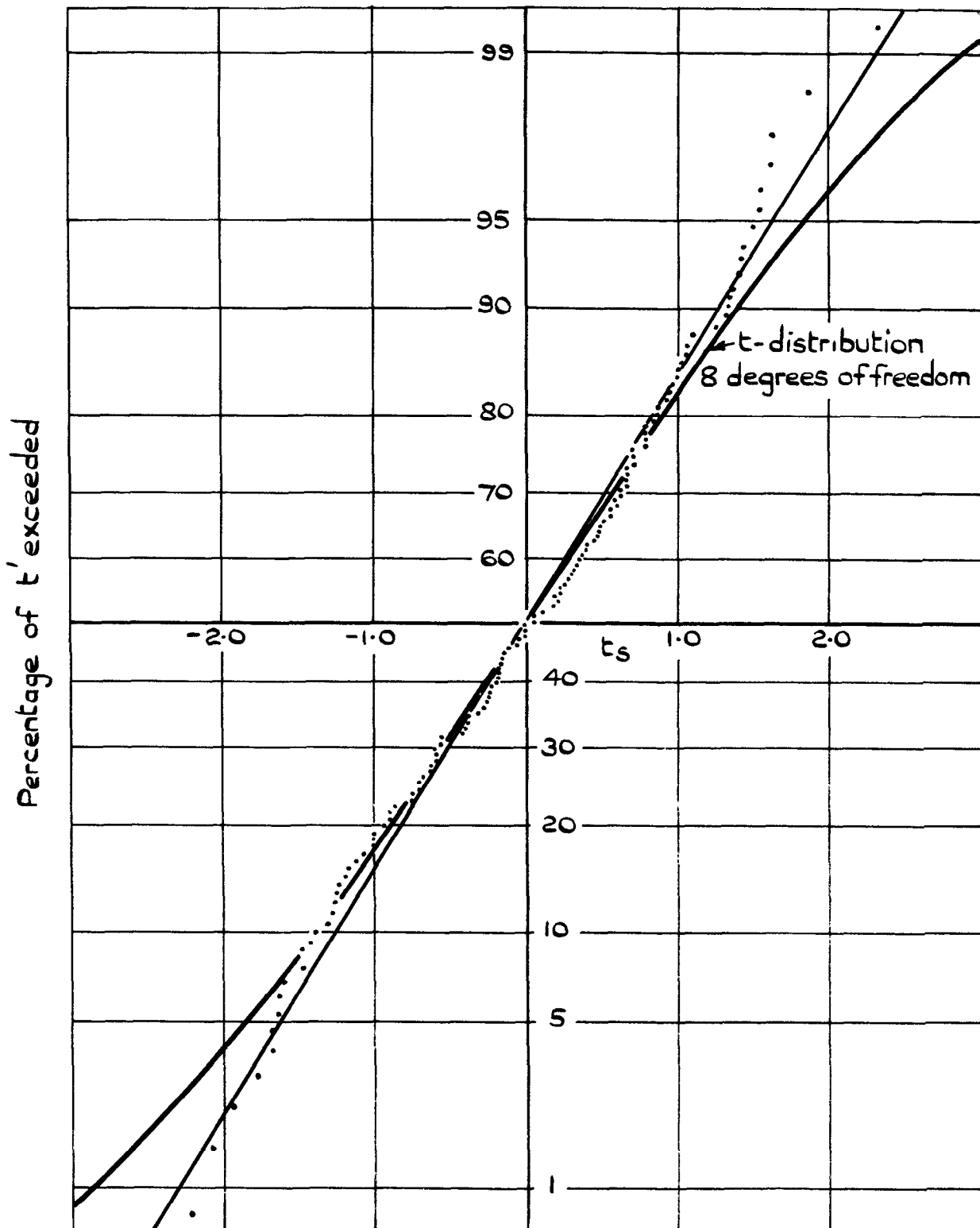


Fig.5 Cumulative distributions of fatigue tests of Mustang wings under random gust loading with and without ground-air cycles Ref (14)

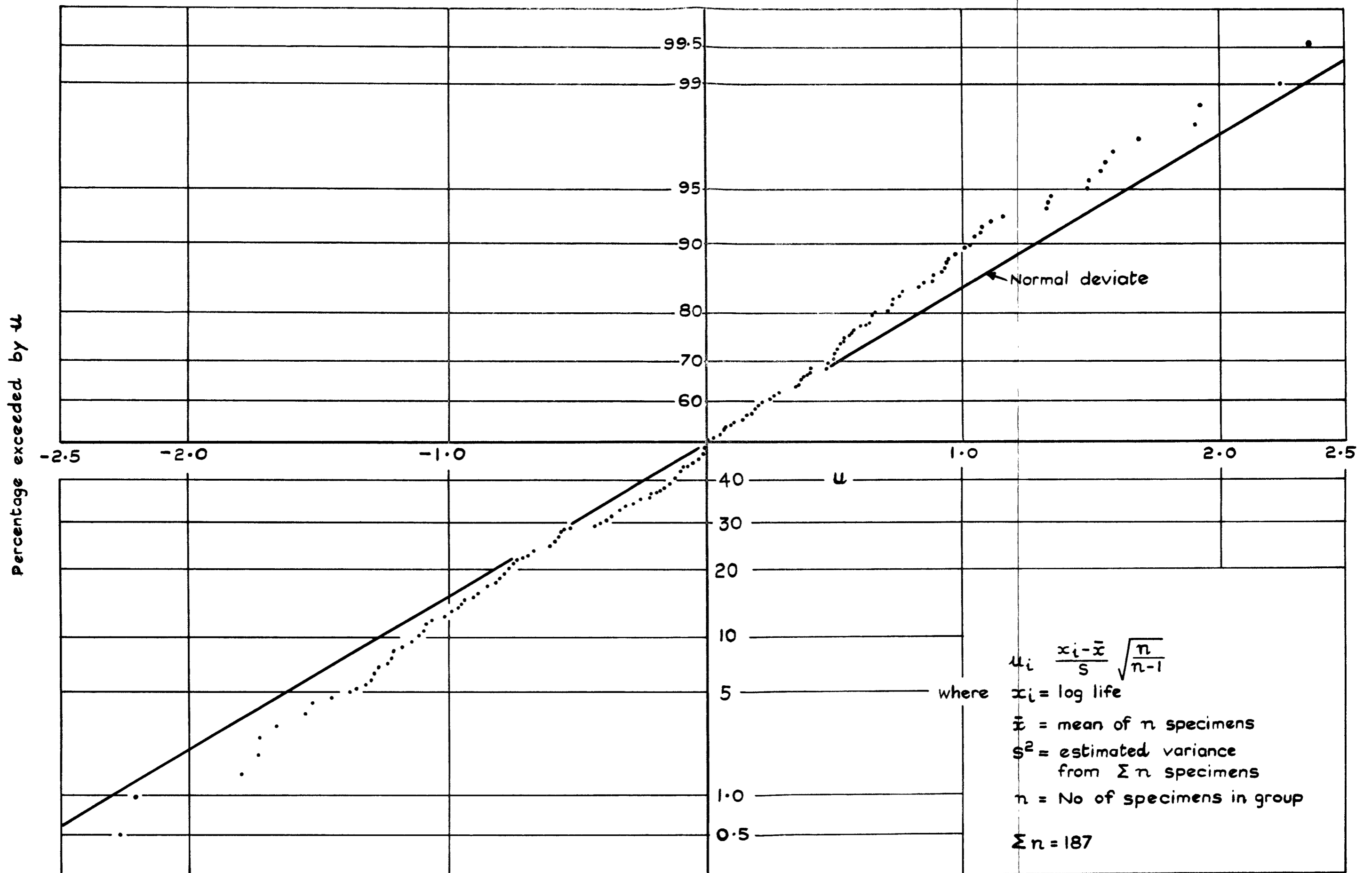


Cumulative distribution of  $t_s$  on normal probability scale

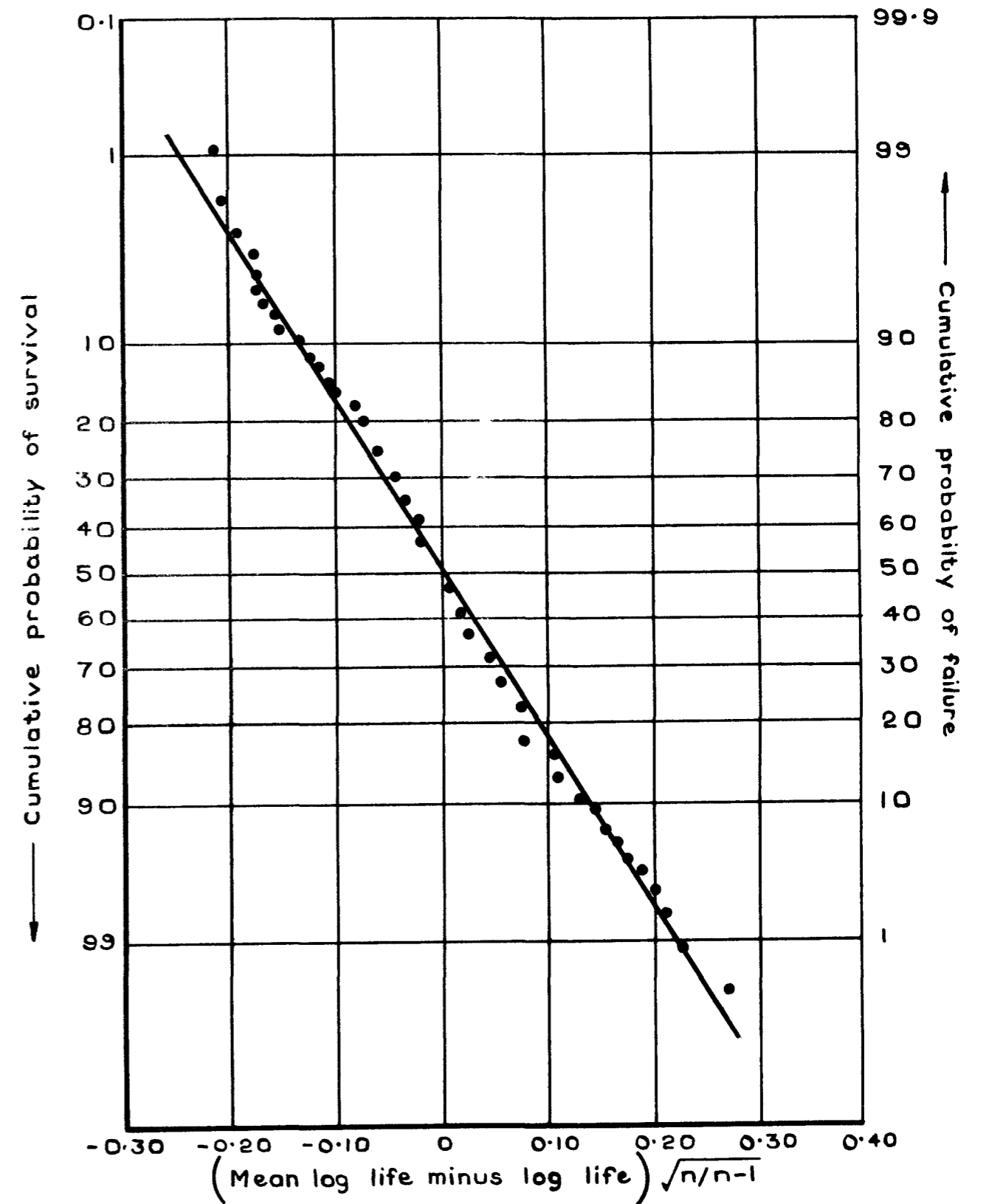
For each sample  $t_s' = \left( \frac{x_i - \bar{x}}{s} \right) \sqrt{\frac{n}{n+1}}$  where  $x_i = \log N_i$

Fig.6 Cumulative distribution of constant amplitude fatigue tests on 130 Mustang wings

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Standardised distribution of log-life complete structures.  
 Fig. 7 Cumulative distribution of 187 constant and variable amplitude fatigue tests of aircraft structures.  
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Probability distribution of log life for 102 specimens from 32 groups of tests on aircraft structures to both constant and variable amplitude loading

Fig.8 Cumulative distribution of 102 constant and variable amplitude fatigue tests of aircraft structures

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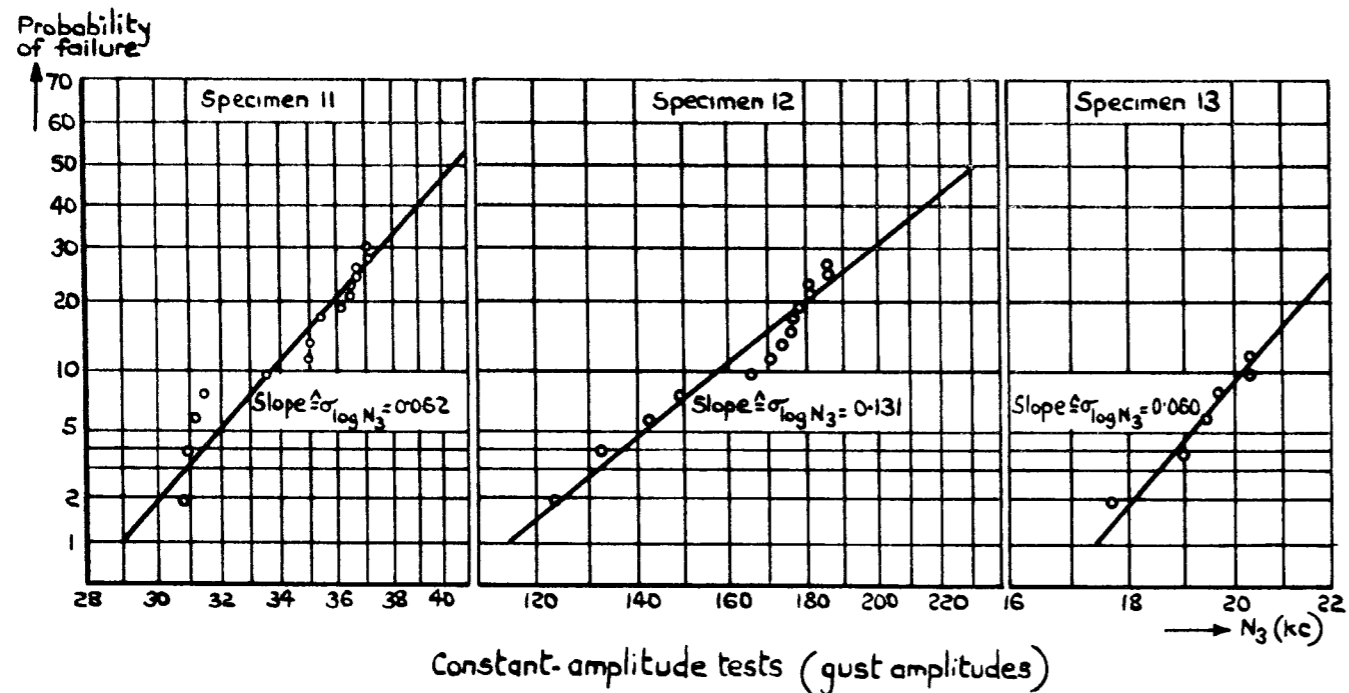
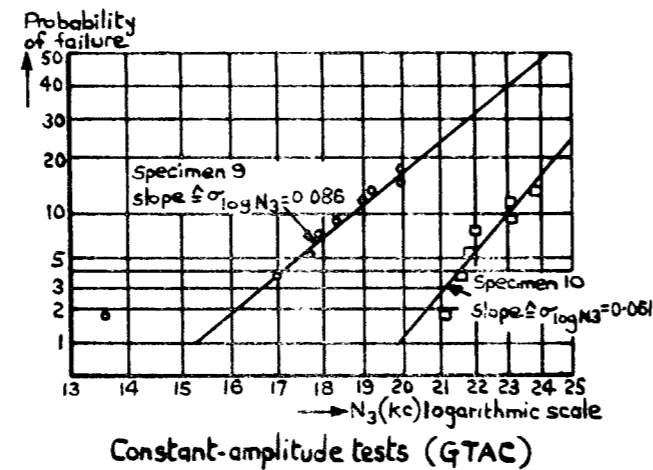
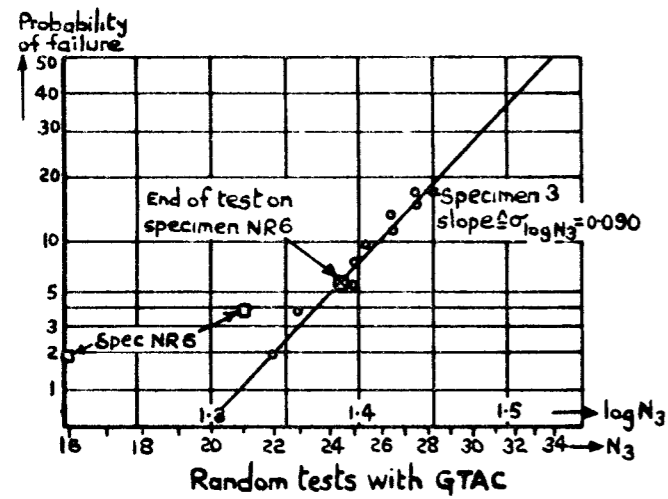
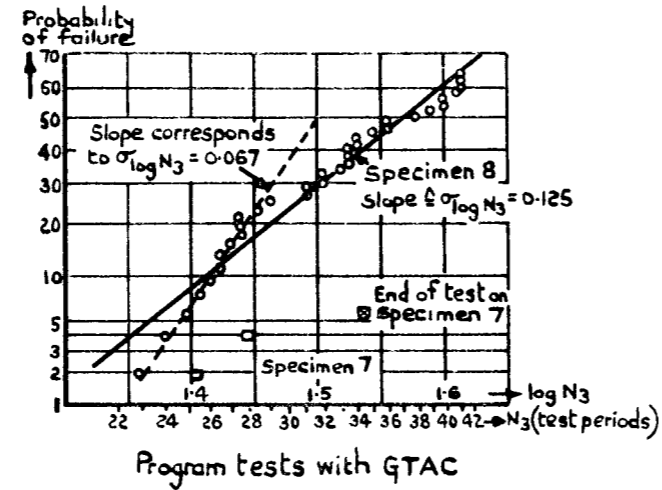
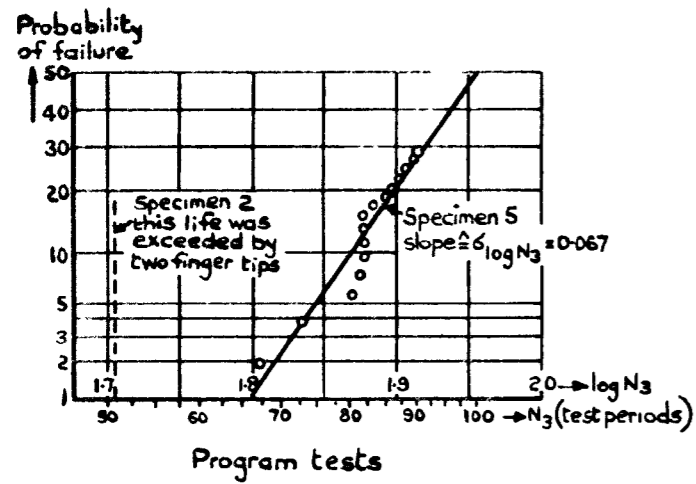
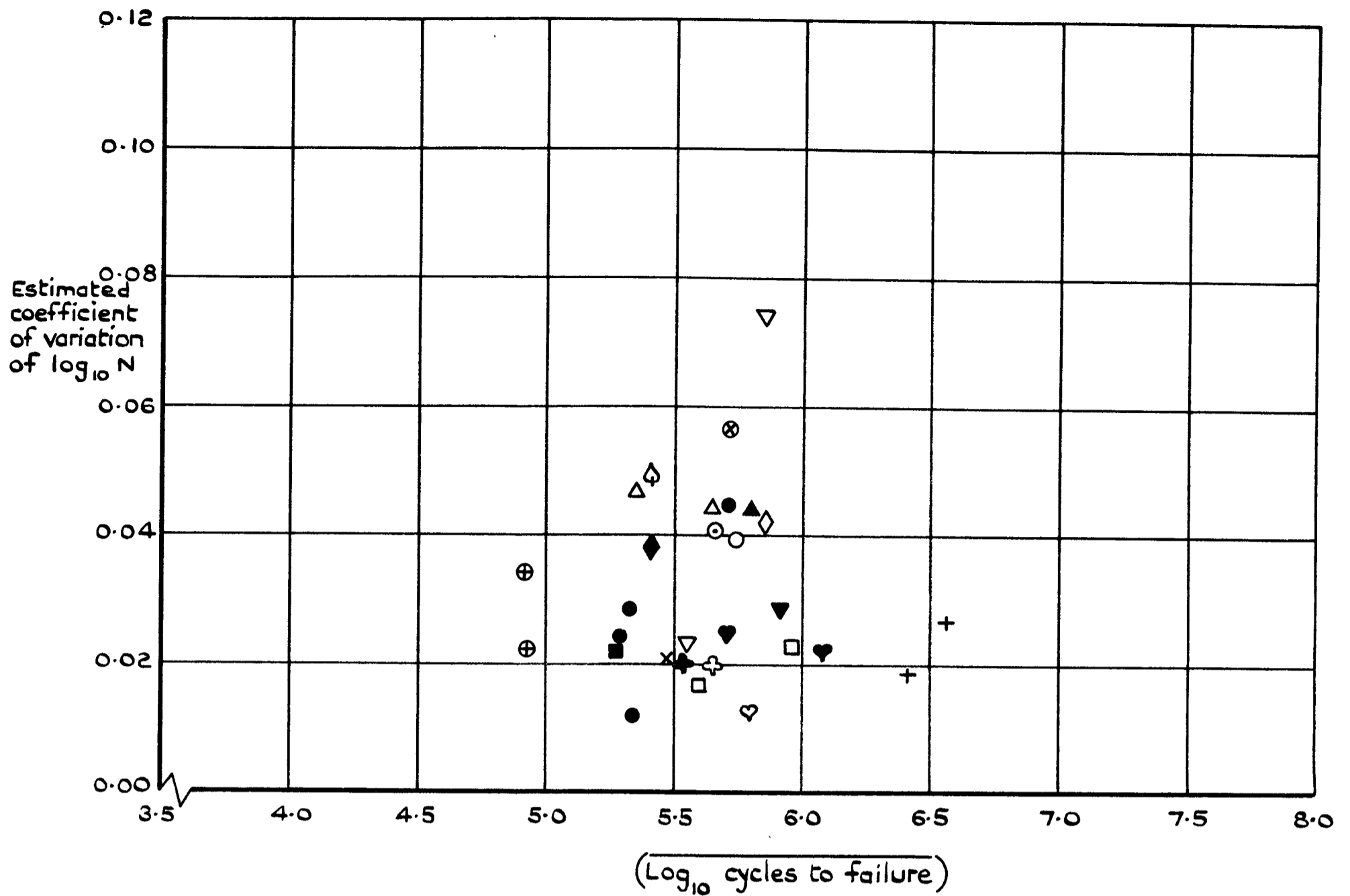


Fig.9 Cumulative distributions of the failures on fatigue tests of F-27 Friendship aircraft. Each distribution includes the failures at the skin finger tips on one specimen only  
(Reproduced from Ref (24) by kind permission of J.Schijve)



Symbol	Specimen	Number of specimens tested	Material
+	Vickers type 800, outer wing joints	Two sets of 4	DTD 363 A
○	Solent, front spar bottom boom root joints	7	DTD 364
△	Twin Pioneer, lift strut attachments	Two sets 5 and 6	DTD 464
▽	Viking, outer spar boom research specimens	Two sets 6 and 17	DTD 363
□	Hastings I, intermediate wing rear spar bottom booms	Two sets 4 and 6	DTD 364
×	Hermes, centre section wing front spar bottom booms	5	DTD 683
●	Hastings, front spar centre and intermediate mainplanes	Four sets 4.45 and 6	DTD 364
◇	Pembroke, front spar bottom boom specimens	6	DTD 364
⊙	Shackleton, rear spar bottom booms	6	DTD 364 A
⊕	Dove, booms	Two sets 8 and 24	DTD 363 A
⊗	Hermes IV, outer wing front spar joints	6	DTD 683
⊕	Hermes IV, outer wing front spar booms	6	DTD 683
♡	Hermes IV, outer wing rear spar booms	6	DTD 683
⊕	Hermes IV, intermediate wing front spar joints	6	DTD 683
▲	Bristol freighter MK 21, front spar bottom booms	12	DTD 364
▼	Meteor, tailplanes	6	L 40
■	Solent, spar booms	6	DTD 364
◆	Viking, inner specimens 'African'	24	DTD 464
♣	Viking, inner specimens 'European'	44	DTD 464
♥	Valetta, wing sections	Two sets 4 and 9	DTD 364

Fig. 10 Constant amplitude fatigue test results for elements and sections of various types of aircraft tested at the R.A.E.



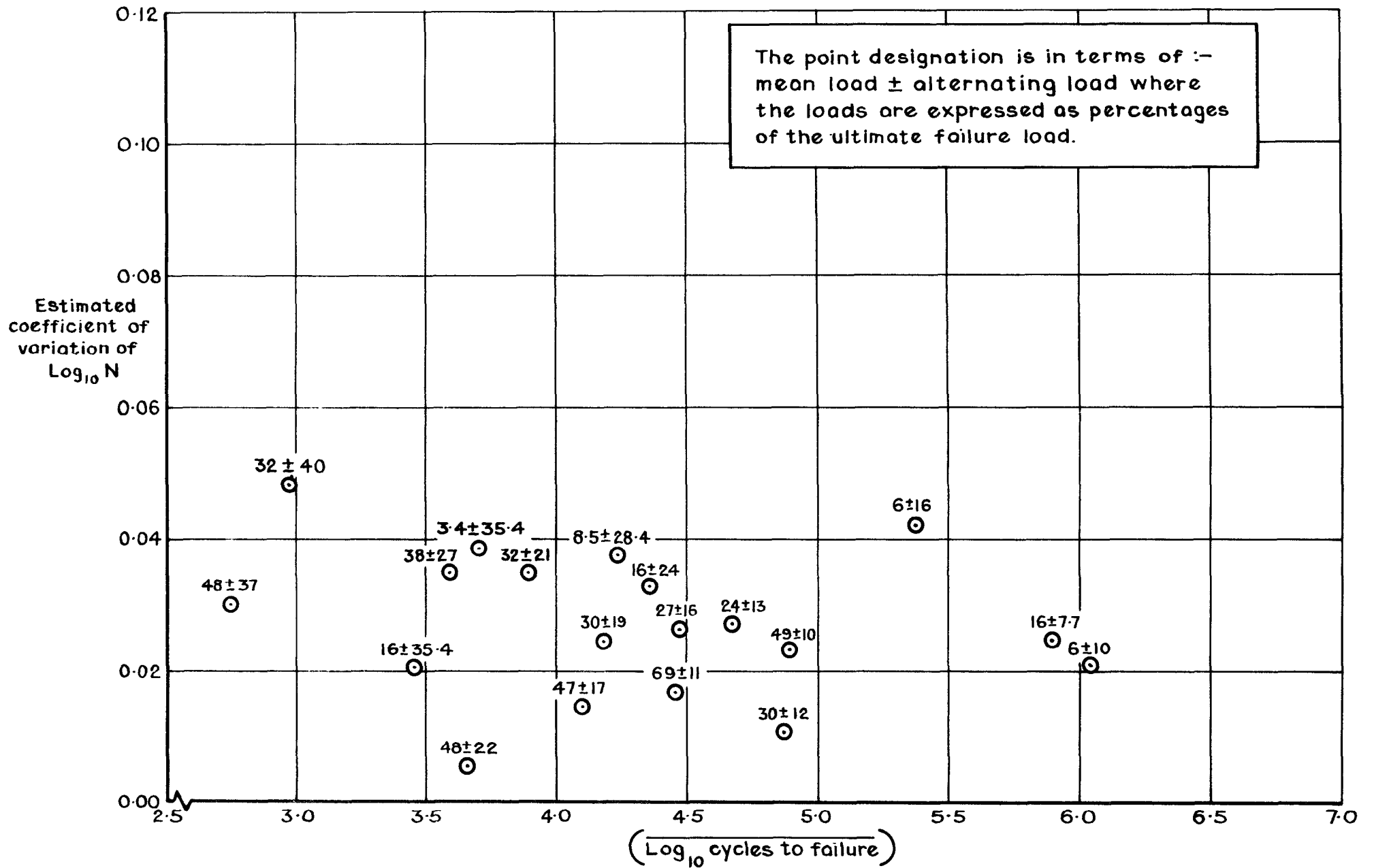


Fig.11 Constant amplitude fatigue test results for Mustang wings tested without a preload (Ref 15)

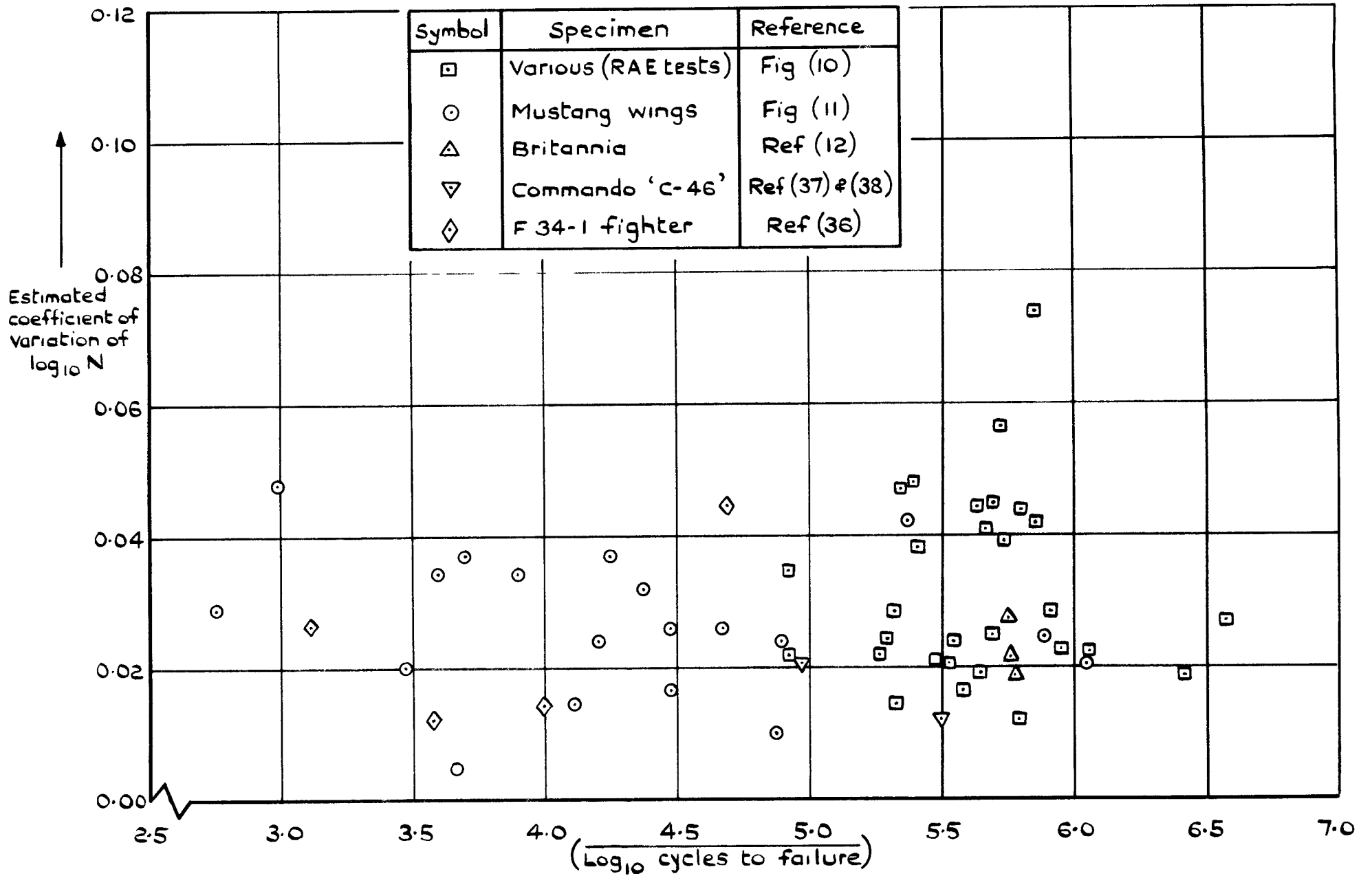


Fig.12 Constant amplitude fatigue test results  
Accumulated data for a variety of aircraft types

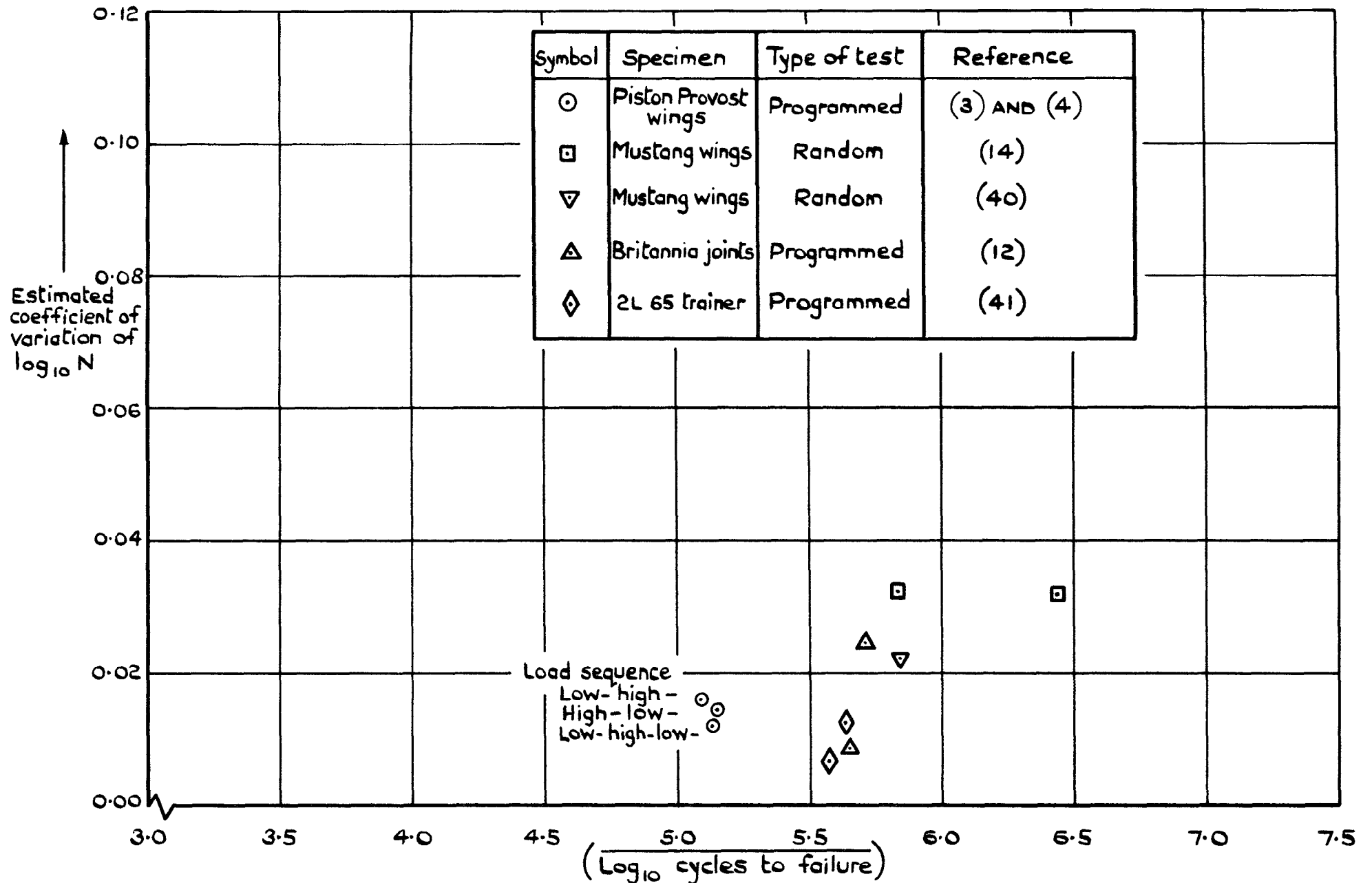


Fig.13 Variable amplitude fatigue test results  
 Accumulated data for a variety of aircraft types

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AIRCRAFT STRUCTURES

The fatigue test results, for elements and sections from the structures of aircraft, obtained by a number of experimenters are analysed in terms of the scatter present in the lives to fatigue failure. Data for structures made of any light alloy material and tested under any form of loading have been included in an attempt to increase the definition of any trends noted in the scatter, which was calculated using a log-normal distribution of fatigue lives as a basis.

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