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Deformation and Failure under Multiaxial Stresses - A Survey of Laboratory Techniques and Experimental Data

by

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1974

PRICE £1.20 NET

C.P. No.1306*
January 1972

Deformation and failure under multiaxial stresses - a survey of laboratory techniques and experimental data

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SUMMARY

A survey has been made of techniques for producing multiaxial stress conditions on a laboratory scale and it is concluded that one of the more suitable systems is combined tension-torsion of thin-walled tubes. Experimentally this technique, although complex, appears to offer several advantages. Furthermore, the shear/normal stress ratio can be conveniently controlled, thus enabling deformation and fracture mechanisms under multiaxial stresses to be studied. A review has also been presented of relevant experimental work, most of which has been concerned with assessment of the various deformation and failure criteria. Although there is some correlation for yield data, ductile or brittle fracture and creep behaviour, the situation is not nearly so well defined for fatigue.

Work on the micro-structural effects of biaxial stresses is also considered. It would appear that although the mechanisms by which dislocations operate remain unchanged from the uniaxial state, modifications can occur to the products of interactions between dislocations. In addition fracture behaviour can be altered by the ratio of tensile to shear stress. Generally, however, little work has been published on this aspect of deformation behaviour. It is concluded that it is important to study micro-structural effects particularly in relation to design criteria.

M.X.A. 22.12.71

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*Replaces NGTE NT 833 - A.R.C.33 753.

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1.0 Introduction

In recent years, particularly since the advent of the gas turbine, a considerable amount of information has been gathered on the creep and fatigue behaviour of a wide range of metals and alloys. Most of this information, however, has been obtained from laboratory experiments involving uni-directional states of stress. In contrast engineering components often operate under more complex loadings. For instance, in the gas turbine, discs, blades and flame tubes, and indeed many other components, can experience an essentially biaxial stress distribution of mechanical or thermal origin. Since such components are continually being pushed to their design limits, it is becoming increasingly important that some account should be taken of multiaxial stresses in the laboratory assessment of materials.

With a view to setting up a facility for producing biaxial stresses on a laboratory scale and to undertake a programme of work on the behaviour of gas turbine materials under such conditions, an extensive survey of the literature has been made. The purpose of this Note is to examine the merits of some of the more widely adopted experimental techniques and to discuss the relevant aspects of the reported data. From these observations it has been possible to define a fairly broad path along which future experimental programmes should be aimed.

2.0 Experimental techniques

The number of techniques proposed for producing biaxial stress distributions on a laboratory scale is relatively large when compared with the rather limited published experimental data on the subject. This diversity probably arises from each experimenter trying to simulate the actual stress pattern found in the particular engineering component that he is considering. For instance, the behaviour of gas turbine discs is often examined by spin tests on small laboratory models[1] whereas, in the pressure vessel field, internal pressurisation of thick walled cylinders is preferred[2]. It is not the purpose of this survey to examine the entire range of techniques but to consider the more widely adopted methods which are relevant to the problems associated with gas turbines. For each technique, the detailed form of the equipment generally varies in the different laboratories although the general principles on which the method is based are often similar. Consideration of each individual apparatus is not relevant to the present survey and consequently only one particular example has been chosen to represent each of the subsequent sections.

2.1 Rotating disc equipment

Although the model disc spinning test is expensive in terms of the material content of the specimens and the sophistication of the experimental rig, it is preferable to full scale engine tests for materials evaluation. By this technique it has been possible to understand material behaviour under conditions leading to disc bursting[3,4] and to derive suitable criteria which can be effectively used for the design of the full scale component[4,5]. More recently low cycle fatigue has also been simulated with model discs. Work has been published accounting for biaxiality effects in terms of a failure criterion and relating elastic and plastic strain behaviour to a linear cumulative damage theory[6].

At NGTE equipment of this type has been in operation for some time[7]. Currently, specimens are about 5 in. diameter by 0.5 in. thick. The standard disc is solid but specimens with a central bore are also used. Notch effects can be produced by drilling holes at relevant positions across the radius. The discs are spun under vacuum by means of an air turbine with speeds of 200,000 rev/min being achieved. Temperatures in the region of 700°C can be attained by the incorporation of electrical elements in the vacuum chamber. Fatigue cycles have been introduced by means of an efficient braking device so that discs may be cycled between the required speeds at a convenient frequency. During testing care is exercised to ensure that the disc is free to contract and expand. The plastic deformation can be determined from strain grids on the surface of the discs either when the disc has been brought to rest or after burst by reassembling the fragments.

In a thick disc the state of elastic stresses is essentially triaxial. However, in a disc which is plain and relatively thin the largely compressive axial component can be neglected, Figure 1. The tangential stress is a maximum at the centre and falls to a minimum at the rim, while, in a solid disc, the radial stress decreases from a value equal to the tangential stress at the centre to zero at the rim, Figure 1(a). If the disc contains a central bore, although the distribution of the tangential stress is similar, the radial stress becomes zero at the bore, Figure 1(b).

Once yielding occurs, however, plastic deformation can result in stress redistribution. This phenomenon has been shown to play an important part in the observed burst characteristics of model discs[3,4]. Baxendale and Bullard[5] have recently demonstrated the effect of speed on the stress distribution in a model bored disc made from Waspaloy, Figure 2. On the basis of such observations, it is possible to predict the manner in which the stresses will alter in a full size disc if the design speed is exceeded[4].

2.2 Cruciform specimen equipment

Several specific forms of biaxial-cross specimens have been used[8-10] although the most useful shape is a cruciform with a reduced central area, Figure 3(a). In the equipment used by Pascoe and de Villiers[8] each arm is gripped in serrated jaws and the loads are applied hydraulically. The reduced centre section ensures a stress peak in that region. The shape of the test area, however, has been so designed to maintain elastic stress uniformity over a considerable area. The elastic strain distribution along each principal axis under uniaxial loading is shown in Figure 3(b). The strain is measured by means of foil gauges bonded at the centre of the test section. In the elastic region there is a definite relationship between the stress in each arm and the strain in the test area. Once yielding has occurred and stress redistribution has taken place the stresses in the centre region are unknown. This is because an unknown proportion of the load is carried in the outer elastic ring. Under these conditions, the only parameter which can be determined is the total strain.

The technique has been used to produce biaxial stress strain curves[9] and to undertake low cycle fatigue[8-10]. The biaxial strain ratios that have been considered are relatively simple largely because of the problems associated with synchronisation of unequal loads applied to each arm. For instance, Pascoe and de Villiers in their fatigue work could only consider uniaxial, equibiaxial and shear loading. Even then, it should be noted that the state of strains with the uniaxial condition was not identical to that found in uniaxial tensile specimens. Thus, for mild steel the strain situation could be represented by (ε, -0.625ε, -0.375ε) while for QT35 steel it was (ε, -0.54ε, -0.46ε) both of which are in contrast to the more usual (ε, -0.5ε, -0.5ε).

Recently Wilson and White have critically assessed biaxial fatigue specimens by finite element analysis and photoelastic techniques[11]. They have introduced a flat bottomed area to the central recess. The purpose of the work was to study elastic and plastic strain distribution and fatigue behaviour in specimens with various d/t ratios where d is the diameter of the flat bottomed region and t is the minimum specimen thickness. The maximum and minimum thickness and the overall diameter of the reduced section were maintained constant but the d/t ratio took values of 12.5, 5, 2.5, 0. The latter corresponded to the specimen used by Pascoe and de Villiers.

Generally, good agreement was obtained between the finite element calculations and the photoelastic measurements. The former showed that irrespective of specimen profile the elastic strain outside the reduced area remained constant. In the central region, the strain distribution was found to be most uniform for the specimen with d/t = 12.5. For the specimen with d/t = 0 it was apparent that the strain continuously varied over the section. However, susceptibility to buckling under high strain fatigue involving equal tensile and compressive cycles occurred in the reverse order. Nevertheless, for low strain cycling the specimen with the largest flat region correlated well with data obtained for push-pull cylindrical specimens. It was concluded that for high endurance fatigue the specimen with d/t = 12.5 should be used, while for strains greater than ±1 per cent specimens with d/t = 5 would be more suitable. A continuously radiused profile, however, should be avoided.

2.3 Bulge test

A technique which has found some support, particularly in the areas of pressure vessel design, involves subjecting a simply supported plate to uniform pressure either on one side or alternating between both sides[12-14]. The strain distribution in the centre of the specimen depends on the shape of the plate. For instance, a $\frac{3}{4}$ in. thick rectangular plate provided a 3.7/1 strain ratio[13], while a 1/16 in. thick circular plate (16 in. diameter) produced an equibiaxial strain field on each surface[14]. Generally, the specimen is so designed that a substantial area is subjected to the strain field at large plastic strains.

The specimen and pressurisation equipment used by Ives et al[14] is shown schematically in Figure 4(a). The pressurising medium was oil and the equipment so designed that the specimen was deflected in either direction through activation of a microswitch. This caused oil to be vented from one side to the other. Strains were determined by means of gauges bonded to each surface.

The static strain analysis of a circular steel specimen is shown in Figure 4(b). Yielding occurs at the centre of the plate over a well defined area of approximately 2 in. diameter. Similarity in the distribution of the circumferential and radial strains was taken to confirm the assumption that the strain field was equibiaxial. For dynamic testing, the cyclic rate was maintained at less than 50 c/m. Fatigue crack initiation and propagation were detected by means of the strain gauges and a specimen was considered to have failed completely when a gauge indicated a marked change in amplitude of the specimen.

2.4 Internal pressurisation equipment

One of the more widely adopted methods of investigating bi- and triaxial stress states is internal pressurisation of thin and thick walled cylindrical tubes. For a thin wall tube closed at one end, the radial stress is equal to the applied pressure at the inside surface. This is relatively small and can be neglected. The axial and tangential stresses, however, are dependent on the wall thickness and the tube radius. They are given by the equations:

$$\sigma_{Tangential} = \frac{Pr}{\delta t} \qquad \ldots(1)$$

$$\sigma_{Axial} = \frac{Pr}{2\delta t}$$

where P is the pressure, r the radius and δt the wall thickness. Pure internal pressurisation, however, restricts the principal stress ratio to 2/1. The range can be extended by simultaneously imposing a tensile or compressive axial load on the specimen.

The thick walled tube, on the other hand, provides a triaxial stress state which can be mathematically reduced to uniform triaxial tension superimposed on a simple shear stress. The well known Lamé equations for the elastic stresses in the walls of a thick cylinder supporting its own end load are[15]

$$\sigma_{rr} = \frac{Pr_o^2}{r_1^2 - r_o^2}\left(1 - \frac{r_1^2}{r^2}\right) \qquad \ldots(2)$$

$$\sigma_{\theta\theta} = \frac{Pr_o^2}{r_1^2 - r_o^2}\left(1 + \frac{r_1^2}{r^2}\right)$$

$$\sigma_{zz} = \frac{Pr_o^2}{r_1^2 - r_o^2}$$

where σ_{rr} is the radial stress at radius r, $\sigma_{\theta\theta}$ the tangential stress at radius r and σ_{zz} the axial stress which is constant across the section, r_1

and r_0 are respectively the outer and bore radii, and P is the internal pressure. Morisson et al have shown[16] that if a volumetric tensile stress equal to σ_{zz} is subtracted from the three components, the two remaining stresses reduce to the shear stress.

$$\sigma_{r\theta} = \frac{P}{(r_1^2 - r_0^2)} \frac{r_1^2 r_0^2}{r^2} \qquad \text{....(3)}$$

This term has a maximum value at the bore

$$\sigma_{r\theta}(\mathrm{MAX}) = \frac{Pr_1^2}{(r_1^2 - r_0^2)} \qquad \text{....(4)}$$

The ratio of triaxial tension to shear stress is related to the ratio, k, of the internal to external radii of the cylinder so that with increasing k the triaxial tension decreases although it never becomes zero.

In the majority of experiments, the pressurising medium is oil[16-18] but water[19,40], steam[2,20] and gases such as nitrogen and argon have also been tried[21-23]. The technique has not only been used for laboratory investigations into the behaviour of materials under tensile[24,25], creep[26,27] and fatigue[28,29] conditions but also for large scale simulation of service conditions in pressure vessels[2]. Frequently, and particularly for the simulation tests, the equipment is housed in a reinforced cell to contain the fragments of a catastrophic failure. This problem is exaggerated by elevated temperatures and by the use of a gaseous pressurising medium due to the enhanced energy content[21].

At NGTE a novel experimental approach to the use of internal pressure was devised by Waldren for simulation of the biaxial stress system found in rotating discs[25]. The apparatus has been designed for use with a Hounsfield Tensometer. Internal pressurisation of a thin walled tube, concurrently with the development of an axial stress, arises from movement of the cross-head. This is due to a small annular reservoir containing the pressurising medium being closed up and the fluid being injected via a narrow tube into the specimen. The equipment is shown schematically in Figure 5. The axial load is measured by means of a load cell and the surface strain by foil gauges. Each stress ratio is controlled by the relative magnitudes of the internal pressure and the axial load and thus is determined by the size of the reservoir. The annular area of the reservoir is given by the equation

$$A = \pi r^2 \left\{ \frac{2}{x} - 1 \right\} \qquad \text{....(5)}$$

where r is the mean radius of the test tube wall and x is the tangential/axial stress ratio. The equipment has been so designed that four tangential/axial stress ratios can be produced (i.e. 0.5, 0.7, 0.85, 1.0). At the present time, the pressurising medium is silicone oil. The low flash point limits operation to approximately 300°C.

Before leaving the topic of internal pressurisation, it is worth noting that not every technique relies on adjustments in the axial load/pressure ratios for variations in the principal stresses. For instance, a similar effect can be achieved by changing the shape of the specimen[30]. This, however, introduces problems through the effects of different geometries on the observed mechanical behaviour.

2.5 Torsion test

Torsion provides a simple shear loading. The shear stress is a maximum at the surface and for a solid cylindrical specimen the maximum elastic shear stress is

$$\tau_{max} = \frac{16T}{\pi D^3} \qquad \text{....(6)}$$

where T is the torsional moment and D is the diameter of the specimen. Frequently a thin tubular specimen is used to avoid the stress gradient across the diameter which causes the surface layers to be restrained by the less highly stressed interior. The shear stress is then given by the equation

$$\tau = \frac{16TD_1}{\pi(D_1^4 - D_2^4)} \qquad \text{....(7)}$$

where D_1 and D_2 are respectively the outside and inside diameters of the tube. Shear strain is given by the relationship

$$\gamma = \frac{r\theta}{L} \qquad \text{....(8)}$$

where r is the radius of the specimen, θ is the angle of twist and L is the gauge length.

Beyond yield, the shear strength over the cross section of a bar is no longer a linear function of the radius. The maximum shear stress can then be derived according to the Nadai analysis as

$$\tau_a = \frac{1}{2\pi r^3}\left(\theta' \frac{dT}{d\theta'} + 3T\right) \qquad \text{....(9)}$$

where θ' is now the angle of twist per unit length (i.e. θ/L).

Shear stress can be represented by two equibiaxial principal stresses, one tensile and the other compressive. In Figure 6 the equivalent pure shear conditions are demonstrated in terms of the Mohr's circle. This situation corresponds to a principal stress ratio of -1. To increase the range of principal stress ratios, the torsion test is often combined with other deformation modes. One of the more commonly used systems is combined bending and torsion[28]. This is usually found in multiaxial fatigue experiments where the two deforming modes have been applied in and out of phase. Bend tests, however, have the disadvantage that the stress distribution is not uniform throughout the cross-section. For instance, if the stress is tensile above the neutral axis of the specimen, then the equivalent stress will be compressive below and vice versa. A more uniform situation occurs where the torsion mode is combined with axial tension and compression. This experimental condition has largely been used for the determination of stress-strain curves[24] and creep data[31]. Under both torsion/bending and torsion/tension the principal stresses are given by

$$\sigma_1 = \tfrac{1}{2}\left\{\sigma_x + \left(\sigma_x^2 + 4\tau_{xy}^2\right)^{\frac{1}{2}}\right\} \qquad \ldots.(10)$$

$$\sigma_2 = 0$$

$$\sigma_3 = \tfrac{1}{2}\left\{\sigma_x - \left(\sigma_x^2 + 4\tau_{xy}^2\right)^{\frac{1}{2}}\right\}$$

where σ_x is the axial stress due to bending or tension (compression) and τ_{xy} is the shear stress due to torsion. The experimental technique involves working at fixed ratios of σ_x/τ_{xy} which, in turn, results in specific ratios of σ_1/σ_3. This ratio is always negative because σ_3 is compressive.

Recently, several machines have been constructed which combine the torsion/tension system with internal pressure[32-34]. Each stress component is individually programmable and capable of operation in or out of phase with the other stresses. Equipment of this type has been used to study the effects of loading path and the shapes of yield surfaces.

3.0 Appraisal of the experimental techniques

Some of the points for and against each technique are summarised in Table I although a fuller consideration is presented below.

Although the spinning disc experiment provides a useful simulation of one of the conditions found in gas turbines, it is essentially a macroscopic approach to the problem of understanding material behaviour under complex stresses. This criticism can be tied down to three related points. In the first place, the size of the specimens tends to make detailed examination difficult by optical, and particularly electron metallographic techniques. Secondly, the variation of the radial/tangential stress ratios across the specimen radius complicates the task of determining the effects of biaxial stresses on deformation and failure characteristics. Thirdly, and perhaps most significantly, fatigue is a localised phenomenon being determined by variations in the magnitude of the normal and shearing stress components and the direction of the shearing stress component on critical planes such as those containing the principal or maximum shearing stress. Consequently a more sensitive and specific laboratory experiment, in which the stress distribution can be accurately measured and controlled, is required to determine the manner in which mechanisms such as the initiation and propagation of cracks are influenced by various biaxial stress situations.

The cruciform experiment appears to offer certain advantages in this respect. However, there are experimental and theoretical features which make it unsuitable. For instance, only a limited number of stress conditions (shear, equibiaxial and uniaxial) can be conveniently produced. Although attempts are being made to extend the range by means of servo control on each of the specimen arms[8], the problem is extremely difficult. Once yielding has occurred with any of the loading states the stress distribution is not known although the strains are measured by resistance gauges. At NGTE calculation[35] and experiment[36] have shown that load or stress controlled experiments, particularly as far as thermal fatigue is concerned, correlate better with service conditions than tests in which strain control is used. In addition, the manner in which stresses redistribute has been shown to be an important factor in the lives of disc and blade materials[4,5]. Consequently, the measurable parameters in the cruciform experiment are at variance with those required for a relevant analysis in terms of gas turbine components. Another feature of some importance is the determination of failure. For fatigue work the initiation and propagation of cracks was followed by optical microscopy[8]. Fracture detection consequently is not particularly convenient at room temperature and would be unreliable at elevated temperatures. Furthermore, failure was arbitrarily taken as the point at which the crack had propagated sufficiently for relative motion to be observed between the two sides of the crack. It is not certain that this criterion is representative of bulk behaviour while the constraining effect of surrounding elastically strained material in the specimen arms might have an effect on the processes of initiation and propagation.

When the various techniques are compared on a cost basis, the bulge test appears to be one of the more economical. Again there are problems with monitoring the fracture area. With increasing temperatures this problem would be accentuated. In addition there might be complications due to thermal stresses arising from a possible uneven temperature distribution across the test-piece. However, a major objection to the technique is the necessity for a different specimen shape to produce each stress or strain ratio. This not only restricts the range of experimental conditions but also introduces complications through the influence of different geometries on the mechanical properties.

A problem where pressurisation is being used, which would apply to the bulge test but more particularly to techniques involving tubular specimens, is the manner in which the pressurising medium may affect the propagation of surface cracks[37]. Morrison et al have shown that the fatigue strength of thick cylinders under internal pressure is about half the expected value[38]. This was partially attributed to the oil used for pressurisation. Frost[39] has pointed out that under pressure the fatigue limit depends on a critical stress to propagate small surface cracks. Since in a thick cylinder the cracks tend to propagate in a direction perpendicular to the tangential stress, the pressure will modify the magnitude of the tangential stress required to propagate the cracks. Thus combination of the tangential stress and the internal pressure (P) gives

$$P + \frac{P(1 + k^2)}{k^2 - 1} = \frac{2k^2P}{k^2 - 1} = 2 \times \text{maximum shear stress in bore (Section 2.4)} \qquad \ldots(11)$$

where k is the ratio of external to internal diameter of the cylinder. This result is consistent with Morrison's experimental data[38]. Oil is not the only medium that can modify fatigue behaviour. Thus Skelton and Crossland also mention that an inert gas such as argon can have a deleterious effect[40]. These observations have largely been made at room temperature. It might be expected that the problem would be exaggerated by increasing temperature.

Combined tension/torsion on the other hand offers some advantages. For instance, most engineering situations involving complex stress can be reduced to tension and torsion by the addition or subtraction of a suitable hydrostatic component. Although hydrostatic stress can possibly influence situations in which a tensile stress is the important controlling parameter, it should not markedly affect behaviour where shear stresses are operating. Thus the growth or propagation of cracks might be affected by a hydrostatic stress whereas mechanisms controlling plastic deformation, provided it is of moderate proportions, should not be altered. One of the problems with the tension/torsion technique, however, has been the lengthy calculation required to convert the torque/twist data measured from the specimen to shear stress and shear strain. The procedure though is ideally suited to computing methods. Another factor has been the unknown effect of the constraining core material in a solid specimen on the mechanical behaviour. Recently, Miller and Chandler have demonstrated that the core material has no effect on fatigue endurance for inside to outside diameter ratios less than 0.5 (Reference 41). Thin tubes tend to have reduced lives probably as a result of faster crack propagation after the removal of constraining elastic core material. It should be remembered, however, that thin shells are necessary if the variation of shear stress across the diameter is to be minimised.

Recently Miller has given an appraisal of torsion testing as a method of evaluating materials properties particularly under cyclic stresses[42]. He points out that since shear predominates in the deformation and fracture processes, the torsion test should be a convenient method of examining the problem. Furthermore, mechanical behaviour can be significantly altered by changes in strain rate[43]. With torsion, the surface strain rates can be controlled during deformation even near the point of failure. Forrest has also shown that experiments in which the principal stresses are of opposite sign are required to separate the criteria relating to fatigue[28]. On the basis of these observations, combined tension/torsion would appear to offer certain advantages for the understanding of material behaviour under complex stresses.

4.0 Literature survey of criteria describing multiaxial stress data

4.1 Yielding and tensile fracture under multiaxial stresses

4.1.1 Yielding behaviour

Combined tension/torsion and tension/internal pressure are probably the most widely adopted methods of investigating yielding and fracture behaviour under complex stresses. For yielding, the criteria most frequently used are the maximum shear stress or Tresca theory i.e.

$$\sigma_0 = (\sigma_1 - \sigma_3) \qquad \text{....(12)}$$

or the von Mises distortion energy theory which states that

$$\sigma_0 = \frac{1}{\sqrt{2}} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 \right]^{\frac{1}{2}} \qquad \text{....(13)}$$

where σ_0 is the uniaxial tensile yield stress and σ_1 σ_2 σ_3 are principal stresses ($\sigma_1 > \sigma_2 > \sigma_3$). The maximum shear stress theory provides a fair agreement with the experimental results tending to be on the safe side in its predictions. It has often been used by designers for ductile materials but, in general, the von Mises theory is preferred. A point of interest here, however, is the physical meaning of this criterion. One of the proposals is that the equation can be regarded as representing the strain energy of distortion i.e. yielding occurs in the multiaxial mode when the strain energy of distortion per unit volume exceeds the equivalent parameter in uniaxial tension or compression[44]. Another proposal is that the criterion is a critical value of the octahedral shear stress. This is the shear stress on the faces of a three dimensional octahedron which form equal angles with the three principal axes[44]. It is also interesting that close agreement with the predictions of Equation (13) can be obtained from a statistical analysis of the behaviour of randomly orientated crystal aggregates[45].

Generally speaking, the von Mises criterion has been found to give the best description of yielding in a number of ductile materials[46,47]. For instance, Lode working on thin walled tubes of iron, copper and nickel, subjected to internal pressure combined with axial tension, demonstrated that the intermediate principal stress influenced yielding[48]. This would not have been found if the shear stress criterion was correct, (Equation (12)). The graphical representation of Taylor and Quinney[46] perhaps best summarises the relative manner in which the Tresca and von Mises criteria have generally been found to describe experimental data, Figure 7. For the purpose of the graph the maximum shear stress theory of yielding has been reduced to

$$\left(\frac{\sigma}{\sigma_0}\right)^2 + 4\left(\frac{\tau}{\sigma_0}\right)^2 = 1 \qquad \text{....(14)}$$

and the distortion energy theory to

$$\left(\frac{\sigma}{\sigma_0}\right)^2 + 3\left(\frac{\tau}{\sigma_0}\right)^2 = 1 \qquad \text{....(15)}$$

where σ and τ are respectively the axial and shear components of stress in a combined tension/torsion test and σ_0 is the uniaxial yield stress.

4.1.2 Fracture

Although yielding can apparently be adequately described in the majority of materials examined by the von Mises relationship, fracture behaviour cannot be so conveniently classified. Dieter[44] has pointed out that this is due to fracture being a function of temperature and prior plastic deformation. Fracture can be considered in terms of a three-dimensional surface, the bounds of which are determined by the three principal stresses. A material is then assumed to fail when this limiting surface is reached. From experimental observations it is possible to deduce that the surface is not fixed but can be altered by prior stress and strain history. Consequently it is not surprising that several criteria have been found which adequately fit biaxial data. For brittle materials, the normal stress criterion or the Mohr theory are frequently used, while in ductile materials the maximum shear stress theory or von Mises criterion are preferred[47].

The normal stress criterion and the Mohr theory are compared graphically for biaxial stresses in Figure 8(a) while the differences between the shear stress and von Mises criteria are illustrated in Figure 8(b). Generally speaking the normal stress criterion is preferred for brittle materials. For brittle cast iron however, although the data follows this criterion in the tension-tension region, the failure strength is greater than the predicted value if one of the principal stresses is compressive[49]. For ductile metals such as aluminium and magnesium alloys[50] and soft steels[51], the maximum shear stress theory is frequently used in preference to the von Mises criterion.

4.2 Fatigue under multiaxial stresses

4.2.1 Theoretical arguments

Fatigue failure under uniaxial conditions can be separated metallographically into two categories[52]. There is the movement of dislocations leading to the initiation of cracks within slip bands, followed by propagation of the cracks until failure finally occurs. Propagation itself can be conveniently divided into two stages. Initially Stage I growth involves a deepening of the original nuclei on planes of high shear stress. Stage II follows which is assumed to be the result of a reduction in the shear to normal stress ratio at the crack tip. The crack therefore tends to propagate perpendicular to the applied stress. It is this stage that is characterised by the formation of striations. Depending on stress and temperature, initiation can occur in less than 10 per cent of the fatigue life, Stage I growth can occupy up to 90 per cent while Stage II may take between 10 and nearly 100 per cent of the cycles to failure.

Based on this metallographic evidence it is possible to arrive at some very general assumptions concerning the mechanics of fracture under more complex stresses. Thus it has been postulated that initiation will be determined by a critical value of the alternating shear stress[53] with possible modification by the magnitude and sign of the normal stress on the plane of maximum shear[54]. On the other hand it has been suggested that propagation might be expected to spread in a direction perpendicular to the direction of maximum tension-compression[55]. However, experimental work[56] involving combined torsion and bending demonstrated that tensile cracking occurred only in preference to shear if the ratio of tensile /shear stress was greater than 1.6.

Blass and Findley[18] have pointed out that the theories on complex stress fatigue can in fact be divided into two groups (a) those where the principal or critical shear stress is important so that the intermediate principal stress has no effect, (b) those where all the principal stresses are implicated.

4.2.2 Multiaxial fatigue in tubular specimens subjected to internal pressure

Experiments on thin and thick walled tubes subjected to an axial load and internal pressurisation have demonstrated that fatigue behaviour is primarily dependent on the maximum shear stress[16,38,57-59]. Some of the data from thick cylinders indicated that the fatigue strength was about half the expected value when compared with torsion experiments[38]. This anomaly was partly attributed to the deleterious effect of the pressurising oil. It has previously been mentioned (Section 2.4) that the experimental conditions used in this work produce a shear stress, which is a maximum at the bore, superimposed on a tensile hydrostatic component. The cylinders in fact failed at the bore with the cracks propagating perpendicularly to the tangential stress. Blass and Findley, using a similar technique, came to the same conclusion that the maximum shear stress provided the best description of fatigue failure[18]. In these experiments, however, they maintained the ratio of the maximum/minimum principal stresses constant and demonstrated that there was no effect of intermediate principal stress on fatigue behaviour, Figure 9(a).

4.2.3 Multiaxial fatigue experiments not conforming to theoretical ideas

In contrast to the internal pressurisation experiments, other techniques for producing multiaxial fatigue do not correlate so neatly with the theoretical assumptions. For instance, in combined bending/torsion, where each loading mode can be varied independently either in or out of phase, ductile materials appear to be best approximated by the von Mises criterion while brittle materials fit in better with the maximum principal stress theory[28]. The discrepancy for the ductile materials from the expected maximum shear stress concept has been attributed to anisotropy and the influence of the normal stress on the plane of maximum shear. Generally, however, none of the criteria have been found to describe the data with great accuracy and no one criterion has been found suitable to describe all the materials examined. This unsatisfactory situation led Gough to introduce his empirical ellipse quadrant formula[60]

$$\frac{S_b^2}{b^2} + \frac{S_t^2}{t^2} = 1 \qquad \text{....(16)}$$

where S_b is the semi-range of direct stress due to bending and S_t is the semi-range of shear stress due to torsion at the fatigue strength of the combination, t and b are respectively the semi-range fatigue strengths in torsion and bending. This relationship has been found suitable for ductile metals and in particular steels but is not applicable for cast iron where the following equation has been used

$$\frac{S_t^2}{t^2} + \frac{S_b^2}{b^2}\left(\frac{b}{t} - 1\right) + \frac{S_b}{b}\left(2 - \frac{b}{t}\right) = 1 \qquad \text{....(17)}$$

It is interesting that Findley was able to reduce the principal stress and principal shear stress criteria to the empirical ellipse form (Equation (16)) by the introduction of suitable assumptions concerning, for instance, a stress concentration factor in the torsional stress term[61]. Cox has deduced Equation (17) from a stress analysis of a material containing holes[62].

However, other experimental techniques have been able to describe fatigue behaviour on the basis of the von Mises criterion. Ives et al[14] used the equivalent strain equation

$$\bar{\varepsilon} = \frac{\sqrt{2}}{3}\left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_1 - \varepsilon_3)^2\right]^{\frac{1}{2}} \qquad \text{....(18)}$$

to relate equibiaxial data from bulge tests on pressure vessel steels to the nearly uniaxial strain situation in wide cantilever bar specimens, where ε_1 ε_2 ε_3 are the principal strains and $\varepsilon_1 > \varepsilon_2 > \varepsilon_3$. Again, Benham[63] has correlated torsion and uniaxial results in cold worked copper on an equivalent stress-strain basis although results for annealed copper could not be so conveniently expressed. Pascoe, on the other hand, has reported that shear, equibiaxial and uniaxial strain data obtained from tests on a cruciform biaxial specimen could not be related through an equivalent strain concept[64]. Instead, he proposed that for design purposes a graphical plot containing contours of equal fatigue life for a given state of strains should be adequate, Figure 9(b).

4.2.4 General fatigue behaviour under multiaxial stresses

Experiments with multiaxial fatigue have not been only concerned with obtaining suitable stress and strain criteria. A number of experiments have been aimed at determining the suitability of the Coffin-Manson relationship under these conditions i.e.

$$\Delta\varepsilon N^{\alpha} = C \qquad \text{....(19)}$$

where $\Delta\varepsilon$ is the total or plastic strain range, N the number of cycles to failure, and α and C are constants.

Several results have shown that the equation is obeyed[42,65,66] but generally it appears that a wide range of values for α and C have been obtained. Pascoe[64] has demonstrated that this variation probably arises because both exponent and constant are related to the state of strain existing in a particular experiment. A similar finding has also been reported for the exponent by Johnson et al from torsional fatigue data for which a relationship was deduced between the strain ratio, strain energy and endurance[67].

Other work has been concerned with application of Miner's cumulative damage law under complex stresses i.e.

$$\sum_{i=1}^{k} \frac{n_i}{N_i} = 1 \qquad \text{....(20)}$$

where n_i and N_i are respectively the number of cycles completed and the number of cycles to failure at the i^{th} stress or strain level[68]. Blatherwick and Viste[69], using reversed axial and torsion loads on thin walled steel specimens, showed that at a fixed maximum to minimum principal stress ratio ($\sigma_1/\sigma_2 = -2$) the behaviour was similar to that observed in uniaxial tests. Deviations from the law occurred so that for increasing stress amplitude the ratio was greater than unity but less for decreasing stress amplitude. Recently Miller has shown that in high strain torsion fatigue the discrepancies could be accounted for by differences in straining rate[70]. In other words, if the straining rate is maintained constant at the two stress or strain levels the sequence effect in the cumulative damage theory is reduced. The constant in Equation (20) however, was dependent on the definition of failure. If failure is the point of crack instability then a value of unity is obtained.

4.3 Multiaxial creep

4.3.1 Analysis of creep and creep fracture

Creep and creep fracture under multiaxial stresses appears to be reasonably well understood for a wide range of materials, largely through the efforts of A. E. Johnson and his colleagues first at NPL and then NEL[31,71,72]. The work was carried out mainly by combined tension/torsion but the results correlate well with data obtained from other techniques[73]. This particular method was chosen because of the assumption that by the addition or subtraction of a suitable hydrostatic stress, complex stress systems generally can be replaced by an equivalent system of simple tension combined with simple torsional stress. Consequently experimental equipment based on these components should produce creep behaviour of a corresponding nature to the original complex stress distribution. This assumption was confirmed during the course of experiments at NEL when it was shown that creep rates were uninfluenced by hydrostatic stress[31]. The tests, which involved comparison of the tension/torsion data with that obtained from biaxial loading equipment[74], also proved the basic premise to complex-stress creep theories that the principal axes of stress and creep strain remain coincident for moderate amounts of strain.

During primary creep in all materials examined, the data appeared to correlate best with the von Mises or octahedral shear stress criterion, Figure 10(a). Even so, primary creep behaviour could be divided into two types depending on the applied stress. At low stresses the materials behaved isotropically and deformation was relatively uncomplicated. A primary creep rate equation based upon the uniaxial expression $\dot{\varepsilon} = A\sigma^n t^m$, was derived and had the form

$$C_{ij} = F(J_2)\, S_{ij}\, t^m \qquad \text{....(21)}$$

where $\dot{\varepsilon}$ is the creep rate, σ the applied stress, t the time, A, n, m are constants, C_{ij} is the creep rate tensor, and S_{ij} is the stress deviator tensor. $F(J_2)$ has the form $B(J_2)^p$ with J_2 the second order stress invariant, A and p are constants. The constant m usually has a value between -0.44 and -0.77 depending on the material and temperature while p often takes values between 0 and 1. At higher stresses, except for Nimonics 75 and 90, considerable anisotropy occurred which complicated the relationship between creep rate and stress, although the general form of Equation (21) was maintained.

Tertiary creep and fracture behaviour, however, were found to depend on the manner in which grain boundary cavities nucleated and grew. In materials such as Nimonics 75 and 90 where a random and continuous crack growth occurred during late tertiary creep, the maximum principal stress determined the time to fracture and, to a large extent, the tertiary creep strain. The relationship between the time to fracture and the maximum principal stress for Nimonic 90 is shown in Figure 10(b). On the other hand, for materials in which the first crack to form produced failure both tertiary creep and fracture were controlled by the octahedral shear stress. Materials exhibiting this behaviour, however, were in the minority.

It has previously been mentioned that most of the reported data on multiaxial creep largely confirms the observations of Johnson. For instance, Kennedy et al found that for Inconel at 1500°F the maximum principal stress is the best criterion of failure[75]. The intergranular cracks were generally distributed in a direction normal to the maximum principal stress. Similarly, Davis correlated rupture data for internal pressurisation of stainless steel tubes with the same criterion[26]. In the thick-walled tubes used in this work, fracture initiated at the outer surface by transverse intergranular cracking. On the other hand, Sawert and Voorhees found in a low ductility cast nickel base alloy (DCM) and a high ductility wrought nickel base alloy (Rene 41) that the shear stress invariant (or maximum shear-stress) theory provided a better description of fracture behaviour[76]. The low ductility material failed catastrophically while the wrought alloy exhibited ductile tearing. However, it would appear that with the latter the cracks, once initiated propagated rapidly thereby indicating that failure possibly occurred at one point only. This behaviour compares with Johnson's work on an aluminium alloy at 200°C, a 0.2 per cent steel at 450°C and a magnesium alloy at room temperature[31].

4.3.2 Creep under changing stress

Johnson and his colleagues also investigated the effect of changing stress during creep and found that the mechanical theories for creep were inadequate[31,77]. For instance, with the increasing stress conditions used in this work the time hardening and superposition theories predicted low values of creep strain, while the strain hardening and combined theories gave high values. The physical significance of the time hardening and strain hardening concepts is given in Figure 11(a). Thus if the creep strain is ε_1 after time t_1 at a stress level σ_1, under the time hardening theory the strain rate on increasing the stress from σ_1 to σ_2 is that at time t_1. The strain hardening theory, however, compares the strain rates at the two stress levels at the same value of ε_1. A combination of both provides a situation somewhere between these two extremes. On the other hand, the principle of superposition involves building up solutions of complicated problems by superposing the stress fields of simpler problems. Effectively, the combined effect of several loads is the same as the sum of the individual effects of the various loads.

The phenomenon of creep relaxation under constant ratios of applied stress was also examined[31,78,79]. For the limited number of materials considered, the time hardening theory predicted the relaxation complex stress/time curves to a reasonable approximation. The other theories tended to overestimate the relaxation time for a particular set of stresses.

Finally, data has been reported on the effects of variable stress on complex stress creep[80]. Basically, in commercially pure copper, the maximum principal stress criterion governed failure irrespective of cyclic stress. There was, though, an increase in life at lower stresses for a given maximum principal stress in proportion to the amount of fatigue, Figure 11(b). This strengthening effect was attributed to the replacement of coarse creep cracking by the formation of fine fissures.

5.0 Structural aspects of deformation and failure under multiaxial stresses

5.1 Introduction

The survey so far has largely dealt with engineering aspects of deformation and failure. In other words, the macroscopic behaviour of various metals and alloys has been examined and interpreted in terms of continuum mechanics whereby the material is assumed to be continuous, uniform and amorphous. In so doing, the tacit assumption has been made that microstructural variability and inhomogeniety approaches some statistical average in the bulk specimen or component. The purpose of this section is to examine the manner in which multiaxial stresses may influence material behaviour on a microscopic scale.

Although a reasonably wide range of materials has been examined, it is apparent that experimental work has been largely concerned with pure metals and simple alloys or with the complex steels generally found in the pressure vessel and generating industries. Recently, several papers have been concerned with the behaviour of ceramics and plastics under complex stress[81-83]. Only a limited amount of information, however, is available on the high temperature titanium, iron and nickel based alloys used in gas turbines[3-6,73]. This is surprising in view of the thermal and mechanical loads encountered by many of the components.

Very little of the published data has been specifically concerned with the manner in which multiaxial stresses might modify deformation and fracture mechanisms. Perhaps this is not surprising if one takes the view that inhomogeneous deformation and interactions between neighbouring grains in a polycrystalline material will mask any differences that might be introduced. However, there is some evidence to suggest that although operating slip systems remain unchanged modifications can occur to the products of dislocation interactions[84]. On this basis, it is possible to speculate that phenomena which depend on interactions, such as strain hardening and recovery, might be affected. Thus recovery rates during high temperature creep are determined by the climb of edge dislocations at jogs on dislocations[85,86] a process which could be altered by the introduction of a second stress system. Again the nucleation and development of cracks could be influenced by the normal/shear stress ratio. The dependence of fatigue failure on this ratio has previously been discussed (Section 4.2). Creep failure could also be so related since it has been proposed that grain boundary cavities are nucleated and grow from ledges formed by slip within grains[87-89]. Changes in the shear stress system could alter the distribution of cavities. It has also been suggested that once cavities attain a stable size further growth could be assisted by diffusion of vacancies[90,91]. This latter mechanism might be dependent on the magnitude and sign of the tensile stress.

Below are detailed some of the limited observations on the effects of multiaxial stresses on deformation and failure mechanisms. The fundamental ideas on the behaviour of dislocations during tensile, creep and fatigue deformation are not considered. There has, however, been several recent reviews on these subjects[52,85,86,92,93].

5.2 Deformation behaviour

5.2.1 Dislocation structures

Pittinato and Frederick examined deformation behaviour of a Ti - 5Al - 2.5 Sn alloy under uniaxial and 1/1 biaxial stress at 20°K (Reference 84). Irrespective of the stress system, deformation was primarily on the prismatic slip plane with pyramidal and basal slip also evident. There was extensive jog and dipole formation but two dimensional dislocation networks only occurred under biaxial stresses. These networks were the result of interactions between screw dislocations on the prism and basal planes, Figure 12. Calculation shows that for the strong basal plane texture in this material network formation is energetically favourable for biaxial stresses but not for the uniaxial state. Twinning was also noted to be more prevalent for the biaxial condition although not extensive enough to be considered a significant deformation mechanism.

In passing, it should be mentioned that a variety of laboratory tests (tension, compression, torsion, rolling etc) have been used for studies of strength and structure under hot working conditions. Although this work is not particularly relevant to the present topic, it is interesting that these techniques encompass a range of uniaxial and multiaxial stress states. Over a wide spectrum of strain rates, stress, strain and temperature, dislocation substructure is independent of the mode of deformation[94]. This would appear to add support to the belief that in general multiaxial stresses do not significantly alter dislocation slip processes.

5.2.2 The interaction between tension and torsion

When fcc metal wires are subjected to combined torsional and tensile deformation, it is possible to obtain plastic strains of up to 300 per cent[95].

Recently, changes in resistivity and work-hardening have been examined in high purity aluminium during this strain accumulation[96]. The wire specimens were subjected to static tensile loads superimposed on the torsional deformation which was applied at a constant rate of twist. Increasing tensile load resulted in an increase of tensile strain per unit torsion but a decrease of the strain coefficient of resistivity, Figure 13(a). The interpretation was in terms of stress dependent dynamic recovery occurring during the deformation. The tensile stress was believed to reduce the activation energy for cross-slip thereby leading to enhanced annihilation of screw dislocations. In addition, the intersection of dislocations, particularly between those produced by each deformation mode, would result in increased jog formation. Jogs however, tend to cluster under stress leading to annihilation and coalescence and a reduction in jog density. Both recovery phenomena increase dislocation mobility, thereby allowing the increase in tensile strain per unit torsion. At high average shear strains and high tensile stresses, the resistivity saturates. This is believed to be due to dislocation annihilation becoming the predominant recovery mechanism. Dislocation annihilation would then be of the same order as dislocation generation.

A similar effect is the observation of apparent creep at room temperature when cyclic torsion is superimposed on a tensile load which would not itself produce deformation[97]. In work by Maria Ronay, cylindrical specimens of super-purity aluminium were subjected to repeated constant torsion angles in conjunction with superimposed static tensile loads[98]. Strain accumulation occurred even without a superimposed tensile stress but was greatly accentuated by increasing tensile stress, Figure 13(b). It is pointed out, however, that a similar effect is produced if a cyclic axial strain is superimposed on steady torsion. In either case, the process always ends in a fatigue failure.

The strain accumulation, it is suggested, is produced during the loading range of each half cycle of the torsional shear stress - shear strain diagram. The tensile load magnifies the plastic strain increment during the loading regions of the torsion cycle. The magnitude of the amplification, however, would suggest strong interaction between axial stress and cyclic torsion. It is possible that as with combined torsion/tensile deformation, dynamic recovery is accentuated leading to increased mobility of dislocations.

5.3 Fracture behaviour

5.3.1 The effect of mean stress on crack propagation

Comparisons have been made between fatigue failures in torsion and bending[99,100]. Metallographic observations showed that slip lines and cracks appeared mainly in the direction parallel to the maximum shear stress. Growth also proceeded in the direction of maximum shear stress particularly in the tangential direction for the torsion mode. Mean stress was found to have no effect on fatigue life in torsion but to accelerate failure in bending. This was interpreted in terms of the relationship between mean stress and the direction of crack propagation. In bending the mean stress acted mainly as a stress component perpendicular to the cracks whereas for torsion it acted in the maximum shear stress direction.

5.3.2 Propagation of fatigue cracks in a biaxial stress field

Recently several investigations have been made into the propagation of fatigue cracks in biaxial stress fields[10,101]. The work evaluates the applicability of fracture mechanics techniques to complex stress situations. It is pointed out that elastic models predict only a second order effect on fatigue properties. In reality there is a plastic zone ahead of the crack which would probably be dependent on the stress state. Although an exact analysis of the stress and strain conditions at the crack tip is not available several approximate models have been proposed. Depending on the model it is possible to show that a stress parallel to the crack should accelerate fracture, have no effect or inhibit failure. Experimental work on biaxial cruciform specimens of aluminium alloy sheet and plexiglas show that biaxial tensile stresses give an increase in fracture toughness and a decrease in the slope of the log da/dN versus ΔK graph, where da/dN is the crack propagation rate, with 'a' the crack length, N the number of cycles, and ΔK is the stress intensity factor range[10]. On the other hand bulge test data on a different aluminium alloy indicate that for a given stress concentration factor, the rate of crack propagation decreased for decreasing biaxiality i.e. for a decrease in the stress parallel to the crack length[101].

5.3.3 Changes in fracture mode with biaxiality

The manner in which fracture mode can be modified by changes in biaxial stress ratio has been demonstrated by Davis for tubes of 0.23 per cent carbon steel subjected to internal pressure[51,102]. For tangential to axial stress ratios less than 0.76 the cracks were circumferential while at larger values they propagated longitudinally. In the former case fracture occurred along planes of maximum shear stress whereas the longitudinal cracks were initiated on planes of maximum shear stress but developed as a brittle fracture in the axial plane. The true maximum shearing stresses at fracture were found to be nearly constant for the two orientations of shear surface. Furthermore, the energy for plastic deformation per unit volume for shear fracture decreases with increasing tangential to axial stress ratio. Nadai points out that the apparent decrease of local ductility is caused by an instability in the uniform mode of deformation[24].

5.3.4 Creep fracture under biaxial stress

Creep rupture under multiaxial stresses has been studied by Johnson and his colleagues (see Section 4.3). Two types of failure were observed. For materials in which the time to failure was determined by the maximum principal stress the development of fracture was gradual and general throughout the specimen. If the von Mises criterion held no cracking was found during tertiary creep. Irrespective of these two broad categories, however, the mode of failure was found to be variable. For instance in the first group of materials molybdenum steel at 550°C exhibited both inter- and transcrystalline fracture whereas copper at 250°C, Nimonic 75 at 650°C and Nimonic 90 at 750°C failed in an intercrystalline manner. In the second group an aluminium alloy at 200°C showed intercrystalline failure while fracture was transgranular for 0.2 per cent C steel at 450°C and a magnesium alloy at 20°C.

Work has also been reported[80] on the effects of a cyclic stress on the creep fracture of copper at 250°C. Irrespective of the cyclic load cracking was general and progressive proceeding in an intercrystalline manner. For creep the cracks started from the interior of the specimen and were rather coarse in nature. Under fatigue conditions cracks were fine and started from the surface. For combined creep and fatigue an intermediate fracture type was observed.

Kennedy has reported work on the interaction between creep and fatigue failure in Inconel at 815°C in which the complex dynamic stress data could be correlated with the creep behaviour[103]. He points out that, although complex creep fracture appears to be often controlled by a maximum principal stress criterion, this observation is inconsistent with failure in specimens having stress gradients that cause the maximum principal stress and maximum shear stress to occur in different regions. Thus a notched specimen has a larger maximum principal stress than a plain test piece but rupture life is increased. This can be related to a decrease in shear stress. Furthermore, although thick-walled tubes fail at the outside surface where the principal stress is largest, medium and thin-walled pressurised tubes failed at the bore where the maximum shear stress occurs. However, he also mentions that shear stresses alone cannot be implicated in creep fracture since torsional data indicates that lives are greater than expected.

On this basis the Hull and Rimmer[104] approach to cavitation failure was adopted to explain the biaxial data. The cavities are considered to nucleate by grain boundary sliding but to grow subsequently by means of boundary diffusion. An expression for failure was derived from Hull and Rimmer's expression for creep failure and Miner's cumulative damage concept. For internal pressure tests the direction and time of failure could be described in terms of damage accumulation occurring independently in each principal stress axis.

It should be mentioned, however, that recent work on creep cavitation has questioned the ideas of diffusion controlled void growth. Grain boundary sliding at ledges or particles in the boundary has been suggested not only as the nucleation mechanism but also the rate controlling process for growth. Important experimental evidence has been put forward supporting these views and the contention that although diffusion processes occur they have only a minor effect on failure[91,105].

6.0 Conclusions

A wide range of techniques commonly used for simulating multiaxial stresses in the laboratory has been examined. One of the most suitable was found to be combined tension/torsion. The various parameters determining deformation and failure behaviour can be accurately controlled while an extensive range of activities can be undertaken from evaluation of design criteria to studies of the effects of different shear stress/tensile stress ratios on mechanical properties. At the same time the technique is not remote from engineering practice since most situations involving complex stresses can be considered in terms of combined tension/torsion through the addition or subtraction of a suitable hydrostatic component.

Although experimentally a wide range of materials has been examined (e.g. metals, alloys, plastics and ceramics) most data has been obtained for relatively simple metals or alloys and the advanced steels used in the power generation and pressure vessel fields. Little information is available on the complex nickel-, iron- and titanium-based alloys found in gas turbines. A large amount of the work has centred on evaluating the various empirical failure criteria. In this respect, a fair correlation has been possible for stress-strain data and creep behaviour but fatigue has not been so conveniently categorised. Some work, however, has also been reported on the effects of biaxial stresses on deformation and fracture mechanisms. Although the operating slip systems are not noticeably different from those observed after uniaxial deformation the products of dislocation interactions can be modified. This has led, for instance, to enhanced dynamic recovery in tension/torsion systems. Crack propagation can also be affected by biaxial stresses. Contradictory data have been produced concerning the effect of multiaxial stress systems on fatigue cracks. In tensile and creep tests involving internal pressure there are critical values of tangential/axial stress ratios where the direction of crack propagation changes from the circumferential to longitudinal directions. For creep, the type of failure could be accounted for by assuming that damage accumulation was independently additive in each principal stress direction.

In spite of the results achieved, the need for extensive experimental programmes involving multiaxial stress systems is becoming increasingly apparent as several recent reviews have indicated[106,107]. Although some work is now underway it is biased towards applications involving steam generating plant and to a phenomenological appraisal of the data for design purposes. An evaluation of the high temperature alloys used in gas turbines is essential while an understanding of structural aspects of deformation and fracture behaviour under complex stress is highly desirable. The areas of importance can be conveniently categorised.

(a) Evaluation of design criteria for low cycle fatigue particularly under conditions where there is an increasing contribution from a creep component.

(b) Determination of the effects of biaxial stresses on relaxation phenomena.

(c) Studies of changes in deformation mechanisms.

(d) Observations on initiation and propagation of cracks under cyclic loads and nucleation and growth of creep cavities at various tensile/shear stress ratios.

ACKNOWLEDGEMENTS

I am grateful to Dr G.P. Tilly for helpful discussions and careful reading of the manuscript and to Mrs M. Crawford for assistance with the diagrams.

REFERENCES

No.	Author(s)	Title, etc
1	G. L. B. Hall	Aero Gas Turbines Engines : Integrity of the high-energy rotating components as governed by low-cycle fatigue. The Metals and Metallurgy Trust, Monograph and Report Series, No. 32, pp. 28-39 1967.
2	W. J. Elder	Stress rupture testing by internal pressure. Design and construction of a multi-point tube bursting facility in South Eastern Region C.E.G.B. C.E.G.B. South Eastern Region, Scientific Services Department, Research Report No. 17/69.
3	N. E. Waldren M. J. Percy P. B. Mellor	Burst strength of rotating discs. Proceedings of the Institution of Mechanical Engineers, Vol. 180, part I, No. 5, pp.111-130, 1965-66.
4	L. Islip	Component design and material selection. Lecture III Bulleid Memorial Lectures, Vol.IV, 1967.
5	J. B. Bullard B. B. Baxendale	Some mechanical design problems of turbine blades and discs. A.G.A.R.D. conference on high temperature turbines No. CP/73/71, 1971.
6	J. L. Mattavi	Low cycle fatigue behaviour under biaxial strain distribution Trans. A.S.M.E., J Basic Engineering Vol. 91, pp.23-31, 1969.
7	A. Graham V.C.H. Bailey	The N.G.T.E. model disc spinning-rig and the effect of bore and bolt-holes on bursting speed. N.G.T.E. Report No. R91. ARC14 664. 1951.
8	K.J. Pascoe J.W.R. de Villiers	Low cycle fatigue of steels under biaxial straining. J Strain Analysis, Vol. 2, pp. 117-126, 1967.
9	S.W. McClaren E.L. Terry	Characteristics of aero-space materials subjected to biaxial static and fatigue loading conditions, A.S.M.E. Paper 63-WA-315, 1963.
10	J.L. Kibler R. Roberts	The effect of biaxial stresses on fatigue and fracture. Trans. A.S.M.E. J Engineering for Industry, Vol. 92, Series B, pp. 727-734, 1970.
11	I.H. Wilson D.J. White	Cruciform specimens for biaxial tests. An investigation using finite element analysis and photoelastic coating techniques. J. Strain Analysis, Vol. 6, pp. 27-37, 1971.

REFERENCES (cont'd)

No.	Author(s)	Title, etc
12	E. Paxson J. Marin L.W. Hu	Mechanical properties of a magnesium alloy under biaxial tension at low temperatures. Proceedings A.S.T.M. Vol. 55, pp.810-822, 1955.
13	R.V. Blaser J.T. Tucker L.F. Kooistra	Biaxial fatigue tests on flat-plate specimens. The Welding Journal, Vol. 31, pp. 161-168, 1952.
14	K.D. Ives L.F. Kooistra J.T. Tucker	Equibiaxial low cycle fatigue properties of typical pressure-vessel steels. Trans. A.S.M.E. J Basic Engineering, Vol. 88, Series D, pp. 745-754, 1966.
15	A.H. Cottrell	The mechanical properties of matter. John Willey & Sons Inc. 1964.
16	J.L.M. Morrison B. Crossland J.S.C. Parry	The strength of thick cylinders subjected to repeated internal pressure. Trans. A.S.M.E. J Engineering for Industry, Vol. 82, Series B, pp. 143-153, 1960.
17	J.L.M. Morrison B. Crossland J.S.C. Parry	Fatigue under triaxial stresses : Development of a testing machine and preliminary results. Proceedings of the Inst. Mech. Engs. Vol. 170, p. 697, 1956.
18	J.J. Blass W.N. Findley	The influence of the intermediate principal stress on fatigue under triaxial stresses. Materials Research and Standards. Vol. 7, pp. 254-261, 1967.
19	A. Cowan K.J. Cowburn	Critical crack-length measurements in hydrided Zircaloy-2 pressure tubes. J Inst. Metals. Vol. 95, pp.302-307, 1967.
20	B. Austin B. Crossland	Equipment for testing high-pressure components. Proceedings Inst. Mech. Engs. Vol. 180, 3A, pp. 118-133, 1965-66.
21	H.L. Cox N.B. Owen	The influence of biaxial mean stress on the failure of tubes by fatigue. The Aeronautical Quart. Vol. 12, pp.1-33, 1961.
22	W. Munro N.A. Hill	The deformation of Beryllium under complex stressing. Metallurgia, Vol. 65, pp.105-110, 1962.

REFERENCES (cont'd)

No.	Author(s)	Title, etc
23	R.J. Parker	Equipment for creep testing pressurised thick-walled metal cylinders. Proceedings Inst. Mech. Engs. Vol. 180, 3A, pp.113-117, 1965-66.
24	A. Nadai	Theory of flow and fracture of solids. McGraw-Hill Book Co. Vol. 1, Ch17, pp. 238-274, 1950.
25	N.E. Waldren	Unpublished work at N.G.T.E., 1963.
26	E.A. Davies	Creep rupture tests for design of high pressure steam equipment. Trans. A.S.M.E., J Basic Engineering, Vol. 82, Series D, pp.453-461, 1960.
27	A.E. Johnson J. Henderson B. Khan	Behaviour of metallic thick-walled cylindrical vessels or tubes subject to high internal or external pressures at elevated temperatures. Proceedings Inst. Mech. Engs. Vol. 175, pp. 1043-1069, 1961.
28	P.G. Forrest	Fatigue of metals. Pergamon Press. Ch IV. pps. 109-113, 1962.
29	A.E. Carden	Bibliography of the literature on multiaxial stress fatigue. Dept. of Engineering Mechanics, College of Engineering, University of Alabama, Report No. MH-67-AEC-Z, 1967.
30	R.E. Schreiber R.J. Allio	Multiaxial strain fatigue in Zircaloy-4. J Electrochemical Technology. Vol. 4, pp. 347-353, 1966.
31	A.E. Johnson J. Henderson B. Khan	Complex-stress creep relaxation and fracture of metallic alloys. H.M.S.O. Edinburgh, 1962.
32	W.N. Findley A. Gjelsvik	A biaxial testing machine for plasticity creep or relaxation under variable principal stress ratios. Proceedings A.S.T.M. Vol. 62, pp. 1103-1118, 1962.
33	H.Ll.D. Pugh W.M. Mair A.C. Rapier	An apparatus for combined stress testing in the plastic range. J Experimental Mechanics. pp. 281-288, 1964.
34	A.E. Carden	Low cycle fatigue under multiaxial stress cycling. Symposium 4-8 Sept. Tokyo 1967, Japan Soc. Mech. Eng., 1967.

REFERENCES (cont'd)

No.	Author(s)	Title, etc
35	P.W.H. Howe	Thermal, Mechanical and Microscopic loading. Lecture II, Bulleid Memorial Lectures. Vol. 4, 1967.
36	G.P. Tilly	Laboratory simulation of thermal fatigue experienced by gas turbine blading. Paper 11 at International Conference on Thermal Stresses and Thermal Fatigue held at C.E.G.B. Berkeley Nuclear Laboratories. 23-26 Sept. 1969.
37	J.S.C. Parry	Further results of fatigue under triaxial stress. Paper 7, Session 2 at the International Conference on Fatigue of Metals held at the Institution of Mechanical Engineers, London 10-14 Sept. 1956.
38	J.L.M. Morrison B. Crossland J.S.C. Parry	Strength of thick cylinders subjected to repeated internal pressure. Proceedings Inst. Mech. Engs. Vol. 74, pp. 95-117, 1960.
39	N.E. Frost	A note on the behaviour of fatigue cracks. J. Mech. Phys. Solids, Vol. 9, p. 143, 1961.
40	W.J. Skelton B. Crossland	Discussion at Session 4 : Pressure Vessels, Proceedings Inst. Mech. Engs. Vol. 180, 3A, pp. 153, 1965-66.
41	K.J. Miller D.C. Chandler	High-strain torsion fatigue of solid and tubular specimens. Proceedings Inst. Mech. Engs. Vol. 184.
42	K.J. Miller	High-strain, low endurance fatigue : a review. University of Cambridge, Dept. of Engineering, Ref. No. CUED/C - Mat/Tri, 1969.
43	D. McLean	Mechanical properties of metals. John Wiley & Sons Inc., Chs 5 and 7, 1962.
44	G.E. Dieter Jr.	Mechanical Metallurgy. McGraw-Hill Book Co. Inc. 1961.
45	H.L. Cox D.G. Sopwith	The effect of orientation on stresses in single crystals and of random orientation on the strength of polycrystalline aggregates. Proceedings Phys. Soc. (London) Vol. 49, p. 134, 1937.
46	G.I. Taylor H. Quinney	The plastic distortion of metals. Trans. Roy. Soc. London. Series A, Vol. 230, pp.323-362, 1931.

REFERENCES (cont'd)

No.	Author(s)	Title, etc.
47	R.E. Peterson	Stress concentration design factors. John Wiley & Sons Inc. (New York), pp.3-8, 1966.
48	W. Lode	Z Physik, Vol. 36, pp.913-939, 1926.
49	W.R. Clough M.E. Shank	The deformation and rupture of gray cast iron. Trans. A.S.M. Vol. 49, pp. 241-262, 1957.
50	J.E. Dorn	Fracturing of metals. Amer. Soc. Metals, Cleveland, Ohio. 1948.
51	E.A. Davis	Yielding and Fracture of Medium-Carbon Steel under combined stress. J. Appl. Mech. Vol. 12, Series A, pp.13-24, 1945.
52	W.J. Plumbridge D.A. Ryder	The metallography of fatigue. Met. Reviews No. 136, 1969.
53	B. Crossland	The effect of large hydrostatic pressures on the torsional fatigue strength of an alloy steel. Session 2, Paper 8. International Conference on fatigue of metals. Inst. of Mech. Engs. 1956.
54	W.N. Findley	Experiments on fatigue ranges of stress in torsion and axial loads from tension to extreme compression. Proceedings A.S.T.M. Vol. 54, p. 836, 1954.
55	A.K. Head	The growth of fatigue cracks. Phil., Mag. Series 7, Vol. 44, p. 925, 1953.
56	H.L. Cox J.E. Field	The initiation and propagation of fatigue cracks in mild steel pieces of square section. Aeronautical Quarterly, Vol. 4, p 1, 1952.
57	G.K. Marikawa Le Van Griffis	Biaxial fatigue strength of low carbon steels. Welding J, Vol. 24, p. 167, 1945.
58	H. Majors B.D. Mills C.W. MacGregor	Fatigue under combined pulsating stress J. Appl. Mech., Vol. 16, p. 269, 1949.
59	R.W. Bundy J. Marin	Fatigue strength of 14S-T4 Aluminium Alloy subjected to biaxial stress. Proceedings A.S.T.M., Vol. 54, p. 755, 1954.

REFERENCES (cont'd)

No.	Author(s)	Title, etc.
60	H.J. Gough H.V. Pollard	The strength of metals under combined alternating stresses. Proceedings. Inst. Mech. Engs. Vol. 3, p. 131, 1935.
61	W.N. Findley J.J. Coleman B.C. Hanley	Theory for combined bending and torsion fatigue with data for S.A.E. 4340 steel. Session 2, Paper 9, International Conference on fatigue of metals, Inst. Mech. Engs, 1956.
62	H.L. Cox	Four studies in the theory of stress concentrations Aero Res. Council Monograph R and M, No. 2704, 1950.
63	P.P. Benham	Torsional strain cycling fatigue of copper at low endurance, J. Inst. Metals, V 91, pp.404-437, 1962-63.
64	K.J. Pascoe	Low cycle fatigue in relation to design. Proceedings Second International Conference on Fracture, Brighton, U.K., 1969.
65	C.T. MacKenzie D.J. Burns P.P. Benham	A comparison of uniaxial low-endurance fatigue behaviour of two steels. Paper 12, Applied Mechanics Convention 1966, Proceedings Inst. Mech. Engs. Vol. 180, Part 3I, 1965-66.
66	S. Taira T. Inoue M. Takahashi	Low cycle fatigue under multiaxial stresses (in the case of combined cyclic tension - compression and cyclic torsion in the same phase at elevated temperatures). 10th Japan Congress on testing materials - Metallic Materials pp. 18-23, 1967.
67	H.A. Abdel-Aziz W. Johnson J.H. Lamble	Low endurance fatigue studies in torsion for steel and aluminium alloy. Proc. Inst. Mech. Engs., Vol. 180, Part 3I, Applied Mechanics Convention, Cambridge, 1966, pp.462-477, 1965-66.
68	M.A. Minor	Cumulative damage in fatigue, J. Appl. Mech. Vol. 12, Series A, pps. 159-164.
69	A.A. Blatherwick N.D. Viste	Cumulative damage under biaxial fatigue stress. Materials Research and Standards, pp. 331-336, 1967.
70	K.J. Miller	An experimental linear cumulative damage law. J. Strain Analysis, Vol. 5, pp. 177-184, 1970.
71	A.E. Johnson	Complex stress creep of metals. Met. Reviews. Vol. 5, No. 20, pp.447-506, 1960.

REFERENCES (cont'd)

No.	Author(s)	Title, etc.
72	J. Henderson	Stress and strain caused by high and low temperatures. European Coal and Steel Community, IVth Congress, Working Party II, 1968.
73	A.E. Johnson B. Khan	Complex-stress creep fracture of Nimonic 90 at 750°C N.E.L. Report No. 179, 1965.
74	A.E. Johnson B. Khan	A biaxial-stressing creep machine and extensometer. N.E.L. Report No. 183, 1965.
75	C.R. Kennedy W.O. Harms D.A. Douglas	Multiaxial creep studies on Inconel at 1500°F. Trans. A.S.M.E., Series D, J Basic Engineering. Vol. 81, pp.599-609, 1959.
76	W. Sawert H.R. Voorhees	The influence of ductility on creep rupture under multiaxial stresses. Trans. A.S.M.E. J Basic Engineering, Vol. 84, Series D, pp. 228-235, 1962.
77	A.E. Johnson J. Henderson V.D. Mathur	Creep under changing complex stress systems. Engineer, Vol. 206, pp. 209-216, 251-257, 287-291, 1958.
78	A.E. Johnson N.E. Frost	Combined tension and torsion machine for relaxation tests. Engineer, Vol. 198, pp. 834-835, 1954.
79	A.E. Johnson J. Henderson V.D. Mathur	Complex stress creep relaxation at elevated temperatures. Aircraft Engineering, Vol. 31, pp. 75-79, 113-118, 1959.
80	A.E. Johnson J. Henderson B. Khan	Complex stress creep fracture of copper at 250°C under vibratory stress. Engineer, Vol. 212, pp. 304-308, 1961.
81	J.S.Y. Lai W.N. Findley	Combined tension torsion creep experiments on polycarbonate in the non-linear range. Polymer Engineering and Science, Vol. 9, pp. 378-382, 1969.
82	K.G. Nolte W.N. Findley	Relationship between the creep of solid and foam polyurethene resulting from combined stresses. Trans. A.S.M.E., J. Basic Engineering, Vol. 92, Series D, pp. 105-114, 1970.
83	L.J. Broutman S.M. Krishinakumar P.M. Mallick	Effects of combined stresses on fracture of alumina and graphite. J. American Ceramic Society, Vol. 53, pp. 649-654, 1970.

REFERENCES (cont'd)

No.	Author(s)	Title, etc.
84	G.F. Pittinato S.F. Frederick	Deformation modes of Ti-5Al-2.5Sn under uniaxial and biaxial stress at 20°K. Trans. A.I.M.E. Vol. 245, pp. 2299-2306, 1969.
85	D. McLean	The physics of high temperature creep. Reports on Progress in Physics, Vol. XXIX, Part 1, 1966.
86	A.K. Mukherjee J.E. Bird J.E. Dorn	Experimental correlations for high-temperature creep. Trans. A.S.M. Vol. 62, pp.155-179, 1962.
87	R.C. Gilkins	A mechanism for the formation of intergranular cracks when boundary sliding occurs, Acta Met, Vol. 4, p. 98, 1956.
88	C.W. Chen E.S. Machlin	On the Mechanism of Intercrystalline Cracking Acta Met, Vol. 4, p. 655, 1956.
89	P.W. Davies B. Wilshire	An interpretation of the relationship between creep and fracture, pp. 101-110, Symposium on Structural Processes on Creep, Iron and Steel Inst. 1961.
90	P.W. Davies R.W. Evans	The contribution of voids to the tertiary creep of gold. Acta Met. Vol. 13, pp. 353-361, 1965.
91	P.W. Davies K.R. Williams	The effect of grain boundary cavities on vacancy diffusion. Phil. Mag., Vol. 18, pp. 137-143, 1968.
92	R.W. Cahn (Editor)	Physical Metallurgy. North Holland Publishing Co., Ch. 15-18, 1970.
93	R.K. Ham	The Metallurgy of transition life fatigue. Thermal and High-Strain Fatigue Conference pp. 55-79. The Metals and Metallurgy Trust. Monograph and Report Series No. 32, 1967.
94	J.J. Jonas C.M. Sellars W.J. McG.Tegart	Strength and structure under hot-working conditions. Met. Reviews. No. 130, 1969.
95	I. Kowacs E. Nagy P. Feltham	Resitivity changes in silver due to plastic deformation. Phil. Mag. Vol 9, p. 797-801, 1964.
96	M.R. Soliman G.A. Hassan F.H. Hammed	The effects of combined torsional and tensile stresses in producing large strains and electrical resistivity changes in aluminium of 99.7 per cent purity. J. Inst. Metals, Vol. 99, pp. 134-136, 1971.

REFERENCES (cont'd)

No.	Author(s)	Title, etc.
97	L.F. Coffin	The stability of metals under cyclic plastic strain. Trans. A.S.M.E., Series D, J. Basic Engineering, Vol. 82, pp. 671-682, 1960.
98	Maria Ronay	On second-order strain accumulation in torsion fatigue. J. Inst. Metals, Vol. 94, pp. 392-394, 1966.
99	S. Taira K. Honda T. Abe	X-ray investigation on fatigue damage of metals under mean stress. Bulletin Japan Soc. Mech. Eng, Vol. 9, pp. 40-47, 1966.
100	S. Taira K. Honda K. Matsuki	X-ray investigation of fatigue damage of metals Trans. J.S.M.E., Vol. 28, p. 1335, 1962.
101	S.R. Joshi J. Shewchuk	Fatigue crack propagation in a biaxial stress field. J. Exp. Mech., pp. 529-533, 1970.
102	E.A. Davis	The effect of size and stored energy on the fracture of tubular specimens, J. Appl. Mech., Vol. 15, pp. 216-221, 1948.
103	C.R. Kennedy	Effect of stress state on high-temperature low cycle fatigue. Symposium on fatigue tests of aircraft structures low cycle, full-scale and helicopters. Fourth Pacific Area Meeting Papers, A.S.T.M. Special Publication No. 338, 1963.
104	D. Hull D.E. Rimmer	The growth of grain-boundary voids under stress. Phil. Mag., Vol. 4, pp.673-687, 1959.
105	P.W. Davies K.R. Williams B. Wilshire	On the distribution of cavities during creep. Phil. Mag., Vol. 18, pp. 197-200, 1968.
106	E.G. Ellison	A review of the interaction of creep and fatigue. J. Mech. Eng. Science, Vol. 11, pp. 318-339, 1969.
107	D.J. White	Some contributions to British Work on thermal and high strain fatigue. Paper 1 International Conference on Thermal Stresses and Thermal Fatigue. Berkeley Nuclear Labs, 23-26 Sept., 1969.

TABLE I

Comparison of experimental techniques

Advantages	Disadvantages
(a) Spinning disc experiment	
Notch effects can be studied.	Expensive in specimen material and rig sophistication.
Operating temperatures can be simulated.	Strain measurement inadequate for a study of biaxial stress effects.
Provides a useful basis for design of actual component.	Essentially a macroscopic approach to the problem of biaxial stresses thereby leading to the following evaluation difficulties
Useful intermediate stage between usual simple laboratory tests and engine discs.	(a) examination of deformation and fracture zones by optical and electron metallographic techniques difficult.
	(b) Principal stress ratios vary over the disc surface thus complicating an assessment of the influence of biaxial stress on deformation and failure.
	(c) Considers problem as a whole whereas fatigue, for instance, is a localised phenomenon.

TABLE I (cont'd)

Advantages	Disadvantages
(b) Cruciform specimen experiment	
Flat bottomed (d/t = 12.5) specimens correlate well with push pull cylindrical specimens for high endurance fatigue.	Strain measurement by foil gauges.
	After yield, stress distribution in working zone is not accurately known.
Wide range of strain ratios theoretically possible.	Only uniaxial, equibiaxial and shear loading can be conveniently obtained.
Convenient specimen shape for microscopy.	
The deformation and failure behaviour at a specific strain ratio can be studied.	The best shape for strain uniformity is most susceptible to buckling under compressive load.
Temperature work possible.	Not suitable for load or stress control.
	Fracture detection unsatisfactory.
(c) Bulge test experiment	
Relatively simple and inexpensive equipment.	Probably expensive in specimen material.
Suitable for assessing plate materials (i.e. for use in pressure vessels).	Various specimen geometries required for different stress ratios thereby
	(a) restricting range of test variables
	(b) Introducing geometrical factors into experimental data
	Strain measurement by foil gauges.

TABLE I (cont'd)

Advantages	Disadvantages
(c) Bulge test experiment (contd)	
	Failure could be modified by pressurising media.
	Satisfactory elevated temperature work difficult and there would probably be uneven temperature distribution over the large specimens.
	Unsuitable for creep work.
(d) Internal pressurisation (combined with axial load) experiments	
Wide range of principal stress ratios possible.	Expensive in equipment and specimens.
Stress ratios can be directly related to those found in discs.	Engineering problems at very high temperatures.
Notch effects can be introduced.	Pressurising media can modify fracture behaviour.
Creep, fatigue and most other deformation modes can be studied.	Strain measurement difficult.
	Possibility of catastrophic failure with gaseous media.
(e) Torsion testing (combined with axial load)	
Control of shear/tensile stress ratios thus allowing a study of deformation and failure mechanisms.	Expensive equipment and specimens
	Problem of converting torque/twist data to shear stress - shear strain.

TABLE I (cont'd)

Advantages	Disadvantages
(e) Torsion testing (combined with axial load) (cont'd)	
Most engineering situations can be thought of as tension/torsion with a superimposed hydrostatic stress. Notch effects can be studied. Specimen shape is amenable to metallographic examination. Temperature work easily achieved. All major deformation modes (i.e. fatigue, creep etc.) can be studied.	Strain measurement problem. Variation of shear stress across radius can be avoided by the use of thin shells but this introduces buckling problems.

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FIG. 1

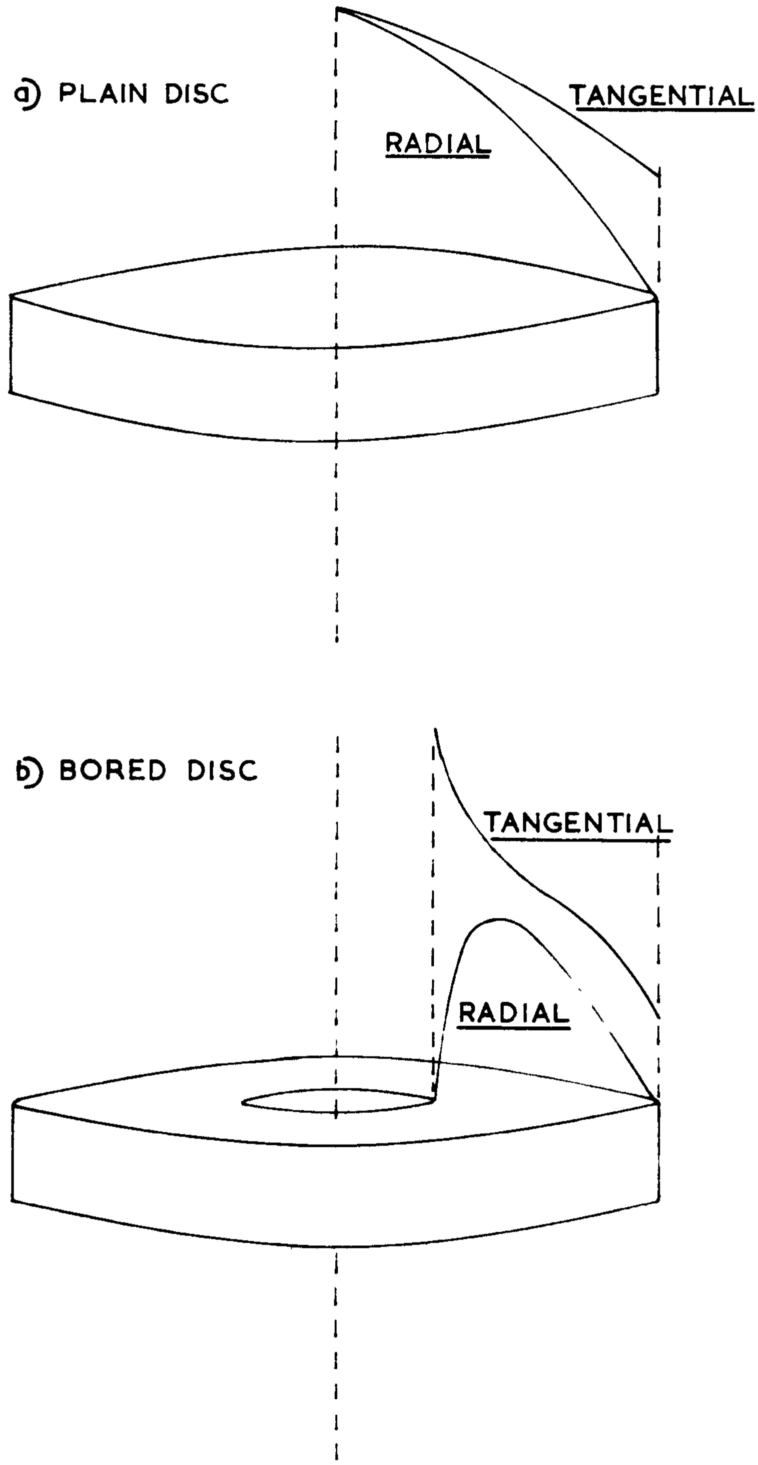

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FIG. 1 STRESS DISTRIBUTION IN A THIN DISC

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FIG 2

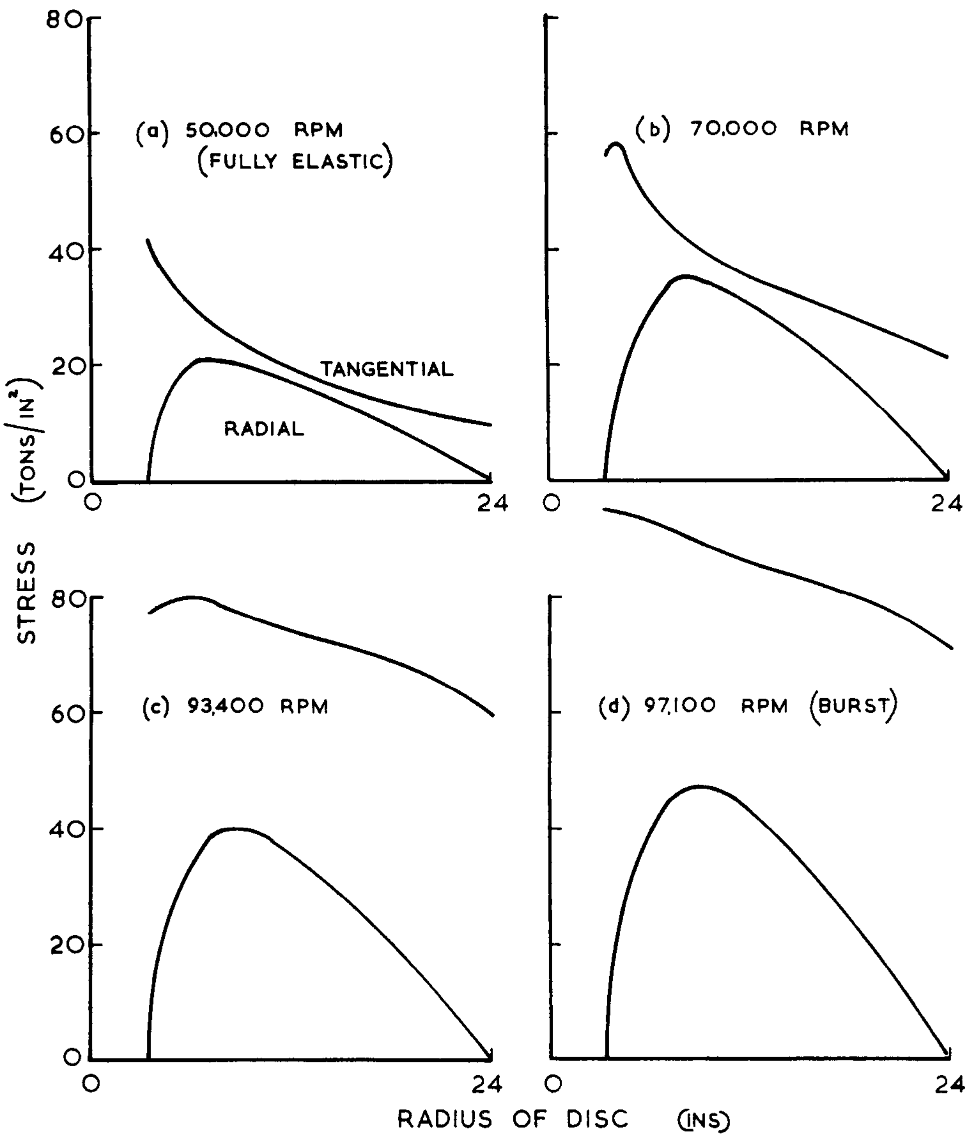

FIG 2 STRESS REDISTRIBUTION IN WASPALDY MODEL DISCS (NOT TO SCALE) (REF 5)

FIG.

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FIG. 3

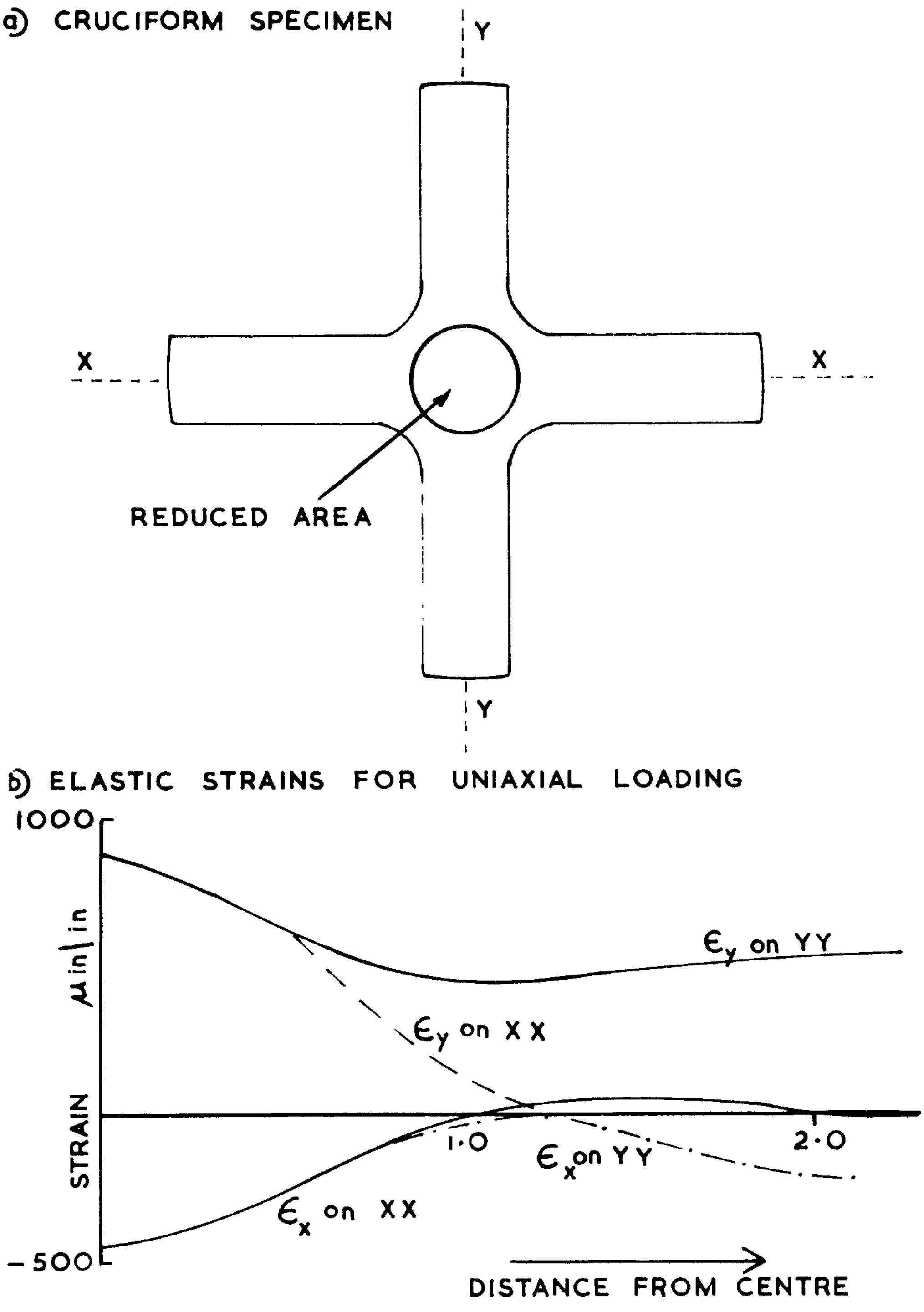
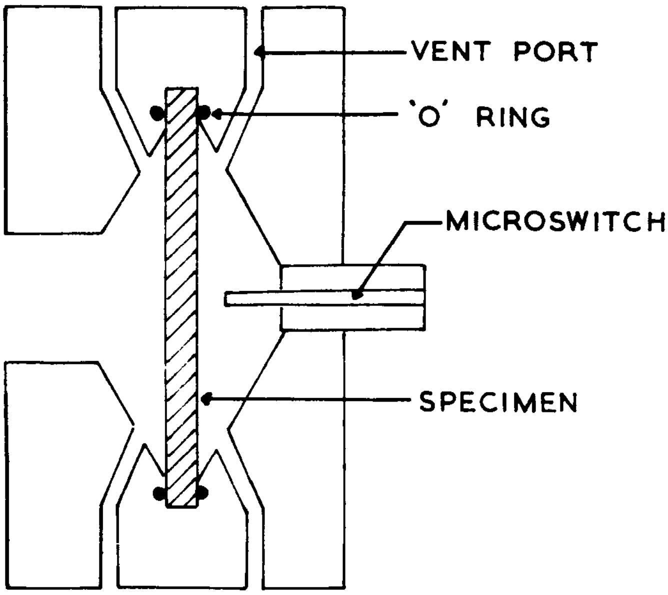


FIG. 3 THE CRUCIFORM EXPERIMENT (REF. 8)

FIG. 4

a) SCHEMATIC OF BULGE TEST EQUIPMENT



b) STATIC ELASTIC STRAIN ANALYSIS

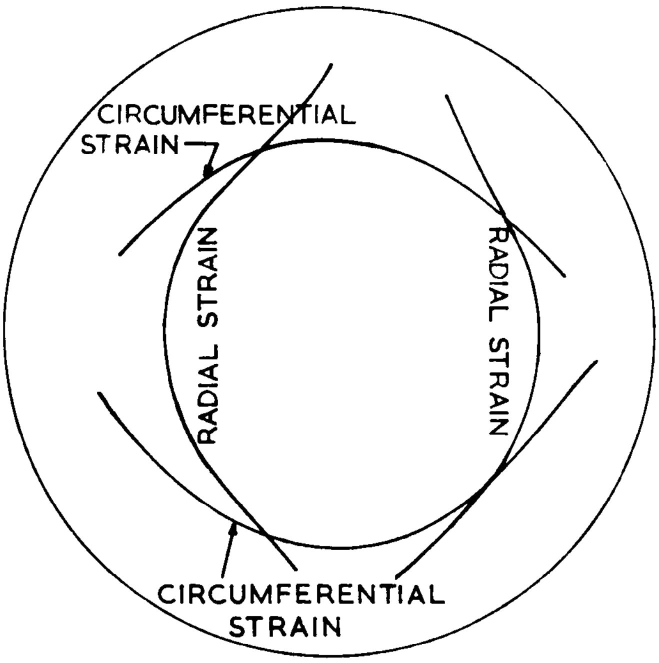


FIG. 4 BULGE TEST EQUIPMENT AND STRAIN ANALYSIS (REF. 14)

FIG.

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FIG 5

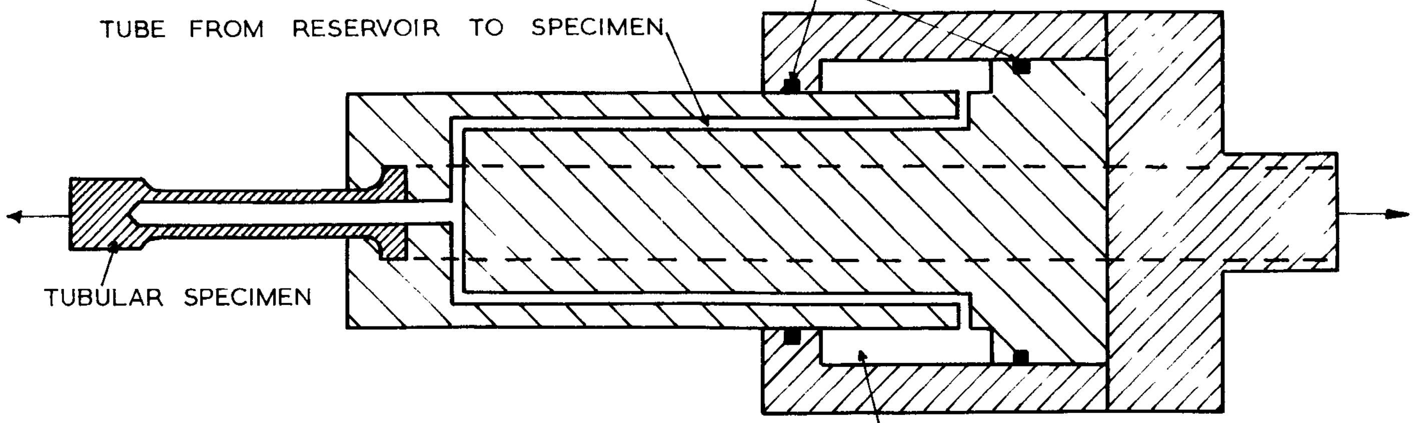

FIG 5 SCHEMATIC OF THE N G T E PRESSURISATION TECHNIQUE

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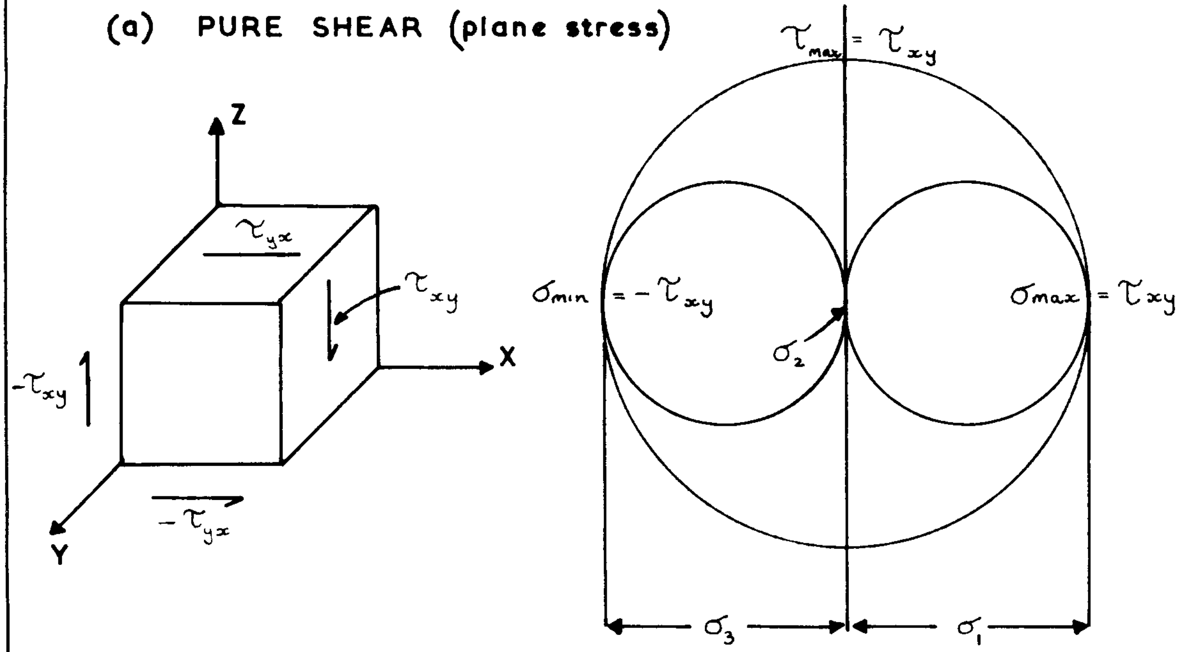
FIG. 6

(a) PURE SHEAR (plane stress)



(b) EQUIBIAXIAL TENSION AND COMPRESSION

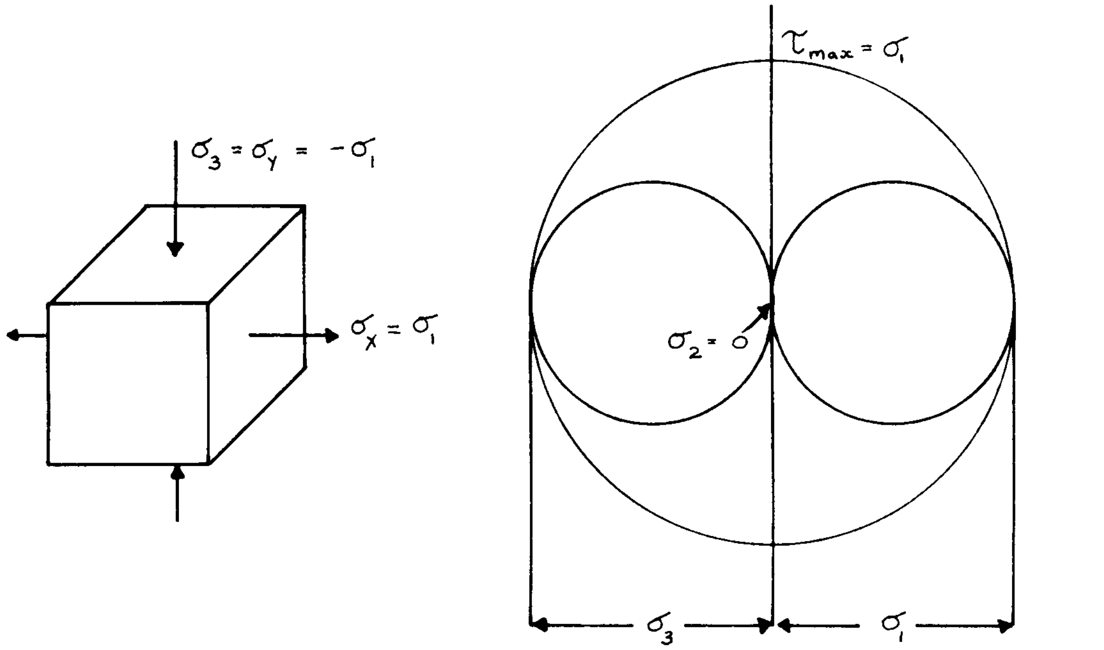


FIG. 6 EQUIVALENT PURE-SHEAR CONDITIONS

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FIG.

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FIG. 7

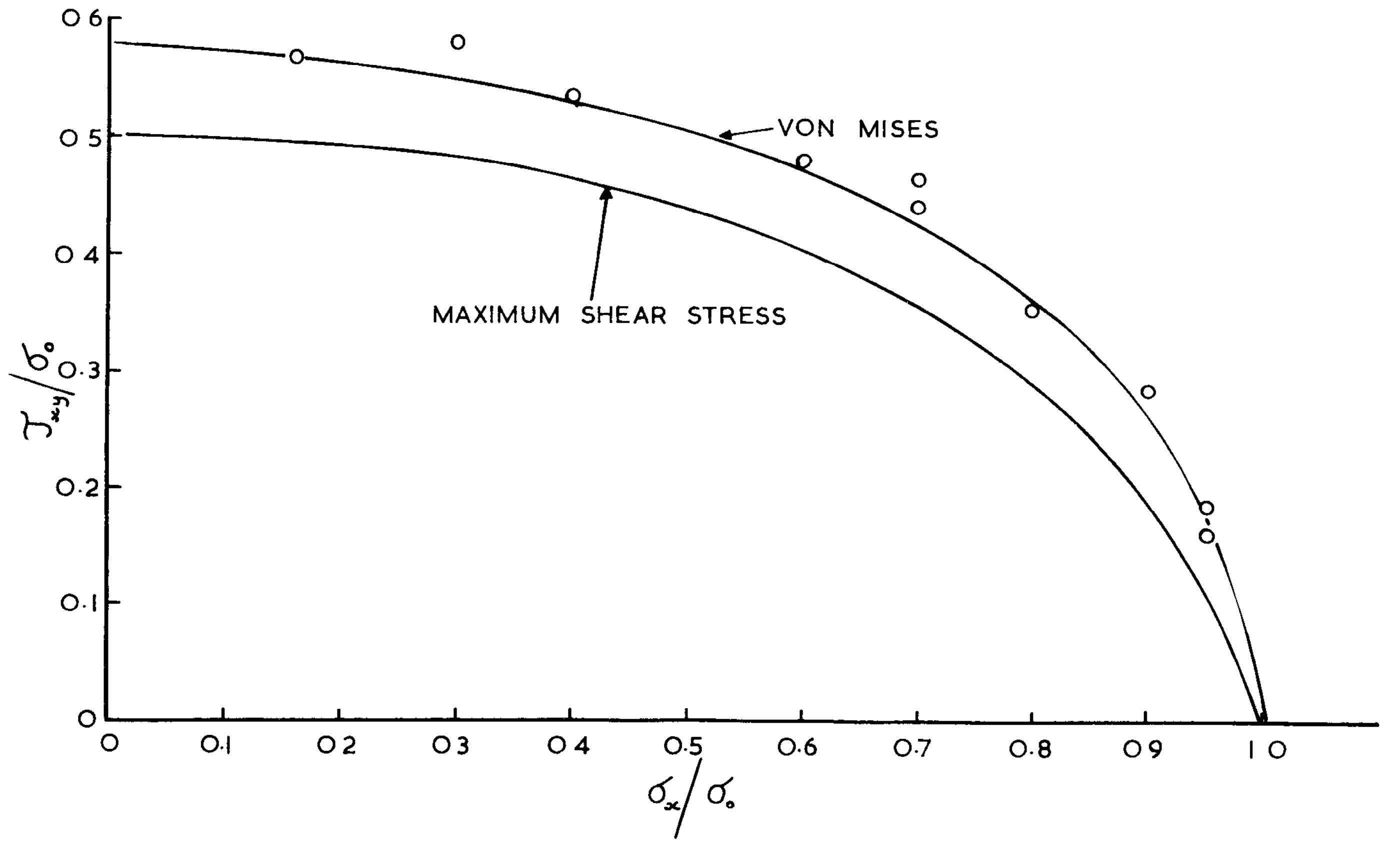

FIG 7 COMPARISON BETWEEN TRESCA AND VON MISES CRITERIA

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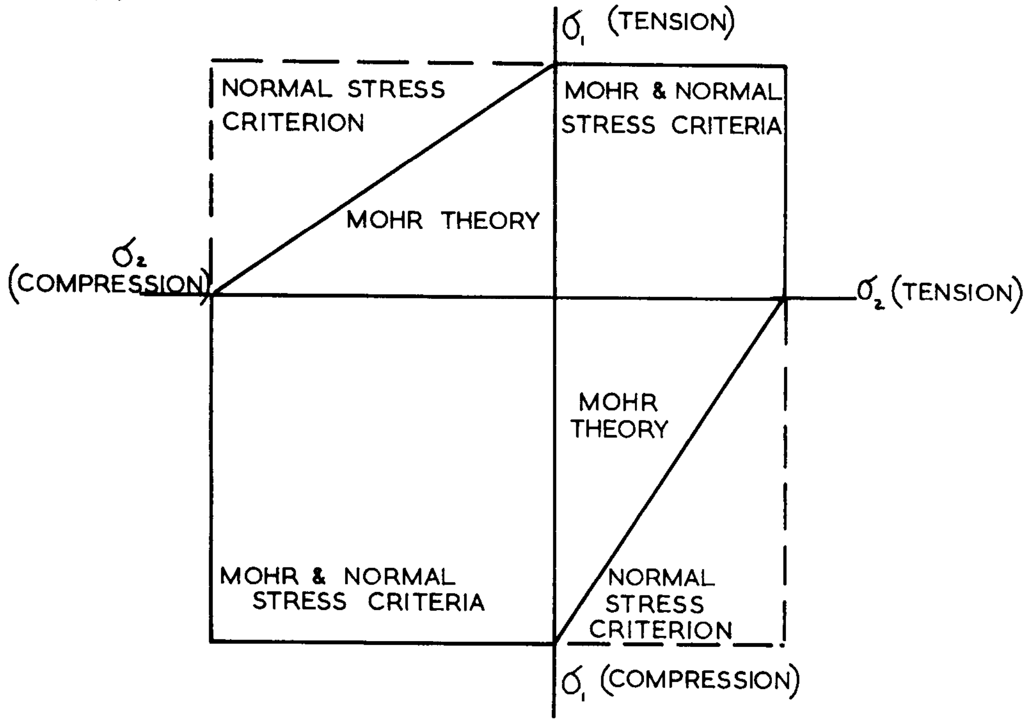
FIG. 8

(a) BI-AXIAL STRENGTH THEORIES FOR BRITTLE MATERIALS



(b) BI-AXIAL STRENGTH THEORIES FOR DUCTILE MATERIALS

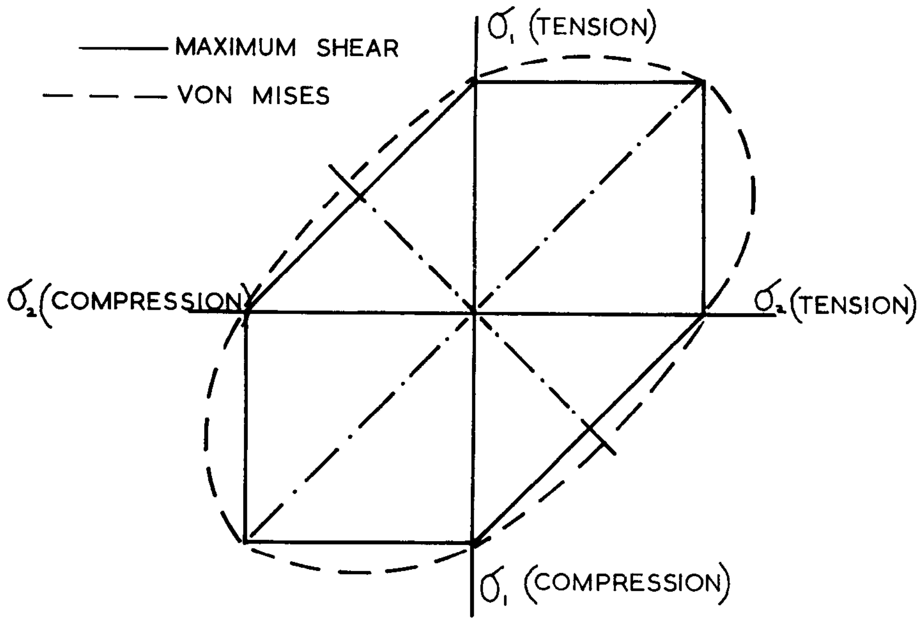


FIG 8 BI-AXIAL STRENGTH THEORIES

FIG.

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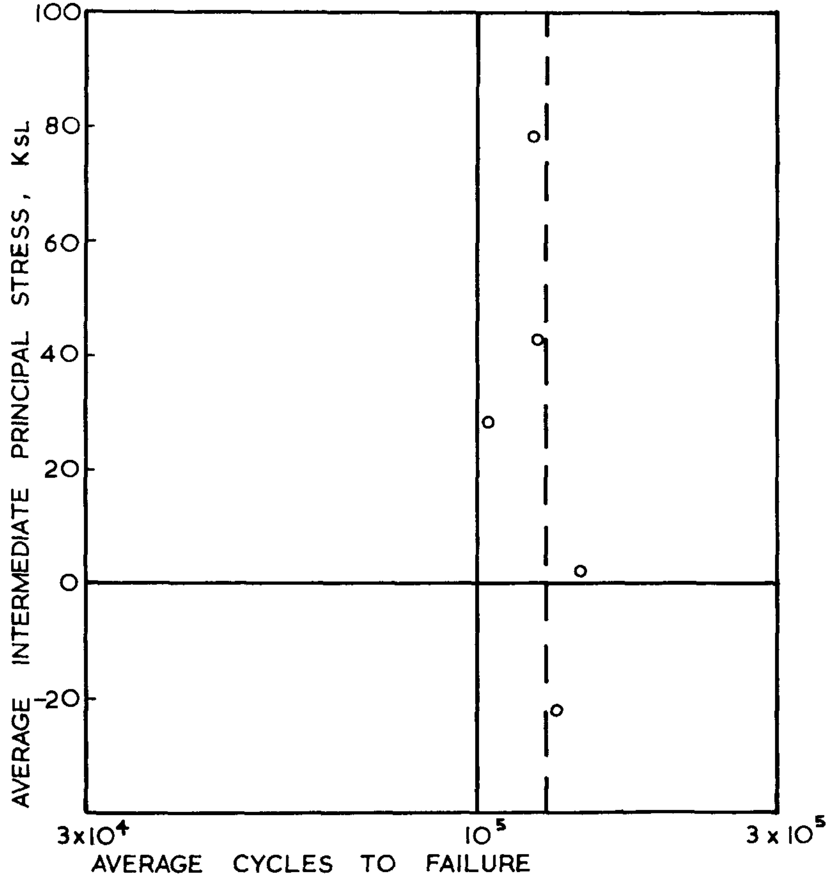
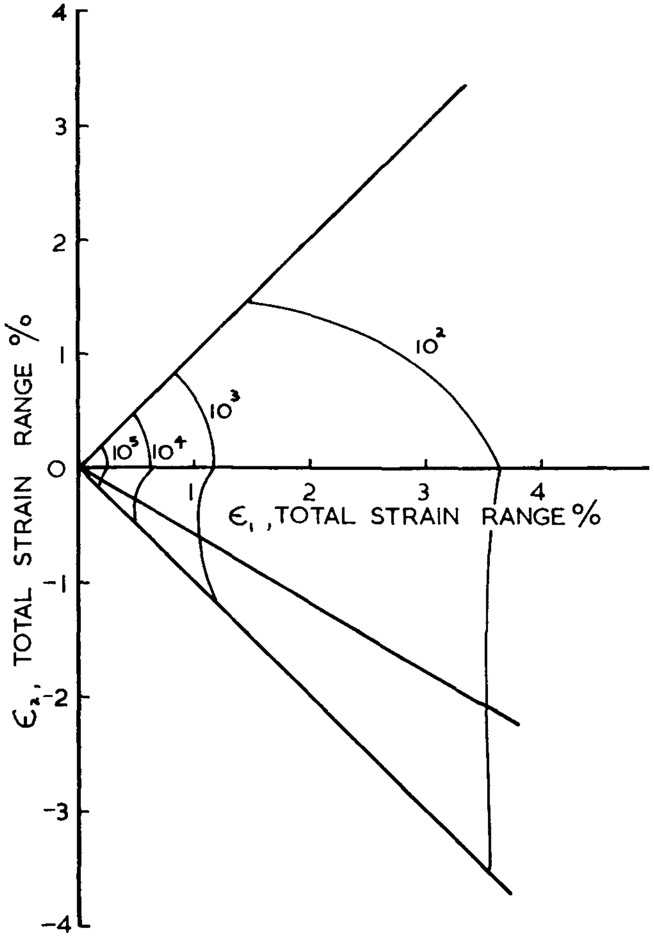
FIG. 9



(a) THE EFFECT OF INTERMEDIATE PRINCIPAL STRESS ON FATIGUE LIFE (ref 18)



(b) CONTOURS OF EQUAL FATIGUE LIFE (ref 64)

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FIG 10

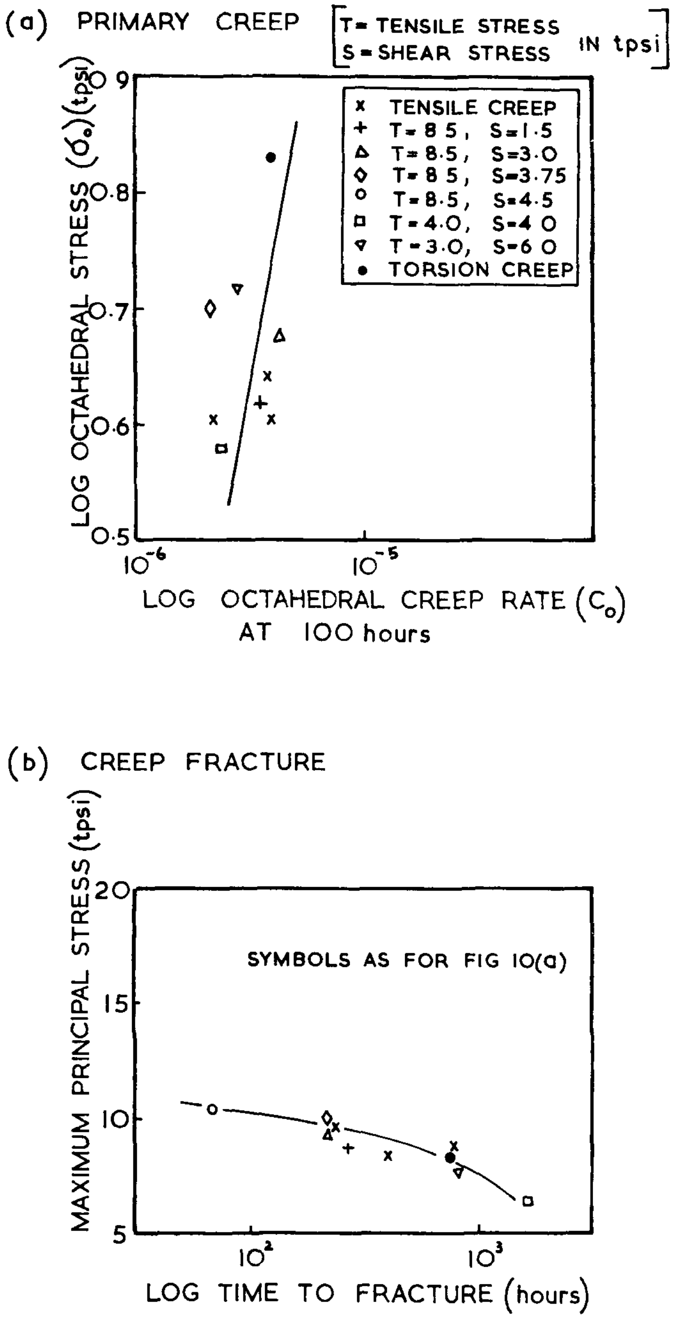

FIG 10 MULTIAXIAL CREEP IN NIMONIC 90 (REF 73)

FIG.

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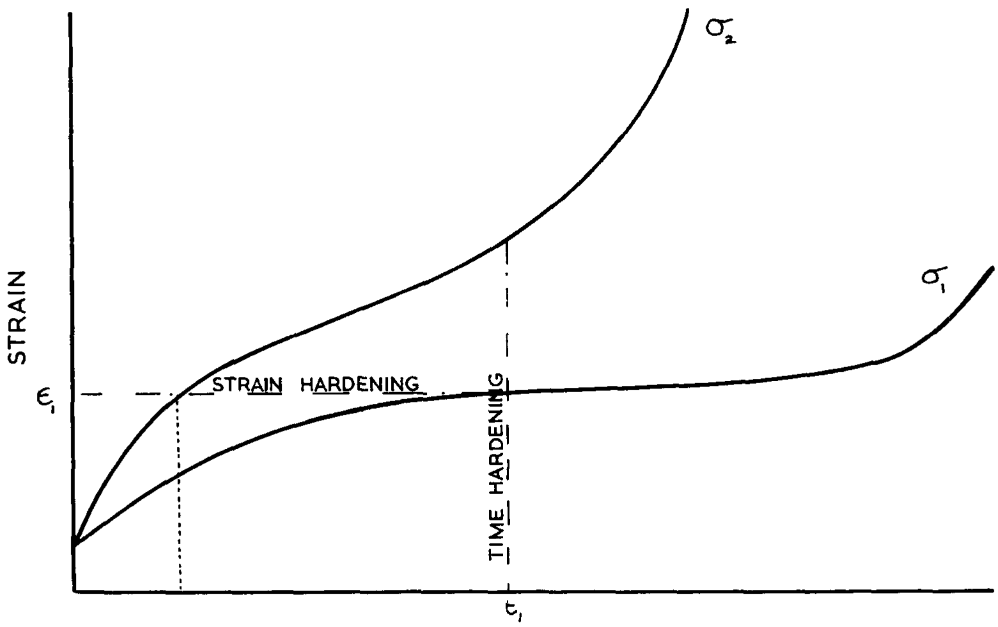
FIG 11

(a) MECHANICAL CREEP THEORIES



(b) THE INTERACTION BETWEEN CREEP AND FATIGUE (ref 80)

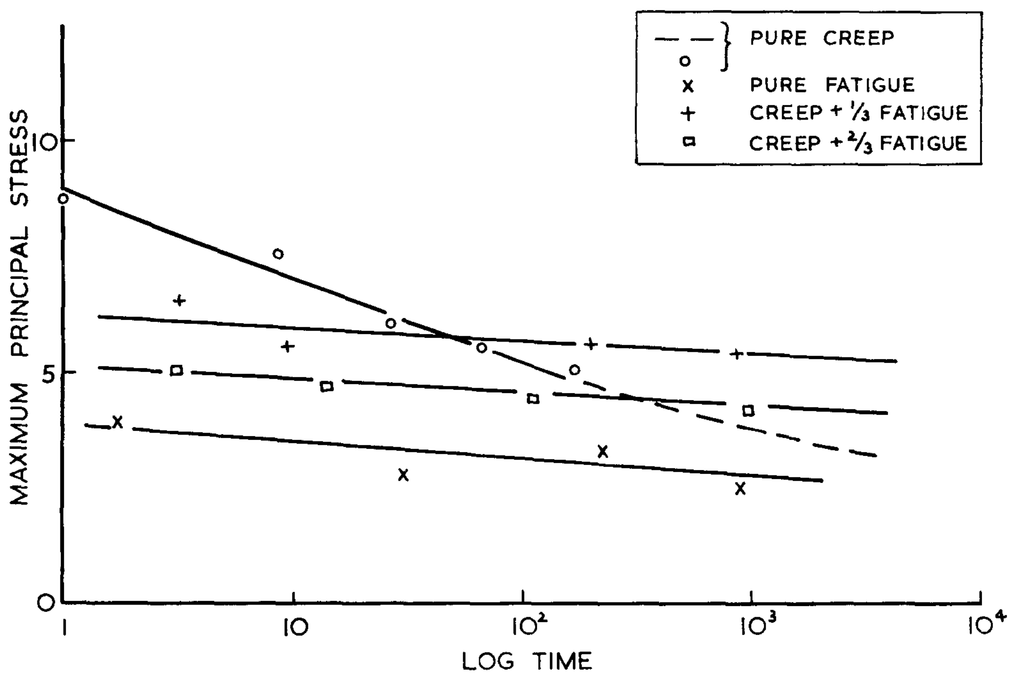


FIG.

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FIG. 12

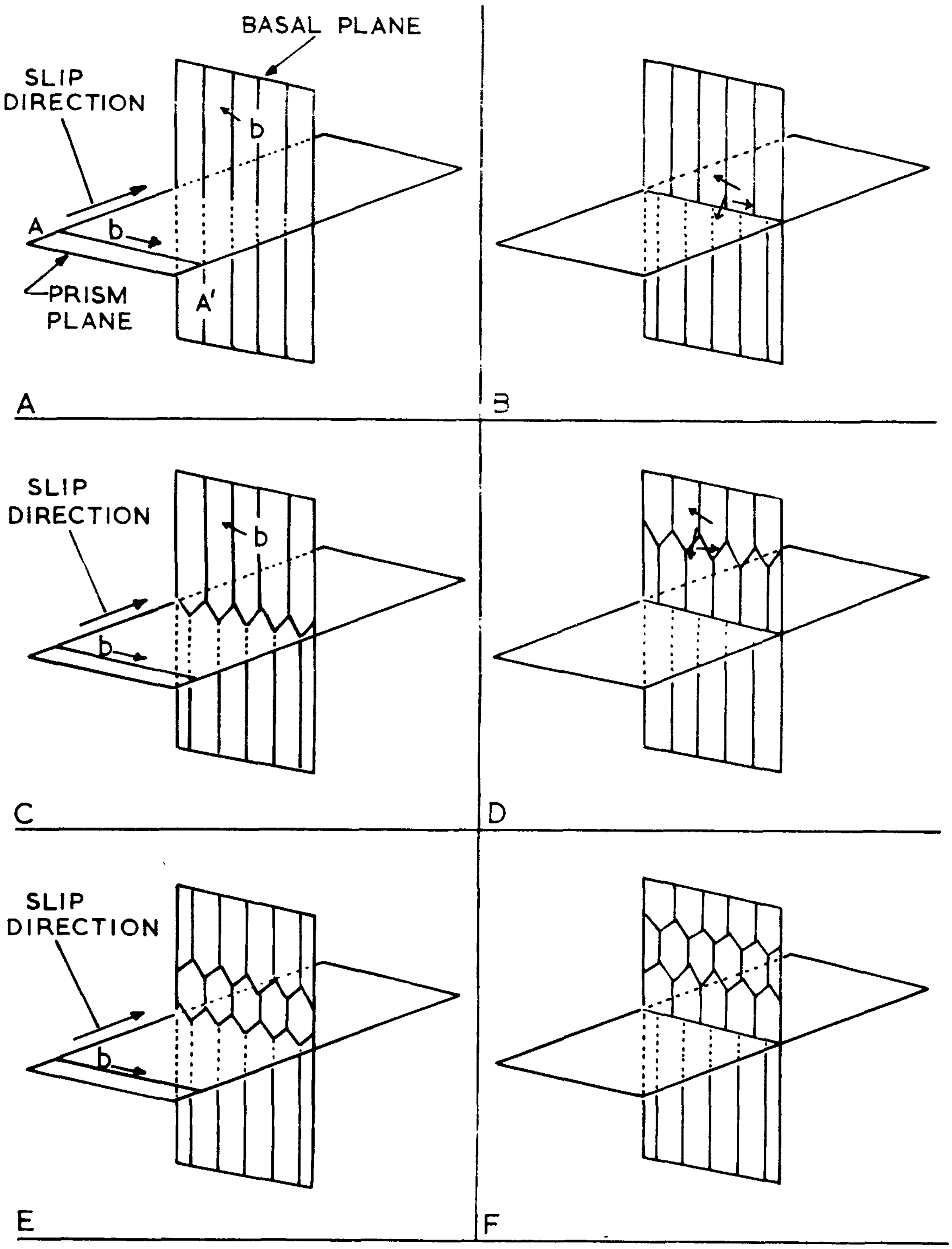

FIG. 12 SCHEMATIC REPRESENTATION OF THE FORMATION OF A HEXAGONAL DISLOCATION NETWORK BY THE CROSS SLIP MECHANISM

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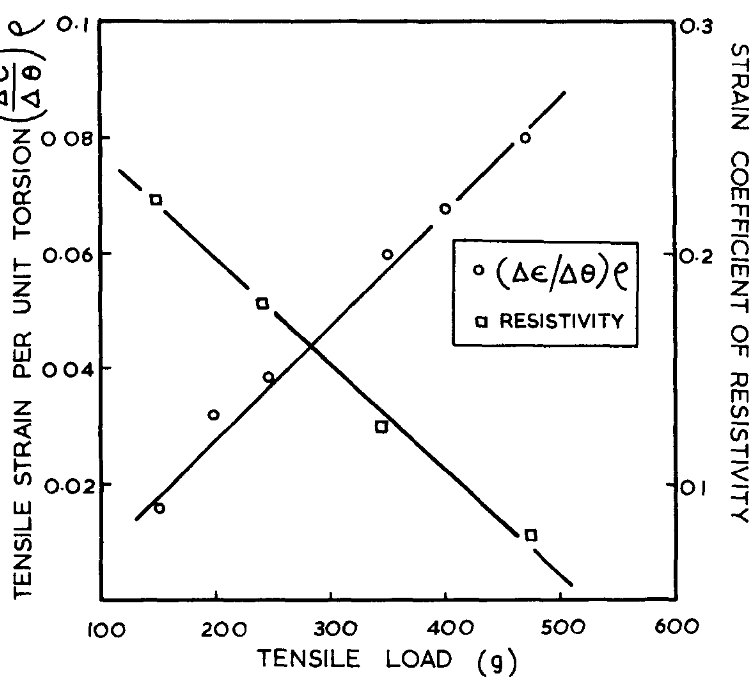
(a) CHANGE IN AXIAL STRAIN AND RESISTIVITY WITH APPLIED LOAD (ref 96)



(b) AXIAL STRAIN ACCUMULATION UNDER TENSILE STRESSES AND CYCLIC TORSION (ref. 98)

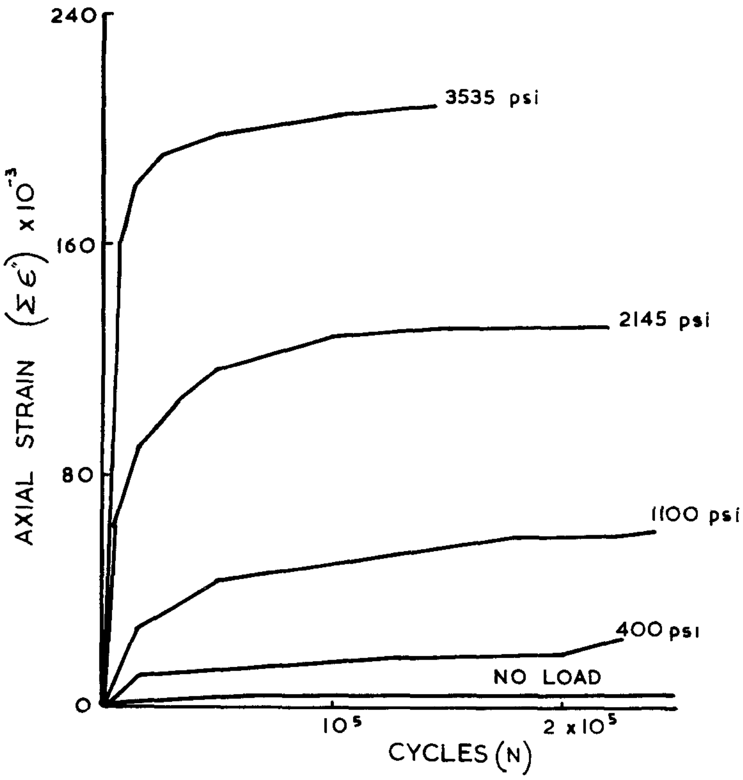


FIG. 13

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A survey has been made of techniques for producing multiaxial stress conditions on a laboratory scale and it is concluded that one of the more suitable systems is combined tension-torsion of thin-walled tubes. Experimentally this technique, although complex, appears to offer several advantages. Furthermore, the shear/normal stress ratio can be conveniently controlled, thus enabling deformation and fracture mechanisms under multiaxial stresses to be studied. A review has also been presented of relevant experimental work, most of

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Work on the micro-structural effects of biaxial stresses is also considered. It would appear that although the mechanisms by which dislocations operate remain unchanged from the uniaxial state, modifications can occur to the products of interactions between dislocations. In addition fracture behaviour can be altered by the ratio of tensile to shear stress. Generally, however, little work has been published on this aspect of deformation behaviour. It is concluded that it is important to study micro-structural effects particularly in relation to design criteria.

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